Connections between Logic and Arithmetic Geometry

Thursday, August 7, 1:45 p.m. – 3:45 p.m., Hilton Portland, Ballroom Level, Grand Ballroom II

In the past few years, ideas from model theory and computability theory, branches of logic, have led to proofs of new results in arithmetic geometry. Sometimes these ideas from logic serve as inspiration by analogy; other times they are directly used in the proofs. The proposed session will consist of survey talks by experts, suitable for a broad audience.

Bjorn Poonen, *Massachusetts Institute of Technology*

Computability Theory at Work: Factoring Polynomials and Finding Roots

1:45 p.m. - 2:15 p.m.

Given a field F, we consider two fundamental questions about polynomials p in $\mathrm{F}[\mathrm{X}]$. First, which ones have proper factorizations in $\mathrm{F}[\mathrm{X}]$? And second, which ones have roots in F? Clearly these questions are related, and initially it may seem that the first one is easier: when p has degree > 3, finding a factorization appears easier than finding a root. However, by the same token, it should then be harder to show that p has no factorization than it is to show that p has no root. So the basic intuitions do not suggest how to compare the difficulty of these questions.

The point of this talk is to show how computability theory (a.k.a $recursion theory$) allows us to address the problem of deciding which of these two questions is more difficult. Working in a computable field F (that is, a countable field in which the field operations can be computed by a Turing machine), we first describe $Turing\ reductions,$ using the above two questions as an illustration. Each question turns out to be Turing-reducible to the other, meaning that, by this measure, they have the same level of difficulty. However, we also describe the very natural notion of an $\mathrm{m}-reduction$. All $\mathrm{m}-reductions$ are Turing reductions, but not vice versa, and we will see that $\mathrm{m}-reducibility$ definitively establishes which of the two questions is more difficult. To find out which it is, come to the talk!

Russell Miller, *Queens College, City University of New York*

The Zilber Trichotomy Principle for Algebraic Dynamics: Hands-On Examples of Deep Notions from Model Theory

2:30 p.m. - 3:00 p.m.

Algebraic dynamics, the study of discrete dynamical systems given by polynomial equations, can be seen as a branch of arithmetic geometry. Some conjectures from arithmetic geometry, such as the Manin Mumford Conjecture and the Mordell-Lang Conjecture), admit natural generalizations in terms of invariant varieties of algebraic dynamical systems.

Ideas from model theory, a branch of mathematical logic, have shed some light on these questions. This talk will not assume any familiarity with any of these.

I will focus on the following concrete question: given several one-variable polynomials f_i, what polynomials $P(x_1, x_2, ..., x_n)$ are invariant under the function $F(x_1, ..., x_n) := (f_1(x_1), ..., f_n(x_n))$? Many simple examples will be used to illustrate deep model-theoretic notions such as ``disintegrated geometry'' and the Zilber Trichotomy Principle.

Alice Medvedev, *University of California at Berkeley*

On the Elementary Theory of Finitely Generated Fields

3:15 p.m. - 3:45 p.m.

The aim of this talk is to give an introduction to the so called "elementary equivalence vs isomorphisms problem" concerning fields K which are finitely generated (over their prime fields). Part of that problem is to give describe by a first order sentence the fact that K has characteristic zero, and/or that K has transcendence degree d. It is actually conjectured that for every such K there exists a first order sentence which characterizes the isomorphism class of K. This is the main open problem in the elementary theory of finitely generated fields.

Florian Pop, *The Pennsylvania State University*