position as a senior scientist for Westinghouse Atomic Power Division, Pittsburgh, Pennsylvania.

Professor Emeritus G. W. Gorrell of the University of Denver died on January 9, 1954. He was a member of the Association for thirty-three years.

Professor Emeritus J. M. Kinney of Wilson Junior College, Chicago, Illinois, died on January 19, 1955. He was a charter member of the Association.

Professor Emeritus H. F. Price of Pacific University died on January 12, 1955.

Mr. A. C. Washburne, Actuary Emeritus of the Berkshire Life Insurance Company, died on August 26, 1954. He was a charter member of the Association.

Assistant Professor Harold Weintaub of Tufts College died on November 7, 1954.

THE MATHEMATICAL ASSOCIATION OF AMERICA

Official Reports and Communications

REPORT OF THE JOINT COMMITTEE OF THE AMERICAN SOCIETY FOR ENGINEERING EDUCATION AND THE MATHEMATICAL ASSOCIATION OF AMERICA ON ENGINEERING MATHEMATICS

This Committee was appointed in the fall of 1953 and charged with:

1. Consideration of how mathematics can be taught in the most effective way for engineers;
2. Acting as liaison between ASEE and the MAA;
3. Keeping in close contact with the MAA Committee on the undergraduate mathematical program under the Chairmanship of W. L. Duren.

The matter of liaison and of keeping in contact with Duren's committee have been effected in the following ways. The first meeting of the Committee was in Baltimore on December 30, 1953 in conjunction with the annual meeting of the MAA. The second meeting of the Committee took place in Urbana on June 17, 1954 at the annual meeting of the ASEE. The MAA Committee on the Undergraduate Program was represented at our first meeting by W. L. Duren and at the second by G. B. Price. At the invitation of R. S. Burington, Chairman of the Mathematics Division of the ASEE, Price also presented to the Division a report on current efforts to revise the undergraduate program in mathematics.

The liaison between the MAA and the ASEE is probably best maintained through the Mathematics Division of the ASEE. Mathematicians engaged in teaching engineers should be encouraged to join and to support the activities of both the ASEE and the MAA. Continued publicity about the activities of the Mathematics Division of the ASEE in the MONTHLY should be encouraged.

Mathematics for Engineers. The Committee has directed its attention to the following three questions in this connection.

1. What mathematics should be taught?
2. Who should teach it?
3. How should it be taught?

What Mathematics should be taught to engineering students? This question is the subject of study and debate from many sides.

The ASEE's Committee on Evaluation of Engineering Education in its Interim Report of June 15, 1954 (p. 10) states: "At the undergraduate level, competence in the theory and use of simple ordinary differential equations and their application to the solution of physical problems lies close to the boundary of minimum acceptability of mathematics in any satisfactory engineering curriculum. For students whose interests will be centered in research, development, or the higher phases of analysis and design, or who contemplate subsequent graduate study in engineering, additional mathematics may be both desirable and necessary."

The MAA's Committee on the Undergraduate Mathematics program has endeavored to formulate a first year mathematics course which would "try to incorporate into collegiate mathematical teaching the valuable results of modern researches in mathematics and logic and to eliminate that which is unnecessary or faulty." While they have in mind a program for *all* students who study mathematics in college, they are thoroughly aware of the fact that "the revision must concentrate upon the needs of the vast majority ('95 per cent') of students who take mathematics to increase their powers for solving problems and grasping the ideas of science. Thus the approach to mathematics at this stage (*i.e.* during the first two years of college) shall be *operational* rather than merely *appreciative.*" They are experimenting with, and currently engaged in writing text materials for, a basic three-hour course for all students who take mathematics for the first year. In the first semester this would take up: graphs; functions; limits; introductory calculus based on polynomials, exponentials, and logarithms. The material in the second semester would include: sets, mathematical language, groups, combinatorics, distributions, approximate sums, probability. The basic course would be supplemented for engineering and physical science students with a two-hour course as follows:

First semester

Numerical trigonometry, graphical analysis, Newton's methods, numerical integration, tables of logarithms and exponential functions, slide rule, log and semi-log paper, rate problems.

Second semester

Worded problems, binary representation, computing machines, centroids and moments of inertia, averages, precision in measure and computation, finite differences, tables of normal probability density, sampling.

For engineering students, this would be continued in the second year into a course in calculus and analytic geometry with geometric and mechanical applications using vector methods, which would also include linear differential equations with constant coefficients. The program has been tried, with success, at Tulane.

There are other experimental mathematics programs being tried at the University of Washington, Haverford College, Carleton College, The Illinois Institute of Technology, Brown University, Yale, and no doubt at many other institutions.

High School Preparation. So far as the undergraduate curriculum in engineering is concerned, the crucial question seems to be "how much useful mathematics can be taught to the engineers in the time now allotted to mathematics in the undergraduate curriculum?" The answer to this question obviously depends upon the previous mathematical preparation of the entering freshmen. Where this preparation is meager, it is now customary for the students to take college algebra, trigonometry, and analytic geometry in the freshman year and to follow this with a course in calculus in the sophomore year. Where the high school preparation includes trigonometry as well as a strong background in algebra, it is possible to give a combined course in calculus and analytic geometry beginning in the freshman year.

But not all entering freshmen engineering students will have had strong preparation in high school mathematics. There is reason to hope that significant improvements can be made in our methods of taking care of these students. Williams College has been successful in taking students without trigonometry directly into the unified calculus, analytic geometry, and differential equations course and developing the necessary analytic trigonometry as it is needed in the calculus. The universal freshman program envisioned by the MAA Committee on the Undergraduate Program will presuppose only one year of high school algebra and one year of geometry. A program in mathematics and physics was devised at Lafayette College for students enrolled in the preinduction program sponsored by the Ford Foundation. Most of these men entered college after two years of high school. They had studied neither physics nor trigonometry and their mathematical background was primarily limited to one year of algebra and to one year of plane geometry. The two year program at Lafayette included the essential parts of trigonometry, analytic geometry, calculus, and differential equations. Experiments along these lines while not conclusive do at least suggest the possibility of by-passing some of the traditional prerequisites for the calculus with a resulting acceleration and expansion of the useful mathematics that can be included in the normal engineering curriculum. However, some serious questions have been raised in connection with these experimental programs. The major items of concern seem to be:

1. that the existence of the experimental programs, with the low level of mathematical pre-requisites for some of them, may result in a further

weakening of the high school mathematics preparation of future engineers and scientists. It is taken as axiomatic that high school students will not learn as much mathematics in two years as they would in four. In the face of increasing demands for mathematical competence, the colleges and universities (so the argument goes) should, if anything, be upgrading their entrance requirements in mathematics for students in engineering and the physical sciences. And the University of Illinois, for one, seems to be moving in this direction.

2. that the experimental courses may attempt to survey so many topics that they will attempt to teach "a little about everything and not much about anything." This may be an exaggerated fear, but it nevertheless exists.
3. that "by far and away the largest portion of engineers get along on and *use* a fair amount of the elements of algebra, plane, solid, and analytic geometry, trigonometry in particular, plus the fundamental concepts of the calculus and copious use of handbooks and tables. Moreover, whatever the shortcomings of the traditional first year mathematics courses in engineering may be, they have at least made a serious attempt to meet the student where his knowledge left off and usually provide time for review and filling in gaps."

This Committee has not had the opportunity to debate the issues raised here. Accordingly, we make no recommendation either for or against any of the experimental programs now under discussion. We do, however, endorse experimentation as such. Moreover, we feel that the prospect for curriculum improvement is enhanced by a lively and serious debate of these issues.

We also wish to call attention to an experiment of a somewhat different nature, designed to stimulate gifted students to make more progress in mathematics at an early age. The Ford Foundation through its Fund for the Advancement of Education has recently sponsored a study called *The School and College Study of Admission with Advanced Standing*. The Mathematics Committee of that study proposed a program in mathematics for the more able students which would include a strong introduction to differential and integral calculus in the *twelfth grade*. And, in a similar vein, we wish to call attention to a recent talk by R. S. Burington on "Mathematics for our time," reported in the Mathematics Teacher, vol. XLVII, No. 5, pp. 295–298, May 1954.

What conclusions, then, has this Committee reached in answer to the question "What mathematics should be taught to engineers"? We wrote personal letters to about forty mathematicians, scientists, and engineers soliciting their comments and advice on the question of what mathematics should be taught to engineers, and how it could be taught most effectively. From the replies received we deduced that:

a. There was a consensus of opinion that all engineers would benefit by studying mathematics through at least the elementary course in ordinary differential equations.

On the question of specific course content we are not prepared to take a stand either for or against the program proposed by the MAA Committee; largely because it is still in the experimental stage and because of certain doubts stated above.

b. Statistics and probability deserve more attention than they now receive. It is not at all obvious how this is to be managed within the present time limitations. One possible solution to the problem has been suggested by the MAA Committee on undergraduate curriculum. This problem deserves further study.
c. It would be highly desirable to provide the opportunity for students to take further electives in mathematics; for example advanced calculus, complex variables, numerical analysis, matrix algebra, and so on.

Who should teach mathematics to engineers? There is, of course, the possibility of a dichotomy with some people answering "the engineering staff, in order to increase motivation" and with others answering "the mathematics staff because of their deeper insight into the logical structure of the subject." In his talk at Urbana, L. W. Cohen referred to mathematics as a language. In order to learn to use this language effectively engineers need teachers who know the language thoroughly and know how to use it. In this connection we quote from the interim report of the Committee on Evaluation of Engineering Education of June 15, 1954: "A minimum level of performance in mathematics should be established whether it be obtained in required mathematics or in engineering courses. However, few engineering courses are taught in a manner to make a significant contribution to the students' knowledge of basic mathematics, nor is time available for this purpose. The engineering sciences and subsequent professional subject matter should be developed by making effective use of such mathematical proficiency, and should be taught by staff members competent to do so."

We are in complete agreement that, in these days when some engineers need far more mathematics than the minimum now required, it is important that mathematics be taught by mathematicians who by their inspiration and enthusiasm can stimulate an interest in mathematics *per se*. Unfortunately, however, the impression exists that some engineering students are taught by mathematicians who have little or no appreciation for the interests and needs of the engineers. We believe that those mathematicians who are responsible for curriculum planning should consult with their engineering colleagues from time to time to try to determine the engineers' current and anticipated needs in mathematics. Then they should try to meet those needs by providing appropriate courses taught by competent, sympathetic teachers.

One experiment which has been tried, and which appears to be worth wider consideration, is for the mathematics department to invite a member of the engineering staff to teach one or more regular mathematics courses. This gives him a chance to become familiar with the structure and timing of the mathematics course, and also puts him in an excellent position to make criticisms and offer suggestions on the course from the point of view of an engineer. This might also be done on a reciprocal basis with a member of the mathematics staff teaching one of the engineering sciences in order to become familiar with the applications of mathematics in it. One member of our Committee has reported that he took part in such an experiment at the University of Illinois and found the experience valuable, particularly the discussions among these involved in the exchange teaching. This sort of an exchange arrangement would be one way of helping a teacher to avoid getting into a rut. Other ways include: individual or group research projects, consulting on mathematics problems arising in industry or governmental laboratories, sabbatical leaves, and so on.

There are two aspects of motivation that bear further mention. On the one hand, mathematics should continually draw upon the physical and engineering sciences for illustrative problems. But, of equal or perhaps greater importance is the fact that motivation is tremendously enhanced if liberal use is made of mathematical techniques in physics and the engineering sciences, both in the first two years and in the later years. We quote again from the interim report previously cited (p. 11): "In the engineering sciences, full use should be made of the prerequisite mathematics, physics, and chemistry, recognizing that repetition is a normal pedagogical necessity, but that it can be most effective only when consciously and purposefully used. Perhaps nowhere else can the qualities of a scholarly engineering faculty be employed so effectively as in the presentation of these engineering sciences with an appropriate mathematical understanding."

How shall mathematics be taught to engineering students? These students are interested in mathematics as a tool. The majority of them are not content to study mathematics for its own sake. They usually can be induced to become interested in "why" a technique works, but only after they are convinced that: (a) it is a useful technique, and (b) they understand how to apply it. Teachers recognize this fact by using a large amount of problem material. In this connection, we should like to call attention to the collection of *Engineering Problems Illustrating Mathematics*, prepared by a committee of the ASEE under the chairmanship of John W. Cell, and published by the McGraw-Hill Book Company in 1943. These problems are divided into categories illustrating the mathematics usually covered in college algebra, trigonometry, analytic geometry, and differential and integral calculus. It has been suggested that more material of this kind would be helpful. In particular, more examples where the emphasis is on the analysis of the problem and the methods for setting it up in mathematical form, would be desirable. A fund of such problems requiring statistical techniques would be particularly welcomed by some teachers. Our deliberations and the replies to our inquiries have indicated needs for:

a. More emphasis on numerical methods.
b. More emphasis on graphical methods.

c. More emphasis on fundamental concepts.
d. More emphasis on translation from physical problems to mathematical problems, and on the interpretation of the results of the solution of the mathematical problems in terms of the given physical problems.
e. More use of mathematics in junior and senior engineering courses.

Much has been published on the question of improvement of teaching methods. The Committee feels it is appropriate to list here, for convenient reference, the following books and articles which are related to the work of the Committee:

BOOKS AND ARTICLES RELEVANT TO THE WORK OF THE COMMITTEE

J. W. Cell, *Engineering Problems Illustrating Mathematics*, McGraw-Hill (1943).
Effective Teaching—McGraw-Hill (1950) (Fred C. Morris, Editor)
Report of the Sub-Committee on Minimum Essentials in Mathematics for Engineering Instruction-California Committee for the Study of Education.
Report of the Committee on Adequacy and Standards of Engineering Education, J. of Engineering Education, vol. 42, No. 5 (1952) pp. 249–254.
Proceedings of the A.S.E.E.

Vol. 39, 1931–32	pp.	299–310
42, 1934–35		149–152, 292–296
45, 1937–38		122–131, 190–194, 548–558
46, 1938–39		716–724
47, 1939–40		394–401, 699–703
49, 1941–42		57– 66, 346–352
50, 1942–43		432–437
51, 1943–44		664–668
52, 1944–45		407–413
54, 1946–47		330–335, 531–535, 536–539, 641–652
55, 1947–48		175–180, 300–307, 308–312, 358–365, 366–373
58, 1950–51		308–310
59, 1951–52		170–172
60, 1952–53		33– 46, 136–144, 472–475

Conclusion. One imagines that the question of "how to teach (anything)" most effectively confronts every conscientious teacher almost constantly. One also feels that the question of *motivation* on the part of the teacher is at least as important as it is for his students. If he feels that *what* he is teaching is interesting, important, esthetic, and useful—then his enthusiasm will almost certainly stimulate his students to *learn*. But no teacher can long maintain a fictitious enthusiasm for subject matter which, to him, is dull, trivial, non-esthetic, and largely useless. It is for this reason that the committee chose to include a consideration of *what* mathematics to teach as well as *how* to teach it. We do not propose any definite answers to either question. But we have pointed out some experiments which seem to us to be significant. In doing so, we hope not only to recognize and encourage those who are conducting these experiments, but also to stimulate others to devise their own or to try some of the existing ones.

Recomendations. We recommend the following to the ASEE and MAA and their respective members involved in the teaching of mathematics to engineers:

1. Use all appropriate means to encourage better mathematics preparation in the high schools.
2. Teach "operational" rather than merely "appreciative" mathematics to engineers in the first two years of college.
3. Understand the engineers' needs and try to meet them; whatever the level of preparation.
4. Debate vigorously the issues involved in curriculum structure.
5. Make a serious attempt to introduce statistics and probability into the curriculum in the first two years.

Respectfully submitted:
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G. B. THOMAS, JR., Chairman

FORD FOUNDATION GRANT

The Ford Foundation has awarded a grant of $25,000 to the Mathematical Association of America for a study by the Association's Committee on the Undergraduate Program in Mathematics. This study will review the status of teaching and research utilization in mathematics, and the relationships of mathematics to the sciences and other fields of knowledge.

The grant will be used by the Committee to facilitate communication among those who are working on these problems. The Committee plans to bring together writing teams which will seek to bridge the gap between modern mathematics and the undergraduate curriculum by writing pilot text materials. However the Committee does not plan to use its funds to subsidize the writing of actual text books.

The present chairman of the Committee on the Undergraduate Program is Professor E. J. McShane of University of Virginia. Other members of the Committee are: J. G. Kemeny, Dartmouth College; G. B. Price, University of Kansas; A. L. Putnam, University of Chicago; A. W. Tucker, Princeton University; W. L. Duren Jr., Tulane University, *ex officio*.

THE JANUARY MEETING OF THE NORTHERN CALIFORNIA SECTION

The seventeenth annual meeting of the Northern California Section of the Mathematical Association of America was held at the University of California, Berkeley, on January 15, 1955. Professor J. G. Herriot, Chairman of the Section, presided at both the morning and the afternoon sessions.