6. Carbon dioxide emission from deforestation.
7. Total carbon dioxide emission functions.

III. Accumulation of carbon dioxide.
8. Increases in atmospheric carbon dioxide concentration.
9. Factors contributing to atmospheric carbon dioxide concentration.

IV. Social factors.
10. People.
11. Money.

V. Save the planet!
12. Changing energy demand.
13. Cost and efficiency of alternative energy sources.
14. What can you do to save the planet?

Environmental Mathematics (Ben Fusaro)

The emphasis is on computational, qualitative, and visual mathematics. All modelling is done by a seven-step process, moving from the visual and qualitative to the computational (calculators and BASIC). The students solve differential equations but they are called "Flow Equations." There is a major project that is done (preferably) by teams of two students. Course outline:

- Systems and Diagrammatics
- Energy and Entropy
- Energy and Growth
- Simulation of Models
- Energy Flow and Money Flow
- Production and Diversity

Appendix B: Mathematicians Develop New Tools to Tackle Environmental Problems

by David L. Wheeler, THE CHRONICLE OF HIGHER EDUCATION

Idling cars spewing fumes, northern spotted owls seeking nesting sites in diminishing plots of old-growth forest, and molecules of sulfur dioxide settling through the branches of the human lung: Such events would not strike most scientists as inherently mathematical. But mathematicians using graphs, equations, and their own brand of abstract thinking have been involved in each of those problems and are seeking a larger role in other environmental research.

"Environmental mathematics is an attempt to get mathematicians to connect again with the natural world," says Ben A. Fusaro, a professor at Salisbury State University and the chairman of the Mathematical Association of America's new committee on mathematics and

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the environment. Mr. Fusaro was an organizer of a series of talks, workshops, and discussions of environmental issues at the Association's joint meeting with the American Mathematical Society here this month.

Models of the Natural World

Mr. Fusaro says mathematicians can help environmental researchers by building models of the natural world and by seeking out both the variables and the things that do not change, or "invariants," in a living system. Most important, he says, mathematicians can help environmental researchers by finding the internal structures that link many different phenomena. One differential equation, for example, describes both the bouncing movements of a weight that is suspended from a mattress spring and the oscillations of an electrical current in a radio.

Robert McKelvey, a professor of mathematical sciences at the University of Montana who created a mathematical model for the northern spotted owl population in the Pacific Northwest, says that mathematics is needed to help set specific environmental policies because old methods of arriving at such decisions have failed. Arriving at a policy decision by placing a dollar value on both the costs and the benefits of an action, for instance, can't work if a dollar value can't be assigned to one or even both sides of cost-benefit calculations, Mr. McKelvey says.

Assessing the Costs

The timber industry is eager to point out the costs in jobs and dollars imposed by a logging ban in the mature forests where the northern spotted owl lives. But environmentalists claim no price can be set on the loss of the reclusive owl, which is protected by federal endangered-species legislation, or of the forests where it lives, which took hundreds of years to form.

Both sides, says Mr. McKelvey, are trying to preserve something that cannot be assessed in dollars: One wants to preserve a way of life tied to logging and the other a forest undisturbed by humans.

Mathematicians, working with economists, psychologists, and others, have developed a formal theory of making decisions with multiple conflicting objectives, known as multiple-criterion decision theory. That method, and others developed by mathematicians, could provide clear outlines of environmental problems, Mr. McKelvey says. "In the end you can't find a magic formula that tells you what to do," he says, "but the trade-offs can be made more explicit."

Mathematicians are accustomed to trying to comprehend uncertainty, Mr. McKelvey says, while many policy makers are afraid of it. He points to the controversy over global warming: Policy makers, he says, "are so frozen by their conservative natures that if they don't know what is going to happen, they don't do anything."

Good and Bad Years

In his own work, Mr. McKelvey has estimated how the portion of old-growth forests that is saved from logging in the Northwest will affect the chances of losing all the northern spotted owls. The owls prefer to nest under the canopy created by the tall trees in the old-growth forest, apparently because they have a better chance of escaping attack from predators there.

With a computer model of the owl population, Mr. McKelvey simulated a series of good and bad years for owls. In good years, owls have plenty of food—chiefly small rodents—and search for new nesting sites and breed. In bad years, the population stays stable or declines. The model randomly creates good and bad years and simulates 250-year periods.

Mr. McKelvey's model used information gathered by biologists, such as the amount of territory a pair of nesting owls requires. After thousands of computer runs simulating various combinations of good and bad years, the model showed that a critical threshold exists for the survival of the owl: When less than 20 percent of the old-growth forest is saved the chances of the owl's survival drops sharply.

Although many may argue about the model's assumptions or the precise location of the threshold, the knowledge of the threshold's existence is a valuable contribution, Mr. McKelvey says. Likewise, he says, the mathematical models can help biologists determine what data are needed to improve such predictions.

Pollutants in the Lungs

Sometimes mathematics is used to model aspects of the biological world that scientists would have difficulty studying in any other way.

At the Center for Mathematics and Computation in the Life Sciences and Medicine at Duke University, mathematicians are trying to determine what happens to pollutants that enter the human lung. The configuration of the lungs in other species is so different from humans' that laboratory animals cannot be used to study the health effects of pollution in humans, says Michael C. Reed, a professor of mathematics and director of the Duke center.

Because experimental surgery on humans is out of the question, Mr. Reed says mathematical models are one of the few tools available to help scientists understand what doses of pollution different parts of the lung will receive when breathing different concentrations of pollutants.

To solve the problem, mathematicians must first understand lung physiology. The sacs at the end of the lung, Mr. Reed says, have an enormous surface area: 80 to 100 square meters, the largest area in the body that is exposed to the outside air. "This is an enormous surface just sitting there and waiting to be injured," Mr. Reed says.

The branches of the lung—tubular bronchioles—are protected by mucus that, in conjunction with the cells lining the lungs, sweeps many pollutants up and out of the lung. The mucus coat thins near the junctions of the lung's branches and is missing completely at the junctions themselves.

Duke researchers have created two-dimensional models that can simulate portions of the human lung, the thickness of the mucus lining, and the motion of the air and the pollutants that it carries into the lung during breathing. The models have helped the scientists discover that the edges of the sacs, near the bronchioles, are likely to receive high concentrations of asbestos fibers when a person is breathing air containing them.

"The equations we are applying have been known for a hundred years, but the techniques of solving those equations are changing all the time," says Satish Anjilvel, a mathematician and an assistant professor of medicine who is working on the lung models. Understanding the deposition of pollutants in the lungs will keep many applied mathematicians busy for at least a decade, says Mr. Reed.

In the lower regions of the lungs, Mr. Anjilvel says, the flow of air is considered to be "laminar" and can be described exactly by standard equations. But in the upper regions of the lungs and in the nose, the flow is turbulent and cannot be simulated exactly by existing equations, he says.

Ending Traffic Jams

At Rutgers University, Fred S. Roberts, a professor of mathematics, conducts research designed to reduce the pollution from automobiles by eliminating traffic snarls. Mr. Roberts uses mathematical tools known as interval graphs to time traffic lights to prevent unnecessary idling of automobile engines.

The interval graphs, originally developed in 1959 to deduce the shape of genes, represent overlapping lines in a figure as points on a graph. Each point stands for an overlap: If two parallel segments don't overlap, there is no point on the graph for them.

In applying the graphs to traffic problems, mathematicians represent traffic flow that can occur simultaneously as points on the interval graphs. Traffic motion that cannot occur simultaneously, such as cars turning left and cars coming from the opposite direction, would not appear as points on the graph.

Mathematicians can search for the largest possible "clique," or cluster of points on an interval graph, to find how to move traffic efficiently.

The mathematical problem then expands to determine how to order the phases of green lights and how long each phase should be. As adjacent lights and surrounding streets are added, the problem becomes an increasingly challenging one for mathematicians.

Another problem on which Mr. Roberts has worked is the design of one-way street patterns. Many cities have adopted one-way streets to move traffic more quickly. But the patterns, which also use graph theory, must be designed without making it too difficult to drive from one place to the other. Transportation officials might, for instance, ask mathematicians to arrange the pattern of one-way streets to make the longest trip that anyone has to take as short as possible.

Mathematicians do not have a way of computing the solution to that problem for all patterns. "Unfortunately, we come very quickly to the forefront of mathematical knowledge," says Mr. Roberts.