From the Editor

Oooh, That Awful Yellow!

The yellow headers and titles in our December issue generated more complaining letters than any other feature of *MAA FOCUS* in the last several years! Let me assure everyone that Carol Baxter and I do retain our sanity. The yellow headers were not the result of some daring experimentation; rather, we both simply weren't paying attention to the interior color until it was too late. Our apologies to everyone, especially those who are colorblind or whose eyes, like mine, are no longer as given to heroic feats as they once were. We have learned our lesson!

To the Editor

We are always happy to hear from our readers. If at all possible, send letters by email to *fqgouvea@colby.edu.* Letters will be edited for publication.

How to Help Students Succeed

In the May/June *MAA FOCUS*, Carmen Latterell asked "How Do Students Study?" I would like to comment on how to help them succeed. Latterell outlines three main lessons that she has learned in years of teaching. I restate each along with techniques I have found useful.

Students do not read the textbook, but they do attend lectures. One interesting way to encourage students to read the textbook is through the use of reading questions. For each section, assign two to three definition-based or open-ended questions that students must turn in at the beginning of class. For instance, in a differential equations class teaching first-order linear DEs, you may ask: 1. What is the standard form of a first-order linear equation? 2. Are all Linear DEs separable? 3. What is the role of the integrating factor?

Students do not work problems that are viewed as extra. One way to get students to work more problems is to collect more than you grade. I reserve a small percentage of the points for completeness. This way, students feel they are rewarded for the problems that are collected, but not graded.

Students think they are doing better than they are. It is important for a student to realize where they stand in a course. If you keep an up-to-date grade book it is very easy to include a students' class total at the top of any assignment. A simple 248/300 at the top of the second test quickly lets the students know that they are currently at a low B.

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More on Induction

In regard to the article "More on Teaching Induction" by David M. Bradley (October 2008) on the overuse of induction as a proof technique: We can certainly construct proofs of combinatorial identities, such as his example—$1 + 2 + 3 + \ldots + n = n(n + 1)/2$—that hide the induction from our students. As mathematicians, though, we should keep in mind that with identities of this type induction is always present, at least in the background. There is an implicit quantifier in front of the identity saying "for every positive integer n…" And in almost every treatment of the foundations of mathematics, the definition of the positive integers is via the recursive Peano axioms or some equivalent recursive method. Induction is often the most natural proof technique when dealing with recursively defined structures.

I have always taught my students that an ellipsis is almost certainly a shorthand notation, a flag shouting "induction is the proper tool to make this concept formal." The left-hand side of the identity above is nonsense when n is 1, 2, or 3, and ambiguous for larger integers. A better, unambiguous expression, valid for every positive integer n, is $\sum_{i=1}^{n} i$, where the $\sum$ notation is defined recursively:

$$\sum_{i=1}^{1} a_i = a_1, \text{and } \sum_{i=1}^{n+1} a_i = \sum_{i=1}^{n} a_i + a_{n+1}.$$

In fact this recursive definition, or something very similar to it, is pretty much forced upon us by the recursive definition of the positive integers and the fact that addition is defined as a binary operation. We should not be surprised, then, when induction turns out to be a natural proof technique for identities that sum over the positive integers.

I am a great fan of the book *Proofs that Really Count*, by Arthur Benjamin and Jennifer Quinn (MAA, 2003). The first chapter, on identities involving sums of Fibonacci numbers, is typical. Some 31 such identities are proved, in a charming and apparently non-

inductive manner, by showing that both sides of a Fibonacci identity count the same set of tiling patterns. But the proofs of all these identities are based on Combinatorial Theorem 1, initially linking Fibonacci numbers to tiling patterns. And the proof of Combinatorial Theorem 1 is inductive. How else can you lay the foundation for proofs involving an integer sequence defined recursively?

There may be times when it is pedagogically advantageous to encourage our students to accept and create non-inductive proofs of combinatorial identities. But let's be aware of the fact that there is something intrinsically recursive about these identities. In such cases, an inductive proof might well be the most natural.

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