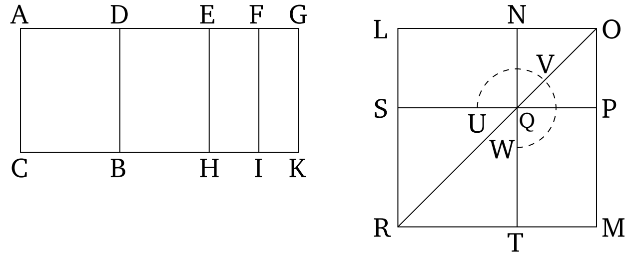
Book 10
Proposition 96

If an area is contained by a rational (straight-line) and a sixth apotome then the square-root of the area is that (straight-line) which with a medial (area) makes a medial whole.



For let the area AB have been contained by the rational (straight-line) AC and the sixth apotome AD. I say that the square-root of area AB is that (straight-line) which with a medial (area) makes a medial whole.

For let DG be an attachment to AD. Thus, AG and GD are rational (straight-lines which are) commensurable in square only [Prop. 10.73], and neither of them is commensurable in length with the (previously) laid down rational (straight-line) AC, and the square on the whole, AG, is greater than (the square on) the attachment, DG, by the (square) on (some straight-line) incommensurable in length with (AG) [Def. 10.16]. Therefore, since the square on AG is greater than (the square on) GD by the (square) on (some straight-line) incommensurable in length with (AG), thus if (some area), equal to the fourth part of square on DG, is applied to AG, falling short by a square figure, then it divides (AG) into (parts which are) incommensurable (in length) [Prop. 10.18]. Therefore, let DG have been cut in half at [point] E. And let (some area), equal to the (square) on EG, have been applied to AG, falling short by a square figure. And let it be the (rectangle contained) by AF and FG. AF is thus incommensurable in length with FG. And as AF (is) to FG, so AI is to FK [Prop. 6.1]. Thus, AI is incommensurable with FK [Prop. 10.11]. And since AG and AC are rational (straight-lines which are) commensurable in square only, AK is a medial (area) [Prop. 10.21]. Again, since AC and DG are rational (straight-lines which are) incommensurable in length, DK is also a medial (area) [Prop. 10.21]. Therefore, since AG and GD are commensurable in square only, AG is thus incommensurable in length with GD. And as AG (is) to GD, so AK is to KD [Prop. 6.1]. Thus, AK is incommensurable with KD [Prop. 10.11].

Therefore, let the square LM, equal to AI, have been constructed. And let NO, equal to FK, (and) about the same angle, have been subtracted (from LM). Thus, the squares LM and NO are about the same diagonal [Prop. 6.26]. Let PR be their (common) diagonal, and let (the rest of) the figure have been drawn. So, similarly to the above, we can show that LN is the square-root of area AB. I say that LN is that (straight-line) which with a medial (area) makes a medial whole.

For since AK was shown (to be) a medial (area), and is equal to the (sum of the) squares on LP and PN, the sum of the (squares) on LP and PN is medial. Again, since DK was shown (to be) a medial (area), and is equal to twice the (rectangle contained) by LP and PN, twice the (rectangle contained) by LP and PN is also medial. And since AK was shown (to be) incommensurable with DK, [thus] the (sum of the) squares on LP and PN is also incommensurable with twice the (rectangle contained) by LP and PN. And since AI is incommensurable with FK, the (square) on LP (is) thus also incommensurable with the (square) on PN. Thus, LP and PN are (straight-lines which are) incommensurable in square, making the sum of the squares on them medial, and twice the (rectangle contained) by medial, and, furthermore, the (sum of the) squares on them incommensurable with twice the (rectangle contained) by them. Thus, LN is the irrational (straight-line) called that which with a medial (area) makes a medial whole [Prop. 10.78]. And it is the square-root of area AB.

Thus, the square-root of area (AB) is that (straight-line) which with a medial (area) makes a medial whole. (Which is) the very thing it was required to show.