MATHEMATICS FOR SOCIAL SCIENTISTS*

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1. Introduction. (Madow). The 1953 Summer Institute of Mathematics for Social Scientists met for eight weeks in the summer of 1953 under the sponsorship of the Social Science Research Council with the help of a grant from the Behavioral Sciences Division of the Ford Foundation.

Approximately twenty hours weekly were devoted to lectures by the four members of the faculty. In addition, ten hours weekly were devoted to supervised study and supplementary lectures.‡ Finally, eight visitors gave eighteen lectures on applications of mathematics in the social sciences that were not discussed in the regular lectures. The Institute was neither a conference nor a discussion group. It was simply a stiff summer session in which topics in mathematics and applications of mathematics in the social sciences were taught intensively.

Although the nature of the student body and the intensity of the program keep the Institute from being considered to be a model for an undergraduate mathematics curriculum, yet the subject matter and level of the curriculum of the Institute were such that we believe the Institute to be a suitable subject for discussion here.

Of late years considerable dissatisfaction has developed with the present fairly standardized first two years undergraduate mathematics curriculum. The reasons for this dissatisfaction are well known and need no repetition. The dissatisfaction is felt not only by social scientists but by others; perhaps most of all, by mathematicians. The Mathematical Association of America has at least one committee working in this area.

We feel that a two year undergraduate curriculum based largely upon the subject matter of the 1953 Summer Institute can be successfully taught not only to majors in social sciences but also, after minor revision, to all students except engineers and physical scientists. Actually, this curriculum might even meet the needs of engineers and physical scientists, but we think it would be practical first to organize this curriculum as an alternative to, rather than replacement of, the present curriculum.

For admittance to the Institute, students were required to be social scientists and to have had at least one semester of college mathematics or its equivalent in independent study. Over 250 persons applied for admission. Thirty persons were admitted and given grants. Twenty-six persons were admitted without

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† The faculty of the 1953 Summer Institute consisted of W. G. Madow, University of Illinois, director, R. R. Bush, Harvard University, Howard Raiffa, Columbia University, and R. M. Thrall, University of Michigan, assisted by R. L. Davis, University of Michigan and R. E. Priest, University of Illinois.

‡ These lectures were given by Davis and Priest.

grants. Of the 56 persons thus admitted 41 attended the Institute.

Twenty-two of the students had received the Ph.D. degree, 12 of them having academic rank of assistant professor or higher. The other 19 were graduate students in a social science.

Twenty-four of the students were between 22 and 30 years of age; the remaining 17 were between 31 and 42 years of age.

The students came from 23 institutions in 13 states. There were 20 psychologists, 11 sociologists, 7 economists, and 3 political scientists.

Twenty-two of the students had studied some calculus. Nineteen had not. But the training in calculus was often weak and had not been used for some years.

The students were highly selected not only by the fact they were admitted to the Institute but also by the fact that they applied. They were very highly motivated and were mature responsible people who had given evidence of achievements in their own fields.

It is still early to state how successful we were. Of the 41, only two did really poorly and only five did fairly poorly. We were well satisfied with the others. But it is only as our students begin using their work of this summer that we will begin to be able to evaluate this summer's work.

No summer institutes are planned for 1954, but we expect to have two Summer Institutes of Mathematics for Social Scientists in the summer of 1955. At each of these, there will be two curricula, one for those who have studied calculus and one for those who have studied college algebra but not the calculus. It is hoped that detailed announcements will be available in the fall of 1954.

In Sections 2, 3, and 4, Bush, Thrall and Raiffa discuss the following five points:

1. Social scientists as students of mathematics
2. Objectives of the Institute
3. Some considerations in selecting the mathematics to be taught to social scientists
4. The mathematics curriculum of the 1953 Institute
5. Some aspects of teaching mathematics to social scientists

In Section 5 will be found comments.

2. The development of mathematics courses for social scientists. (Bush). I will discuss two central problems that arise in developing mathematics programs for social scientists. First I will comment on the *kinds* of mathematics which I think should be included in such a program, and second, I will express my views on how such mathematics should be taught.

In planning a mathematics curriculum for social scientists, one can adopt many different criteria for including or excluding any particular topic in mathematics. At one extreme, one might argue that the program should be a survey of *all* kinds of mathematics known to man and let the social scientists of a few decades hence decide which kinds are most useful to their fields. A person cynical about what has been done in quantitative social science might very well take such an extreme position. But the difficulty with this position is that the program is necessarily too lengthy or too superficial.

Another possible view that is held by some people is that social scientists should learn the kinds of mathematics which are proven tools in physics, chemistry, and astronomy. The trouble with this proposal is that it excludes very little and probably makes a faulty assumption that the social sciences are fundamentally similar to the natural sciences.

An attempt to evaluate what kinds of mathematics are most appropriate for social science problems is another possible approach. But here one can find only a small area of agreement among people who have thought about the problem. The difficulty is simple: to make a sound prognosis about the "future social science," one must have a thorough knowledge of the several social sciences and at the same time know a good deal about all fields of mathematics. No such man exists as far as I know. The people who attempt to draft a program of study on this basis are either competent mathematicians who have had little real experience with social science research, or social scientists who have rather limited mathematical skills but who are anxious to use those skills.

The only satisfactory criterion, I maintain, for selecting content of a program in mathematics for social scientists is the following: Select and weight topics in accordance with what has already been done in applying mathematics to social research. Unfortunately, relatively little has been done in this direction outside of statistics, but the list of papers and books is long enough to allow us to make some inferences. One advantage of this criterion is that the teacher has useful illustrations of the mathematics being taught. This is important, as I will argue later.

On the basis of this criterion I would propose the following outline as desirable for any program of the type we are discussing.

1. *Probability theory with special emphasis on stochastic processes.* The argument for this topic hinges upon the importance of statistics and the growing interest in stochastic models for learning, communication, and small group behavior.
2. *Calculus and differential equations.* Not only is calculus used in most treatments of mathematical statistics, but it also arises in many models for social science phenomena. Some work on finite calculus should be included because of its use in stochastic models and in statistics.
3. *Algebra and axiomatics.* Various uses of matrix algebra, set theory, and axiomatics can be found in the social science literature. I would argue that matrix theory is the most important topic in this category.

The three main categories I have listed should be given about equal weight I believe—and this belief is based upon my acquaintance with what has been done in applying mathematics to social research. I should point out that the list of topics just given is roughly the list of topics included in the program at the

Social Science Research Council's Summer Institute on Mathematics for Social Scientists, held at Dartmouth College last summer.

The other major problem I want to discuss is that of *how* mathematics should be taught to social science students. One part of this problem concerns how much detail and rigor should be included in the program, and this is clearly determined mainly by the time available and the capacities of the students. Concerning this latter matter—the capacity of the social science student for learning mathematical material—let me state some observations. First of all, a great many undergraduates in our universities drift into social science departments because they have had trouble with courses in mathematics and the natural sciences or because they know they dislike such subjects. The clinical psychologist can supply many reasons for such behavior, but prominent among those reasons is unpleasant experiences with mathematics in elementary and secondary schools. Hence, to solve this problem we need to improve secondary education in mathematics or recruit our social science students from a different population. For many years to come, I suspect, we will merely have to accept as a boundary condition that students in the social sciences on the whole do not like mathematics, find it difficult, and indeed have serious psychological blocks against learning mathematics. This fact creates some special problems that do not exist in the teaching of mathematics to engineers, chemists, and physicists.

A closely related observation is that few social scientists will bother with mathematics unless they are highly motivated—have found their lack of mathematical training a serious handicap in their own work or have been convinced by their elders that it will become a serious handicap. Therefore in optional programs or courses in mathematics for social scientists, we can depend on a high level of motivation, a real eagerness to learn, even though these same students have serious psychological problems in this connection. But it is very important, I'm convinced, to keep in mind the source and direction of this strong motivation. In a word, it is directed at finding security in his own field. Consequently, the man who teaches mathematics to social scientists must recognize his dual role of teacher and psychotherapist.

Because of the rather special and intense motivation in the student to acquire skills useful in his own field, the method of teaching is critical. One simply cannot teach mathematics to social scientists of today the way he teaches it to mathematics students; the motivation, the system of needs, the goals of these two kinds of students are entirely different. I think the social science student of mathematics must have frequent reinforcement from manipulative skill at working easy problems—problems he previously could not handle. Such reinforcement will maintain his motivation.

My main point on how mathematics should be taught to social scientists is this. It should be problem-oriented. Each topic in mathematics should be introduced by displaying concrete, non-trivial problems in social research and indicating how some mathematical machinery will be useful. Elegance in mathematical presentation must be sacrificed at times. Social science problems should be stated, some mathematics taught, and the original problems solved by using that mathematics.

To develop a course such as I propose, a mathematician must necessarily learn a good deal about current problems in the several social sciences, and this is not easy. I have one suggestion: mathematicians should collaborate with social scientists in developing programs and giving courses. By such joint efforts, mathematicians can seriously contribute, I believe, to the development of social science.

3. The curriculum of the institute. (Thrall). The 1953 Summer Institute in Mathematics for Social Scientists had four major goals:

A. *Communication*. This is primarily communication with mathematicians; however, experience shows that expressing facts in mathematical language also facilitates communication between social scientists from different areas. One of the bars to communication between mathematician and social scientist is the different meanings attached by them to such words as *relation*, *function*, *variable*, *vector*, *dimension*, *continuum*. If the meanings were entirely different the discrepancy would be quickly discovered and cared for; the real difficulty is that the meanings have enough in common to obscure the difference until the damage is done. Since terms such as these are used in social science for essentially mathematical purposes, it seems desirable that the social scientist should learn and then use their mathematical meanings (at least when speaking to mathematicians).

A second bar to communication is lack of technical acquaintance by the social scientist with such basic mathematical concepts as order relation, partial order, real number, continuous functions.

B. *Preparation for Further Courses*. In a single eight-week session one cannot cover all of the mathematics needed by a social scientist. However, a reasonable goal would be to lay a foundation which would enable the student to continue his mathematical education with courses such as advanced calculus, linear algebra, mathematical statistics.

C. *Reading Knowledge*. The amount of mathematics appearing in papers and books in the social sciences is steadily increasing. The mathematical material sometimes appears in appendices and is usually summarized or paraphrased in word form. However, the critical and thorough reader will wish to be able to follow the mathematical arguments. Most of the mathematics used is statistical in nature, but a reading knowledge requires familiarity with such concepts and areas as integral, derivative, order relation, matrix, vector, linear transformation, stochastic process, linear programming, game theory.

D. *Model Building*. The mathematical background needed for model building in the social sciences is much the same qualitatively as for reading knowledge except for a considerably greater emphasis on axiomatics and foundations. Quantitatively, there is a great difference between reading and creating and one cannot expect an eight-week course to produce full-blown experts in the use of mathematics for model building. However, this goal should be kept in mind in constructing the course.

The goals discussed above would apply to any mathematics course designed for social scientists, but here we are thinking primarily of a coures designed for social scientists who are at or beyond the Ph.D. level when they realize the need for mathematics. At this level it is not practical to undertake the traditional undergraduate mathematics sequence, and even when new undergraduate sequences become available, there will still be a need for special courses to care for mature social scientists. The topics included in the 1953 Summer Institute were selected after consideration of the goals listed above and also keeping in mind the nature of the participants. The topics are not listed in order of importance or in the order in which they should be presented. The letters following each topic indicate the goal (or goals) for which it is important.

1. Set algebra, relations, functions, one-to-one correspondence, equivalence relations, partitions, order relations. A.
2. Axiomatic development of number system including the concept of limit of a sequence. Careful definitions, but proofs limited to heuristic discussions. A, C.
3. Differential and integral calculus. Emphasis on polynomials, but also logarithms and exponential functions. Some analytic geometry included. B, C.
4. Selected topics from advanced calculus, including partial derivatives and multiple integrals. B.
5. Axiomatics, some simple system in detail and the general principles of model building. D.
6. Linear algebra, vector spaces, matrices, linear transformations. B, C.
7. Introduction to probability, sample spaces, stochastic processes, Markov chains. B, C.
8. Models from social science situations; this should include numerous small illustrative examples and also some large scale models such as linear programming, utility theory, learning theory, social choice, game theory, measurement theory. A, C.

4. Two aspects of a mathematics program for social scientists. (Raiffa). I would like to divide my comments into two parts: first, the general teaching approach that I would advocate for a program similar to the one given at Dartmouth this past summer; second, the importance of abstract thought (in contradistinction to mathematical technique) to the social scientist.

The points I raise here are, I believe, non-controversial in broad outline—indeed, in broad outline they are trivial. However, the stress given to these issues is more problematical, and since my own viewpoints have changed over the past summer, I would like to outline my subjective opinions at this point.

It is often said that the art of good lecturing is first to say what you are going to say, say it, and then say what you have said. The price in subject matter covered for following this advice is well worth it *for the social scientist studying mathematics* (for the mathematics student it is not so necessary). In particular, I interpret this advice as follows: Motivation is well worth the expense —this includes motivation of mathematical subject matter, motivations of definitions, motivation of hypotheses, motivation of proofs (rigorous proofs play an important role in such a course—as I see it!)

As regards motivation of subject matter, I think that broad areas should be outlined before plunging into detail. For example, I would not advocate proceeding from a discussion of real numbers to a definition of limit (assuming, of course, that limits were not introduced formally in defining—or should I say in "talking about"—real numbers). I think it would be more appropriate to start off with a series of problems, perhaps related to the social sciences, to abstract these to problems involving maxima and minima, areas, sums of series, *etc.*, and then to show in turn that these have a common abstraction in terms of limits of sequences. Then one could introduce the limit notion, keeping in mind the diverse examples which motivated the abstract subject matter, and using these examples as test cases for the developing theory. Another point I wish to emphasize for the social scientist is that it is a mistake to build up to a principal theorem or application without disclosing our aims during the development of the theory. Of course it might not be possible to point meaningfully to our goal without the necessary machinery at hand, but I think one should not use this as an excuse for postponing the motivation indefinitely.

As regards motivation of definitions, I think it desirable for the student to know where he is going. Tentative definitions should be tried and shown to be faulty. Thus the ϵ-δ definition of limit should crystallize only after some preliminary fumbling with "definitions" which "really don't capture the idea which we are after." Note that the idea should precede the definition. The non-uniqueness of ways of defining some concept should be indicated.

As regards motivation of hypotheses of a theorem, I think it desirable for the lecturer to check the theorem's validity by means of special cases before discussing its proof. Numerous counter-examples should be given to theorems when hypotheses are altered or relaxed. With this preliminary discussion of the theorem, the student should begin to appreciate its meaning, why certain hypotheses are needed, and what cases are carefully ruled out by the hypotheses. By attempting to find counter-examples to the theorem itself and finding it to no avail the student often gains an insight into the crux of the proof. Incidentally one should not always formulate true theorems; theorems should be thought of as intelligent guesses or conjectures.

As regards motivating the formal proof itself, I wish to point out that understanding each step of a proof is no indication of understanding the proof looked at as a whole. Similarly, if one understands each theorem it does not mean one understands the subject matter as a whole. We should not lose sight of the *Gestalt* by looking too much at details. Learning mathematics by understanding in turn each sentence in a formal style of presentation can become a cook-book style of learning. Hence I would stress the "idea" of a proof as much as the formal proof itself, and the "idea" should precede the formal proof. However, one should emphasize that the "idea" falls short of the requirements of a formal proof and these shortcomings should be explicitly pointed out. When the formal proof is completed one should tie in loose ends by checking the places where the hypotheses were needed in the proof. If time is pressing, as it always is, I would much prefer to see the formal proof omitted rather than omitting the motivating remarks and/or the "idea" of the proof (especially since the formal proof can be found in texts while the motivation is not often enough in print). However, if this program is too time-consuming for the lecturer, part of it could be assigned explicitly as a problem for the student.

In the realm of motivation one should stress the creativity of the mathematician and one should indulge in heuristic arguments. It should be pointed out that mathematical creativity is largely a trial-and-error procedure (similar to empirical research) and that the formal proof is usually quite different from the pioneering attack on the problem.

The technique of starting a lecture by summarizing the pertinent information from previous lectures helps the student to see the main trend, but what I consider more important is that it affords an opportunity for the lecturer to repeat the material in a more symbolic and abstract form, unencumbered by motivation and examples, and thus far more compact; and then there is the secondary effect of having the student aware of all the innuendos and the rich meaning involved in a pithy abstract mathematical statement. Hence I would recommend repeated summaries of material covered in order to raise the mathematical maturity level of the students.

I concur wholeheartedly with Bush that, psychologically speaking, the student has to have some sense of accomplishment and should not become overawed with the extensive scope of the material. To this end, there should be numerous graded exercises. We should play along with the game that if one can manipulate and substitute in formulas then one "understands" the theory. We should not destroy this false sense of security. Where one should draw the line is hard to say, but personally, after the summer's experience, I would say that there should be more manipulation than we gave this summer. Any program such as the Summer Institute must face the problem of weighing the emphasis between abstract and concrete presentation of the subject matter (*e.g.*, vectors *vs.* n-tuples, linear transformations *vs.* matrices, stress on limit notions *vs.* more topics in maximization, *etc.*). If approach A is easier to get across than approach B, is it better? Not necessarily. I feel that another indicator that one should take into account is whether A or B is more conducive to the general "mathematical maturity level" of the student, and I would go so far as to introduce material which is primarily intended to increase mathematical maturity. Of course, I admit that I do not know how to do this effectively, and I do not deny that solving a series of differential equations may contribute to the mathematical maturity of the student.

In order to defend retaining some moderate degree of abstract presentation in programs of this kind for the social scientist, I would like to say something about axiomatics in general. I choose this topic because I think it is defended too often for reasons which I do not consider important; but, on the other hand, I think it is important to include this topic in our program. Some people argue that for approaches to the social sciences to be sophisticated they must be axiomatic in nature; they point to game theory as an example. On the contrary, I think that premature emphasis on the axiomatic development of a subject matter—learning theory, for example—can be detrimental. If applied to large areas it can cause sterility of mathematical ideas. The fact that Von Neumann axiomatized game theory is not the reason that game theory is interesting to some mathematicians. The reason is rather that the theory presents well-formulated unsolved problems which would exist regardless of the axiomatic formulation of the subject matter. Certainly I would agree that it is sometimes beneficial for the social scientist to try to axiomatize an area—mainly because this directs his attention to the structure and to some of the basic notions of the area. However, axiomatic-type thinking has played and will play in the future an increasingly important role in mathematical work in the social sciences. Often in the behavioral sciences, when one grapples with some nebulous material (*e.g.*, cohesiveness of a group, so-called "rational" behavior, "socially desirable" welfare functions, *etc.*), definition after definition is discarded or discredited because it does not fit the bill in certain situations (*i.e.*, does not fulfill the hazy desiderata one has formed in one's own intuition). It then behooves one to pay more attention to these subjective desiderata and to formulate conditions one wishes the definitions to satisfy. These conditions should be viewed as axioms, and one should then determine whether they are consistent, independent, and categorical. If a set of mutually inconsistent conditions is prescribed, then one must review one's intuition. As a case in point, Arrow, in his *Social Choice and Individual Value*, formulates some historically important and intuitively palatable conditions for a social welfare function to satisfy, only to show that these conditions are inconsistent. This plays the valuable role of directing our attention to our inconsistent intuitions rather than to the shortcomings of specific proposals. A similar example can be pointed out in the area of decision making under uncertainty, where a set of reasonable but inconsistent conditions has been given (Milnor, Chernoff) for a rule of inductive behavior. These conditions are then studied further to show that various subsets lead to various principles (minimax principle of Wald, equally likely *a priori* principle of Laplace, *etc.*). Thus the investigations of these conditions serve to resolve certain philosophical difficulties for oneself. Another area in which there has been much thinking in axiomatic terms is in the solution concept of the n-person game. If a set of conditions or requirements turns out to be consistent and categorical in the sense that it uniquely characterizes a concept, then this procedure implicitly serves as a definition of that concept; if a set of conditions turns out to be consistent but permits a subset of possible interpretations then one has localized one's consideration to this well-defined subset and one hopes that various arguments will hold for all the interpretations of that subset (*i.e.*, have an argument remain invariant over the subset characterized by our initial desiderata). Instances of these variants have turned up in the social sciences, and they are becoming increasingly popular. I thus conclude that the study of axiomatics in general is pertinent to the study of mathematics for social scientists.

5. Discussion. (Madow). On the whole, the curriculum of the 1953 Summer Institute was chosen in the way outlined by Bush, and, indeed, this is reflected by the rather close agreement between the list of topics recommended by Bush and that used in the 1953 Institute.

I do believe that it is an exaggeration to say that one should select and weight topics in accordance with what has already been done in applying mathematics to social research. We all know that certain lines of approach are developed and then an impass reached. For that reason, in reviewing what has been done, one must also take into account what is being done and what is likely to be done in the future. Thus, twenty years ago, one would have said that it was quite unnecessary to study algebra and set theory in economics and statistics. Just about that time, the importance of both of these subjects was beginning to be recognized and today the topics are far more important than they were twenty years ago. On the other hand, the importance of the calculus, particularly in statistics, has been sharply reduced. Thus, as in so many other fields, merely to consider the past would result in a biased program.

On the other hand, to consider only one's own personal research or the research activities of a small group would be likely to be very much biased also. It is a question of judgment and one can not avoid some forecast.

A similar comment applies to Bush's statement that he could argue that matrix theory is the most important topic among matrix algebra, set theory, and axiomatics. All three are needed, and the precise balance will depend on the individuals and needs concerned.

On the whole, I agree with Bush's characterization of social scientists, namely, that if they study mathematics, they will turn out to be highly motivated but also that many will have had some bad experiences with mathematics. A further difficulty faced by the social scientist as an undergraduate, when he starts studying mathematics, will be that he will not find the mathematics used in courses in his own subject matter. This is a disadvantage not faced by the physical scientist or engineer who, after studying some mathematics finds that it is referred to in other courses. Thus, the social scientist comes with high motivation but the knowledge that he has had previous difficulties and the feeling that he could probably get along without the mathematics. Then, too, he is not part of a captive audience, as are the students from the physical sciences and engineering. The result is that he definitely requires a higher level of teaching than do the usual students of mathematics courses. In this connection, it is important to realize that social scientists believe that learning is better advanced through reinforcement and reward than through challenge, whereas in our mathematics courses, we tend to challenge the student. Here, again, we need better mathematics teaching. It is worth noting that the great interest of mathematicians in the type of curriculum we suggest may well produce a higher level of teaching than now exists.

Bush's discussion of how to teach mathematics to social scientists should be read in conjunction with the comments of Raiffa on the same subject. Perhaps I can summarize the situation by saying that we think that mathematics should be taught, not presented.

In our announcement of the Institute, we listed as the goals in order: model building, reading knowledge, and preparation for further courses. It was our feeling that all but the very poorest students would be able to communicate, that many of the students would be able to take further courses and read mathematical literature in their own fields, and that a few would be able to create models. Of the various goals listed by Thrall, I would say that the most of our students and the faculty were in agreement that the most important single result would be an increase in reading knowledge, and that this was attainable during the eight-week session.

Although the course we gave at Dartmouth was designed primarily for mature students, we believe that with very little change it would be a more suitable mathematics course for undergraduate social scientists than one or two years of the classical undergraduate curriculum in mathematics. I even think that the student would benefit far more from taking this course one year and repeating it the second than he would from the current first two years of college mathematics. Of course, there are administrative reasons why this may not be practicable, but I mention it to stress the point of view.

I think that there are many differences of opinion concerning Thrall's identification of mathematical topics and goals. To mention just one, to the economist and to the statistician, the reasons for studying the partial derivative far transcend the preparation for further courses in mathematics. But the precise identifications are not essential.

Whether the axiomatic development of the number system was needed is doubtful. If needed, it is hard to see why it is important for the goal of communication. It should not be understood that each of the topics by Thrall has equal importance. Thus, far more space would be given to item 6 and to item 7 than, let us say, to item 2.

All of us are in general agreement with the distribution of time mentioned by Bush. The only differences that would occur would be of the order of replacing one-third by one-quarter, or by one-fifth, neither of which would be great changes.

I should like to stress my agreement with Raiffa's comments on the teaching of mathematics, and to add that they really go beyond the teaching of mathematics to social scientists, and apply to teaching of mathematics in general. This is particularly true with respect to his statement that understanding each step of a proof is no indication of understanding the proof looked at as a whole, and that understanding each theorem does not mean that one understands the subject matter as a whole. Of course, it is not easy to achieve the full understanding, but this is a challenge to those who teach mathematics, as well as to those who study it.

It seems likely that axiomatic *versus* non-axiomatic treatments will continue to be a source of controversy. In general, social scientists will not find axiomatic approaches too difficult. Considerable gains to them will result from the consciousness that comes from the axiomatic approach, of the way in which mathematics and the world of experience interact.

After a lapse of many centuries, axiomatic approaches have again become important in many parts of mathematics. Here, as in many other questions of education, we must be certain that our preferences, be they for axiomatic or non-axiomatic approaches, are based on rational reasons rather than on what we happen to have studied.

I have just two more comments. At the present time, when a student has had undergraduate mathematics and decides to continue the study of mathematics beyond the junior year, he often feels as though the subject matter he is studying has changed completely. With the curriculum that we taught in the summer of 1953, this would not be true. I think, therefore, that that curriculum would be superior to the present undergraduate curriculum not only for the social scientist but also for students of mathematics, and indeed, for students from all fields other than those such as the physical sciences and engineering in which it is important that certain technical processes be known at certain times. Even for such students, I think that a curriculum such as we are discussing would be better than the present curriculum, but this needs to be worked out more.

Doubtless, there will be many questions raised concerning the inclusion or exclusion of certain topics. For example, if differential equations are taught, should one include differential equations of the second order? I suggest that we must avoid this type of question at present. The mathematics that is used in social science papers depends on the mathematical background that the researcher happens to have had. It would be impossible to cover all points, and it will be impossible to satisfy all people on the amount of time devoted to different topics. What we need is to have more courses, more experiments. Then, gradually, a curriculum will evolve.