Preface

Two things are distinctive about this book: the low prerequisites (multi-variable calculus and linear algebra) and the development via problems (nearly 200 are provided). This is a text for active learning and one that can be used by students with a range of backgrounds. The material can be covered in a single semester.

In 1976–77, at the University of Oregon on my first sabbatical, I sat in on a course on Lie groups taught by Richard Koch. Now, I had studied Lie algebras and groups of Lie type as a graduate student (I am an algebraist steeped in the finite classical groups and their geometries), so I knew how to start with Lie algebras and define groups, but Koch's course was a revelation to me: the groups came first! I became so excited that I was determined to teach a course to Mount Holyoke undergraduates that would expose them to some of these beautiful ideas. My department indulged me, and the following year I tried. It was a dismal flop. I spent an entire semester trying to help my students understand left-invariant vector fields on a manifold. I learned a lot, even if perhaps they didn't. I realized that I needed to suppress the differential geometry and topology and start with matrix groups. I wasn't the only one who thought so. In 1979, Morton Curtis published *Matrix Groups*, although he included topology. In 1983, Roger Howe made an eloquent argument that Lie groups belong in the undergraduate curriculum in "Very Basic Lie Theory" in the *American Mathematical Monthly*. But no one took as extreme a view as I did: trying to create a course that students could take right after multi-variable calculus and linear algebra. (This is no longer quite true: John Stillwell's *Naive Lie Theory*, published in 2008, has similar prerequisites but is aimed more specifically at juniors and seniors.)

Why so extreme? At that time, my entire department was having conversations about the role of prerequisites and our desire to bring more advanced topics into the undergraduate curriculum. We had two concerns. One was that our students were choosing a major before they had seen much of the breadth and richness of our discipline. Another was that, since most of our majors did not go on to graduate study in mathematics, most of our students *never* saw many important mathematical ideas. We were audacious enough to propose to the U.S. Department of Education's Fund for the Improvement of Post-Secondary Education (FIPSE) the creation of seven reduced-prerequisite courses on advanced topics, and FIPSE was bold enough to fund us to make them happen. The FIPSE funding not only paid for release time for course developers, it also — perhaps more importantly— funded a series of sabbatical visitors who sat in on and critiqued the first offerings of our courses. In my own case, I was helped by Ken Rogers of the University of Hawaii, who sat in on an early matrix group version. More recently, I benefitted from comments by David Murphy, who used my materials in a Linear Algebra II course at Kalamazoo College.

In order to make this material accessible to students who have taken just multivariable calculus and linear algebra, I have concentrated on the algebraic ideas, with just enough analysis to define the tangent space and the differential and to make sense of the exponential map. Topology is excluded except for informal references to connected and simply connected sets and to closed sets. I also exclude quotient structures, except in asides for students who have studied abstract algebra. On the other hand, I do include a brief discussion of groups of Lie type over finite fields, the algebraist's Lie algebras first and groups second approach, very beautiful in its own right.

Over the years I've taught Lie groups many times at Mount Holyoke, and the enthusiasm and achievements of my students have consistently inspired me. This text, consisting mostly of problems, represents the form my course took when it stabilized. It's always true that students learn best when they are actively engaged, but it seems to me that when mathematically inexperienced students encounter something very different, it's especially important that they work through the new ideas for themselves. When I teach this material I lecture very little; most class time is spent with students either working on the problems or presenting and discussing their solutions. Sometimes, especially when we begin a new topic, my students work on problems all together, with me as writer at the board; other times, they work in pairs or in small groups.

My students are usually quite diverse in preparation, and often a number of science majors are among them. (In fact, the most recent time I taught Lie groups, just half the students in my class were math majors.) Many of my students have had only a very mild exposure to reading and writing formal proofs. The workshop-like format permits me to offer these students the extra support that they need. This text includes many routine problems, which my less experienced students have found helpful, as well as a good number of more challenging ones. In particular, a few problems ask students to "explain why" a statement is true for a statement that, with more sophisticated students, one would simply regard as obvious. Some problem sequences repeat very similar ideas, giving these students extra practice.

What about students who have already studied abstract algebra or real analysis? My experience has been that these students are also both engaged and challenged by this material. Obviously I steer them to the more substantial problems, and I often suggest supplementary reading. When students work in groups during class, I'm mindful of the students' backgrounds in helping them form groups that will function well. On the other hand, students who go on to abstract algebra after Lie groups (and some who hadn't originally intended to go on in fact do) carry with them a rich store of concrete examples to motivate and illuminate the ideas they encounter when systematically studying groups, rings and fields. In particular, my students acquire a deep understanding of the concept of an algebraic structure and its morphisms. Perhaps more important, they also develop an appreciation of the inter-connectedness of mathematics and the sometimes surprising ways in which it provides a language to describe and understand the physical world.

Because so much of the content of the book is scattered among the problems, every chapter has a section called *Putting the pieces together* in which all definitions and results are collected for reference. More advanced theorems of which the problems are special cases and references to other applications are sometimes given in this section as well. It also includes suggestions for further reading.

One of my favorite books is *Introduction to Affine Group Schemes* by William Waterhouse (Graduate Texts in Mathematics, Springer-Verlag, 1979). One of the (many) things I admire about it is the way the author occasionally includes a “vista” that puts the text material in a larger context. I’ve adopted this strategy with a section at the end of every chapter called *A broader view.* Sometimes the context is mathematical and sometimes historical (or both). These sections are not detailed, but I hope they give a flavor of the terrain the reader glimpses.

Appendix I includes the specific facts from linear algebra that I expect students to know and be able to use. I assume they will consult their own linear algebra books to fill any gaps.

There is now a small but growing body of material that students at this level can profitably read. Some of it I assign for supplementary reading. I also require my students to write a short expository paper based on reading about something not covered in class or in the text. This is another opportunity to tailor the assignment to students’ different levels of preparation. The problems in Chapter 6 are less demanding than in the other chapters, so students have time to work on their papers. In fact, I treat the material in Chapter 6, especially in 6.2, largely as an opportunity to review. Appendix II includes my paper assignment and list of possible topics.

Finally, Appendix III includes several suggestions for more extensive reading, building on some of the ideas in this text. Here I hope the interested student (or instructor) will find ideas for a second semester of study, and students with more background will find enriching and challenging collateral reading.

The Bibliography includes the sources in Appendices II and III, in addition to those cited in the text or mentioned as suggested readings.

Notational Conventions

Throughout the text, x.y denotes section y in Chapter x (referred to as just section y within Chapter x). The label (x.y.z) means problem z in x.y. The notation **(S)** following a problem number (or letter) means that a solution appears in the back of the text. (The solutions to all problems are in a supplement for instructors.) A few of the most challenging problems are marked **(C)**. I use the word “result” to mean a theorem, proposition, corollary or lemma. Ordinarily, Result x.y means result y in Chapter x. An exception is in Chapter 6: the results in the (fifth) section *A broader view* are numbered Result 6.5.x for result x in 6.5.

An asterisk on a result number means its proof is not obtainable using the elementary methods in this book. With few exceptions, the proof of an un-starred result is either in the problems in that chapter or is sketched in the text. Proofs or sketches of proofs are concluded with the symbol □. The symbol $\Leftrightarrow$ means “if and only if.” I use the symbol $\mathbb{R}$ for the set of all real numbers, $\mathbb{Z}$ for the integers and $\mathbb{Q}$ for the rational numbers. Ordinarily, matrices are denoted by capital letters (A), vectors by boldface letters ($\boldsymbol{a}$), and scalars by lower case (a). Linear transformations are typically denoted by capital letters also (T). Other functions are represented in a variety of ways, depending on context.