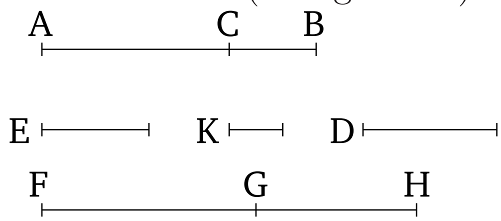
Book 10
Proposition 50

To find a third binomial (straight-line).



Let the two numbers AC and CB be laid down such that their sum AB has to BC the ratio which (some) square number (has) to (some) square number, and does not have to AC the ratio which (some) square number (has) to (some) square number. And let some other non-square number D also be laid down, and let it not have to each of BA and AC the ratio which (some) square number (has) to (some) square number. And let some rational straight-line E be laid down, and let it have been contrived that as D (is) to AB, so the (square) on E (is) to the (square) on FG [Prop. 10.6 corr.]. Thus, the (square) on E is commensurable with the (square) on FG [Prop. 10.6]. And E is a rational (straight-line). Thus, FG is also a rational (straight-line). And since D does not have to AB the ratio which (some) square number has to (some) square number, the (square) on E does not have to the (square) on FG the ratio which (some) square number (has) to (some) square number either. E is thus incommensurable in length with FG [Prop. 10.9]. So, again, let it have been contrived that as the number BA (is) to AC, so the (square) on FG (is) to the (square) on GH [Prop. 10.6 corr.]. Thus, the (square) on FG is commensurable with the (square) on GH [Prop. 10.6]. And FG (is) a rational (straight-line). Thus, GH (is) also a rational (straight-line). And since BA does not have to AC the ratio which (some) square number (has) to (some) square number, the (square) on FG does not have to the (square) on HG the ratio which (some) square number (has) to (some) square number either. Thus, FG is incommensurable in length with GH [Prop. 10.9]. FG and GH are thus rational (straight-lines which are) commensurable in square only. Thus, FH is a binomial (straight-line) [Prop. 10.36]. So, I say that (it is) also a third (binomial straight-line).

For since as D is to AB, so the (square) on E (is) to the (square) on FG, and as BA (is) to AC, so the (square) on FG (is) to the (square) on GH, thus, via equality, as D (is) to AC, so the (square) on E (is) to the (square) on GH [Prop. 5.22]. And D does not have to AC the ratio which (some) square number (has) to (some) square number. Thus, the (square) on E does not have to the (square) on GH the ratio which (some) square number (has) to (some) square number either. Thus, E is incommensurable in length with GH [Prop. 10.9]. And since as BA is to AC, so the (square) on FG (is) to the (square) on GH, the (square) on FG (is) thus greater than the (square) on GH [Prop. 5.14]. Therefore, let (the sum of) the (squares) on GH and K be equal to the (square) on FG. Thus, via conversion, as AB [is] to BC, so the (square) on FG (is) to the (square) on K [Prop. 5.19 corr.]. And AB has to BC the ratio which (some) square number (has) to (some) square number. Thus, the (square) on FG also has to the (square) on K the ratio which (some) square number (has) to (some) square number. Thus, FG [is] commensurable in length with K [Prop. 10.9]. Thus, the square on FG is greater than (the square on) GH by the (square) on (some straight-line) commensurable (in length) with (FG). And FG and GH are rational (straight-lines which are) commensurable in square only, and neither of them is commensurable in length with E.

Thus, FH is a third binomial (straight-line) [Def. 10.7]. (Which is) the very thing it was required to show.