New Directions for the Calculus Track

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The college calculus sequence has been, and continues to be, the mathematical bedrock upon which most STEM disciplines are built. It is where many students start, and often end, their college mathematics careers. To bend the famous quote of Lynn Steen, we want calculus to serve as a pump, but we are aware that it sometimes serves as a filter. For the past three decades, the mathematics community has engaged in a great deal of experimentation and innovation to improve both the content and the pedagogy of the calculus sequence. In recent years, several new directions for calculus have emerged in this continued quest for improvement.

Connections with the Partner Disciplines

Many initiatives strive to more closely link the calculus sequence with the partner disciplines by including more mathematical modeling, using examples drawn from a wider variety of contexts, and adding applied projects. Two national MAA reports—*Curriculum Foundations Project: Voices of the Partner Disciplines* and *Partner Discipline Recommendations for Introductory College Mathematics and the Implications for College Algebra* (both freely available at http://bit.ly/2aYMMxT)—suggest that many of the partner disciplines want similar changes in calculus:

Curriculum Renewal Across the First Two Years (CRAFTY) is a subcommittee of the MAA's Committee on the Undergraduate Program in Mathematics (CUPM). This is the fourth in a series of articles about general recommendations to renew mathematics course work and instruction.

- Increase the emphasis on conceptual understanding, problem-solving skills, mathematical modeling, and communication of mathematical ideas;
- Provide a better balance of perspectives (such as exact and approximate); and
- Make appropriate use of technology.

These recommendations also call for an increased emphasis on multivariable calculus, vectors, and three-dimensional geometry.

For example, the University of Michigan has upended its usual calculus sequence to support students building their procedural fluency online, thereby allowing class time (and exam problems) to focus on applications from engineering and science. Wright State University has created a one-semester engineering mathematics course taught by engineers that uses hands-on laboratories to bring to life the concepts students will subsequently see in the usual calculus and differential equations sequence.

Louisiana Tech takes a different twist, blending the calculus, physics, and engineering into an integrated course sequence in which students are Living with the Lab. One project that emphasizes the learning of fundamental multivariate and vector calculus concepts via geometric reasoning is the Vector Calculus Bridge project out of Oregon State University that produced the free online text *The Geometry of Vector Calculus* (math.oregonstate.edu/bridge/). The U.S. Military Academy (West Point) has long taught an applied, problem-rich sequence to cadets.

An important new emphasis is connecting calculus to the life sciences. The national reports *Bio 2010: Transforming Undergraduate Education for Future Research Biologists* (http://bit.ly/2aOo9Fd), *Math & Bio 2010: Linking Undergraduate Disciplines* (http://bit.ly/2aNlm1Y), and *Scientific Foundations for Future Physicians* (http://bit.ly/2aUYCvD) all call for increased attention to the connections of calculus to life sciences, modeling skills, and using data/statistics. One response is the University of Minnesota at Rochester's aptly named, yearlong sequence in calculus, modeling and data that uses *Calculus for Biology and Medicine* by Neuhauser. Other calculus texts for life-sciences students include *Calculus for the Life Sciences* by Schreiber, Smith, and Getz, and *Mathematics for the Life Sciences* by Bodine, Lenhart, and Gross. At smaller institutions where it is not feasible to offer different "flavors" of calculus, many standard calculus texts are now including more examples and applications from the life sciences, and instructors can easily mine these specialized textbooks for more examples.

Resequencing Calculus Topics
A recent promising phenomenon, especially at smaller institutions, has been to reconsider the timing of the topics within the calculus sequence based on the reality that while mathematics, physical science, and engineering majors typically complete the full calculus sequence, students majoring in the life sciences, economics, finance, or other partner disciplines may well stop after only one or two semesters. Such students benefit from an earlier introduction to multivariable geometry and calculus and

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an increased emphasis on modeling (especially differential equations) and numeric methods, both to better align with their major courses and to provide earlier entry to advanced mathematics courses such as probability, differential equations, linear algebra, and statistical modeling.

The textbook from the Re-sequencing Calculus Project restructures the course sequence, bringing partial derivatives, multiple integrals, vectors, and matrices into the second semester along with a hearty serving of differential equations and postponing infinite series and more engineering-focused applications of the integral to the third semester (resequencingcalculus.com).

The applied calculus sequence at Macalester College recognizes that many mathematics and physics majors enter college with at least one semester's credit for calculus; thus, the college's revamped first semester of calculus focuses heavily on applications relevant to the students who need it—often life sciences and non-STEM majors; a description appears as chapter 5 in *Undergraduate Mathematics for the Life Sciences: Models, Processes, and Directions* edited by Ledder, Carpenter, and Comar (http://bit.ly/2aM6Isn). Macalester's sequence also gives an earlier treatment of multivariate topics, uses the program R for both data analysis and calculus, and develops infinite series concepts more gradually over the three terms.

A recent *PRIMUS* article describes how Nazareth College took a less-comprehensive but simple approach: just cover Calculus 2 and 3 topics in a different order (http://bit.ly/2bjcdiE).

Active Learning + Calculus
New research on how students learn calculus suggests that active learning plays a critical role in both learning and retention, especially for women and students

know what a homomorphism is. So then I'll start trying to explain what a homomorphism is, but then I'll remember that they haven't learned what a group is. So then I'll start trying to explain what a group is, but by the time I finish writing the group axioms on my napkin, they've already forgotten why I was talking about groups in the first place. And then it's 1 p.m., people need to go places, and I can't help but think: Man, if I had 40 hours instead of 40 minutes, I bet I could actually have explained this all.

This book is my attempt at those 40 hours.

Actually, my list of "things to write" and "things to learn" are the same list: nowadays if I really want to understand a topic well, I'll write a blog post about it.

What has been your favorite course thus far?
I took Math 55a last year at Harvard, which I enjoyed a lot: it was a one-semester tour of lots of abstract algebra. Tons of the material in *Napkin* is based off things I learned in that class.

What would you like to write about next?
The next thing on my list of things to write is on duality with respect to a triangle, an advanced topic in Euclidean geometry that I've heard much of but don't know anything about. Actually, my list of "things to write" and "things to learn" are the same list: nowadays if I really want to understand a topic well, I'll write a blog post about it.

What book do you wish someone else would write?
I wish someone would explain to me why I should care about scheme theory. Despite having finally learned the definition of a scheme and some of the major theorems, I still don't at all understand what this structure does for you. ◈

Evan Chen was interviewed by Jennifer R. Bowen, an associate professor and chair of the department of mathematics at the College of Wooster in Ohio. She also is a member of the MAA Problem Books Editorial Board.

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from traditionally underrepresented minorities. The meta-analysis *Active Learning Increases Student Performance in Science, Engineering, and Mathematics* (pnas.org/content/111/23/8410.full), by Freeman et al., suggests that any level of active learning tends to produce greater pass rates than traditional lectures do. The comprehensive MAA National Studies of College Calculus by Bressoud et al. studied calculus at primarily research universities. Among many findings, results suggest that successful programs are often incorporating more active learning. Their results are published in the MAA Notes volume 84, *Insights and Recommendations From the MAA National Study of College Calculus,* which is free at http://bit.ly/25cxNqs; in Bressoud's MAA *Launchings* blog (http://launchings.blogspot.com); as well as in the series that has been running in this magazine.

Recent textbooks emphasize incorporating active learning into calculus. Examples include the open source *Active Calculus* by Boelkins, Austin, and Schlicker and *Calculus: An Active Approach with Projects* by Hilbert, Schwartz et al. The Academy of Inquiry Based Learning and the *Journal of Inquiry-Based Mathematics* host IBL problem sets for calculus (inquirybasedlearning.org; jiblm.org).

Preparation for Calculus
Assessing students' readiness for calculus, including just-

in-time review within the calculus sequence, and rethinking precalculus curricula are also under examination (and will be the topic of a future article in this series).

Several online communities and resources are geared toward all aspects of calculus instruction, particularly the MAA's Course Communities (http://bit.ly/2bjcKkH). The MAA's online Classroom Capsules Project also posts a range of engaging articles about calculus topics from the *College Mathematics Journal* and other MAA journals (http://bit.ly/2bkUKT1).

In short, there's something for everyone. Whether you hope to create a new flavor of calculus course, to rebuild or resequence your existing calculus to better meet the needs of the partner disciplines, to include more active learning or applied projects, or to reenergize your calculus sequence with new examples, it is well worth taking the time to draw inspiration from the diverse resources available. ◈

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