

RECOMMENDATIONS ON THE UNDERGRADUATE
MATHEMATICS PROGRAM FOR ENGINEERS AND PHYSICISTS

A Report of
The Panel on Mathematics for the Physical Sciences
and Engineering

Revised January 1967

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BACKGROUND (1962)

One reason for the current effort on the undergraduate program is the rapid change in the mathematical world and in its immediate surroundings. Three aspects of this change have a particular effect on undergraduate curricula in the physical sciences and engineering. The first is the work being done in improving mathematics education in the secondary school. Several programs of improvement in secondary school mathematics have already had considerable effect and can be expected to have a great deal more. Not only can we hope that soon most freshmen expecting to take a scientific program will have covered precalculus mathematics, but, perhaps more important, they will be accustomed to care and precision of mathematical thought and statement. Of course, not all students will have this level of preparation in the foreseeable future, but the proportion will be large enough to enable us to plan on this basis. Students with poorer preparation may be expected to take remedial courses without credit before they start the regular program.

This improved preparation obviously means that we will be able to improve the content of the beginning calculus course since topics which take time in the first two years will have been covered earlier. More than that, however, it means that the elementary calculus course will have to take a more sophisticated attitude in order to keep the student from laughing at a course in college which is less careful mathematically than its secondary school predecessors.

The second aspect of change in mathematics which confronts us is the expansion in the applications of mathematics. There is a real "revolution" in engineering--perhaps "explosion" is an even better description than "revolution," because, as it turns out, several trends heading in different directions are simultaneously visible. One is a trend toward basic science. The mathematical aspect of this trend is a strengthening of interest in more algebraic and abstract concepts. An orthogonal trend is one toward the engineering of large systems. These systems, both military and nonmilitary, are of ever-increasing complexity and must be optimized with regard to such factors as cost, reliability, maintenance, etc. Resulting mathematical interests are linear algebra and probability-statistics. A further trend, in part a consequence of the preceding two, is a real increase in the variety and depth of the mathematical tools which interest the engineer. In general, engineers are finding that they need to use new and unfamiliar mathematics of a wide variety of types.

A third factor is the arrival of the electronic computer. It is having its effect on every phase of science and technology, all the way from basic research to the production line. In mathematics it has, for one thing, moved some techniques from the abstract to the practical field; for example, some series expansion, iterative techniques, and so forth. Then too, computers have led people to tackle problems they would never have considered before, such as large systems of linear equations, linear and nonlinear programming, and

Monte Carlo methods. Many of these new techniques require increased sophistication in mathematics.

An additional factor entering from another direction must also be mentioned. Mathematicians in the United States have in recent years become much more closely involved with areas adjacent to their own research. Of the many factors which enter here, we may mention the greatly increased interest of mathematicians at all levels in education, the rapid growth of mathematical employment in industry, the spread of research and consulting contracts into the universities, and the development of a number of mathematical disciplines, such as information theory, that have many applications but are not classical applied mathematics. There is thus a real desire among mathematicians and scientists to cooperate in matters of education.

The conclusions above and the recommendations that constitute the body of this report were formulated by the Panel after extensive consultation with mathematicians, physicists, and engineers. In engineering, in particular, representatives of many fields and many types of institutions were consulted, as well as officials of the American Society for Engineering Education.* The recommendations for physicists were drawn up in close collaboration with the Commission on College Physics.

In considering the recommendations which follow, it is crucial to examine what has been our attitude toward certain ideas which inevitably occupy a central position in any discussion of mathematical education. Among these are mathematical sophistication and mathematical rigor, motivation, and intuition. Now it is a fact that mathematical rigor--by which we mean an attempt to prove essentially everything that is used--is not the way of life of the physicist and the engineer. On the other hand, mathematical sophistication--which means to us careful and clear mathematical statements, proofs of many things, and generally speaking a broad appreciation of the mathematical blocks from which models are built--is desired by, and desirable for, all students preparing for a scientific career. How does one choose what is actually to be proved? It seems to us that this is related to the plausibility of the desired result. It is unwise to give rigor to either the utterly plausible or the utterly implausible, the former because the student cannot see what the fuss is all about, and the latter because the most likely effect is rejection of mathematics. The moderately plausible and the moderately implausible are the middle ground where we may insist on rigor with the greatest profit; the great danger in the overzealous use of rigor is to employ it to verify only that which is utterly apparent.

* Some of the results of a conference with engineers are embodied in four addresses delivered at a Conference on Mathematics in the Engineering Curriculum, held under the auspices of this Panel in March, 1961. These addresses were published in the Journal of Engineering Education, 52 (1961), pp. 171-207.

Let us turn next to the subject of motivation. Motivation means different things to different people and thus requires clarification. One aspect of motivation is concerned with the difference between mathematics and the applications of mathematics, between a mathematical model and the real world. For many engineers and physicists motivation of mathematical concepts can be supplied by formulating real situations which lead to the construction of reasonable models that exhibit both the desirability and the usefulness of the mathematical concept. Thus, motion of a particle or growth of a bacterial culture may be used as physical motivation for the notion of a derivative. It is also possible, of course, to give a mathematical motivation for a new mathematical concept; the geometric notion of a tangent to a curve also leads to the notion of derivative and is quite enough motivation to a mathematician. Since each kind of motivation is meaningful to large groups of students, we feel that both should appear wherever relevant. It is certainly a matter of individual taste whether one or both motivations should precede, or perhaps follow, the presentation of a mathematical topic. In either case, however, it is necessary to be very clear in distinguishing the motivating mathematical or physical situation from the resulting abstraction.

Physical and mathematical examples which are used as motivation, as well as previous mathematical experience, help to develop one's intuition for the mathematical concept being considered. By "intuition" we mean an ability to guess both the mathematical properties and the limitations of a mathematical abstraction by analogy with known properties of the mathematical or physical objects which motivated that abstraction. Intuition should lead the way to rigor whenever possible; neither can be exchanged or substituted for the other in the development of mathematics.

A mathematics course for engineers and physicists must involve the full spectrum from motivation and intuition to sophistication and rigor. While the relative emphasis on these various aspects will forever be a subject for debate, no mathematics course is a complete experience if any of them is omitted.

INTRODUCTION TO THE REVISION (1967)

In the five years that have elapsed since the first publication of these recommendations, several factors have emerged to affect the teaching of mathematics to engineers. The most striking of these is the widespread application of automatic computers to engineering problems. It is now a commonplace that all engineers must know how to use computers and that this knowledge must be gained early in their training and reinforced by use throughout it. We have, accordingly, included an introductory course in computer science as a

requisite for all engineering students and have increased the amount of numerical mathematics in other courses wherever possible.

A second factor is the fairly general acceptance of linear algebra as part of the beginning mathematics program for all students. In the engineering curriculum this is tied in to the expansion in computing, since linear algebra and computers are precisely the right team for handling the large problems in systems analysis that appear in so many modern investigations. Five years ago there were only a handful of elementary texts on linear algebra; now treatments are appearing almost as fast as calculus books (with which they are often combined).

A development of particular interest to these recommendations is the appearance of the CUPM report A General Curriculum in Mathematics for Colleges (1965), referred to hereafter as GCMC. It is too early to judge how widely the GCMC will be adopted, but initial reactions, including those of teachers of engineering students, have been generally favorable. GCMC makes considerable use of material in the first version of these recommendations, and now we, in turn, borrow some of the courses in GCMC.

Minor changes in the content of courses and some rearrangement and changes of emphasis are the result of experience and discussions over the years.

Relatively little change has been made in the program for physicists. The only major one has been the inclusion of Introduction to Computer Science in the required courses. We do this in the conviction that all scientists (if not, indeed, all college graduates) should know something about the powers and limitations of automatic computers.

Applications of Undergraduate Mathematics in Engineering, written and edited by Ben Noble, published in January, 1967, by the Mathematical Association of America and the Macmillan Company, is based on a collection of problems assembled as a joint project of CUPM and the Commission on Engineering Education. The book has five parts: Illustrative Applications of Elementary Mathematics, Applications of Ordinary Differential Equations, Applications to Field Problems, Applications of Linear Algebra, Applications of Probability Theory.

INTRODUCTION TO THE RECOMMENDATIONS

This report presents a program for the undergraduate mathematical preparation of engineers and physicists.

Since obviously no single program of study can be the best one for all types of students, all institutions, and all times, it is important that anyone expecting to make use of the present recommendations understand the assumptions underlying them. The following comments should make these assumptions clear and also explain some other features of the recommendations.

1. This is a program for today, not for several years in the future. Programs somewhat like this are already being given at various places, and the sample courses we outline are patterned after existing ones. We assume a good but not unusual background for the entering freshman.

Five or ten years from now the situation will undoubtedly be different--in the high schools, in research, in engineering practice, and in such adjacent areas as automatic computation. Such differences will necessitate changes in the mathematics curriculum, but a good curriculum can never be static, and it is our belief that the present proposal can be continually modified to keep up with developments. However, the material encompassed here will certainly continue to be an important part of the mathematical education needed by engineers and physicists.

2. The program we recommend may seem excessive in the light of what is now being done at many places, but it is our conviction that this is the minimal amount of mathematics appropriate for students who will be starting their careers four or five years from now. We recognize that some institutions may simply be unable to introduce such a program very soon. We hope that such places will regard the program as something to work toward.

3. Beyond the courses required of all students there must be available considerable flexibility to allow for variations in fields and in the quality of students. The advanced material whose availability we have recommended can be regarded as a main stem that may have branches at any point. Also, students may truncate the program at points appropriate to their interests and abilities.

4. The order of presentation of topics in mathematics and some related courses is strongly influenced by two factors:

- a. The best possible treatment of certain subjects in engineering and physics requires that they be preceded by certain mathematical topics.
- b. Topics introduced in mathematics courses should be used in applications as soon afterwards as possible.

To attain these ends, coordination among the mathematics, engineering, and physics faculties is necessary, and this may lead to course changes in all fields.

5. The recommendations are, of course, the responsibility of CUPM. In cases where it seems of interest and is available, we have indicated the reaction of the groups of engineers and physicists who were consulted. For convenience we refer to them as "the consultants."

LIST OF RECOMMENDED COURSES

It is desirable that all calculus prerequisites, including analytic geometry, be taught in high school. At present it may be necessary to include some analytic geometry in the beginning analysis course, but all other deficiencies should be corrected on a non-credit basis.

The following courses should be available for undergraduate majors in engineering and physics:

1. Beginning Analysis. (9-12 semester hours)

As far as general content is concerned, this is a relatively standard course in calculus and differential equations. There can be many variations of such a course in matters of rigor, motivation, arrangement of topics, etc., and textbooks have been and are being written from several points of view.

The course should contain the following topics:

- a. An intuitive introduction of four to six weeks to the basic notions of differentiation and integration. This course serves the dual purpose of augmenting the student's intuition for the more sophisticated treatment to come and preparing for immediate applications to physics.
- b. Theory and techniques of differentiation and integration of functions of one real variable, with applications.
- c. Infinite series, including Taylor series expansion.
- d. A brief introduction to differentiation and integration of functions of two or more real variables.
- e. Topics in differential equations, including the following: linear differential equations with constant coefficients and first-order systems--linear algebra (including eigenvalue theory, see 2 below) should be used to treat both homogeneous and nonhomogeneous problems; first-order linear and nonlinear equations, with Picard's method and an introduction to numerical techniques.

- f. Some attempt should be made to fill the gap between the high school algebra of complex numbers and the use of complex exponentials in the solution of differential equations. In particular, some work on the calculus of complex-valued functions of a real variable should be included in items b and c.
- g. Students should become familiar with vectors in two and three dimensions and with the differentiation of vector-valued functions of one variable. This material can obviously be correlated with the course in linear algebra (see below).
- h. Theory and simple techniques of numerical computation should be introduced where relevant. Further comments on this point, applying to the whole program, will be found below (under course 3).

We feel that the above comments on beginning analysis sufficiently describe a familiar course. The remaining courses in our list are less generally familiar. Hence the brief descriptions of courses 2 through 12 are supplemented in the Appendix [or elsewhere in this COMPENDIUM] by detailed outlines of sample courses of the kind we have in mind.

2. Linear Algebra. (3 semester hours)

A knowledge of the basic properties of n -dimensional vector spaces has become imperative for many fields of applications as well as for progress in mathematics itself. Since this subject is so fundamental and since its development makes no use of the concepts of calculus, it should appear very early in the student's program. We recommend a course with strong emphasis on the geometrical interpretation of vectors and matrices, with applications to mathematics (see items 1-e and 1-g above), physics, and engineering. Topics should include the algebra and geometry of vector spaces, linear transformations and matrices, linear equations (including computational methods), quadratic forms and symmetric matrices, and elementary eigenvalue theory.

It may be desirable, for mathematical or scheduling purposes, to combine beginning analysis and linear algebra into a single coordinated course to be completed in the sophomore year.

For outlines of a Beginning Analysis sequence, see the courses Mathematics 1, 2, and 4 described in Commentary on A General Curriculum in Mathematics for Colleges, page 44. The course Mathematics 3 (Elementary Linear Algebra) of the GCMC Commentary (page 55) approximates the linear algebra course described here, but does not contain the recommended material on quadratic forms and elementary eigenvalue theory. This Panel's recommended courses on functions of several variables, functions of a complex variable, real variables, and algebraic structures coincide with those of the GCMC Commentary (Mathematics 5 [alternate version], 13, 11, and 6M, respectively).

3. Introduction to Computer Science. (3 semester hours)

The development of high-speed computers has made it necessary for the appliers of mathematics to know the path from mathematical theory through programming logic to numerical results. This course gives an understanding of the position of the computer along this path, the manner of its use, its capabilities, and its limitations. It also provides the student with the basic techniques needed in order to use the computer to solve problems in other courses.

An even more important part of the path must be provided by the student's program as a whole. All the courses discussed here should contain, where it is suitable and applicable, mathematical topics motivated by the desire to relate mathematical understanding to computation. It is especially desirable that the student see the possibility of significant advantage in combining analytical insight with numerical work. Indications of such opportunities are scattered throughout the recommended course outlines.

4. Probability and Statistics. (6 semester hours)

Basic topics in probability theory, both discrete and continuous, have become essential in every branch of engineering, and in many engineering fields an introduction to statistics is also needed. We recommend a course based on the notions of random variables and sample spaces, including, inter alia, an introduction to limit theorems and stochastic processes and to estimation and hypothesis testing. Although this should be regarded as a single integrated course, the first half can be taken as a course in probability theory. For ease of reference we designate the two halves 4a and 4b.

5. Advanced Multivariable Calculus. (3 semester hours)

Continuation of item 1-d. A study of the properties of continuous mappings from E_n to E_m , making use of the linear algebra in course 2, and an introduction to differential forms and vector calculus based on line integrals, surface integrals, and the general Stokes theorem. Application should be made to field theory, elementary hydrodynamics, or other similar topics, so that some intuitive understanding can be gained.

6. Intermediate Ordinary Differential Equations. (3 semester hours)

This course continues the work on item 1-e into further topics important to applications, including linear equations with variable coefficients, boundary value problems, rudimentary existence theorems, and an introduction to nonlinear problems. Much attention should be given to numerical techniques.

7. Functions of a Complex Variable. (3 semester hours)

This course presupposes somewhat more mathematical maturity than courses 5 and 6 and so would ordinarily be taken after them,

even though they are not prerequisites as far as subject matter is concerned. In addition to the usual development of integrals and series, there should be material on multivalued functions, contour integration, conformal mapping, and integral transforms.

8. Partial Differential Equations. (3 semester hours)

Derivation, classification, and solution techniques of boundary value problems.

9. Introduction to Functional Analysis. (3 semester hours)

An introduction to the properties of general linear spaces and metric spaces, their transformations, measure theory, general Fourier series, and approximation theory.

10. Elements of Real Variable Theory. (3 semester hours)

A rigorous treatment of basic topics in the theory of functions of a real variable.

11. Optimization. (3 semester hours)

Linear, nonlinear, and dynamic programming, combinatorics, and calculus of variations.

12. Algebraic Structures. (3 semester hours)

An introduction to the theory of groups, rings, and fields.

13. Numerical Analysis.

14. Mathematical Logic.

15. Differential Geometry.

The last three courses are topics that might well be of interest to special groups of students. Their lengths and contents may vary considerably. For a sample outline of a course in Numerical Analysis, see Mathematics 8 (Introduction to Numerical Analysis) in Commentary on A General Curriculum in Mathematics for Colleges, page 83.

The above list of courses is the result of careful consideration by the Panel and the consultants. The brief description given here and the detailed sample outlines found in the Appendix [or elsewhere in this COMPENDIUM], while based on the mathematical structure of the topics themselves, reflect strongly the expressed interests of engineers and physicists. We realize that the nature of the institution and the requirements of other users of mathematics as well as of the mathematics majors may influence the specific offerings.

RECOMMENDED PROGRAM FOR ENGINEERS

- A. Courses to be required of all students.
1. Beginning Analysis. This recommendation needs no comments.
 2. Linear Algebra. The great majority of the consultants felt that this is important material that all engineers should have during the first two years.
 3. Introduction to Computer Science. Developments of the last few years make it clear that engineering is strongly dependent on a knowledgeable use of computers.
 - 4a. Probability. All students should have at least a 3-semester-hour course in probability. The consultants agreed on the value of probability to an engineer, but there was considerable disagreement among the consultants as to the advisability of requiring it of all students. However, the members of our Panel are unanimously and strongly of the opinion that this subject will soon pervade all branches of engineering and that now is the time to begin preparing students for this development.
- B. Courses recommended for students intending to go into research and development.
- 4b. Statistics.
 5. Advanced Multivariable Calculus.
 6. Intermediate Ordinary Differential Equations.
 7. Functions of a Complex Variable.

The consultants agreed to the value of the material in courses 5, 6, and 7, and some preferred that it be completed within the junior year. The Panel is convinced that an adequate presentation requires a minimum of nine semester hours, which could, of course, be taken in one year if desired. The order in which courses 5 and 6 are taken is immaterial except as they may be coordinated with other courses. If they are to be presented to the students in a fixed order, the instructor may wish to adjust the time schedules and choice of topics.

- C. Courses which should be available for theoretically minded students capable of extended graduate study.
8. Partial Differential Equations.
 9. Introduction to Functional Analysis.

10. Elements of Real Variable Theory.

Presumably a student would take either 9 or 10 but not both; 9 is probably more valuable but 10 is more likely to be available.

11. Optimization.

D. Courses of possible interest to special groups.

12. Algebraic Structures.

13. Numerical Analysis.

14. Mathematical Logic.

15. Differential Geometry.

RECOMMENDED PROGRAM FOR PHYSICISTS

A. Courses to be required of all students.

1. Beginning Analysis.

2. Linear Algebra. Like the engineers, the physicists felt that this material is essential.

3. Introduction to Computer Science.

5. Advanced Multivariable Calculus. This course should be taken in the sophomore year if possible, and in any event no later than the first part of the junior year.

6. Intermediate Ordinary Differential Equations.

B. Additional courses, in order of preference. Students contemplating graduate work should be required to take a minimum of three to nine semester hours of these courses.

7. Functions of a Complex Variable.

9. Introduction to Functional Analysis.

4a. Probability. The value of requiring this course in the undergraduate program of all physicists is not as well established as it is for engineers.

12. Algebraic Structures.

10. Elements of Real Variable Theory.
8. Partial Differential Equations.

Appendix

DESCRIPTION OF RECOMMENDED COURSES

While we feel strongly about the spirit of the courses outlined here, the specific embodiments are to be considered primarily as samples. Courses close to these have been taught successfully at appropriate levels, and our time schedules are based on this experience. Some of these courses are sufficiently common that approximations to complete texts already exist; others have appeared only in lecture form.

2. Linear Algebra. (3 semester hours)

The purpose of this course is to develop the algebra and geometry of finite-dimensional linear vector spaces and their linear transformations, the algebra of matrices, and the theory of eigenvalues and eigenvectors.

The course Mathematics 3 (Elementary Linear Algebra) of Commentary on a General Curriculum in Mathematics for Colleges (page 55) approximates the linear algebra course which this Panel has in mind. Mathematics 3 does not, however, contain the recommended material on quadratic forms and elementary eigenvalue theory.

3. Introduction to Computer Science. (3 semester hours)

This course serves a number of purposes:

- (1) It gives students an appreciation of the powers and limitations of automata.
- (2) It develops an understanding of the interplay between the machine, its associated languages, and the algorithmic formulation of problems.
- (3) It teaches students how to use a modern computer.
- (4) It enables instructors in later courses to assign problems to be solved on the computer.

For an outline of such a course, see C1 (Introduction to Computing) in Recommendations for an Undergraduate Program in Computational Mathematics (page 563).

4. Probability and Statistics. (6 semester hours)

This is a one-year course presenting the basic theory of probability and statistics. Although the development of the ideas and results is mathematically precise, the aim is to prepare students to formulate realistic models and to apply appropriate statistical techniques in problems likely to arise in engineering. Therefore new ideas will be motivated and applications of results will be given wherever possible.

First Semester: Probability.

a. Basic probability theory. (4 lectures) Different theories of probability (classical, frequency, and axiomatic). Combinatorial methods for computing probability. Conditional probability, independence. Bayes' theorem. Geometrical probability.

b. Random variables. (5 lectures) Concept of random variable and of distribution function. Discrete and continuous types. Multidimensional random variables. Marginal and conditional distributions.

c. Parameters of a distribution. (4 lectures) Expected values. Moments. Moment-generating functions. Moment inequalities.

d. Characteristic functions. (4 lectures) Definition, properties. Characteristic functions and moments. Determination of distribution function from characteristic function.

e. Various probability distributions. (6 lectures) Binomial, Poisson, multinomial. Uniform, normal, gamma, Weibull, multivariate normal. Importance of normal distribution. Applications of normal distribution to error analysis.

f. Limit theorems. (6 lectures) Various kinds of convergence. Law of Large Numbers. Central Limit Theorem.

g. Markov chains. (4 lectures) Transition matrix. Ergodic theorem.

h. Stochastic processes. (6 lectures) Markov processes. Processes with independent increments. Poisson process. Wiener process. Stationary processes.

Second Semester: Statistics.

- a. Sample moments and their distributions. (5 lectures)
Sample, statistic. Distribution of sample mean. Student's distribution. Fisher's Z distribution.
- b. Order statistics. (4 lectures) Empirical distribution function. Tolerance limits. Kolmogorov-Smirnov statistic.
- c. Tests of hypotheses. (5 lectures) Simple hypothesis against simple alternative. Composite hypotheses. Likelihood ratio test. Applications.
- d. Point estimation. (5 lectures) Consistent estimates. Unbiased estimates. Sufficient estimates. Efficiency of estimate. Methods of finding estimates.
- e. Interval estimation. (6 lectures) Confidence and tolerance intervals. Confidence intervals for large samples.
- f. Regression and linear hypotheses. (4 lectures) Elementary linear models. The general linear hypothesis.
- g. Nonparametric methods. (5 lectures) Tolerance limits. Comparison of two populations. Sign test. Mann-Whitney test.
- h. Sequential methods. (5 lectures) The probability ratio sequential test. Sequential estimation.

5. Advanced Multivariable Calculus. (3 semester hours)

For an outline of this course, see Mathematics 5 (Multivariable Calculus II--alternate version) in Commentary on A General Curriculum in Mathematics for Colleges, page 77.

6. Intermediate Ordinary Differential Equations. (3 semester hours)

The presentation of the course material should include: (1) an account of the manner in which ordinary differential equations and their boundary value problems, both linear and nonlinear, arise; (2) a carefully reasoned discussion of the qualitative behavior of the solution of such problems, sometimes on a predictive basis and at other times in an a posteriori manner; (3) a clearly described awareness of the role of numerical processes in the treatment of these problems, including the disadvantages as well as the advantages--in particular, there should be a firm emphasis on the fact

that numerical integration is not a substitute for thought; (4) an admission that we devote most of our lecture time to linear problems because (with isolated exceptions) we don't know much about any nonlinear ones except those that (precisely or approximately) can be attacked through our understanding of the linear ones. Thus, a thorough treatment of linear problems must precede a sophisticated attack on the nonlinear ones.

The distribution of time among items d through f cannot be prescribed easily or with universal acceptability. Only a superficial account of these topics can be given in the available time, but each should be introduced.

a. Systems of linear ordinary differential equations with constant coefficients. (6 lectures) Review of homogeneous and non-homogeneous problems; superposition and its dependence on linearity; transients in mechanical and electrical systems. The Laplace transform as a carefully developed operational technique without inversion integrals.

b. Linear ordinary differential equations with variable coefficients. (10 lectures) Singular points, series solutions about regular points and about singular points. Bessel's equation and Bessel functions; Legendre's equation and Legendre polynomials; confluent hypergeometric functions. Wronskians, linear independence, number of linearly independent solutions of an ordinary differential equation. Sturm-Liouville theory and eigenfunction expansions.

c. Solution of boundary value problems involving nonhomogeneous linear ordinary differential equations. (7 lectures) Methods using Wronskians, Green's functions (introduce δ functions), and eigenfunction expansions. Numerical methods. Rudimentary existence and uniqueness questions.

d. Asymptotic expansion and asymptotic behavior of solutions of ordinary differential equations. (3 lectures) Essentially the material on pp. 498-500 and pp. 519-527 of Methods of Mathematical Physics by Harold Jeffreys and Bertha S. Jeffreys (third edition; New York, Cambridge University Press, 1956).

e. Introduction to nonlinear ordinary differential equations. (6 lectures) Special nonlinear equations which are reducible to linear ones or to quadratures, elliptic functions (pendulum oscillations), introductory phase plane analysis (Poincaré).

f. Numerical methods. (7 lectures) Step-by-step solution of initial value problems for single equations and for systems. Error analysis, roundoff, stability. Improper boundary conditions, discontinuities, and other pitfalls.

7. Functions of a Complex Variable. (3 semester hours)

For an outline of this course, see Mathematics 13 (Complex Analysis) in Commentary on A General Curriculum in Mathematics for Colleges, page 97.

8. Partial Differential Equations. (3 semester hours)

This course is suitable for students who have completed a course in functions of a complex variable. The emphasis is on the development and solution of suitable mathematical formulations of scientific problems. Problems should be selected to emphasize the role of "time-like" and "space-like" coordinates and their relationship to the classification of differential equations. (It seems very useful to introduce the appropriate boundary conditions motivated by the physical questions and be led to the classification question by observing the properties of the solution.) The student should be led to recognize how few techniques we have and how special the equations and domains must be if explicit and exact solutions are to be obtained; he particularly must come to realize that the effective use of mathematics in science depends critically on the researcher's ability to select those questions which both fill the scientific need and admit efficient mathematical treatment. To accomplish this realization, the instructor should frequently introduce a realistic question from which he must retreat to a related tractable problem whose interpretation is informative in the context of the original question.

a. Derivation of equations. (2 lectures) The derivation of mathematical models associated with many scientific problems. Review of heat conduction to a moving medium, the flow of a fluid in a porous medium, the diffusion of a solute in moving fluids, the dynamics of elastic structures, neutron diffusion, radiative transfer, surface waves in liquids.

b. Eigenfunction expansions. (5 lectures) Eigenfunction expansions in both finite and infinite domains (Titchmarsh).

c. Separation of variables. (7 lectures) The product series solutions of partial differential equation boundary value problems.

Integral transforms such as the Laplace, Fourier, Mellin, and Hankel transforms and their use. Copious illustration of these techniques, using elliptic, parabolic, and hyperbolic problems.

d. Types of partial differential equations. (5 lectures)

The classification of partial differential equations, characteristics; appropriate boundary conditions. Domains of influence and dependence in hyperbolic and parabolic problems. The use of characteristics as "independent" coordinates.

e. Numerical methods. (8 lectures) Replacement of differential equations by difference equations; iterative methods; the method of characteristics. Convergence and error analysis.

f. Green's function and Riemann's function. (9 lectures)

Their determination and use in solving boundary value problems. Their use in converting partial differential equation boundary value problems into integral equation problems.

g. Similarity solutions. (3 lectures)

h. Expansions in a parameter. (3 lectures) Perturbation methods in both linear and nonlinear problems.

9. Introduction to Functional Analysis. (3 semester hours)

The purpose of this course is to present some of the basic ideas of elementary functional analysis in a form which permits their use in other courses in mathematics and its applications. It should also enable a student to gain insight into the ways of thought of a practicing mathematician and it should open up much of the modern technical literature dealing with operator theory.

Prerequisite to this course is a good foundation in linear algebra and in the concepts and techniques of the calculus of several variables. The material of this course should be presented with a strong geometrical flavor; undue time should not be spent on the more remote and theoretical aspects of functional analysis. Topics should be developed and first employed in mathematical surroundings familiar to the student. It would be very much in keeping with the intention of the course to emphasize the relationship between functional analysis and approximation theory, discussing (for example) some aspects of best uniform or best L^2 approximation to functions, and some error estimates in integration or interpolation formulas.

While some knowledge of measure theory and Lebesgue integration is needed for an understanding of this material, it is not

intended that the treatment be as complete as that in a standard real analysis course. The intended level is that to be found in the treatment by Kolmogorov and Fomin (Kolmogorov, A. N. and Fomin, S. V. Elements of the Theory of Functions and Functional Analysis. Vol. 2: Measure, the Lebesgue Integral, Hilbert Space. Baltimore, Maryland, Graylock Press, 1961.) If there is additional time, students might be introduced to some of the elementary theory of integral equations, or to applications in probability theory, or to the study of a specific compact operator, or to distributions.

For an outline of such a course, see Mathematics Q (Functional Analysis) in A Beginning Graduate Program in Mathematics for Prospective Teachers of Undergraduates, page 125.

10. Elements of Real Variable Theory. (3 semester hours)

For an outline of this course, see Mathematics 11 (Introductory Real Variable Theory) in Commentary on A General Curriculum in Mathematics for Colleges, page 93.

11. Optimization. (3 semester hours)

Attempts to determine the "best" or "most desirable" solution to large-scale engineering problems inevitably lead to optimization studies. Generally, the appropriate methods are highly mathematical and include such relatively new techniques as mathematical programming, optimal control theory, and certain combinatorial methods, in addition to more classical techniques of the calculus of variations and standard maxima-minima considerations of the calculus.

The 3-semester-hour course outlined below is planned to provide a basic mathematical background for such optimization studies. Another outline for a course in optimization, utilizing methods of programming and game theory, can be found in the report Applied Mathematics in the Undergraduate Curriculum, page 722.

a. Simple, specific examples of typical optimization problems. (3 lectures) Minimization with side conditions (Lagrange multipliers, simple geometrical example). Linear program (diet problem). Nonlinear program (least squares under inequality constraints, delay line problem). Combinatorial problem (marriage or network). Variational problem (brachistochrone). Control problem (missile). Dynamic program (replacement schedule).

b. Convexity and n-space geometry. (6 lectures) Convex regions, functions, general definition (homework: use definition

to show convexity [or nonconvexity] in nonobvious cases, such as Chebychev error over simple family of functions). Local, global minima. Convex polyhedra (review matrix, scalar product geometry). Geometric picture of linear programming.

c. Lagrange multipliers and duality. (6 lectures) Classical problem with equality constraints. Kuhn-Tucker conditions for inequality constraints. Linear programs. Dual variables as Lagrange multipliers. Reciprocity, duality theorems.

d. Solution of linear programs--simplex method. (3 lectures)

e. Combinatorial problems. (6 lectures) Unimodular property. Assignment problem (Hall's theorem, unique representatives). Networks (min-cut max-flow).

f. Classical calculus of variations. (7 lectures) Stationarity. Euler's differential equation, gradient in function space. Examples, especially Fermat's principle and brachistochrones.

g. Control theory. (8 lectures) Formulation. Pontryagin's maximum principle (Lagrange multipliers again).

12. Algebraic Structures. (3 semester hours)

For an outline of this course, see Mathematics 6M (Introductory Modern Algebra) in Commentary on A General Curriculum in Mathematics for Colleges, page 68.