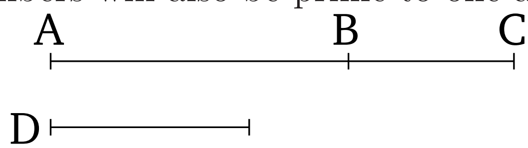
Book 7
Proposition 28

If two numbers are prime to one another then their sum will also be prime to each of them. And if the sum (of two numbers) is prime to any one of them then the original numbers will also be prime to one another.



For let the two numbers, AB and BC, (which are) prime to one another, be laid down together. I say that their sum AC is also prime to each of AB and BC.

For if CA and AB are not prime to one another then some number will measure CA and AB. Let it (so) measure (them), and let it be D. Therefore, since D measures CA and AB, it will thus also measure the remainder BC. And it also measures BA. Thus, D measures AB and BC, which are prime to one another. The very thing is impossible. Thus, some number cannot measure (both) the numbers CA and AB. Thus, CA and AB are prime to one another. So, for the same (reasons), AC and CB are also prime to one another. Thus, CA is prime to each of AB and BC.

So, again, let CA and AB be prime to one another. I say that AB and BC are also prime to one another.

For if AB and BC are not prime to one another then some number will measure AB and BC. Let it (so) measure (them), and let it be D. And since D measures each of AB and BC, it will thus also measure the whole of CA.

And it also measures AB. Thus, D measures CA and AB, which are prime to one another. The very thing is impossible. Thus, some number cannot measure (both) the numbers AB and BC. Thus, AB and BC are prime to one another. (Which is) the very thing it was required to show.