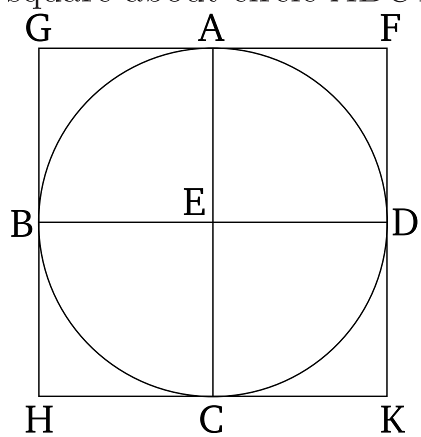
Book 4
Proposition 7

To circumscribe a square about a given circle.

Let $ABCD$ be the given circle. So it is required to circumscribe a square about circle $ABCD$.



Let two diameters of circle $ABCD$, AC and BD, have been drawn at right-angles to one another. And let FG, GH, HK, and KF have been drawn through points A, B, C, and D (respectively), touching circle $ABCD$.‡

Therefore, since FG touches circle $ABCD$, and EA has been joined from the center E to the point of contact A, the angles at A are thus right-angles [Prop. 3.18]. So, for the same (reasons), the angles at points B, C, and D are also right-angles. And since angle AEB is a right-angle, and EBG is also a right-angle, GH is thus parallel to AC [Prop. 1.29]. So, for the same (reasons), AC is also parallel to FK. So that GH is also parallel to FK [Prop. 1.30]. So, similarly, we can show that GF and HK are each parallel to BED. Thus, GK, GC, AK, FB, and BK are (all) parallelograms. Thus, GF is equal to HK, and GH to FK [Prop. 1.34]. And since AC is equal to BD, but AC (is) also (equal) to each of GH and FK, and BD is equal to each of GF and HK [Prop. 1.34] [and each of GH and FK is thus equal to each of GF and HK], the quadrilateral $FGHK$ is thus equilateral. So I say that (it is) also right-angled. For since $GBEA$ is a parallelogram, and AEB is a right-angle, AGB is thus also a right-angle [Prop. 1.34]. So, similarly, we can show that the angles at H, K, and F are also right-angles. Thus, $FGHK$ is right-angled. And it was also shown (to be) equilateral. Thus, it is a square [Def. 1.22]. And it has been circumscribed about circle $ABCD$.

Thus, a square has been circumscribed about the given circle. (Which is) the very thing it was required to do.