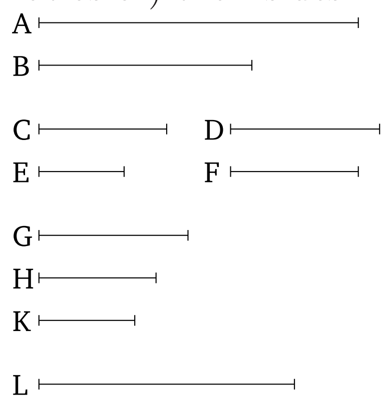
Book 8
Proposition 5

Plane numbers have to one another the ratio compounded† out of (the ratios of) their sides.



Let A and B be plane numbers, and let the numbers C, D be the sides of A, and (the numbers) E, F (the sides) of B. I say that A has to B the ratio compounded out of (the ratios of) their sides.

For given the ratios which C has to E, and D (has) to F, let the least numbers, G, H, K, continuously proportional in the ratios C E, D F have been taken [Prop. 8.4], so that as C is to E, so G (is) to H, and as D (is) to F, so H (is) to K. And let D make L (by) multiplying E.

And since D has made A (by) multiplying C, and has made L (by) multiplying E, thus as C is to E, so A (is) to L [Prop. 7.17]. And as C (is) to E, so G (is) to H. And thus as G (is) to H, so A (is) to L. Again, since E has made L (by) multiplying D [Prop. 7.16], but, in fact, has also made B (by) multiplying F, thus as D is to F, so L (is) to B [Prop. 7.17]. But, as D (is) to F, so H (is) to K. And thus as H (is) to K, so L (is) to

B. And it was also shown that as G (is) to H, so A (is) to L. Thus, via equality, as G is to K, [so] A (is) to B [Prop. 7.14]. And G has to K the ratio compounded out of (the ratios of) the sides (of A and B). Thus, A also has to B the ratio compounded out of (the ratios of) the sides (of A and B). (Which is) the very thing it was required to show.