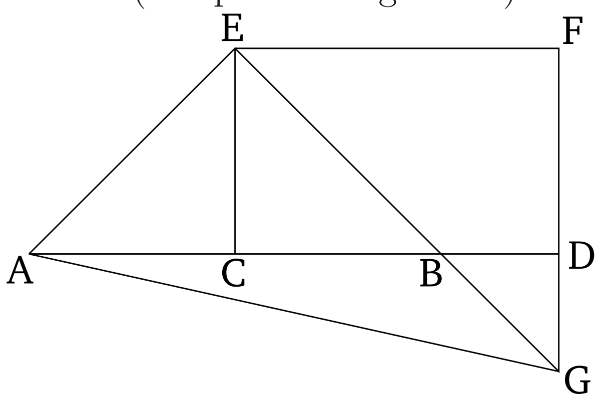
Book 2
Proposition 10

If a straight-line is cut in half, and any straight-line added to it straight-on, then the sum of the square on the whole (straight-line) with the (straight-line) having been added, and the (square) on the (straight-line) having been added, is double the (sum of the square) on half (the straight-line), and the square described on the sum of half (the straight-line) and (straight-line) having been added, as on one (complete straight-line).



For let any straight-line AB have been cut in half at (point) C, and let any straight-line BD have been added to it straight-on. I say that the (sum of the) squares on AD and DB is double the (sum of the) squares on AC and CD.

For let CE have been drawn from point C, at right-angles to AB [Prop. 1.11], and let it be made equal to each of AC and CB [Prop. 1.3], and let EA and EB have been joined. And let EF have been drawn through E, parallel to AD [Prop. 1.31], and let FD have been drawn through D, parallel to CE [Prop. 1.31].

And since some straight-line EF falls across the parallel straight-lines EC and FD, the (internal angles) CEF and EFD are thus equal to two right-angles [Prop. 1.29]. Thus, FEB and EFD are less than two right-angles. And (straight-lines) produced from (internal angles whose sum is) less than two right-angles meet together [Post. 5]. Thus, being produced in the direction of B and D, the (straight-lines) EB and FD will meet. Let them have been produced, and let them meet together at G, and let AG have been joined. And since AC is equal to CE, angle EAC is also equal to (angle) AEC [Prop. 1.5]. And the (angle) at C (is) a right-angle. Thus, EAC and AEC [are] each half a right-angle [Prop. 1.32]. So, for the same (reasons), CEB and EBC are also each half a right-angle. Thus, (angle) AEB is a right-angle. And since EBC is half a right-angle, DBG (is) thus also half a right-angle [Prop. 1.15]. And BDG is also a right-angle. For it is equal to DCE. For (they are) alternate (angles) [Prop. 1.29]. Thus, the remaining (angle) DGB is half a right-angle. Thus, DGB is equal to DBG. So side BD is also equal to side GD [Prop. 1.6]. Again, since EGF is half a right-angle, and the (angle) at F (is) a right-angle, for it is equal to the opposite (angle) at C [Prop. 1.34], the remaining (angle) FEG is thus half a right-angle. Thus, angle EGF (is) equal to FEG. So the side GF is also equal to the side EF [Prop. 1.6]. And since [EC is equal to CA] the square on EC is [also] equal to the square on CA. Thus, the (sum of the) squares on EC and CA is double the square on CA. And the (square) on EA is equal to the (sum of the squares) on EC and

CA [Prop. 1.47]. Thus, the square on EA is double the square on AC. Again, since FG is equal to EF, the (square) on FG is also equal to the (square) on FE. Thus, the (sum of the squares) on GF and FE is double the (square) on EF. And the (square) on EG is equal to the (sum of the squares) on GF and FE [Prop. 1.47]. Thus, the (square) on EG is double the (square) on EF. And EF (is) equal to CD [Prop. 1.34]. Thus, the square on EG is double the (square) on CD. But it was also shown that the (square) on EA (is) double the (square) on AC. Thus, the (sum of the) squares on AE and EG is double the (sum of the) squares on AC and CD. And the square on AG is equal to the (sum of the) squares on AE and EG [Prop. 1.47]. Thus, the (square) on AG is double the (sum of the squares) on AC and CD. And the (sum of the squares) on AD and DG is equal to the (square) on AG [Prop. 1.47]. Thus, the (sum of the) [squares] on AD and DG is double the (sum of the) [squares] on AC and CD. And DG (is) equal to DB. Thus, the (sum of the) [squares] on AD and DB is double the (sum of the) squares on AC and CD.

Thus, if a straight-line is cut in half, and any straight-line added to it straight-on, then the sum of the square on the whole (straight-line) with the (straight-line) having been added, and the (square) on the (straight-line) having been added, is double the (sum of the square) on half (the straight-line), and the square described on the sum of half (the straight-line) and (straight-line) having been added, as on one (complete straight-line). (Which is) the very thing it was required to show.