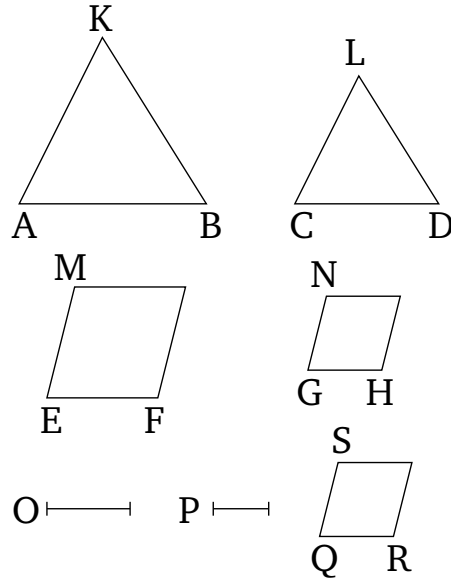# Book 6
Proposition 22

If four straight-lines are proportional then similar, and similarly described, rectilinear figures (drawn) on them will also be proportional. And if similar, and similarly described, rectilinear figures (drawn) on them are proportional then the straight-lines themselves will also be proportional.

Let AB, CD, EF, and GH be four proportional straight-lines, (such that) as AB (is) to CD, so EF (is) to GH. And let the similar, and similarly laid out, rectilinear figures KAB and LCD have been described on AB and CD (respectively), and the similar, and similarly laid out, rectilinear figures MF and NH on EF and GH (respectively). I say that as KAB is to LCD, so MF (is) to NH.

For let a third (straight-line) O have been taken (which is) proportional to AB and CD, and a third (straight-

line) P proportional to EF and GH [Prop. 6.11]. And since as AB is to CD, so EF (is) to GH, and as CD (is) to O, so GH (is) to P, thus, via equality, as AB is to O, so EF (is) to P [Prop. 5.22]. But, as AB (is) to O, so [also] KAB (is) to LCD, and as EF (is) to P, so MF (is) to NH [Prop. 5.19 corr.]. And, thus, as KAB (is) to LCD, so MF (is) to NH.

And so let KAB be to LCD, as MF (is) to NH. I say also that as AB is to CD, so EF (is) to GH. For if as AB is to CD, so EF (is) not to GH, let AB be to CD, as EF (is) to QR [Prop. 6.12]. And let the rectilinear figure SR, similar, and similarly laid down, to either of MF or NH, have been described on QR [Props. 6.18, 6.21].

Therefore, since as AB is to CD, so EF (is) to QR, and the similar, and similarly laid out, (rectilinear figures) KAB and LCD have been described on AB and CD (respectively), and the similar, and similarly laid out, (rectilinear figures) MF and SR on EF and QR (resespectively), thus as KAB is to LCD, so MF (is) to SR (see above). And it was also assumed that as KAB (is) to LCD, so MF (is) to NH. Thus, also, as MF (is) to SR, so MF (is) to NH [Prop. 5.11]. Thus, MF has the same ratio to each of NH and SR. Thus, NH is equal to SR [Prop. 5.9]. And it is also similar, and similarly laid out, to it. Thus, GH (is) equal to QR.[†] And since AB is to CD, as EF (is) to QR, and QR (is) equal to GH, thus as AB is to CD, so EF (is) to GH.

Thus, if four straight-lines are proportional, then similar, and similarly described, rectilinear figures (drawn) on them will also be proportional. And if similar, and

similarly described, rectilinear figures (drawn) on them are proportional then the straight-lines themselves will also be proportional. (Which is) the very thing it was required to show.