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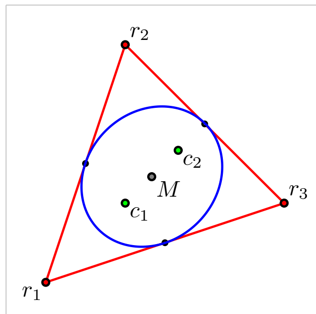
1. **The Geometry of Polynomials**

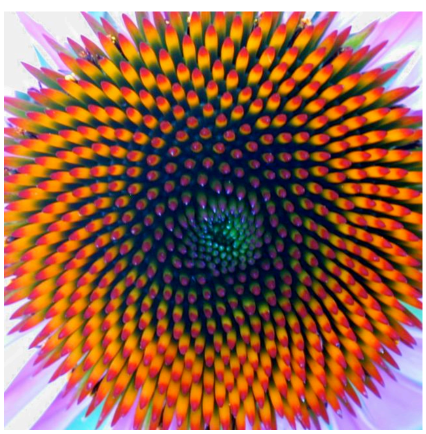
Abstract. In the *geometry of polynomials*, we seek to understand relationships among certain sets connected to polynomial functions. For instance, given a member of a family of polynomials, we may be interested in how the set of critical numbers is related to the corresponding set of the polynomial's zeros, with the goal of making general observations that apply to the entire family regarding the relationship between these sets. This subject area goes back at least to C. F. Gauss (1777-1855); Morris Marden (1905-1991) popularized its study through his classic text (1966).



In this talk, we'll discuss several fundamental historical results from the geometry of polynomials, including the Gauss-Lucas Theorem and Marden's Theorem. We will also survey some recent developments centered on the idea of ***polynomial root-dragging***, the study of how continuously changing one or more roots of a polynomial function affects various properties of the function. Along the way, we'll consider a few results proved by undergraduate students and witness beautiful interplay between Euclidean geometry and calculus in the context of cubic polynomials.

2. **Fibonacci's Garden**

Abstract. Why do so many spectacular spirals appear in the coneflower and sunflower pictured below? Why are the seeds in each packed so efficiently?



A marvelous combination of plant biology, mathematics, and a computer model enables us to generate seeds according to a fixed angle of rotation, experiment with different angles, and explore patterns in the numbering of seeds. In so doing, we discover some startling connections between a famous number and the ubiquitous Fibonacci sequence that help explain these spiral patterns and the evident beauty that captures our eye.

Along the way, we'll also discuss the nature of mathematics and think about some of the deep observations and questions about mathematics posed by a few of humankind's greatest minds.

"*How can it be that mathematics, being after all a product of human thought independent of experience, is so admirably adapted to the objects of reality*?"

-Albert Einstein

3. **Is it totally irrational to ask what $\zeta(3)$ is?**

   Euler's famous result that

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

   is one of the great theorems of mathematics. The so-called Basel Problem challenged several famous mathematicians and was ultimately solved by one of the greatest mathematical minds ever. While Euler's original solution may have been insufficiently rigorous by today's standards, a long line of alternative proofs have emerged in the almost three centuries since he first discovered the value $\frac{\pi^2}{6}$ in 1734. Euler even went so far to determine values of

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s},$$

   where $s = 2k$ is any even integer. And yet, the odd case eluded him, and essentially remains an unsolved problem to this day.

   In 1978, R. Apéry proved a landmark theorem, showing that $\sum_{n=1}^{\infty} \frac{1}{n^3}$, the sum of the reciprocals of the cubes, is irrational. A much simpler proof was published soon after by F. Beukers, whose argument rests on a a lemma that may be easily used to establish the irrationality of $\zeta(2)$, the irrationality of $\zeta(3)$, that the value of $\zeta(2)$ is $\pi^2/6$. Furthermore, if mathematicians can ever discover a way to evaluate a complicated double integral, the exact value of $\zeta(3)$ (which is as yet unknown) will be found.

   In this talk we'll consider the history of the equation $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ and discuss an overview of several different proofs. In addition, we will look at a more general approach to several different related results and see commonalities among arguments used to establish the irrationality of π, $\ln(2)$, φ, and $\zeta(2)$.

4. **Calculus 2020**

   Abstract. Calculus is one of the great intellectual achievements of humankind, and the subject can reasonably be considered an essential part of a liberal education, particularly in mathematics. At the same time, the study of calculus in the 21st century should be less about acquiring a body of knowledge, and instead be focused on a student experience filled with rich opportunities for problem-solving, communicating, and working hard. Indeed, in an age when Wolfram|Alpha can solve almost any procedural problem in calculus, students of the subject need to build skills that go well beyond basic calculations.

   In this talk, we revisit the early years of calculus reform, discuss an overview of our current understanding of both the state of collegiate calculus instruction and the effects of pedagogical choices on student learning, and cast a vision for calculus instruction in the coming decade. In particular, we will consider a 1997 MAA report that followed the first decade of calculus reform, and called for calculus instruction to (a) be valuable to students' goals, and not a barrier to their success; (b) emphasize direct experience with methods and processes of inquiry; (c) acknowledge that students learn by doing, by applying, by working with peers, and by writing about their work; and (d) involve application, technology, and multiple points of view. From there, we'll discuss an overview of the results of the MAA's recent Characteristics of Successful Programs in College Calculus, as well as some major studies regarding student learning outcomes that result from different teaching approaches. Finally, we'll state some natural goals and actions that apply to both individual instructors and our community at large.

5. **Young, Wild, and Free: Open-Source Math Textbooks**

   Abstract. University textbook publishing is an appromixately $7-billion-a-year industry. For comparison, Party & Event Planning, Halloween, and Greeting Cards are each industries with comparable revenues (±$2B). The average cost of four popular single-variable calculus textbooks[1] is $178.50. While a 500-page printed for-profit calculus text may sell for $200 or more new, a 500-page .pdf file can be turned into a softcover, bound text through a print-on-demand service for under $20. Moreover, of course, the internet can be used to freely transmit the .pdf file.

   In this talk, we discuss multiple ways in which the internet is disrupting traditional textbook publishing, share news and information regarding a wide range of available free and open-source mathematics textbooks, and argue that mathematics faculty should at least *consider* the possibilities presented by this new and emerging market. Along the way, we will pose questions and challenges for free and open texts as part of a conversation about the pros and cons of free books for students.

[1] New, via Amazon, as of 1/11/2016. Books sampled: Stewart's *Concepts and Contexts, 2010* ($240.57), Hughes-Hallett's *Calculus, 2012* ($118.32), Larson and Edwards's *Calculus of a Single Variable, 2013* ($234.18), and Rogawski's *Calculus, 2011* ($120.70)