CLASSROOM CAPSULES

EDITOR

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Classroom Capsules consists primarily of short notes (1–3 pages) that convey new mathematical insights and effective teaching strategies for college mathematics instruction. Please submit manuscripts prepared according to the guidelines on the inside front cover to the Editor, Michael K. Kinyon, Indiana University South Bend, South Bend, IN 46634.

Algebra in Respiratory Care

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In the early summers of 1995 to 2000, I served as faculty on the NIH-funded Health Careers Opportunity Program at Southwest Texas State University (now Texas State University, San Marcos). Students in this program were two-year college students indicating an interest in continuing their studies in a health science career (such as pre-med or respiratory care or radiology); all students came from disadvantaged backgrounds (poor or minority or both). My role was to prepare students for the mathematical demands at the University. For some, this was as 'simple' as preparing them for College Algebra, for others preparing them for Calculus or Statistics. Other faculty in the program, health sciences specialists, often guided me in what mathematical skills were needed by their students. One application from the respiratory therapists that is of life-or-death concern (especially in emergency cases) is the calculation of total flow and fraction of inspired oxygen in the total flow in an oxygen delivery system.

Background. Oxygen delivery devices are used to correct hypoxemia, acute myocardial infarction (heart attack), and severe trauma. They also are used extensively in post-anesthesia recovery. Devices may be low-flow (not meeting all the inspiratory flow demands of the patient), such as the simple mask, or high-flow (guarantees a set flow of inspired oxygen, no matter the patient's breathing pattern), such as the Venturi mask. High flow devices typically deliver more than 30 liters of air per minutes (lpm) to the patient, at least enough to meet patient inspiratory demand. The systems we consider here are the air-entrainment oxygen delivery systems: the ambient air is drawn into the stream of oxygen (or vice versa). In the Venturi mask a jet stream of ambient

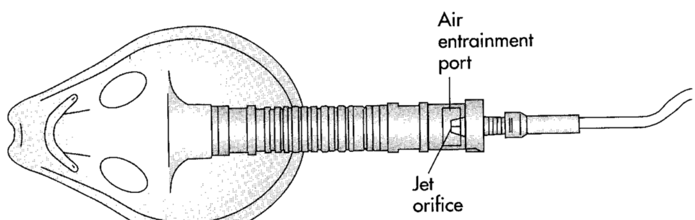

Figure 1. A Venturi mask.

air mixed with pure oxygen pulls ambient air through the air entrainment port into the mask's total flow. Ambient air is approximately 21% oxygen, though in practice 20% is used to make quick estimations.

The respiratory therapist's primary concern is to deliver an appropriate amount of oxygen to the patient. A principal measure of dosage is the fraction of inspired oxygen (FIO2). The oxygen delivery device typically has preset values of FIO2 for the respiratory therapist to choose from; this setting is in practice described as a ratio of the entrained air to the oxygen streamed from the oxygen tank. However, to be assured that the correct FIO2 is actually delivered, it is necessary that the total flow through the oxygen delivery device is, at minimum, meeting the inspiratory needs of the patient. If the total flow is too low, then the patient will draw in extra ambient air from the room, diminishing the FIO2 and thereby hazarding the health of the patient. Another way to put it: the respiratory therapist *must* be able to compare the device's flow to the patient's. In order to do this, they first must be easily able to calculate the air-to-oxygen ratio.

Calculating the air-to-oxygen ratio. Ambient air contains approximately 21% oxygen. Thus the minimal FIO2 setting possible is 21%, when the oxygen stream is cut off and only air is allowed through the delivery system. This is a $1:0$ air-to-oxygen ratio. At the other extreme, an FIO2 setting of 100% gives an air-to-oxygen ratio of $0:1$ (pure oxygen, no entrainment). Let x represent the FIO2 as a percent, so $21 \leq x \leq 100$. The percentage of oxygen that is to be supplied by the ambient air is 21% so that leaves $y = x - 21$ as the percentage of oxygen to be supplied by the delivery device.

We approached this problem by modeling it as a mixture problem. Since the air-to-oxygen ratio is usually given with respect to 1 part oxygen, we let a represent the units of air mixed with 1 unit of oxygen. So the total flow when supplying 1 unit of oxygen is $a + 1$. From the last paragraph, we get the total oxygen being $.21a + 1 = x(a + 1)$. Solving for a, we get $a = (x - 1)/(0.21 - x) = (1 - x)/(x - 0.21)$. The air-to-oxygen ratio is then $a : 1$.

For example, if the FIO2 is set to 41%, then the air-to-oxygen ratio at this setting is $2.95 : 1$, since $a = (1 - .41)/(.41 - .21) = 0.59/0.2 = 2.95$.

Computing the total flow. After determining the correct air-to-oxygen ratio that will deliver the desired concentration of oxygen to the patient, the respiratory therapist must then ensure that the concentration is actually delivered by setting the oxygen flow of the delivery device to meet the inspiratory needs of the patient. Typically, the oxygen flow on the delivery device is set so that the total flow (the oxygen flow plus the entrained air flow) is at or above 30 liters per minutes (30 lpm). Given the oxygen flow setting and the desired FIO2, what is the total airflow delivered to the patient? Letting f denote the oxygen flow in units of lpm and assuming an air-to-oxygen ratio of $a : 1$, the total airflow is $a \cdot f + f = (a + 1) \cdot f$ lpm. So we have the constraint $(a + 1) \cdot f \geq 30$. Continuing our example FIO2 setting of 41% and minimizing the total flow, we would set the oxygen flow at $30/3.95 \approx 7.6$ lpm.

Conclusion. In conclusion, respiratory therapists must be adept at solving the constrained system

$$\begin{cases} a = (1 - x)/(x - .21) \\ (a + 1) \cdot f \geq 30 \\ 0.21 \leq x \leq 1 \end{cases}$$

The three general algebraic, yet practical, questions that I had the students work on, and derive a general method of solution for are (in order of difficulty for them):

- Given the FIO2 and the flowmeter settings, what is the total flow?
- Given the total flow $(a + 1) \cdot f$ and the flowmeter setting, determine the delivered FIO2.
- Given the total flow and the desired FIO2, what flowmeter setting should be selected?

On a Common Mnemonic from Trigonometry

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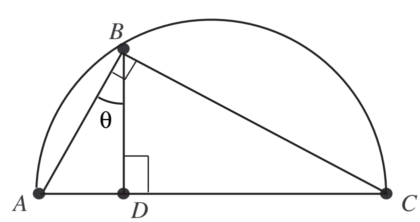
A commonly used mnemonic for the sine of the special angles 0, $\pi/6$, $\pi/4$, $\pi/3$, and $\pi/2$ is

$$\sin\left\{0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}\right\} = \frac{\sqrt{\{0, 1, 2, 3, 4\}}}{2}.$$

That is, we take the square roots of the numbers in the sequence 0, 1, 2, 3, 4 and divide by 2. With these numbers we can compute any trig function of any of the special angles between 0 and 2π via elementary identities.



But how is it that the sines of these angles can be obtained via such a simple arithmetic sequence? The angles themselves occur in a pattern to be sure, but not such a simple one.

We will show that θ (see the figure) is one of the special angles, 0, $\pi/6$, $\pi/4$, $\pi/3$, or $\pi/2$, precisely when the ratio $\frac{|AD|}{|AC|}$ is 0, 1/4, 1/2, 3/4, or 1, respectively. To put it another way, if we think of the point D as moving in a straight line from A to C then θ successively reaches each of the special angles $\pi/6$, $\pi/4$, and $\pi/3$ precisely when D is 1/4, 1/2, 3/4, or all of the way to C. (Clearly $\theta = \{0, \pi/2\}$ when $D = \{A, C\}$.)

For clarity and without loss of generality we assume that $|AC| = 1$. Observe that triangles ABD and BCD are similar to each other since both are similar to triangle ABC. Thus by proportionality of similar triangles we have $\frac{|BD|}{|AD|} = \frac{1-|AD|}{|BD|}$ or

$$|BD| = \sqrt{|AD|\,(1 - |AD|)}$$

