

*Mathfest 2002*

*Burlington, VT*



*MAA and Pi Mu Epsilon  
Student Paper Sessions  
August 1 - 2, 2002*



## PI MU EPSILON

Pi Mu Epsilon is a national mathematics honor society with over 275 chapters throughout the nation. Established in 1914, Pi Mu Epsilon is a non-secret organization whose purpose is the promotion of scholarly activity in mathematics among students in academic institutions and among staffs of qualified non-academic institutions. It seeks to do this by electing members on an honorary basis according to their proficiency in mathematics and by engaging in activities designed to provide for the mathematical and scholarly development of its members.

Pi Mu Epsilon regularly engages students in scholarly activity through its *Journal* which has published student and faculty articles since 1949. In addition, the society awards monetary prizes for mathematics contests and awards established by chapters.

Since 1952, Pi Mu Epsilon has been holding its annual National Meeting in conjunction with the summer meetings of the Mathematical Association of America (MAA).



## MAA Student Chapters

The MAA Student Chapters program was launched in January 1989 to encourage students to continue study in the mathematical sciences, provide opportunities to meet with other students interested in mathematics at national meetings, and provide career information in the mathematical sciences. The primary criterion for membership in an MAA Student Chapter is "interest in the mathematical sciences." Thus, the Student Chapter program supplements, but does not compete with, the chapters of Pi Mu Epsilon. Currently there are approximately 225 active Student Chapters on college and university campuses nationwide. Students are also members of the MAA Sections in their geographic region. Many of the MAA Sections provide special activities for students at their regularly scheduled meetings.

## **J. Sutherland Frame Lecture**

Friday, August 2, 2002  
8:00 - 8:50 pm  
Emerald Grand Ballroom, Sheraton

**SOAP BUBBLES: OPEN PROBLEMS**

**Frank Morgan**

Williams College

Despite much recent progress by many mathematicians, including undergraduates, many simple open problems remain. This presentation will also include a little contest with demonstrations, explanations, and prizes. No prerequisites.

*The J. Sutherland Frame Lecture is named in honor of the ninth President of Pi Mu Epsilon, who served from 1957 to 1966 and passed away on February 27, 1997. In 1952, Sud Frame initiated the student paper sessions at the annual Pi Mu Epsilon meeting, which is held at the Summer Mathfests. He continually offered insight and inspiration to student mathematicians at these summer meetings.*

**Student Activities  
Schedule of Events**

**Wednesday, July 31**

5:30 pm - 6:30 pm      MAA/PME Student Reception      Courtyard, Sheraton

**Thursday, August 1**

9:00 am - 5:00 pm	Student Hospitality Center	Exhibit Hall, Sheraton
1:00 pm - 2:57 pm	MAA Session #1	Kingsland Room
1:00 pm - 2:55 pm	PME Session #1	Willsboro Room
1:00 pm - 2:57 pm	MAA Session #2	Shelburne Room
1:00 pm - 2:55 pm	PME Session #2	Meeting Room 12
3:00 pm - 4:57 pm	MAA Session #3	Kingsland Room
3:00 pm - 4:35 pm	PME Session #3	Willsboro Room
3:00 pm - 4:57 pm	MAA Session #4	Shelburne Room
3:00 pm - 4:35 pm	PME Session #4	Meeting Room 12
5:30 pm - 6:20 pm	MAA Modeling Contest Winners	Amphitheater, Sheraton

**Friday, August 2**

9:00 am - 5:00 pm	Student Hospitality Center	Exhibit Hall, Sheraton
1:00 pm - 2:57 pm	MAA Session #5	Kingsland Room
1:00 pm - 2:35 pm	PME Session #5	Willsboro Room
1:00 pm - 2:57 pm	MAA Session #6	Shelburne Room
1:00 pm - 2:35 pm	PME Session #6	Meeting Room 12
3:00 pm - 5:15 pm	MAA Session #7	Kingsland Room
3:00 pm - 4:35 pm	PME Session #7	Willsboro Room
3:00 pm - 5:15 pm	MAA Session #8	Shelburne Room
3:00 pm - 4:35 pm	PME Session #8	Meeting Room 12
6:15 pm - 7:45 pm	PME Banquet	Diamond Ballroom
8:00 pm - 8:50 pm	J. Sutherland Frame Lecture <b>Frank Morgan</b> , Williams College <i>Soap Bubbles: Open Problems</i>	Emerald Grand Ballroom

**Saturday, August 3**

9:00 am - 3:00 pm	Student Hospitality Center	Exhibit Hall, Sheraton
1:00 pm - 2:50 pm	MAA Student Workshop <b>Patti Frazer Lock</b> , St. Lawrence University <i>Topics in Graph Theory</i>	Emerald Ballroom I
3:00 pm - 3:50 pm	MAA Student Lecture <b>Colin Adams</b> , Williams College <i>"Blown Away: What Knot to do When Sailing"</i> by Sir Randolph "Skipper" Bacon	Emerald Ballroom III
4:00 pm - 4:50 pm	Student Problem Solving Competition	Shelburne Room

## MAA Session #1

**Kingsland Room (Burlington Sheraton Hotel)**

**1:00 P.M. – 2:57 P.M.**

The first four speakers in this session are students of Professor Frank Morgan and the last three of Professor Cesar Silva in the REU Program at Williams College.

1:00-1:15

A HUNT FOR THE OCTAGON-SQUARE AND SOME OTHER NASTY COMPETITORS

**Eric Schoenfeld**  
Williams College

The notorious octagon square tiling is the most efficient way to enclose and separate two areas on the two-dimensional torus. We hope to expose it for the fraud it truly is. Some other questionable characters arise already in  $\mathbf{R}^2$  if bubbles are allowed to overlap themselves.

1:17-1:32

DOUBLE BUBBLES IN SPHERICAL AND HYPERBOLIC 3-SPACE

**George Lee, Jr.**  
Harvard University

The double bubble problem seeks the least-area way to enclose two regions of prescribed volume. We suspect that the solution in spherical and hyperbolic three-space is similar to a familiar double soap bubble. One major difficulty in proving this is showing that no volume is split into multiple components.

1:34-1:49

BUBBLES ON CONES

**Tracy Borawski**  
Williams College

The most efficient way to enclose a single area on the surface of a cone is a circle about the vertex. We seek the most efficient way to enclose and separate two areas. Numerous possibilities are reduced to two: a variation of the standard double bubble and something new.

1:51-2:06

SYMMETRIES AND COMPARISONS OF MINIMIZING DOUBLE BUBBLES

**Robert Lopez**  
Williams College

The Double Bubble problem seeks to find the least-perimeter way to fence off two areas. On certain surfaces many types of minimizing double bubbles appear. This is a relatively new finding and can lead to some rather interesting solutions to the double bubble problem.

2:08-2:23

DYNAMICAL SYSTEMS AND THEIR EIGENFUNCTIONS

**Brian Katz**  
Williams College

Here is a functional relationship between the study of dynamical systems and their eigenfunctions: Many of the seemingly purely dynamical questions have a precise rewording as eigenvalue questions. I will illuminate this relationship in the finite measure case and introduce some of the issues in the infinite measure case.

2:25-2:40

THE DYNAMICS OF  $\mathbf{R}^2$  ACTIONS

**Sarah Iams**  
Williams College

We will examine group actions in  $\mathbf{R}^2$  on the measure space and define some dynamical properties of these systems. Our  $\mathbf{R}^2$  examples will lead to an understanding of tilings as dynamical systems.

2:42-2:57

ON DYNAMICS OF  $\mathbf{R}^2$  ACTIONS AND TILINGS

**Kirsten Wicklegren**  
Harvard University

Given a finite set of tiles, the space of all tilings of the plane can be viewed as a dynamical system. We will investigate the dynamical properties of tiling spaces, a particular example of an  $\mathbf{R}^2$  action.



## PME Session #1

Willsboro Room (Burlington Sheraton Hotel)

1:00 P.M. – 2:55 P.M.

1:00-1:15

FACTORIZATION AND  $PSL_2(13)$   
**Tom Wakefield**  
 Youngstown State University - Ohio Xi

The group  $G$  is factorizable if  $G = HK$ , where  $H$  and  $K$  are nontrivial subgroups. Although trivial examples of non-factorizable groups are easy to generate, it is more difficult to prove that nontrivial groups do not factor. An overview of the proof that  $PSL_2(13)$  is not factorizable will be presented.

1:20-1:35

CARRY GROUPS  
**Lorne Fairbairn**  
 SUNY Potsdam – New York Phi

A carry group  $G \circ^x H$  is defined on  $G \times H$  with  $x$  in the center of  $G$ . The group's operation is given by  $(g_1, h_1) \bullet (g_2, h_2) = (g_1 g_2 x^i, h_1 h_2)$  where  $i = 0$  or  $1$  depending on  $h_1$  and  $h_2$ . The  $x^i$  can be thought of as  $i$  carries of  $x$  from  $H$  into  $G$ . This method of joining groups allows for a distinct way of seeing isomorphisms.

This is joint work with Michael Tsiang, UC Santa Cruz, and Grant Clifford, Harvey Mudd College. All are students of Professor Blair Madore in the REU program at SUNY Potsdam.

1:40-1:55

THE EVOLUTION HOMOMORPHISMS AND CLASSIFICATION OF CELLULAR AUTOMATA  
**Nicole Miller**  
 Salisbury University – Maryland Zeta

This paper describes cellular automata generated by group multiplication. We discuss conjectures pertaining to long-term effects of finite cellular automata where time evolution induces a homomorphism of the underlying automata into itself. Partitioning the state transition diagram modulo the kernel of this homomorphism into cosets produces a quotient machine.

2:00-2:15

SEARCH FOR CONSTRUCTIONS OF PARTIAL DIFFERENCE SETS  
**Ed Kenney**  
 University of Richmond – Virginia Alpha

We investigate the connection between partial difference sets and projective planes of several different orders in an effort to locate a family of partial difference sets in new groups. We first show several Galois Ring constructions, and then describe work utilizing quadratic forms and Mathon-constructed maximal arcs to provide insight into their geometry and structure. (Preliminary report.)

2:20-2:35

HOW TO COLOR A GRAPH  
**Eric C. Polley**  
 St. John's University – Minnesota Delta

Complete edge-colored graphs with loops that have the geometric property of 2-point homogeneity provide a natural generalization of distance transitive graphs. Several of my results display the relationship between the direct product and the wreath product of these graphs. The long-term goal of this research is to classify finite two-point homogeneous graphs, up to isomorphism.

2:40-2:55

ANALYSIS OF THE CLOSURE AND INTERIOR OF TOPOLOGICAL SPACES  
**Christopher Jones**  
 Youngstown State University – Ohio Xi

The closure operator, and its dual, the interior, are concepts fundamental to topological spaces. For a set  $A$  in a topological space  $X$ , let  $\alpha(A)$  be the number of sets produced using the closure and interior operators on  $A$ , and let  $\bar{\alpha}(X)$  be the maximum number of sets that can be produced in the space. An upper bound is known for  $\bar{\alpha}(X)$ ; in this talk, that upper bound and other values for  $\bar{\alpha}$  will be presented, along with other results on this topic. These results build primarily on the work of Kuratowski in the 1920s.

## MAA Session #2

Shelburne Room (Burlington Sheraton Hotel)

1:00 P.M. – 2:57 P.M.

The speakers in this session are students of Professor Robert Strikartz in the REU Program at Cornell University.

1:00-1:15

## THE FINITE ELEMENT METHOD ON FRACTALS

**Kevin Coletta**

Rensselaer Polytechnic Institute

The finite element method is a powerful tool for approximating solutions to differential equations. An analogous method has been developed on the Sierpinski gasket and other fractals. I will discuss some refinements of this method, as well as applications to the wave equation on fractals.

1:17-1:32

## FOURIER SERIES METHODS ON FRACTALS

**Kealey Dias**

SUNY Stony Brook

Ordinary Fourier series may be thought of as eigenfunction expansions for the Laplacian ( $2^{\text{nd}}$  derivative) on an interval. There is an analogous expansion on certain fractals, including the Sierpinski gasket, based on the Laplacian defined by Kigami. I will discuss this type of expansion and some applications.

1:34-1:49

## POLYNOMIAL TYPE FUNCTIONS ON FRACTALS

**Jonathan Needleman**

Oberlin College

On certain fractals, including the Sierpinski gasket, Kigami has defined the analog of the Laplacian. Functions annihilated by a power of the Laplacian are analogous to polynomials on the line. I will discuss properties of these polynomial type functions.

1:51-2:06

## POWER SERIES ON FRACTALS

**Po-lam Yung**

Chinese University of Hong Kong

Power series provide a powerful tool for understanding functions defined on the line. On certain fractals, including the Sierpinski gasket, it is possible to define the analog of power series. I will discuss properties of this type of expansion.

2:08-2:23

## CONSTRUCTING A P-LAPLACIAN ON THE SIERPINSKI GASKET

**Carto Wong**

Chinese University of Hong Kong

I will discuss the construction of a non-linear differential operator on the Sierpinski gasket. This construction is the analog of the p-Laplacian on Euclidean space.

2:25-2:40

## PDE'S ON THE OCTAGASKET FRACTAL

**Daniel G. Treat**

University of Missouri-Rolla

The study of PDE's on fractals provides interesting examples of phenomena that run counter to intuition. In this talk, the various properties of electrical resistance on the octagasket fractal will be discussed, along with their implications in PDE fractal theory.

2:42-2:57

## MOCK FOURIER SERIES FOR THE STANDARD CANTOR MEASURE

**Matthew Hirn**

Cornell University

Jorgensen and Pedersen show that orthonormal bases of exponential functions exist for certain special Cantor measures, but not for the usual one-third Cantor measure. In the talk I will describe a weaker kind of basis of exponential functions that has some of the properties in common with ordinary Fourier series.

## PME Session #2

Meeting Room 12 (Burlington Sheraton Hotel)

1:00 P.M. – 2:55 P.M.

1:00-1:15

## IMPLEMENTATION OF ERROR CORRECTING CODES

**Conrad Miller**

Southwestern University – Texas Pi

Through the use of the Pentium/8086 assembly language, we show how it is possible to implement a Hamming error correcting/detecting code by using the most basic functions of the computer's architecture.

1:20-1:35

## CHECK, PLEASE!

**James Sloan**

Southwestern University – Texas Pi

I will discuss the IBM, Verhoeff, and ISBN check digit schemes, and their implementation in Java.

1:40-1:55

## THE ILLUSTRATED ANALYST: GRAPHING FUNCTIONS, CONVOLUTIONS, AND FOURIER TRANSFORMS

**Michael B. Henry**

Augustana College – Illinois Eta

This project develops interactive software in OpenGL and Java to visualize theorems in analysis pertaining to signal processing. It will help you understand how the transform of a convolution is the product of the transforms.

This project was developed at the NSF-VIGRE supported REU program, illiMath2002, at the University of Illinois, under the leadership of Professor George Francis.

2:00-2:15

## BINARIZING TEXT IMAGES, AND RELATED ISSUES

**Elizabeth Fite**

Hendrix College – Arkansas Beta

Image processing has a wide variety of applications; one such application is the retrieval of textual data from paper sources for input into a database. This presentation will focus on the issues associated with preparing text images for use by an OCR.

2:20-2:35

## MATHEMATICS OF HIGH PERFORMANCE COMPUTER GRAPHICS

**Igor Crk**

Carthage College – Wisconsin Epsilon

We will be presenting the results of our summer research on high performance computer graphics.

2:40-2:55

## MATHEMATICAL ANALYSIS OF COMPUTING ALGORITHMS

**Noorie Hanum**

San Angelo State University – Texas Zeta

We analyze the basic computing algorithms using mathematical tools, such as the Big O notation, to evaluate their efficiency in computing. Some of the algorithms studied are Quick Sort, Heap Sort, Selection Sort, Binary Search and Sequential Search. Mathematics is used to determine the time taken by these algorithms and to rate and compare them. The use of mathematics as an essential tool in mathematics is established.



## MAA Session #3

Kingsland Room (Burlington Sheraton Hotel)

3:00 P.M. – 4:57 P.M.

3:00-3:15

CHAOS IN THE SOLAR SYSTEM

**Carolyn Staples**

Claremont McKenna College

It is known that Hyperion, one of the satellites of Saturn, does not follow the standard behavior of all satellites. I will discuss the chaotic behavior of Hyperion and compare it to the chaotic behavior of a forced pendulum.

3:17-3:32

SARAH HUGHES COMES FROM NOWHERE?

**Thomas Breunig**

University of Wisconsin Oshkosh

Many articles claimed that Sarah Hughes pulled off a huge upset to win the 2002 Women's Olympic Figure Skating Gold Medal. I will explain and model the method used by the Olympic Committee to calculate the medal rankings as a social welfare procedure, and examine some of the oddities that occur from this procedure.

3:34-3:49

WHERE TO PARK?

**Sarah Grove**

Youngstown State University

After touring several different parking lots of various sizes, the ultimate question of where to park the car is randomly decided. Once all of the available spots are filled, the spatial distribution of spaces with and without cars is noted. Interesting correlations to real world problems are explored.

3:51-4:06

HIGH PERFORMANCE SHOELACE TIGHTENING

**Michael Piatek**

Duquesne University

A shoelace with ends connected is one example of a mathematical knot. An evolving problem in knot theory is optimizing conformations with respect to different definitions of complexity such as rope length. In this introductory talk, we will investigate complexity measures and methods of speeding up associated computational optimization tasks.

4:08-4:23

MTBI PROJECT: MATHEMATICS AND BIOLOGY

**Thela Morales**

Mesa State College

A presentation of some of work performed at Cornell University's 2002 Summer Mathematical and Theoretical Biology Institute.

4:24-4:40

BENFORD'S LAW: THE SURPRISING DISTRIBUTION

**Abigail Fleming**

Mesa State College

When looking at the first digits of a set of numbers, it would seem natural that the probability of any given first digit would be one in nine. However, the occurrence usually follows a different pattern: Benford's Law. We will look at characteristics of Benford's Law as well as its history, applications and an overview of its proof.

4:42-4:57

BALANCING THE BALANCE

**Tammy Bastion**

St. Norbert College

Let  $n$  be a positive integer. Using a balance scale and masses of known weights, you want to be able to weigh an object of weight  $1, 2, 3, \dots, n$ . What is the minimum number  $m$  of known masses, and what are their weights, that will allow you to do this?

## PME Session #3

Willsboro Room (Burlington Sheraton Hotel)

3:00 P.M. – 4:35 P.M.

3:00-3:15

## RECURSIVE METHOD FOR SOLVING THE MANY-BODY QUANTUM PROBLEM

**Jonathan Moussa**

Worcester Polytechnic Institute – Massachusetts Alpha

The quantum many-body problem in the Born-Oppenheimer approximation is recast in a recursive form. The electrons are added into the system sequentially, where the  $n^{\text{th}}$  electron is solved in terms of a basis consisting of the  $(n-1)$ -electron and single electron wave functions. When compared to the configuration-interaction method, this method is computationally more favorable. Costly Slater determinants are not needed for anti-symmetrization, and the basis size needed to represent the same  $n$ -electron space is exponentially reduced. As an example, the energy eigenvalues of multiple electrons in a one-dimensional infinite square well are calculated.

3:20-3:35

## KICKING THE SYSTEM: THE EFFECT OF 4:1 FORCING ON STABLE PULSE LENGTH

**Ben Blaiszik**

Elmhurst College – Illinois Iota

In systems that exhibit a bistability between a nonlinear (nonzero) state and the basic (zero) state, fronts can form connecting these two states. Furthermore, two fronts can bind to create a pulse. The Ginzburg-Landau equation is used as a mathematical model to investigate this behavior in travelling waves. A differential equation is obtained describing the length of a pulse, and a phase line analysis is used to determine stability. Recent work by Crawford and Riecke has shown that periodically forcing the system at twice the natural frequency of the waves can result in stable pulses. We investigate the effect of additionally forcing the system at four times the natural frequency. We find that there is a critical value for the amplitude of the 4:1 forcing where the behavior of the stable pulse length changes.

3:40-3:55

## DELAYED RESONANCE

**Valerie Kunde**

Aquinas College – Michigan Lambda

A second order differential equation build to display resonance in its solution can have a surprising solution curve: The amplitude decreases before it increases. In this talk we will explore why this happens and discuss the model interpretation which reflects this phenomenon.

4:00- 4:15

## DYNAMICS OF POPULATION MODELING

**Joel Lepak**

Youngstown State University – Ohio Xi

Modeling populations using differential equations can in many cases accurately describe systems found in nature, such as species competing for resources, or species feeding off each other. Advantages and drawbacks of various modeling techniques will be discussed, as well as some important features found in population models.

4:20-4:35

## NUMERICAL SOLUTIONS OF PDES

**Robert Shuttleworth**

Youngstown State University – Ohio Xi

Partial differential equations are a valuable tool for modeling many physical problems in today's changing world. However many PDEs cannot be easily solved analytically. One remedy is to use finite difference methods to approximate the solutions. The Forward, Backward, and Central Difference Methods provide means for approximation, but with either poor orders of convergence or horrendous stability concerns. The Crank Nicholson Method is one method with a good rate of convergence but without concerns relating to stability. With this in mind, the Crank Nicholson was expanded to cover solutions of two-dimensional PDEs. One application is to provide insight into solutions of the Black-Scholes Equation, which is used to price options in the financial world.