11608. Proposed by D. Aharonov and U. Elias, Technion - Israel Institute of Technology, Haifa, Israel. Let f and g be functions on \mathbb{R} that are differentiable n + m times, where n and m are integers with $n \ge 1$ and $m \ge 0$. Let A(x) be the $(n + m) \times (n + m)$ matrix given by

$$A_{j,k}(x) = \begin{cases} (f^k(x))^{(j-1)}, & \text{if } 1 \le j \le n; \\ (g^k(x))^{(j-1-n)}, & \text{if } n < j \le n+m, \end{cases}$$

Let $P = \prod_{r=1}^{n-1} r! \prod_{q=1}^{m-1} q!$. Prove that

$$\det A(x) = Pf(x)^n g(x)^m [g(x) - f(x)]^{mn} f'(x)^{n(n-1)/2} g'(x)^{m(m-1)/2}.$$