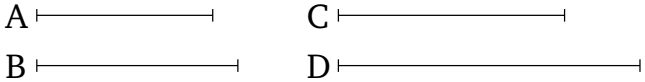
Book 9
Proposition 18

For two given numbers, to investigate whether it is possible to find a third (number) proportional to them.



Let A and B be the two given numbers. And let it be required to investigate whether it is possible to find a third (number) proportional to them.

So A and B are either prime to one another, or not. And if they are prime to one another then it has (already) been show that it is impossible to find a third (number) proportional to them [Prop. 9.16].

And so let A and B not be prime to one another. And let B make C (by) multiplying itself. So A either measures, or does not measure, C. Let it first of all measure (C) according to D. Thus, A has made C (by) multiplying D. But, in fact, B has also made C (by) multiplying itself. Thus, the (number created) from (multiplying) A, D is equal to the (square) on B. Thus, as A is to B, (so) B (is) to D [Prop. 7.19]. Thus, a third number has been found proportional to A, B, (namely) D.

And so let A not measure C. I say that it is impossible to find a third number proportional to A, B. For, if possible, let it have been found, (and let it be) D. Thus, the (number created) from (multiplying) A, D is equal to the (square) on B [Prop. 7.19]. And the (square) on B is C. Thus, the (number created) from (multiplying) A, D is equal to C. Hence, A has made C (by) multiplying D. Thus, A measures C according to D. But (A) was, in fact, also assumed (to be) not measuring (C). The very

thing (is) absurd. Thus, it is not possible to find a third number proportional to A, B when A does not measure C. (Which is) the very thing it was required to show.