Kaity Parsons, Peter Tingley and Emma Zajdela

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"So you want to win at poker?" Thus begins this exciting article. The authors work through strategies based on the hands that are dealt, their probabilities of winning, random behavior of players, bluffing, and slow-play.

To simplify things, the article focuses on a game of poker involving only two players. To further set rules that allow the authors to analyze the game's outcomes, they require both players to write a computer program specifying how they will play. Player 1 writes their program first, and Player 2 gets to see Player 1's program while deciding on their own program! Though this may seem less than fair, the authors make a case that poker players who have known each other a long while are each likely to know the other's typical strategies, and so seeing another player's computer program is not so far from the reality of competing against each other. As a fascinating outcome, if Player 1 proceeds in a totally straightforward way, betting only on hands that are likely to win, then the best outcome for Player 1 is to break even. However, by introducing bluffing, Player 1 can win money from Player 2.

From here, the article takes the logical next step of assuming that Player 1 also knows the entire strategy for Player 2. Therefore, each player knows what to expect from the other, similarly to what might happen between two people who have played poker together many times before. Both players can then determine their best system of play, based on full information about the other player's decision-making strategies, which can lead to a Nash equilibrium.

In developing the many possible outcomes they consider, this article's authors use a tried-and-true teaching technique: they begin with a simplified set of rules, from which they build intuition with their readers, and then they progress to wide-ranging and much more abstract ideas. In particular, our authors initially allow only six possible poker hands, determined randomly by the roll of a die. With only six hands, the table of outcomes fits nicely onto a journal page. Once readers understand these outcomes, the authors introduce the idea of infinitely many outcomes, having all possible probabilities from 0 to 1, and this concept appears completely natural and quickly understandable. The betting possibilities also expand dramatically throughout the article.

"When to Hold 'Em" is lively and feels conversational throughout. In this inviting format, the authors carefully and fully develop several approaches to evaluating poker play. Their explanations are approachable, and the writing style welcomes us to continue reading and thinking. They have put together a wonderfully readable examination of mathematical ideas.

Response from Kaity Parsons

I am both honored and surprised to receive this award. This paper was my greatest achievement of my undergraduate career at Loyola University Chicago. When Dr. Tingley first approached me with this project, I was thrilled at the concept. I grew up playing card games, poker included. Though I have yet to win millions in Vegas, the theories outlined in this paper have served me well. However, the experience of this project was the most vital in developing my love of playing with numbers. The many hours spent sitting with a simple question; How can you win, or rather, lose the least, at poker?; was pivotal in my math journey. Now, I try to do the same for my own students. Math has a bad reputation and I, like Dr. Tingley and this project did for me, am determined to make math fun.

Response from Peter Tingley

It is a great honor and pleasure to accept this award, thank you! The writing of "When to Hold 'Em." played out over several years and involved many people. It really began in 2012, when I saw an amazing and inspiring talk on poker and math by Yan X Zhang. It continued when Nick Barron convinced me to teach game theory, which was not at all my field, but which I greatly enjoyed. Kaity Parsons and Emma Zajdela were both students in that class, and both did undergraduate research projects on poker with me, leading to the first draft of this paper. We kept in touch after they graduated, and the paper slowly evolved. It has been used in my game theory classes ever since, and many students have commented on and improved it — most notably Emily Danning He who in 2017 gave who gave it a super thorough proofreading. The whole process has been a wonderful experience, which I am grateful to have shared with these amazing students. It was its own reward, but winning an actual award is certainly a nice addition!

Response from Emma Zajdela

I am honored and delighted to receive the Carl B. Allendoerfer award for our paper on game theory and poker. Historically, people have become interested in mathematical problems through games of chance (think of Pascal in the 17th century), and this holds true today —workshops based on this paper have been successful in several outreach programs designed to introduce high-school students to mathematical research. It also sparked a fascination for me with the idea that we can use math to understand human behavior. This research was the impetus for me to pursue a PhD in applied math, with a focus on modeling complex social systems, for which I received the NSF Graduate Research Fellowship.

Biographical Sketches

Kaity Parsons received her BS in mathematics at Loyola University Chicago in 2017. Since then she has continued to teach all levels of math to children of all ages. Currently, Kaity is running her own tutoring service in McCall, Idaho. She loves making math fun for those who ordinarily struggle with the subject. When Kaity isn't busy juggling all her students, she enjoys reading mystery novels, painting, and hiking through the stunning national parks Idaho has to offer.

Peter Tingley received a PhD in mathematics from the University of California, Berkeley in 2008. He spent short periods at the University of Melbourne (Australia) and MIT, and since 2012 has been at Loyola University Chicago. He also helps run the Chicago Math Teachers' circle, which he co-founded in 2015. When not doing math he enjoys camping and playing with his kids—inspired by them, he has recently taken up circus lessons, and has been learning to do cartwheels!

Emma Zajdela is a PhD candidate in applied math at Northwestern University and National Science Foundation Graduate Research Fellow. She received a BS in math and physics from Loyola University Chicago in 2016 and an MS in math from the University of Illinois Chicago in 2018. Since 2016, she has served as senior assistant to the president of the Malta Conferences Foundation, a nonprofit that uses science as a bridge to peace in the Middle East.