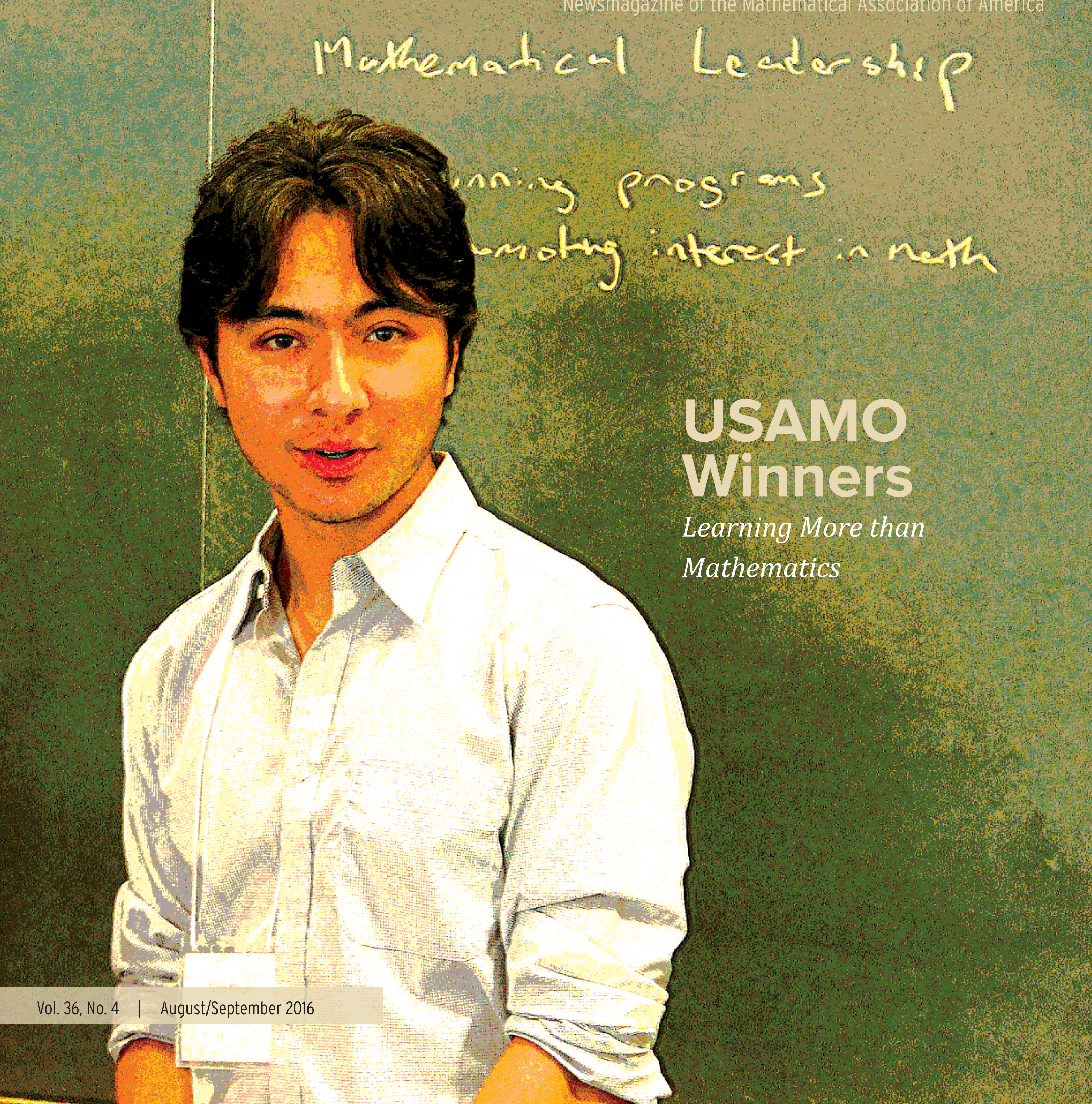
MAA FOCUS

Newsmagazine of the Mathematical Association of America



USAMO Winners

Learning More than Mathematics

Vol. 36, No. 4 | August/September 2016

MAA 100
MATHEMATICAL ASSOCIATION OF AMERICA
CELEBRATING A CENTURY OF ADVANCING MATHEMATICS

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From the Editor

Summer is flying by. MAA MathFest marks a bittersweet moment for me—signifying the end of summer with its long list of not-yet-accomplished projects, but bringing excitement from seeing so many friends and getting fired up for another academic year!

What are you looking forward to in the coming year? We hope that the Toolkit columns are useful, that the Section Happenings are sparking ideas as you plan meetings, and that our features are keeping you informed and engaged. But what would you like to see in *MAA FOCUS* this year? Submit an article about something amazing you are doing, or let us know what topics you’d love to read about. We look forward to hearing from you!

I hope the rest of the summer is great for you and that your preparations for fall go well, whether you are starting a new research program, greeting a classroom of fresh faces for a course you’ve taught for years, or looking forward to sabbatical! Stay tuned for a special issue highlighting the life of Paul Halmos, coming in October/November.

Jacqueline Jensen-Vallin

Corrections to the June/July Issue

Quote: The words attributed to Benjamin Franklin (“Tell me and I forget. Teach me and I remember. Involve me and I learn.”) more likely came from Xun Kuang, a Chinese philosopher (312–230 B.C.).

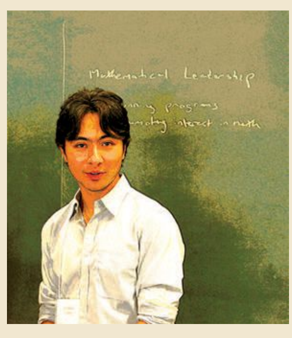
Putnam: On page 4 it was stated that there were 3,092 participants on the 2015 Putnam exam who scored 0 and the median score was 1. The correct numbers were 2,367 participants and median score 0.

Zeta: In the review of *The Riemann Hypothesis,* at the bottom of the first column on page 33, the plus sign in the denominator of Euler’s product formula for the zeta function should be a minus sign. At the bottom of the second column, on the righthand side of Riemann’s functional equation, the $(z + 1)$ should be $\zeta(z + 1)$.

ON THE COVER

Jacob Klegar, one of the dozen winners of the 2016 USA Mathematical Olympiad.

Photo: Alexandra Branscombe



MAA FOCUS

Mathematical Association of America

Vol. 36, No. 4 | AUGUST/SEPTEMBER 2016

Dear MAA: What can I do if half of my class is failing?

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New Online Bookstore Open for Business

In June the MAA opened its newly renovated online bookstore, found at store.maa.org. The store offers new titles, best-sellers, award winners, textbooks, videos, and information on soon-to-be-published titles. Books are arranged by topic, making the site easy to navigate.

The first time you come to the new store, you'll need to create an account with a new username and password. The MAA member discount code (MMBR2016) is valid in the new store until the end of 2016. Members must use this code during the checkout process to receive their 25 percent discount on books.

FOUND MATH: *Francis Su getting hands-on with the spinning mathematical sculpture at the University of Michigan.* The Cube "Endover," *was created by Tony Rosenthal in 1968. Submit photos to* foundmath@maa.org.

Call for Nominations

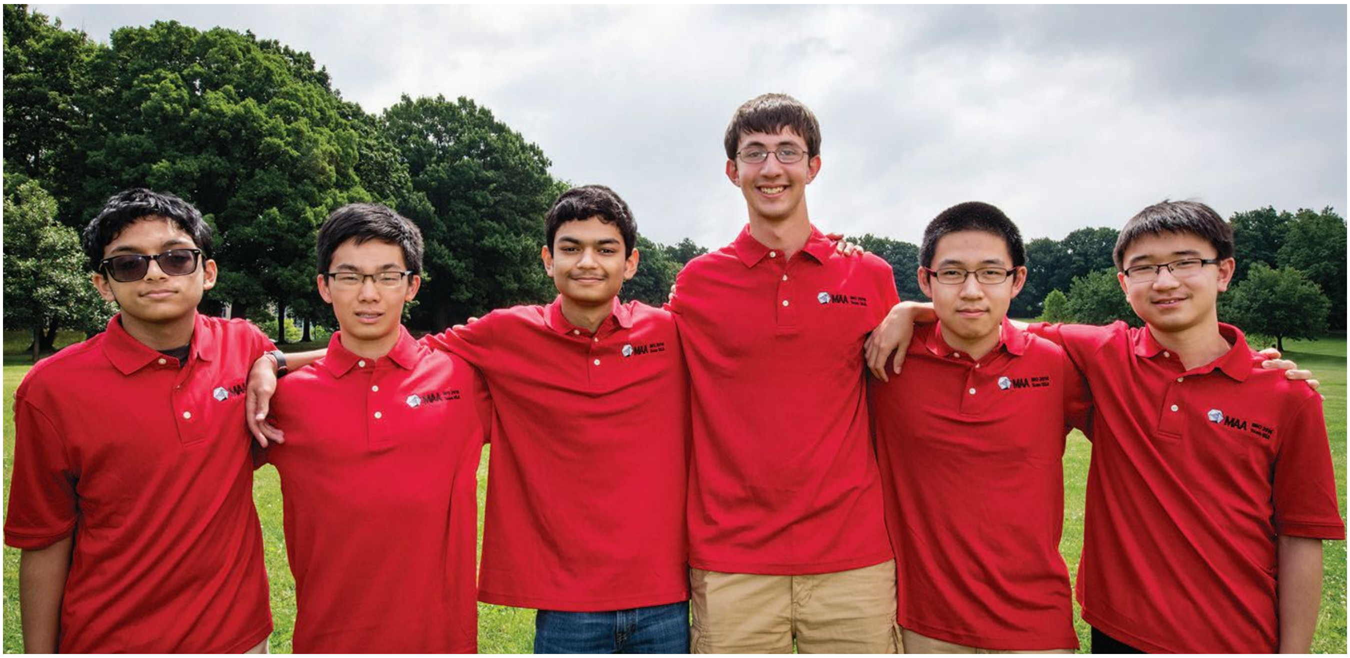
DANIEL SOLOW AUTHOR'S AWARD

This annual award is to recognize the author or authors of undergraduate mathematics teaching materials. The primary criteria for selection will be the material's impact on undergraduate education in mathematics and/or the mathematical sciences (operations research, statistics, computer science, applied mathematics). Eligible materials, written in English, may come from the mathematical sciences, broadly construed, and be published or developed within 15 years of the date of submission for the award. For materials to be considered, the nominators must reference faculty at at least three institutions where the materials have had a positive impact on students during at least the last three years. Details on the award are given at the MAA website (*maa.org/awards*). Nominations should be sent to Barbara Faires, secretary of MAA, *secretary@maa.org*, by October 1.

Call for Nominations

HENRY L. ALDER AWARD
for Distinguished Teaching by a Beginning College or University Faculty Member

The Alder Award honors beginning college or university faculty whose teaching has been extraordinarily successful and whose effectiveness in teaching undergraduate mathematics is shown to have influence beyond their own classrooms. Each year up to three college or university teachers who are members of the MAA are honored with this national award. Details on the award are given at the MAA website (*maa.org/awards*). Nominations for the award may be made by any member of the MAA or by any section of the MAA and should be sent to Barbara Faires, secretary of MAA, *secretary@maa.org*, by October 1.



U.S. Wins First Place at International Competition, Again

For the second year in a row, the U.S. team won first place at the International Mathematical Olympiad (IMO) in Hong Kong, July 6-16. This was the 57th year for the competition.

The IMO is the World Championship Mathematics Competition for High School students, where the brightest mathematics students from more than 100 countries compete. The winning U.S. team score was 214 out of a possible 252, ahead of the Republic of Korea (207) and China (204).

"We are very excited to bring home another first-place IMO award, which serves as a recognition for the the high standard of mathematical creativity and problem-solving capabilities we have in our country," said Po-Shen Loh, lead coach for the U.S. team and associate professor of mathematics at Carnegie Mellon University.

The six U.S. team members were selected through a series of competitions organized by the Mathematical Association of America (MAA), culminating with the USA Mathematical Olympiad. The six team members joined 70 of their peers at Carnegie Mellon University in June to immerse themselves in problem solving for three weeks at MAA's Mathematical Olympiad Summer Program.

"We have been running the U.S. Olympiad training program with a focus on the long-term development of our country's talent, and it's great to see that reflected in the continued team success a second year in a row," said MAA Executive Director Michael Pearson.

Members of the winning 2016 U.S. team were Ankan Bhattacharya, Michael Kural, Allen Liu, Junyao Peng, Ashwin Sah, and Yuan Yao, all of whom were awarded gold medals for their individual scores. Team members Liu and Yao each earned perfect test scores. The team was accompanied by Loh and deputy coach Razvan Gelca, professor of mathematics and statistics at Texas Tech University.

IMO scores are based on the number of points scored by individual team members on six problems. On each day of the two-day competition, the teams have 4.5 hours to work on three problems. Liu and Kural are the only returning team members from last year's winning U.S. team.

The mission of the MAA's American Mathematics Competitions is to increase interest in mathematics and to develop problem-solving skills through participation in a fun competition. Major donors to the MAA Competitions program include the Akamai Foundation and the Simons Foundation. Robert P. Balles provides cash awards to team members.

—Alexandra Branscombe

ABOVE: Ankan Bhattacharya, Michael Kural, Allen Liu, Junyao Peng, Ashwin Sah, and Yuan Yao.

Applause, Applause

Awards Scheduled for MAA MathFest 2016

Carl B. Allendoerfer Awards

The Carl B. Allendoerfer Awards, established in 1976, are made to authors of articles of expository excellence published in Mathematics Magazine. *The awards are named for Carl B. Allendoerfer, a distinguished mathematician at the University of Washington and MAA president, 1959-1960.*

Julia Barnes, Clinton Curry, Lisbeth Schaubroeck, and Elizabeth Russell

"Emerging Julia Sets," *Mathematics Magazine* 88, no. 2, April 2015.

Ben Klein and Irl Bivens

"The Median Value of a Continuous Function," *Mathematics Magazine* 88, no. 1, February 2015.

Trevor Evans Award

The Trevor Evans Award, established by the MAA Board of Governors in 1992 and first awarded in 1996, is made to authors of expository articles accessible to undergraduates and published in Math Horizons. *The award is named for Trevor Evans, a distinguished mathematician, teacher, and writer at Emory University.*

Joshua Bowman

"The Way the Billiard Ball Bounces," *Math Horizons,* February 2015.

Paul R. Halmos - Lester R. Ford Awards

The Paul R. Halmos-Lester R. Ford Awards recognize authors of articles of expository excellence published in the American Mathematical Monthly. *The awards were established in 1964 as the Ford awards, named for Lester R. Ford Sr., a distinguished mathematician, editor of the* American Mathematical Monthly, *1942-1946, and MAA president, 1947-1948. In 2012, the MAA Board of Governors designated these awards as the Paul R. Halmos–Lester R. Ford Awards to recognize the support for the awards provided by the Halmos family and to recognize Paul R. Halmos, a distinguished mathematician and editor of* The Monthly, *1982-1986.*

Alex Chin, Gary Gordon, Kellie MacPhee, and Charles Vincent

"Pick a Tree—Any Tree," *American Mathematical Monthly* 122, no. 5, May 2015.

Zhiqin Lu and Julie Rowlett

"The Sound of Symmetry," *American Mathematical Monthly* 122, no. 9, November 2015.

Annie and John Selden Prize

In November 2004, the MAA Board of Governors approved the Annie and John Selden Prize for Research in Undergraduate Mathematics Education honoring a researcher who has established a significant record of published research in undergraduate mathematics education and who has been in the field at most 10 years. The prize is designed to be an encouragement to such researchers and at most one is awarded every other year.

Pablo Mejia-Ramos

Mejia-Ramos completed his PhD in mathematics education in 2008, at the University of Warwick. He is an associate professor at Rutgers University jointly appointed in the Department of Learning and Teaching (within the Graduate School of Education) and the Department of Mathematics.

Mejia-Ramos has published 26 journal articles—unusually high for such a young scholar. His research is mostly recognized in his works on proof comprehension. However, he has also contributed to research on student evaluation of proofs, proving practices of undergraduate mathematics students, the efficacy of instructional recommendations from mathematics education research, and understanding of mathematicians' practice. His work, most in collaboration with K. Weber and M. Inglis, has had a significant influence on the direction of research in undergraduate mathematics education in general, and proof education research in particular. His coauthors praise him for raising the standards of their studies and developing additional studies to further test the emerging hypotheses.

George Pólya Awards

The George Pólya Awards, established in 1976, are made to authors of articles of expository excellence published in the College Mathematics Journal. *George Pólya was a distinguished mathematician, well-known author, and professor at Stanford University.*

Hassan Boualem and Robert Brouzet

"To Be (a Circle) or Not to Be," *College Mathematics Journal* 46, no. 3, May 2015.

Gordon Hamilton, Kiran S. Kedlaya, and Henri Piccotto

"Square-Sum Pair Partitions," *College Mathematics Journal* 46, no. 4, September 2015.

Certificates of Meritorious Service

Certificates for Meritorious Services are presented, on the recommendation of the sections of the association, for service at the national level or for service to a section of the association. The first such awards were made in 1984. Each year, honorees from several sections are recognized.

EPADEL: **Nancy Hagelgans**

Iowa: **Ruth Berger**

Metropolitan New York: **Raymond N. Greenwell**

North Central: **Deanna Haunsperger** and **Stephen Kennedy**

Pacific Northwest: **Stuart Boersma**

Southwestern: **Tom Gruszka**

Henry L. Alder Awards for Distinguished Teaching by a Beginning College or University Mathematics Faculty Member

The Alder awards were established in January 2003 to honor beginning college or university faculty whose teaching has been extraordinarily successful and whose effectiveness in teaching undergraduate mathematics is shown to have influence beyond their own classrooms. An awardee must have taught full time in a mathematical science in the United States or Canada for at least two, but not more than seven, years since receiving a PhD. Henry Alder was MAA president, 1977-78, and served as MAA secretary from 1960 to 1974.

Benjamin Galluzzo, Shippensburg University

Jana Gevertz, College of New Jersey

Dandrielle Lewis, University of Wisconsin–Eau Claire

Combating Education Inequity: MAA Distinguished Lecture

Talithia Williams.

—Katharine Merow

"Question for you," Talithia Williams (Harvey Mudd College) said to her audience in the MAA Carriage House on June 8. "As of 2008, what percent of white Americans between the ages of 25 and 34 had at least an associate's degree?"

"Twenty-four percent," ventured one listener.

"Sixty," countered another, laughing at his own optimism.

After entertaining a smattering of other guesses, Williams disclosed the statistic: "Thirty-eight percent," she said. "What percentage of black Americans?"

Here estimates clustered around 10 percent, though the actual figure is 26 percent.

"That is a wonderful example of lowered expectations," remarked Ezra "Bud" Brown (Virginia Tech).

And so transpired perhaps the most interactive installment of MAA's Distinguished Lecture Series, funded by the National Science Foundation. In "A Mathematical Mind-Set: Mitigating America's Achievement Gap," Williams not only presented information about and ideas for combating education inequity in the United States, but also encouraged audience members to offer their own perspectives.

The Gap(s)

Williams defined an achievement gap as "any persistent disparity in academic performance between different groups of students, or the unequal distribution of educational results," and she prompted attendees to name some commonly discussed gaps.

Gaps exist between American students and those from other countries, between students from low- and high-income households, between native speakers of English and those for whom it is a foreign language. There are performance disparities between male and female students, between Asian/Caucasian students and those in other groups.

You can affect that one person, and in doing that, you will affect the generations that come behind them.

"What do you think the number one cause of the achievement gap is?" Williams asked.

"Poverty," piped up several listeners in unison.

Correct. Other contributing factors include inferior educational resources, familial instability, flawed assessments, and those lowered expectations Bud Brown mentioned.

Stereotypes of minorities often lead schools to expect less of African American and Hispanic students and therefore to enroll them in less challenging courses.

"Oh sweet darling, it's just a blessing that you're not in a gang," Williams said, mimicking the message sent to countless minority students across the country. "You don't need a degree. You don't need to be college-ready. You just do what you can. I'm just so proud of you for showing up at school today."

Growth Mind-Set

A first step in resolving the achievement gap, Williams said, is promoting a growth mind-set.

Drawing on the work of Carol Dweck, Williams contrasted a growth mind-set with a fixed mind-set, in which intelligence and talent are viewed as inborn, immutable traits.

"In a growth mind-set, people believe that these abilities can be developed through dedication and hard work," Williams explained.

The idea that intelligence can be developed motivates students to embrace challenges, Williams said, to persist despite obstacles, to learn from criticism, and to see effort as the path to mastery.

To encourage a growth mind-set, you don't let a student off with, "Great effort! You tried your best." That comment accepts less-than-optimal performance. A better tack is to tell a struggling student that the point isn't to get it all right away. "The point is to grow your understanding step by step," you might say. "What can you try next?"

Math Mind-Set

Williams cited work by the U.S. Department of Education's Clifford Adelman showing that the more mathematics a student studies in high school, the greater his or her chance of earning a bachelor's degree (in any subject).

Why the correlation? Williams attributed it to the assumption that, if you do math, you must be smart and the effect this has on students who shine in mathematics but don't immediately excel across the board.

"When I would go to those other classes where I wasn't as good, you know what I would think?" Williams asked. "'I'm in trigonometry. I must be smart! How hard can this essay be?' I pushed myself in these other courses because I was told I was so smart in math."

Success in math forces a growth mind-set, Williams said.

What You Can Do

Williams closed her talk by describing the Sacred SISTAHS Math and Science Conference that brings African American girls ages 12 to 18 to the Harvey Mudd campus each year to meet African American women working in STEM fields.

"This event can be transformative because it gives you an image of what you can become," Williams said, remembering how the late Claudia Alexander of NASA's Jet Propulsion Laboratory did that for her as a teen. "Often I would hold on to her image when I would struggle in grad school. I was like, 'Well, Claudia did it.' I didn't have to call her. I didn't have to email her. I just knew that I could do it because of her."

And even if you don't have the wherewithal to organize an annual conference, Williams assured her Carriage House audience, you too can help chip away at the achievement gap.

"Often we think, 'Oh my gosh, I can't affect the achievement gap. It's so big,'" Williams said. "But you know somebody who is probably in poverty or first-generation. You can affect that one person, and in doing that, you will affect the generations that come behind them."

Katharine Merow is a freelance writer and editor.

A. BRANSCOMBE

USAMO Winners Celebrated

With mathematical excellence comes great responsibility—and even a call to lead and innovate. That was the theme of this year's celebration of the 2016 USA Mathematical Olympiad (USAMO), honoring the winners of the highest level of high school mathematics tests administered by the MAA's American Mathematics Competitions program (AMC).

Each year, the top 12 USAMO scorers are invited to the MAA headquarters in Washington, D.C., to meet with mathematical leaders from academia, industry, and government. This year's USAMO winners are Ankan Bhattacharya, Ruidi Cao, Hongyi Chen, Jacob Klegar, James Lin, Allen Liu, Junyao Peng, Kevin Ren, Mihir Singhal, Alec Sun, Kevin Sun, and Yuan Yao.

MAA President Francis Su and Talithia Williams, both professors of mathematics at Harvey Mudd College, led a morning workshop for the students. Their presentation, "Mathematical Leadership," highlighted the advice from past USAMO and International Mathematical Olympiad contestants who are leaders in the mathematical community today.

As young achievers, the students in the room may enter leadership opportunities very soon, advised Williams and Su, and maybe from unexpected places. They then read from the remarks sent in from past math olympians.

"Leadership is less a matter of knowing what to do and telling others as it is an ability to be part of a shared endeavor, and help others to guide you and each other in reaching a shared destination," read Williams, from the advice of Paul Zeitz, mathematics professor at the University of San Francisco and IMO team member in 1974. "In other words, 'leadership' is very difficult, and like mathematics, rarely is direct or without setbacks," Zeitz wrote.

Continuing the mathematical celebration, the students, accompanied by their parents, mentors, and other mathematical attendees, gathered for the evening ceremony at the U.S. Department of State. There, the 12 honorees were each awarded with a USAMO medal and financial prizes for their accomplishment.

Representing the White House, D. J. Patil, the U.S. chief data scientist and deputy chief technology officer for data policy at the White House Office of Science and Technology Policy, attended the event to honor the U.S. students.

During dinner, Patil read aloud a letter from President

A. BRANSCOMBE

Top USAMO 2016 Scorers

Ankan Bhattacharya (International Academy East, Troy, Michigan)
Ruidi Cao (Missouri Academy, Maryville, Missouri)
Hongyi Chen (Fairview High School, Boulder, Colorado)
Jacob Klegar (Choate-Rosemary Hall, Wallingford, Connecticut)
James Lin (Winchester High School, Winchester, Massachusetts)
Allen Liu (Penfield Senior High School, Penfield, New York)
Junyao Peng (Princeton International School of Mathematics and Science, Princeton, New Jersey)
Kevin Ren (Torrey Pines High School, San Diego, California)
Mihir Singhal (Palo Alto Senior High School, Palo Alto, California)
Alec Sun (Phillips Exeter Academy, Exeter, New Hampshire)
Kevin Sun (Phillips Exeter Academy, Exeter, New Hampshire)
Yuan Yao (Phillips Exeter Academy, Exeter, New Hampshire)

Awards and Prizes

Samuel L. Greitzer/Murray S. Klamkin Award for Mathematical Excellence:
Allen Liu

Robert P. Balles U.S.A. Mathematical Olympiad Prize:
USAMO Winners

Akamai Foundation Scholarship Awards:
First Place – Allen Liu
Second Place – Junyao Peng
Third Place – Kevin Ren

Barack Obama. "Our nation's youngest minds possess the ingenuity and curiosity that will help write the next great chapters of our story," read Patil. "When the time comes, society will look to doers and dreamers like you to help us reach for unbounded heights and greater possibilities."

Among the mathematical leaders who followed Patil in offering their congratulations to the evening's honorees, were Ken Baron from Two Sigma; Michael Catalano-Johnson, associate director of Susquehanna International Group; Katherine Sorensen, vice president of the D. E. Shaw Group; and Noelle Faris, president of the Akamai Foundation. Many of them mathematicians in their own right, they offered sage words and a glimpse into the challenges ahead of the young problem-solvers.

"You have a tough task ahead of you," Faris said. "It has been said that this generation will be filling jobs that don't currently exist. . . . How does one prepare for a job that doesn't currently exist?" The answer, she said, is mathematics.

—*Alexandra Branscombe*

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The MAA acknowledges the generous support of donors who help sustain the MAA American Mathematics Competitions and Olympiads.

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Sustainer's Circle – Academy of Applied Science, American Mathematical Society, Ansatz Capital, Army Educational Outreach Program
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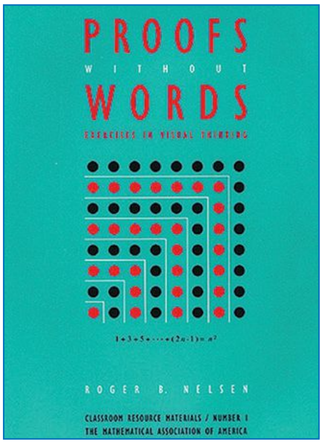
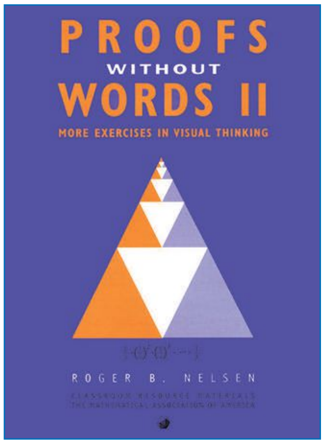
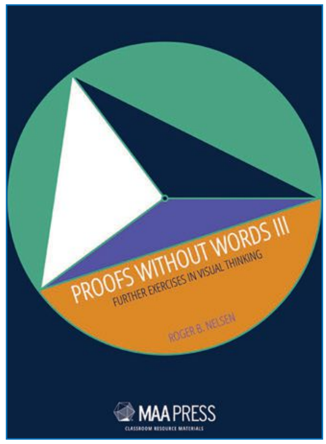
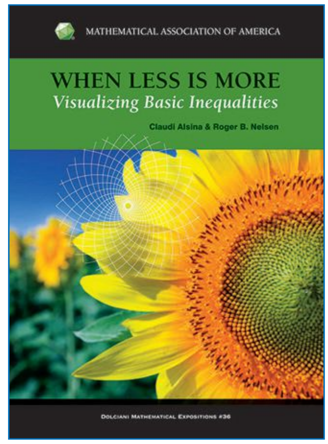
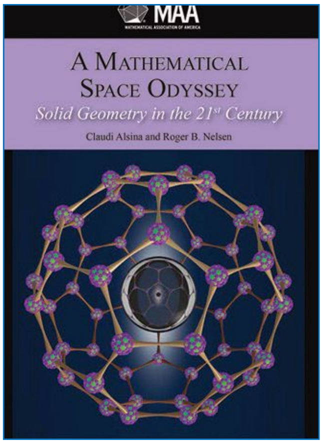
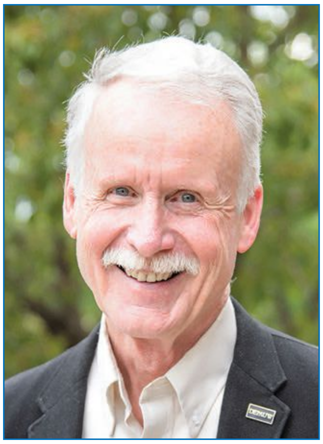


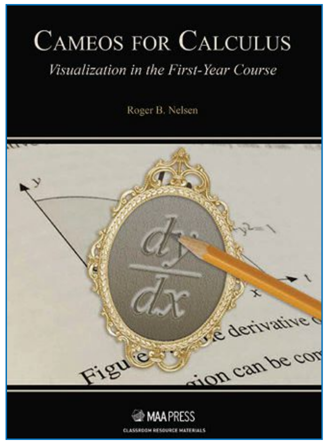


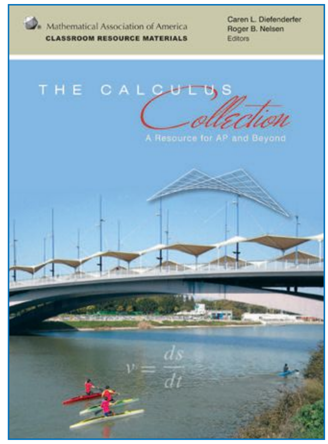












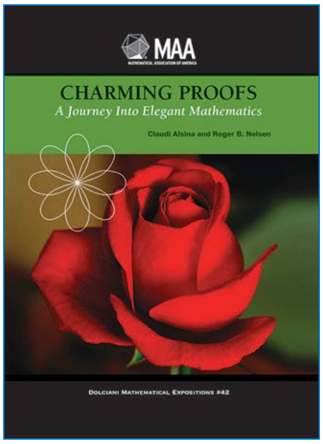
Roger Nelsen:

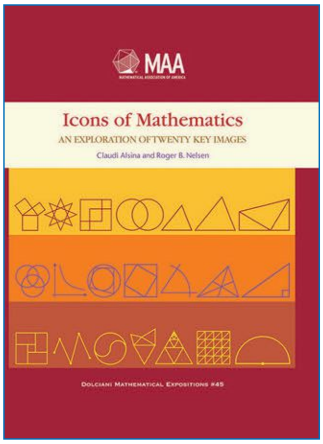
On Being an MAA Author

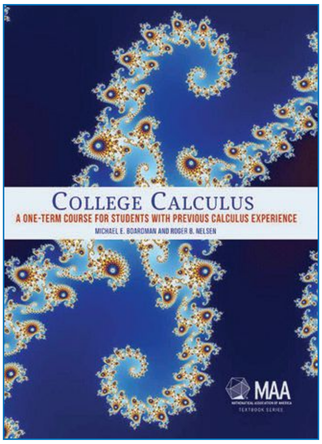
Roger Nelsen, professor emeritus of Lewis & Clark College, has written 11 books for the MAA since 1993. Perhaps his most well-known (and best-selling, at number 2,869 on Amazon's Science and Mathematics Textbooks list) is Proof Without Words. *Wondering how someone has time to write that much? Nelsen says that it is a matter of finding a topic that really interests and engages you, finding great co-authors who are also friends, and being retired. We can't all make the last of these happen (at least right now), but finding the right topic and coauthor is great advice for any aspiring writer. He was interviewed by Jacqueline Jensen-Vallin, editor of* MAA FOCUS.

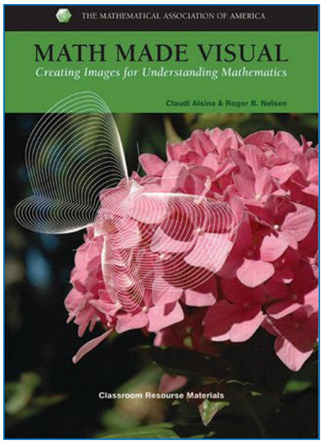
How did you get involved with MAA?

As an undergraduate in the early '60s, I occasionally read the [*American Mathematical*] *Monthly* at the college library, but I knew little about its publisher. That changed soon after I arrived at Lewis & Clark in 1969. The department chair, Elvy









Frederickson, was a member of the MAA and would share her copies of the *Monthly* and *Mathematics Magazine* with me. Soon I found myself borrowing her copies each month to read some of the articles and try my hand at the problems in the Problems and Solutions column.

After a few years Elvy told me it was high time that I "support the profession" (and obtain my own copies of the *Monthly*, the *Magazine*, and the *College Mathematics Journal*) by becoming a member of the MAA, which I did in 1973. I continued to attempt the problems in all three journals, which eventually led to a 10-year stint (1989-1998) with several of my L&C colleagues as coeditors of the Problems and Solutions column in the *College Mathematics Journal.* That led to a term as chair of the editorial board for the MAA's Problem Books series.

How did you get started writing books?
In the fall of 1975 the first "proofs without words" (PWWs) appeared in *Mathematics Magazine*, and soon PWWs were appearing regularly as end-of-article filler. Like lots of readers, I enjoyed the mental challenge of figuring out how a PWW illustrates the truth of a statement and hints at its more formal proof. I thought that perhaps I could create PWWs, and after a number of disappointing efforts, I succeeding in having my first PWW appear in the *Magazine* in 1987, when Jerry Alexanderson was its editor.

After a few more successes, Jerry began to send PWW submissions to me to referee. To do so, I made copies of every published PWW I could find and placed them in file folders so I'd know which submissions were new and which ones weren't. Refereeing the PWWs taught me two things: PWWs were popular with the readers, and many submissions (more than half) were rediscoveries of previously published PWWs.

At one of the January meetings, I asked Don Albers, then the MAA's director of publications, if the MAA would be interested in a PWW book, which I could create from those growing file folders of PWWs. Don encouraged me to send him a manuscript, which I duly did sometime later, and *Proofs Without Words: Exercises in Visual Thinking* appeared in 1993 as the inaugural volume in the Classroom Resource Materials series.

That was perhaps a good choice for a first book, rather easy to write as it had so very few words! That has also made it easy to translate, and translations have appeared in Greek, Spanish, Japanese, Persian, French, Czech, and German.

How have textbooks changed since you started writing them?
That's a hard question to answer! In mathematics, I think it depends on the course for which the text is written. For example, in calculus (the only course for which I've coauthored a text), the texts look quite different than they did, say, 20 years ago. They have more applications, more use of color, more graphics, and more pages. However, I'm not sure the texts have kept up with the changes in the audience for calculus. I believe there are more students today taking calculus in high schools than in all the two- and four-year colleges and universities combined. As a consequence, the college calculus sequence is not really a sequence for many of our students, but texts are often written as if it were.

What are the components of a good book (or textbook)?
Another tough question! I can only answer from the perspective of an author, rather than of a reader or an adopter. For me a "good" book project is one that (a) will be enjoyable to write, because I'm interested in the topic and believe I have something worth saying about it, and (b) I'll learn more about the topic in the process of researching and writing. Of course, I always hope that a reader will experience some of the enjoyment I've had writing when he or she reads the book.

You have published 11 books with MAA. How do you manage to write as much as you do?
Well, being retired certainly helps. Of the 11 books I've authored or coauthored with the MAA, four were published prior to retirement in 2009, and seven since. For me, it's a lot of fun to write, and I learn quite a bit in the process, due in no small measure to the fact that I've had great coauthors with which to work.

The text we ultimately wrote is designed for well-prepared students to replace Calc II and prepare them for further college mathematics.

Tell me about your coauthors—how did you end up working with each of them?
I have been blessed to have had the opportunity to work with three really good friends as coauthors: Claudi Alsina at the Universitat Politècnica de Catalunya in Barcelona, Caren Diefenderfer at Hollins University in Virginia, and Mike Boardman at Pacific University in Oregon.

I met Claudi at an international symposium on functional equations in August of 1986 at Mount Holyoke College, and we became better acquainted coauthoring a 1993 paper with a colleague at UMass–Amherst. One of the benefits of teaching at L&C has been the opportunity to accompany students on overseas study programs to Spain in 1999, 2003, and 2009.

After each program I visited Claudi in Barcelona, in the process discovering a common interest in exploring visualization in mathematics. This ultimately led to our first three MAA books, *Math Made Visual: Creating Images for Understanding Mathematics* in 2006, *When Less is More: Visualizing Basic Inequalities* in 2009, and *Charming Proofs: A Journey Into Elegant Mathematics* in 2010. Retirement in 2009 ended the overseas programs with the students, but Claudi and I have continued our collaboration, mostly electronically now, yielding *Icons of Mathematics: An Exploration of Twenty Key Images* in 2011 and *A Mathematical Space Odyssey: Solid Geometry in the 21st Century* in 2015.

I met both Caren and Mike through the Advanced Placement Calculus program, where each of us attended the annual "reading" [grading] of calculus exams each summer for over 20 years. Both Caren and Mike served as the AP Calculus chief reader, Caren from 2004 through 2007, and Mike from 2008 through 2011. At the 2005 reading at Colorado State University, Caren and a representative of the College Board convened a meeting of several readers (of which I was one) to discuss the possibility of making some college-level calculus material more available to high school AP teachers.

Over the years a great many articles on calculus have been published in MAA journals, but most are not read by high school teachers since only a small fraction of them are MAA members. So Caren and I decided to edit a collection of articles, originally published in MAA journals, for teachers of the AP Calculus course. We were aided admirably in the project by a team of six veteran high school teachers who helped us select the 123 articles that appear in our volume *The Calculus Collection: A Resource for AP and Beyond.*

Both Caren and Mike will tell you that being an AP chief reader takes up a lot of one's time. Nearing the end of his term in 2011 and knowing that he'd no longer have to commit so much time to AP, Mike suggested we write what he vaguely called a "calculus book" together.

At our schools (Pacific U. and Lewis & Clark) many future math majors have studied AP calculus in high school and performed well on the AP calculus exam. But the syllabus for the Calculus AB exam doesn't mesh well with the syllabi for first year (single-variable) calculus in many colleges—the AB syllabus covers all of Calc I and about 40 percent of the standard Calc II at many colleges. So the text we ultimately wrote—*College Calculus: A One-Term Course for Students With Previous Calculus Experience*—is designed for well-prepared students (as measured by their scores on the AP exam) to replace Calc II and prepare them for further college mathematics.

Mike and I also hope it helps in the current conversation about how to better serve the students who studied calculus in high school when they come to college.

What would you like to write about next?
My favorite math course as an undergraduate was number theory, and I thought I'd continue in that field in graduate school. Then I discovered stochastic processes and changed my direction. However, the number theory course at L&C was one of my favorites to teach. But a cursory examination of number theory texts reveals how few illustrations many of them have. A number can represent many things—the cardinality of a set, the length of a line segment, or the area of a plane region. Such representations naturally lead to a variety of visual arguments, so number theory texts should have more pictures. Perhaps my coauthor Claudi Alsina and I will work on such a supplement for the number theory course.

When Prison and Graph Theory Meet

Jenny Switkes and Devonna Alatorre

We show up at the California Rehabilitation Center every Thursday morning. For seven weeks we teach a volunteer graph theory class to a group of prison inmate students. Has there ever before been a prison graph theory course? A quick Google search yields graph theory questions about prisoners, but no graph theory work being done by prisoners. How did this graph theory class happen? And what exactly happens in a graph theory class for prisoners?

Teaching through the Prison Education Project

The California Rehabilitation Center is a Level II correctional facility located in the city of Norco in suburban Southern California, east of Los Angeles. It is one of the facilities served by the Prison Education Project (prisoneducationproject.org), a great organization led by political science professor Renford Reese of California State Polytechnic University, Pomona. The PEP has provided academic programming in 11 California correctional facilities, with an overarching objective to create a "Prison-to-School Pipeline."

Our inmate students are enrolled in community college through old-school correspondence courses offered by two local schools. The students are supplementing their formal

ABOVE: Renford Reese, founder of the Prison Education Project, speaking to a group of inmate students.

coursework with classes offered through PEP. After teaching an introduction to calculus course for two terms, I asked the students if they would like to learn some different mathematics. They liked my description of graph theory, and so our prison graph theory course was born. Devonna Alatorre took my graph theory class in fall 2015, and she enthusiastically agreed to team teach this prison graph theory class with me—her very first teaching experience. Thus began an experience that is transforming both us and our inmate students.

Mathematics on the Inside

We arrive at the prison each equipped with an ID, car keys, notes, and a whistle. We go through security, take a golf cart up to the classroom area, and then wait while prison educational employees ensure that all is in order. Then our inmate students are released into the classroom. Depending on the week, we have between 10 and 20 inmate students in our class. They come into the room wearing their prison garb, hand over their identification cards, and sit down at desks. They are diverse ethnically and in age.

These students are generally upbeat, enthusiastic, and ready to do some hardcore mathematical thinking. They enjoy our class, and we enjoy the opportunity each week to encourage them and think mathematically along with them. We tell them that our time with them is a highlight of our week and that we are proud of them. Our students are not oblivious to their situation, but they are choosing to look forward and to learn.

Often, at the end of a class meeting, our inmate students applaud for us. We try not to cry, and let them know that *we* applaud *them*. And we struggle to understand the complex reality of inmate students—the same people who have committed serious crimes and are incarcerated for years—who are smart, kind, and insightful.

Thinking Together

Recently, we introduced these students to the concept of a complete bipartite graph, $K_{m,n}$. We helped the students count the number of edges, mn. Then somehow we started talking about tripartite graphs—almost certainly because a student asked if there is such a thing. So, we asked the students to guess how many edges there are in the complete tripartite graph, $K_{m,n,p}$. At first a student guessed mnp, the first natural guess at an extension from the complete bipartite case.

We explored this hypothesis together as a class using the complete tripartite graph $K_{2,3,4}$. Together, using this example, we figured out the correct result: $mn + mp + np$. As we ran out of time, we all began bringing up other questions, thoughts, and examples. We agreed to come back the next week with more thoughts on all of this.

The next Thursday, we showed the students that the original guess, mnp, does count something—it counts the number of distinct triangles in $K_{m,n,p}$. Meanwhile, this whole discussion got us all into a discussion of what it means for two graphs to be isomorphic. An inmate student then asked what $mnpq$ would count in the complete quadripartite graph, $K_{m,n,p,q}$. This led us to the following statement:

In the complete quadripartite graph, $K_{m,n,p,q,}$ the number of distinct subgraphs isomorphic to K_4 is given by $mnpq$.

We then stood back in awe of the reality that a group of inmate students had just enthusiastically explored and understood a technical graph theory statement. And then, to round out a pretty much perfect morning, after class one of our inmate students presented us with a write-up he had done by himself during the week in counting the number of edges in the complete quadripartite graph $K_{m,n,p,q}$. He correctly obtained the result $mn + mp + mq + np + nq + pq$, which he wrote in a less simplified but perfectly accurate form and tested out on $K_{1,2,3,4}$, written up formally.

What Is Next?

A few of our inmate students will be released soon. They have a tough road ahead of them, but we will reassure them that they can succeed. Some of them will get a sense of community and support through the Prison Education Project's Reintegration Academy (reintegrationacademy.org). Others will be in prison for many more years. We will do our best to encourage them, too, during the time we have with them.

From our graph theory course through the PEP, we have seen a glimpse of just how far passion for mathematics can take a student, no matter their educational or personal background. Our students have shown us how important it is to appreciate the art of learning through their enthusiasm toward graph theory. After our students conclude their graph theory course with us, we hope that they will continue to experience and share the beauty of mathematics.

Meanwhile, we have the pure joy of working together with these students who, like mathematics students everywhere, are fearfully and wonderfully made.

Jenny Switkes teaches mathematics at Cal Poly Pomona. Devonna Alatorre is just-graduated mathematics major from Cal Poly Pomona. Both of them are almost speechless about their experience teaching mathematics to inmate students through the Prison Education Project.

Insights about (Good and Ambitious) Teaching

Sean Larsen and Vilma Mesa

This article is the fifth report on findings from the MAA's NSF-supported studies to determine characteristics of successful programs in college calculus. Beginning in 2009, the MAA undertook a national survey of Calculus I instruction and conducted multiday case study visits to 20 colleges and universities with interesting and, in most cases, successful calculus programs. For further details, see the MAA Notes volume Insights and Recommendations from the MAA National Study of College Calculus *(Bressoud, Mesa, and Rasmussen, 2015) or visit* maa.org/cspcc.

The MAA's national study of college calculus defined success for a Calculus I program as a combination of student persistence in calculus; program passing rates; and changes in students' attitude toward mathematics (a composite of enjoyment of math, confidence in mathematical ability, and interest to continue studying math).

The survey and case study data allowed us to understand how such success played out in the participating institutions. An analysis of student survey data identified three factors, two of which had a pedagogical dimension, "good teaching" and "ambitious teaching" (see chapters 7 and 8 in the MAA calculus study, maa.org/cspcc). (Technology was the third factor.) Only good teaching was positively correlated with changes in students' affect (see chapter 2 in the study). But what exactly does that label cover? To answer that question, a further factor analysis of the 22 items in the "good teaching" factor revealed three components:

- **Classroom interactions that acknowledge students** included items such as "My instructor listened carefully to my questions and comments."

piranka/iSTOCK

■ The **Encouraging and Available Faculty** component included items such as: "My instructor acted as if I was capable of understanding the key ideas of calculus."

■ Finally, the **Fair Assessments** component included items such as: "My Calculus I exams were a good assessment of what I learned" (Mesa, Burn, and White, 2015).

Note that these three components have ample support in the literature, both in K-12 mathematics education and in higher education (Chickering and Gamson, 1991; Kuh, 2005; Leinhardt and Steele, 2010; Rendon, 1994; Staples and Truxaw, 2010).

These three components were explored in the case study data, which revealed two striking instructional features of these calculus programs: interactive lecture and predictable exams.

The first factor, interactive lecture, includes many exchanges between instructors and students (Burn, Mesa, and White, 2015). The exchanges were quick, in the form of questions with short answers, and involved more than a handful of students. When supportive instructors do this in the context of mathematically challenging activities, with the expectation that students can handle the content, students' attitudes are likely to remain positive.

The second feature was that students were rarely surprised by their exams. The students said their teachers were very explicit about their demands. Exams and assignments were said to be fair even though students also reported sometimes not doing well on them. Overall, during visits to the selected institutions for the MAA study, most of the administrators and students we talked to explicitly said that their calculus program was successful because of their teachers.

Challenging but Worth It

Ambitious teaching was the label that Sonnert and Sadler (2015) used to describe another of the three factors they identified with the student survey data. This factor included instructional characteristics such as using group projects, employing unfamiliar problems both in homework and on exams, requiring students to explain how they arrived at their answers, and cutting back on lecturing as the primary mode of instruction.

The research literature suggests that ambitious teaching can have important benefits. In fact, based on their meta-analysis of 225 studies, Freeman et al. (2014) argue that a promising way to reduce failure rates in STEM courses is to deploy any form of active learning in instructional practices.

The Characteristics of Successful Programs in College Calculus (CSPCC) survey analysis indicated that ambitious teaching was rare nationally, but somewhat more common at the schools selected as case study schools. The analysis also indicated that ambitious teaching had a small negative effect on student attitudes. However, the story does not end there. A complementary analysis indicated that ambitious teaching was associated with lower rates of students switching out of the calculus sequence, especially when paired with good teaching (Rasmussen and Ellis, 2013). These findings suggest a complex relationship between ambitious teaching and students' attitudes and beliefs (Larsen, Glover, and Melhuish, 2015).

Analysis indicated that ambitious teaching was associated with lower rates of students switching out of the calculus sequence, especially when paired with good teaching.

The primary goal of ambitious teaching is to develop strong conceptual knowledge and problem-solving ability in students. This goal is indeed ambitious, and efforts to achieve it put a good deal of pressure on both students and teachers. Thus, ambitious teaching can be challenging to implement and sustain. Our case study institutions offer some clues as to how it can be done.

Two of the institutions we selected as case study sites featured large-scale ambitious approaches to teaching calculus.

The first was an established program that emphasized conceptual understanding and student engagement using the Harvard Consortium text (Hughes-Hallett, Gleason, McCallum et al., 2012). The second was a newer initiative focused on flipping instruction to create time to engage students actively during class time that continues a tradition of technology-related instructional innovation at the institution.

Larsen et al. (2015) discuss the factors that supported these departmentwide ambitious teaching efforts. Perhaps the most important factor was buy-in. In each case, the faculty who were most invested in these ambitious efforts were well aware of the importance of convincing administration, other instructors, and students of the value of the effort.

The more established program used the Calculus Concept Inventory (Epstein, 2007) to demonstrate success in order to fend off pressures from administrators to use less expensive approaches. The program also featured a robust graduate-instructor training program that simultaneously promoted and explained the key features of the pedagogical approach (see Ellis, 2015). In the case of the newer flipped-calculus approach, a comparison study is being conducted so that, if the program is successful, the resulting data will promote buy-in on the part of more skeptical instructors.

It is crucial to staff calculus courses with encouraging and supportive instructors.

Program Buy-In

The findings from the MAA calculus study make it clear that good teaching is an essential part of a successful program in college calculus. Specifically, it is crucial to staff calculus courses with encouraging and supportive instructors. The study also suggests that programs looking to boost students' conceptual understanding using ambitious teaching approaches should carefully attend to the issue of buy-in on the part of administrators, instructors, and students, and that careful collection of data related to student outcomes is an excellent way to promote such buy-in.

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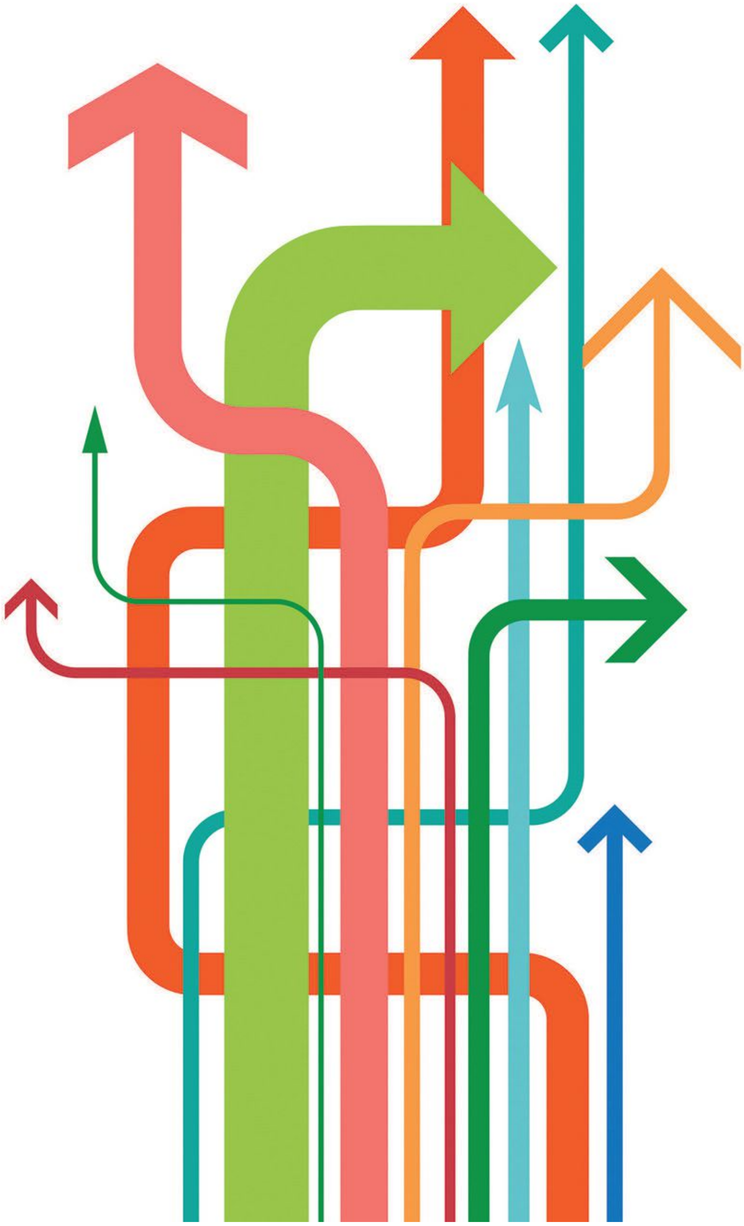

Alternative Approaches to Developmental Math

Bruce and Katherine Yoshiwara

Postsecondary education is the ticket to upward mobility for many Americans. But a large fraction of college students are placed into developmental (or remedial) mathematics programs from which they never emerge. In particular, 59 percent of incoming community college students are placed into developmental math courses (CCRC, http://bit.ly/1QbXpAh), but fewer than one-fifth of these students move past developmental math to earn degrees.

Nationwide, four broad areas are being addressed to increase student success through developmental mathematics: (1) placement, (2) pedagogy, (3) curriculum, and (4) noncognitive factors. In this article we describe some of the more promising reforms.

Improving Placement

Failing a class is not the only barrier to gaining a degree. More developmental students drop out of college before completing their first mathematics class than do those who fail a class. In addition, the length of the developmental math path defeats many students, who exit the sequence, which may include three or more courses, before reaching a college-level course. ("Student Progression Through Developmental Sequences in Community Colleges," Thomas Bailey, Dong Wook Jeong, and Sung-Woo Cho, http://bit.ly/1Uk6Hc9). To reduce the number of exit points, every effort should be made to place students into the highest appropriate math level.

Many entering students need only a brief review to improve their algebra skills. Some colleges now offer a summer program called Math Jam, typically an intensive two-week review scheduled just before placement exams are administered. Cañada College found that 62 percent of Math Jam students improved their math placement by at least one level, and Math Jam students showed both higher retention rates (93 percent vs. 77 percent stay enrolled until the end of the semester) and success rates (77 percent vs. 53 percent achieve grades of A, B, C, or Satisfactory) in their math courses, compared with other students (http://bit.ly/CCEmathjam).

The placement instrument itself, typically a machine-graded standardized test, could be improved. The "multiple measures" of high school GPA, how recently previous math courses were completed, and the student's weekly work hours and total course load have better predictive value than a single test (http://bit.ly/25MCOJP). Some schools have abandoned placement entirely and, instead, offer supplementary resources, such as extra study sessions or corequisite review courses, for students in credit-bearing classes.

Modifying Pedagogy

Many institutions are also changing the way material is presented. The methods we mention here have had some success in moving students through developmental math.

So-called compressed courses can be used to reduce attrition. Students enroll in two consecutive math courses during one term and, by meeting for twice the usual contact hours, are essentially immersed in math. Instructors lecture for short intervals, then guide students practicing in small groups. And the increased contact hours help build community among developmental students, who typically commute and leave campus after class (Pierce College ASAP, http://bit.ly/1sGOHzs).

Modularized courses offer another way to shorten the developmental sequence, by allowing students to spend time only on the topics they need to study. However, these courses often require a dedicated testing center and detailed administrative oversight, resources not available to many community colleges.

Institutions are also trying to use technology to facilitate remedial math courses, with mixed results. The Emporium Model relies on software to provide instruction and testing, with human interaction largely limited to one-on-one tutoring in a computer lab. Of necessity, such courses concentrate on mastering skills (http://wapo.st/1PJh5v6).

Technology also plays a key role in MOOCs (massive open online courses). The "massive open" aspects of MOOCs, however, appear not to improve student success, compared with existing online developmental math courses (http://bit.ly/SanJoseMOOC; Pamela Burdman, "Changing Equations," http://bit.ly/1HPbKJ8).

Other colleges and universities use software to produce flipped or hybrid classes. In a flipped course, students view the lectures online before coming to class, and class time is used to answer questions and work on assignments. In a hybrid course, students complete drill and practice problems on a computer, either individually or in a lab, and work on more significant and conceptual problems in class (http://bit.ly/1VN35BA; http://bit.ly/1tarbec; http://bit.ly/1tarvJQ).

Adjusting the Curriculum

A 2014 position paper from the American Mathematical Association of Two-Year Colleges (AMATYC) states, "Prerequisite courses other than intermediate algebra can adequately prepare students for courses of study that do not lead to calculus" (http://bit.ly/1PrTEFS).

Numerous pathways have been created to reduce the developmental sequence for non-STEM students. Such courses omit some of the topics traditionally included in intermediate algebra, but lead to a transferable math course, typically statistics or quantitative reasoning. Here are four promising examples:

CRAFTY (Curriculum Renewal Across the First Two Years) is a subcommittee of the MAA's Committee on the Undergraduate Program in Mathematics (CUPM). This is the third in a series of articles about general recommendations to renew mathematics course work and instruction. The first two stories are in the *MAA FOCUS* archive.

- Path2Stats is part of the California Acceleration Project, based on a program developed at Los Medanos College (http://bit.ly/263sSs9).
- Statway and Quantway are projects of the Carnegie Foundation for the Advancement of Teaching (http://bit.ly/1U9YDbG).
- The Dana Center has developed Math Pathways for both STEM and non-STEM students (http://bit.ly/1KUqb5o).
- Mathematical Literacy for College Students and Algebraic Literacy grew out of an AMATYC project to develop alternative pathways (http://bit.ly/23bTS6Z).

Addressing Noncognitive Factors

There are other strategies for increasing student success beyond modifying course content or methods of presentation (Core Principles for Transforming Remediation within a Comprehensive Student Success Strategy, core-principles.org). Here are a few notable ones:

- Carol Dweck's research indicates that students with "growth mindsets" persist and succeed better than their peers with "fixed mindsets." And, importantly, students can learn to move from one to the other (http://n.pr/1wZf679).
- David Yeager's research suggests that the performance gap in math suffered by women and other underrepresented groups can be eliminated by specific brief interventions (http://bit.ly/23bTZzE).
- City University of New York's Accelerated Study in Associate Programs (ASAP) helps students design a program of study based on their career goals. ASAP stipulates full-time enrollment and provides participants with academic advisement, career services, tutoring, financial supports, and specially blocked or linked courses (http://bit.ly/1dOprzL).

Remedial mathematics classes have become a barrier preventing college students from graduating and moving upward economically. Although postsecondary institutions are trying to remedy the situation, more work needs to be done. Institutions can take advantage of recent educational and cognitive research to design more effective developmental math programs.

Bruce Yoshiwara is professor emeritus from Pierce College in Los Angeles and former vice president of AMATYC. Katherine Yoshiwara, also retired from Pierce College, serves on the American Institute of Mathematics (AIM) Editorial Board for the Open Textbook Initiative. Bruce and Katherine have both received teaching awards from the MAA and AMATYC.

PRESIDENT'S MESSAGE

Proof School

—Francis Edward Su

I serve on an advisory board for Proof School, an exciting new school in San Francisco that serves grades 6–12. As the name suggests, mathematics is a central focus, but there is a full curriculum that will give kids an opportunity to rediscover what they love about school. Here is my recent interview with some key leaders.

Paul Zeitz, a math professor at the University of San Francisco and an MAA Haimo Award recipient, is the founder of Proof School. Head of School and Dean of the Mathematical Sciences Sam Vandervelde was recognized with the MAA's Alder Award during his career as a college mathematics professor. Kathy Lin is director of community outreach and Maker Studio teacher, and Zachary Sifuentes is dean of humanities and lead humanities teacher at Proof School.

The school opened in 2015 with 45 students and will have 60 in the fall. It hopes to enroll 150 eventually.

Francis Su: *Please tell us what Proof School is, how it got started, and what your hopes are for the school.*

Paul Zeitz: Anyone who has worked in a math circle, or read *A Mathematician's Lament,* knows that mathematics is an art, and people who really want to learn this art need practicing artists to guide them in an environment where the mentor gives the student quite a lot of freedom and time to explore. Proof School is an attempt to make this a reality.

Our mission to serve "kids who love math" is not the same as being a "math school." We've seen Proof School succeed beyond expectations. We have wonderful students who work hard at learning—not just at math—and who treat each other well and work well with each other. Our gender balance, while not one-to-one, is good. Thanks to some very generous donors, we have been completely need-blind in our admissions. We expect the school to grow steadily, serving students in grades 6–12 from all over the Bay Area and even beyond.

It is essential that Proof School transcend its role as a mere school, since no matter how innovative it is, a small school is just a boutique. We would like Proof School to evolve toward becoming a mathematical cultural center, sharing with all. We

An exhibit of student work at the Proof School in San Francisco.

hope that Proof School will become part of the mathematical landscape, landing somewhere on the continuum between the Moscow Center for Continuous Mathematical Education and MoMath [the Museum of Mathematics in New York City].

FS: *Seems like there's some exciting stuff happening. What's innovative or unique about the math program at Proof School?*
Sam Vandervelde: This past April every student in the school spent four consecutive afternoons working in teams to investigate different open-ended problems, then spent the following week preparing a poster or talk in which to present their results at our Math Symposium. We gave our students the opportunity to explore, make conjectures, prove results, and share their findings. It was a tremendous success.

This experience is replicated on a smaller scale in our math classrooms every day. In particular, we live up to our name by incorporating mathematical writing—at an appropriate level and format—into every math class we teach.

Our scheduling is unique in that we teach all of our math courses simultaneously for over two hours each afternoon, which permits students to take the course that is best suited for their interest and background, regardless of their grade level. We divide the year into five blocks and offer a different set of math courses within each block, beginning with combinatorics/problem solving, followed by algebra, geometry, analysis, and number theory/programming.

FS: *Can you talk about the role of the humanities at Proof School?*
Zachary Sifuentes: We teach making, writing, and speaking in a problem-solving context. Those problems are more physical in our studio courses and ethical in our literature courses. In Latin, we've designed board games and have studied Roman culture; in history, we've visualized networks that emerged alongside civilization. Writing happens across the curriculum, from literature to science and math, where principles like structure are practiced and reinforced.

The humanities curriculum really reflects and challenges the best in our students. They are drawn to puzzles, conundrums, and questions, but they're also drawn to deductive thinking, methodical process, and clarity. They're deep thinkers and they are natural communicators. Our humanities classes tap into these strengths and challenge students to think independently while working collaboratively. The humanities are truly a core element of our school culture.

FS: *Beyond mathematics, what are some noteworthy features of Proof School's curriculum?*
Kathy Lin: In starting a small school from scratch, we have the luxury of building the programs that we believe will best serve our students. For example: programming is a powerful skill that is intrinsically compelling to kids who love math. Our sixth- through eighth-graders take programming all year round, which is extremely unusual for a middle school.

We integrate making—and the ability to turn ideas into physical reality—across our curriculum, from our middle school Maker Studio course to our cooking club to one-time activities like constructing an inflatable planetarium. Every year students design and tackle independent projects that allow them to pursue their particular interests while developing the ability to drive their own learning.

It is a place where you discover, learn, and create mathematics joyfully with your peers, guided by your mentors.

FS: *How will what's happening at Proof School shape the broader conversation about math education?*
Vandervelde: I believe that Proof School will serve as a proof of concept in several important respects. To begin, the school is founded on the premise that bringing together students who love math under a single roof for a complete secondary education in the liberal arts is more than just worthwhile; [it] in fact permits a learning environment that is nothing short of transformative. Our students' feedback and accomplishments thus far indicate that this is a very sound premise.

The math faculty at Proof School is comparable to that of a small liberal arts college, both in preparation (all continuing faculty hold a PhD) and in scholarship. By the latter I mean that our math faculty are active mathematicians who enjoy creating, solving, and writing about math.

The most important by-product of this faculty culture is that our students learn in an environment where mathematics is a living subject. However, I also expect that it is only a matter of time until one of our faculty publishes a paper in a peer-reviewed journal.

DREW BIRD PHOTOGRAPHY

FS: *Some may worry that a school with such a focus would attract only people who already have had opportunities to love math. What are you doing to expand the admission pool? Are you attracting girls and underrepresented groups?*

Vandervelde: It's something we are thinking carefully about. We're still developing pipelines and extending our reach to attract a wide pool, but in order to get our school off the ground, we have mainly been targeting kids via established channels, which has primarily reached kids who have already had opportunities to love math. Girls will make up one-third of our students next year, and our goal is to improve on that. And our girl culture is alive and well, even in year one, which poises us for success in this respect. We would welcome financial support to improve our diversity efforts.

FS: *What would you most like college faculty and students to know about Proof School?*

Zeitz: If you are a mathematician, unless you had the good fortune to attend a place like School No. 2 or No. 57 in Moscow, Proof School is the school that you wish you went to. It is a place where you discover, learn, and create mathematics joyfully with your peers, guided by your mentors.

FS: *Starting a new school must be exciting! What's been the most rewarding and most challenging parts of this experience for you?*

Zeitz: The most rewarding thing is watching the students interact with each other and their teachers. The most challenging thing is worrying about money and real estate. We are committed to a location in San Francisco that is close to public transportation, so that students can come from all over the region (and they do!). But that makes rent very high, and SF is a very difficult place to find appropriate locations. Furthermore, in order to stay need-blind, we need generous donations. So far, we have been successful in raising money. But it is an ongoing endeavor.

FS: *Thank you all for taking the time to chat with me. I wish you all the best in your second year!*

If you'd like to learn more about Proof School, visit its website at proofschool.org.

Francis Su, MAA president, is the Benediktsson-Karwa Professor of Mathematics at Harvey Mudd College, su@math.hmc.edu. He's on *Twitter at* @mathyawp.

TOOLKIT

Name That Tune: Teaching Predicate Logic with Popular Culture

—John Quintanilla

Before they graduate, math majors should be able to read, understand, and negate statements like

$\forall \varepsilon > 0 \exists \delta > 0 \left(0 < |x-c| < \delta \Rightarrow |f(x) - L| < \varepsilon\right)$ and

$\forall g \in G \exists h \in G\left(gh = e \wedge \forall h' \in G\left(h' \neq h \Rightarrow gh' \neq e\right)\right)$.

In preparation for this, math majors at my university are first introduced to $\forall$, $\exists$, and other symbols of logic in our freshman-level course in discrete mathematics. As I prepared to teach this course last year, I was convinced that my students would be uninspired when translating the utterly boring examples and homework problems normally provided by textbooks.

Typical Textbook Example. Let $F(x,y)$ = "x and y are friends" and $P(x)$ = "x is perfect."

(a) Translate $\exists x \neg P(x)$ and $\forall x\left(P(x) \Rightarrow \forall y \neg F(x,y)\right)$ into your own words.

(b) Write "I have a friend who has a friend who's perfect" as a logical expression.

Although I agreed that my students should be able to translate such examples, I sought a more engaging way for students to develop proficiency with logic.

When I was in high school, I really enjoyed deciphering Donald Knuth's translations of "Ninety-Nine Bottles of Beer on the Wall" and "Old MacDonald Had a Farm" into mathematical notation ("The Complexity of Songs," *Communications of the Association for Computing Machinery* 27, no. 4 (1984): 344–346; accessible at bit.ly/1tuxdXN). So that my students could have a similar experience, I began mining the depths of popular culture, looking for propositional and predicate logic in the lyrics of songs and quotes from movies. I'm a middle-aged math nerd and not an expert on popular culture, which in an odd way makes me a decent judge of what's popular. After all, if even I know some one-liner, then a whole lot of other people probably know it too! Once engaged, my students also learn standard topics like truth tables and the different types of conditional statements.

An early example that I'll show students uses only $\neg$ and $\wedge$ and sets the tone for my students' introduction to the language of logic.

Example 1. Let p = "Billie Jean is my lover," q = "Billie Jean is a girl," r = "Billie Jean claims I am the one," and s = "The kid is my son." Translate the expression $\neg p \wedge q \wedge r \wedge \neg s$.

Nearly all my students will chuckle and recognize the chorus of Michael Jackson's "Billie Jean." Inevitably, someone will comment that the straightforward translation of $\neg p \wedge q \wedge r \wedge \neg s$ doesn't perfectly match the chorus. When this happens, I tell my class that translations from one language to another are often imperfect, and so it's acceptable when translations to and from logical statements omit some of the nuances of ordinary language.

Example 2. Let $X(x)$ = "x is my ex" and $T(x)$ = "x lives in Texas." Translate the logical statement $\forall x\left(X(x) \Rightarrow T(x)\right)$, where the domain is all people.

The very definitions of $X(x)$ and $T(x)$ suggest the George Strait song "All My Exes Live in Texas." (Also, I couldn't resist the wonderfully alliterative definition of $X(x)$.) This example illustrates that a direct translation (in this case, "For all people, if they're my ex, then they live in Texas") sometimes can be more succinctly stated. Furthermore, this statement is quite different than $\forall x\left(X(x) \wedge T(x)\right)$, or "All people are my exes living in Texas."

My acid test of whether students really get it is if they can perform the converse of the previous two examples and translate a sentence into a logical statement. I may also ask them to negate the statement using De Morgan's Laws and then translate the negation back to a sentence.

Example 3. Translate the sentence "Someday, I'll be living in a big old city" (taken from "Mean" by Taylor Swift) into a logical statement.

As a first attempt, my students will usually suggest $\exists t L(t)$, where $L(t)$ = "I am living in a big old city at time t." That's almost correct, except the unspoken sentiment of the song is "Someday, I'll be *permanently* living in a big old city." With some discussion, students are usually able to develop $\exists t \forall s \geq t L(s)$. Though I wouldn't necessarily mention this to freshmen math majors who are still learning calculus, I'm pleased that this musical example is analogous to a key portion of the definition of a limit to infinity: $\exists N \forall n \geq N(|a_n - L| < \varepsilon)$.

My students have been highly engaged while developing their proficiency with logic, but occasionally a student will playfully complain his or her favorite songs or movies weren't included. Indeed, the whims of popular culture may render today's best examples obsolete in only a few years. Since I'm no expert on popular culture, I'll invite my students to volunteer their own favorite one-liners that I might use in future classes. To get their creative juices flowing, I'll ask them to think of song lyrics or movie quotes containing words like (but not limited to) all, none, everywhere, something, always, forever, and never. Sentences with repetitive wording are also good candidates. I'm confident that this dialogue with my students will result in a replenishing supply of fresh examples that fairly reflect the diverse cultural tastes of my students. I'm also confident that this engaging introduction to propositional and predicate logic will prepare my students for their more advanced courses in mathematics.

I invite you to translate and identify some additional logical statements. In these examples, the domains for x and y are all people, the domain for t is all times (where $t = 0$ is now), and the domains for u and v are all things. Also, where it appears, "I" is the first-person singular pronoun (and not a variable!).

John Quintanilla is a professor of mathematics and a University Distinguished Teaching Professor at the University of North Texas in Denton, Texas. He maintains a daily blog on mathematics and mathematics education at meangreenmath.com*.*

Answers to the following are at maa.org/maa-focus-supplements.

1. Movie 1980: $p \wedge q$, where p = "It's dark outside" and q = "We're wearing sunglasses."
2. Pop music 1986: $\neg(q \Rightarrow p) \wedge \neg (s \Rightarrow r)$, here p = "You are rich," q = "You are my girl," r = "You are cool," and s = "You rule my world."
3. Pop music 2015: $\neg M(\text{you}) \wedge \forall x M(x)$, where $M(x)$ = "My momma likes x."
4. Movie soundtrack 2013: $\forall t \leq 0 \neg C(t)$, where $C(t)$ = The cold bothers me at time t."
5. Literature c. 1600: $\exists u(R(u) \wedge D(u))$, where $R(u)$ = "u is rotten" and $(D(u)$ = "u is in the state of Denmark."
6. Movie soundtrack 2006: $\neg\exists u(S(u) \wedge \neg R(u))$, where $S(u)$ = "u is a star in heaven" and $R(u)$ = "We can reach u."
7. Musical 1986 and movie 2014: $\exists x S(x) \wedge \neg\exists y A(y)$, where $S(x)$ = "x is on your side" and $A(y)$ = "y is alone."
8. Movie 2001: $\forall x(W(x) \Rightarrow \neg L(x) \wedge \neg \mathrm{E}(x))$, where $W(x)$ = "x is a wizard," $L(x)$ = "x is late," and $E(x)$ = "x is early."
9. Pop music 1987: $\forall t \geq 0 \neg(G(\text{give you up}, t) \vee G(\text{let you down})\ t) \vee G(\text{desert you}, t))$, where $G(u,t)$ = "I am going to do u at time t."
10. Country music 1990: $\exists x \exists y(F(x) \wedge F(y) \wedge x \neq y)$, where $F(x)$ = "I am friends with x" and $L(x)$ = "x is in low places."
11. Pop music 1990: $\exists t > 0 \exists x(\mathrm{T}(x,t) \wedge S(x,t))$, where $T(x,t)$ = "At time t, x makes you turn around" and $S(x,t)$ = "At time t, x makes you say goodbye."
12. Folk rock music 1965: $\forall u \exists t S(u,t)$, where $S(u,t)$ = "t is the season for u."
13. Pop music 2011: $\neg S(\text{you}) \wedge \forall x((x \neq \text{you} \wedge R(x)) \Rightarrow S(x))$, where $S(x))$ = "x can see it" and $R(x)$ = "x is in the room."
14. Country music 2010: $(T = 1{:}15) \wedge \neg(\exists y \neq \mathrm{I}(W(\mathrm{I},y,T)) \wedge N(\mathrm{I},\text{you},T)$, where T is the current time, $W(x,y,t)$ = "x is with y at time t," and $N(x,y,t)$ = "x needs y at time t."
15. Country music 2015: $\exists t_1 < 0(K(\mathrm{I},t_1) \wedge \forall \mathrm{t} < \mathrm{t}_1 \forall x \neg K(x,t)) \wedge \exists t_2 > 0(K(\text{she},t_2) \wedge \forall t > t_2 \forall x \neg K(x,t)$, where $K(x,t)$ = "He kisses x at time t."
16. New wave music 1983: $\forall x(F(\text{you},x) \Rightarrow \neg D(x)) \wedge \forall x(\neg D(x) \Rightarrow \neg F(\mathrm{I}, x))$, where $F(x,y)$ = "x and y are friends" and $D(x)$ = "x dances."

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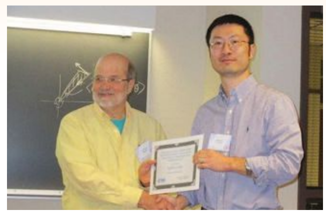

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Nebraska/SE South Dakota - no award
Texas - no award

Matthew Haines photo taken by Stephen Geffre

DEAR MAA

This spring one of my classes has only five students passing with grades of A, B, or C out of 18 students enrolled. The rest of the students probably aren't going to make it. What can I do? Should I be worried about getting in trouble with my department?

Sincerely,
Wanting to Help Them

Dear Wanting,

The first thing to say is that you want to check with your department and make sure that your standards for the course are in line with the standards of other faculty. In particular, is this pass rate typical of the course? Are you covering the material on the syllabus? Are your colleagues covering the material on the syllabus? Knowing if what you are experiencing is typical for the students (or course or department) will help you decide how to proceed.

If it turns out that you are covering the typical material and that your standards are similar to those of your colleagues (or, at the very least, that your standards are appropriate for the course), then you can ask if you have a typical group of students and if there is anything you can do to improve their performance.

For instance, take a semester where the average number of absences among students who were earning Ds and Fs is 15 (out of 45 class days). If students are missing that many classes, the teacher needs to answer two questions: Are there ways that I could make class more engaging, and that would encourage students to attend more often? How I am to help students who never attend?

In answer to the first question, are there active learning strategies you could use that would increase student participation and interest? How can you create a community environment in the class to encourage social bonds and facilitate learning? How can assessments or exams in the class reflect the learning you want to encourage in class?

In answer to the second question, is there a good way to reach out to students who are not attending? If they are not attending, no one can help them learn, but first students can be encouraged to return to class. In particular, do you use a course management system where you could send messages to the students who are struggling? Is there a center on campus in place to support failing students? Can you refer your students to that center?

In a class this small, try having an earnest conversation with the class about what happened. What does the class believe is going on here? If the dynamic is too uncomfortable in the classroom, anonymous feedback could be used.

Another alternative would be to bring someone in who can do a classroom interview, such as a head of the teaching and learning center. In a classroom interview, the person asks the questions and summarizes the major ideas for the professor.

It's important to learn students' perception of the barrier to success so that these can be addressed in the future—and in your review file. How you respond to the problem is likely to be more important than the fact that this happened.

In the end, the students need to do their part by putting in the work and get any help that required to be successful, and we all occasionally have classes with high fail rates. When that happens, be ready to explain to your department chair or administration why it occurred and what you've done to try to address the underlying issues. If this is happening every semester, you should speak with your department or center for teaching on your campus for advice about how to increase student success.

Also, there are many great ideas shared in *MAA FOCUS* about ways to redesign your course—you might try one of those to see if you have different results. All in all, do your best, and hope that your students will do the same. Good luck!

Dear MAA is our regular column offering advice and information. Please send us questions, large or small, regarding the MAA and life as a mathematician. "Dear MAA" will answer as honestly as possible. Address questions to the attention of DearMAA@maa.org.

PUZZLE PAGE

Fence in the Z-Pentomino

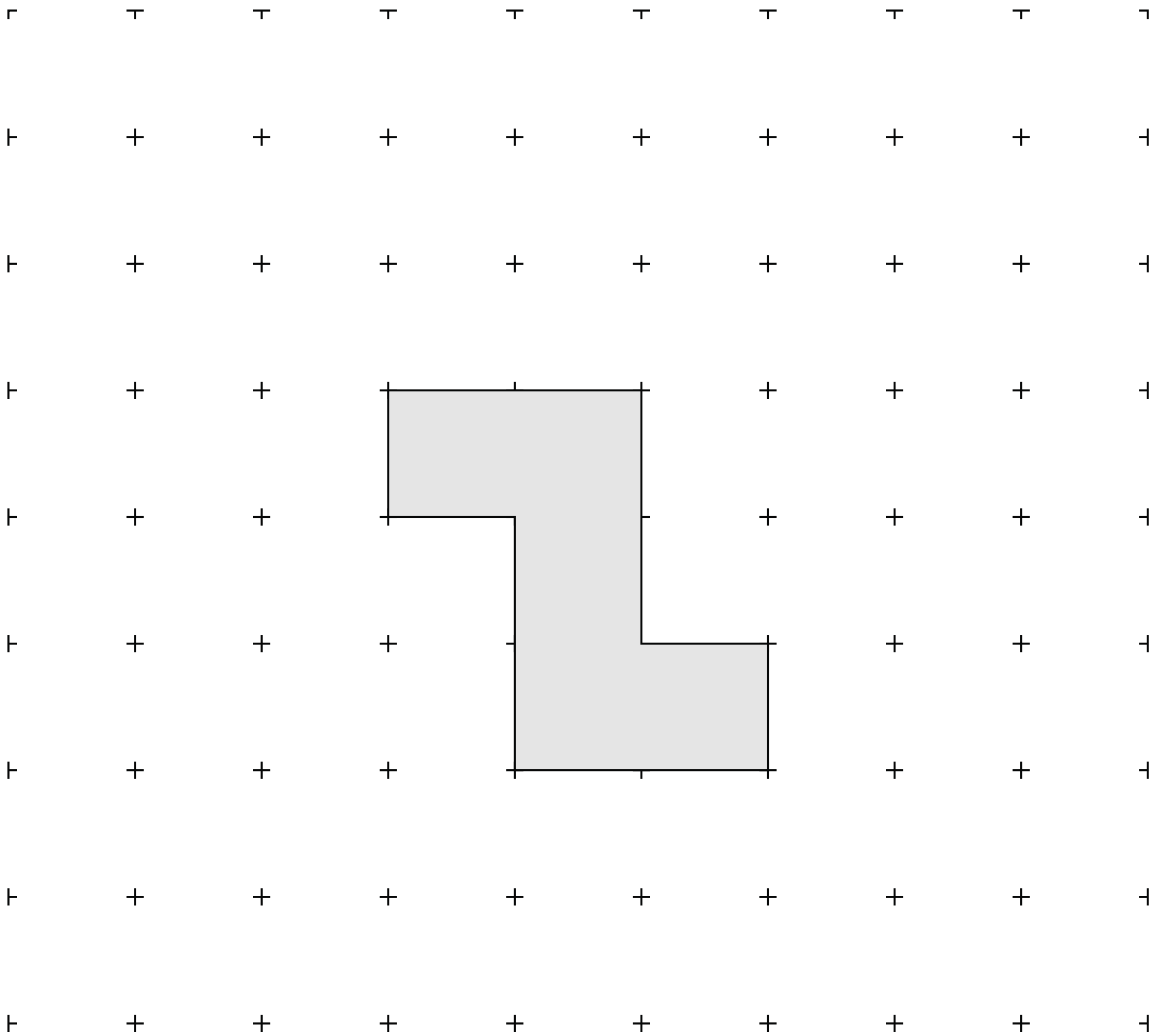
—Craig S. Kaplan

The last issue's puzzle was so much fun, we're running a variation this time. In the grid below, build a "good fence" around the given Z-pentomino. You must surround the shape with copies of itself, where each copy has at least one point in common with the original. Copies may be rotated, reflected, or both, and they may not overlap each other except at their boundaries. What is the maximum number of copies you can lay down this way, all touching the original pentomino?

Because this puzzle might involve a healthy amount of erasing and redrawing, we've made a pdf that you can print out and work on at maa.org/maa-focus-supplements.

An interactive version of "Good Fences" is available for iOS and Android. For more information, please see isohedral.ca/goodfences.

Craig S. Kaplan is a professor at the University of Waterloo in Waterloo, Ontario, Canada.



MAA BOOKS BEAT

Common Sense Mathematics

—Stephen Kennedy

Testifying before Congress in 2007, in response to a question about the number of Americans' phone calls that may have been overheard through wiretaps on foreign phone lines, Admiral Michael McConnell, then director of National Intelligence, said, "I don't have the exact number . . . considering that there are billions of transactions every day." Did he mean there were literally billions of international phone calls involving Americans every day? Or was he speaking loosely and just meant there are lots of such calls?

That's the opening question in Ethan Bolker and Maura Mast's *Common Sense Mathematics.* We move on to estimating the number of heartbeats in a human lifetime, how many glasses of orange juice are drunk in the United States each morning, and whether Baba Brinkman might have planted 1 million trees during the course of 10 summers' work.

Mast and Bolker asked themselves what, exactly, they wanted their students to remember 10 years after finishing their course. They describe their subsequent thinking as "sobering." But their answer isn't timid or lacking in ambition: they wish to change the way their students' minds work, the way their students approach a problem, even the way they approach the world. That's all.

They propose to do this by cultivating their students' common sense. Sure, they want to teach some mathematics, but really they commit themselves fully to making their quantitative literacy (QL) or quantitative reasoning (QR) course an experience in encountering the world quantitatively.

The book is not arranged by mathematical topic. Instead every chapter begins with a story, usually taken from a newspaper or magazine, that prompts the thoughtful reader to ask quantitative questions. Most of the exercises are structured the same way. There are, literally, hundreds of references to the mass-market media and science and social science literature. In the preface, the authors promise that "you'll find very few problems invented just to teach particular mathematical techniques." That is, your students will not be wondering—and asking you—"What is this good for?" The answer to that question will always be obvious to everyone.

Describing some chapter-starting scenarios will give you an idea of the mathematical content:

- The chapter on units and conversions begins by asking who would save more energy: the SUV driver who gets 12 miles per gallon (mpg) and upgrades to a model that gets 14 mpg, or the sedan driver who gets 28 mpg and goes to a hybrid that gets 40 mpg?

- The chapter on percentages begins by asking the students to consider the impact of Red Sox ticket price hikes. Who should be more upset: the grandstand dwellers whose average ticket price increased (between 2003 and 2008) from \$42.26 to \$52.16, or the box-seat plutocrat whose layout went from \$275 to \$325?

Common Sense Mathematics

Ethan D. Bolker, Maura B. Mast
MAA, 2016
Hardbound, 326 pages
\$60 list price; \$45 member price

- The next chapter asks us to consider how these increases compare when inflation is taken into account. The chapter on descriptive statistics starts by considering data on income inequality. Linear functions are introduced by analyzing an electric bill and probed by examining climate change data. Probability is introduced by way of the lottery.

Any number of other features of this text make it an attractive choice for your QL course. Every chapter ends with scores of exercises, most drawn from the mass media or scientific literature (the signature Mast-Bolker move). Nearly every exercise calls for an extended solution with commentary, context, and interpretation. Your students can't just compute a number—they must think about what they are doing and what it means. The exposition is friendly, engaging, and never condescending. Every chapter begins with an explicit, numbered list of modest, sensible, and clear goals (and almost every exercise is labeled by the goal[s] it supports).

In *Mathematics and Democracy,* Lynn Steen argued passionately and persuasively that numeracy is distinct from mathematics and that in the 21st century we are “awash with numbers” and “deluged by data” and numeracy of the population is essential to effective functioning of society (Woodrow Wilson National Fellowship Foundation, 2001).

To read half the front page of the *New York Times* on June 23, I needed to understand: the effect of affirmative action on the racial composition of the undergraduate enrollment of the University of Texas; the probability of being killed in a mass shooting versus being struck by lightning; the comparative costs of pain management therapy versus opioid use with its consequent contribution to the opioid addiction problem; the logic of apportionment and the U.S. Electoral College—among other things! As the examples indicate, and as Steen argued, numeracy is not about deep mathematical abstractions; it is about applying relatively elementary mathematical ideas in “subtle and sophisticated contexts.”

Whether we call it numeracy, quantitative literacy, or quantitative reasoning, the case has been securely made that we need to teach it to our students. As a community, we are still figuring out how best to do that. Mast and Bolker have presented a compelling model for doing it effectively and engagingly by showing our students numeracy in action.

Stephen Kennedy (Carleton College) is senior acquisitions editor at MAA. Contact him if you’re interested in writing a book for the MAA, at kennedy@maa.org*.*

BOOK REVIEW

Torture and Game Theory

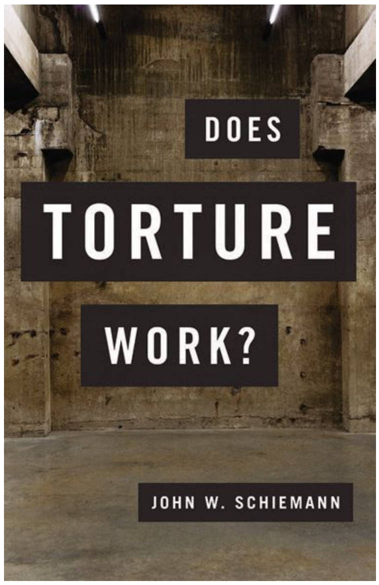
—Reviewed by Mark Hunacek

It may at first seem strange to see this provocatively titled book included for review in a mathematics-oriented column like this one. Certainly, the efficacy and morality of torture have for years (especially since the tragic events of September 11, 2001) been the subject of heated discussion at all levels of American society, from barrooms to political debates to academic conferences and university lecture halls. But these conversations are not usually mathematical in nature.

The interesting idea motivating this book is to try to use the mathematical discipline of game theory to determine whether torture really does work. (The author concludes that it does not.) No prior knowledge of game theory or any kind of advanced mathematics is assumed, and although some technical details are in the book, the difficult ones are tucked away in several easily omitted appendixes so as not to frighten off the lay reader. The author makes very clear that it is not his intent to make any original contribution to the subject of game theory: "My goal is to say something about interrogational torture, and game theory is just a tool to that end."

Of course, some people will be offended by the very idea of reducing an issue like torture to mathematical analysis, particularly when the mathematics being used has the word "game" in it. These people will argue that torture is morally wrong and illegal and therefore should not be used, whether it works or not, and that it is offensive to refer to torture as a "game." On the other extreme, there are those who are perfectly willing to torture even if it turns out to be not useful.

The author acknowledges both sides, but points out that there is a



Does Torture Work?
John W. Schiemann
Oxford University Press, 2015
336 pages, $34.95

"third group of Americans for whom torture is justified only because, and insofar as, it is effective, and who would oppose it otherwise." These people (the author calls them Pragmatists) may, according to Schiemann, "be open to reasoned and logical arguments evaluating the supposed effectiveness of interrogational torture." Hence, this book.

The use of mathematics to say something about issues like this is not without some precedent in the current literature. The second half of Jeff Suzuki's book *Constitutional Calculus,* for example, attempts to use mathematics to address issues arising under the Bill of Rights. (The first half discusses issues like voting theory and apportionment.) Suzuki (who, unlike Schiemann, is a mathematician) does not discuss torture but does address other hot-button political issues such as detention, "three strikes" laws, the death penalty, and jury composition.

Schiemann's text does not seem to make biased or slanted assumptions. Nevertheless, I am not wholly convinced by the mathematical arguments and conclusions reached herein. This is not to say that I think Schiemann's conclusions are in fact incorrect; I merely think that they do not follow inexorably from mathematics. In fact, I have some doubt that issues like the one tackled by Schiemann are really suited to mathematical analysis in the first place. Before explaining why, I'll quickly summarize the contents of the book.

Schiemann begins with what he calls the Bush Interrogation Torture (BIT) model, a simple model consisting of two players, the Detainee and the Interrogator, each with two strategies: the Detainee's strategies are "Talk" or "Don't Talk" and the Interrogator's are "Torture" or "Don't Torture." Over the course of two chapters, this model is defined and analyzed, and certain defects in the model are observed: the equilibria "solutions" seem odd and at variance with real life.

This is quickly followed by a discussion of a more sophisticated model, which Schiemann calls the Realistic Interrogational Torture (RIT) game. The RIT model allows for several different types of players (for example, the interrogator can be pragmatic or sadistic) and also takes into account the fact that neither player knows for sure what type of person the other player is. Much of the rest of the book is devoted to the development of this game and the analysis of its equilibria. The conclusions reached, as noted earlier, are that torture does not work: it produces false information or no information at all.

Although I do feel that Schiemann took pains to avoid cooking the books, I am, as noted above, not convinced that game theory is particularly well suited to situations like this. Game theory, with its emphasis on analysis of strategic decisions, necessarily presupposes free and voluntary choice on the part of the players, and it is awfully hard to say that a certain strategic choice is voluntary if it is induced by torture. The issue of whether there is rational choice in this context is a controversial one; see, for example, the article "Torture is Not a Game: On the Limitations and Dangers of Rational Choice Methods" by Dustin Howes (*Political Research Quarterly* 65 [2012]: 20–27). (This article predates the book, but is subsequent to, and in response to, a previous article by Schiemann on this issue.)

Schiemann, to his credit, acknowledges and addresses this concern. He relies on the fact that some people have refused to divulge information even under torture, concluding from this and first-person accounts of torture victims that people being tortured for information do exercise some choice. "Many victims, by their own accounts, think of themselves as having at least two actions, confessing or not confessing, providing information or not. . . . Finally, both sides understand, even if imperfectly and under uncertainty, how their actions lead to outcomes." Thus, the author views the decision to be tortured as a form of cost in a cost/benefit analysis.

I remain skeptical, however—and so do the courts in the U.S. judicial system. In *Ashcroft v. Tennessee,* for example, the U.S. Supreme Court considered a situation where a person confessed to a crime after five days of prolonged interrogation, including a 36-hour period during which the suspect was not allowed sleep or rest. There was not even any suggestion in that case of the kind of physical abuse that is commonly associated with the word "torture," yet the Supreme Court found the confession to be "not voluntary, but compelled."

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As the Court said, "We think a situation such as that here shown by uncontradicted evidence is so inherently coercive that its very existence is irreconcilable with the possession of mental freedom." The point is that undergoing torture is not just strategic choice with outcomes of its own. Even if a person literally has a "choice" to make, that choice is, by virtue of the torture, made so unpleasant that it is not in any realistic sense a voluntary one.

I see another potential problem with the use of game theory in situations like this. There is definitely a certain allure in attempting to justify a particular point of view not by reference to opinion and moral values but instead by appeal to the cold logic of mathematics. This gives the result a certain indicium of reliability that an opinion-based conclusion may not have. But at the same time there are, it seems to me, certain issues, particularly issues that are at the forefront of public policy, that cannot be divorced from things like individual value systems. We live on planet Earth, not Vulcan; logic can take us only so far. As noted earlier, there are many very logical people, including, I am sure, lots of professional mathematicians, who take the view that, regardless of its efficacy, the government should not torture people because torture is contrary to the values of this country. This point of view strikes me as entirely reasonable, and I certainly do not wish to be seen as disparaging it, but the point is that at some point the analysis inevitably diverges from mathematics.

Indeed, Schiemann himself eventually appears to fall back on a values-driven point of view. Chapter 13 of the text begins with a summary of what has come before; after stating that torture does not work, Schiemann goes on to say: "Does saying it does not work mean it can never work? No. It can work. Under conditions that we hardly ever obtain in the real world, it can work (but only if we're willing to torture innocent detainees)."

And a little later in this chapter, Schiemann makes even more explicit his reliance on ingrained values:

> Some will say we must torture even if there is a small chance that it will work. No. There are some things we cannot do because they run too much against the grain of our character. As Senator John McCain said on the floor of the Senate, the CIA torture program "stained our national honor."

This point of view is reasonable, but it is important to realize that the ultimate conclusions reached here are based to some extent on considerations other than mathematics.

These concerns notwithstanding, I think this is a useful and thought-provoking book. The idea of using game theory to analyze this problem is creative, and the resulting analysis, even if problematic (the author, of course, does not claim to be proving any theorems here; game theory is used as a tool, and mathematical models frequently are simplified versions of real life) is interesting. Moreover, the game-theoretic analysis, while not airtight, at least can be viewed as clarifying what assumptions are necessary to ensure that torture, at least as an institutional practice, can be made to work.

The book is also skillfully written. The author takes pains to make the development of material accessible; each chapter, for example, ends with a recap of what has been done to date and explains what is coming next. The mathematics in the book is slowly and clearly explained. It should be largely comprehensible to a lay audience.

From the standpoint of a professional mathematics educator, I can see this book being used productively as the springboard for an interesting discussion on the use (and limitations) of game theory. From the standpoint of a citizen, I see this book as raising questions and offering ideas that merit intelligent discussion.

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