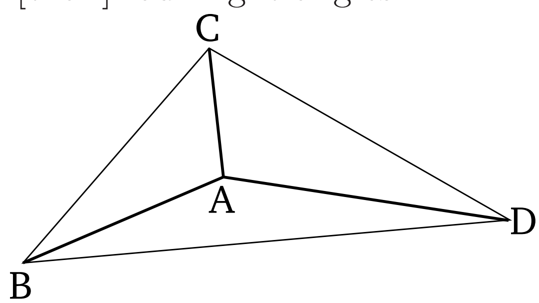
Book 11
Proposition 21

Any solid angle is contained by plane angles (whose sum is) less [than] four right-angles.†



Let the solid angle A be contained by the plane angles BAC, CAD, and DAB. I say that (the sum of) BAC, CAD, and DAB is less than four right-angles.

For let the random points B, C, and D have been taken on each of (the straight-lines) AB, AC, and AD (respectively). And let BC, CD, and DB have been joined. And since the solid angle at B is contained by the three plane angles CBA, ABD, and CBD, (the sum of) any two is greater than the remaining (one) [Prop. 11.20]. Thus, (the sum of) CBA and ABD is greater than CBD. So, for the same (reasons), (the sum of) BCA and ACD is also greater than BCD, and (the sum of) CDA and ADB is greater than CDB. Thus, the (sum of the) six angles CBA, ABD, BCA, ACD, CDA, and ADB is greater than the (sum of the) three (angles) CBD, BCD, and CDB. But, the (sum of the) three (angles) CBD, BDC, and BCD is equal to two right-angles [Prop. 1.32]. Thus, the (sum of the) six angles CBA, ABD, BCA, ACD, CDA, and ADB is greater than two right-angles. And since the (sum of the) three angles of each of the triangles ABC, ACD, and ADB is equal to two right-angles, the (sum of the) nine angles CBA, ACB, BAC, ACD, CDA, CAD, ADB, DBA, and BAD of the three triangles is equal to six right-angles, of which the (sum of the) six angles ABC, BCA, ACD, CDA, ADB, and DBA is greater than two right-angles. Thus, the (sum of the) remaining three [angles] BAC, CAD, and DAB, containing the solid angle, is less than four right-angles.

Thus, any solid angle is contained by plane angles (whose sum is) less [than] four right-angles. (Which is) the very thing it was required to show.