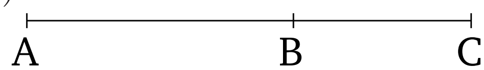
# Book 10
# Proposition 36

If two rational (straight-lines which are) commensurable in square only are added together then the whole (straight-line) is irrational—let it be called a binomial (straight-line).



For let the two rational (straight-lines), $AB$ and $BC$, (which are) commensurable in square only, be laid down together. I say that the whole (straight-line), $AC$, is irrational. For since $AB$ is incommensurable in length with $BC$—for they are commensurable in square only—and as $AB$ (is) to $BC$, so the (rectangle contained) by $ABC$ (is) to the (square) on $BC$, the (rectangle contained) by $AB$ and $BC$ is thus incommensurable with the (square) on $BC$ [Prop. 10.11]. But, twice the (rectangle contained) by $AB$ and $BC$ is commensurable with the (rectangle contained) by $AB$ and $BC$ [Prop. 10.6]. And (the sum of) the (squares) on $AB$ and $BC$ is commensurable with the (square) on $BC$—for the rational (straight-lines) $AB$ and $BC$ are commensurable in square only [Prop. 10.15]. Thus, twice the (rectangle contained) by $AB$ and $BC$ is incommensurable with (the sum of) the (squares) on $AB$ and $BC$ [Prop. 10.13]. And, via composition, twice the (rectangle contained) by $AB$ and $BC$, plus (the sum of) the (squares) on $AB$ and $BC$—that is to say, the (square) on $AC$ [Prop. 2.4]—is incommensurable with the sum of the (squares) on $AB$ and $BC$ [Prop. 10.16]. And the sum of the (squares) on $AB$ and

$BC$ (is) rational. Thus, the (square) on $AC$ [is] irrational [Def. 10.4]. Hence, $AC$ is also irrational [Def. 10.4]—let it be called a binomial (straight-line). (Which is) the very thing it was required to show.