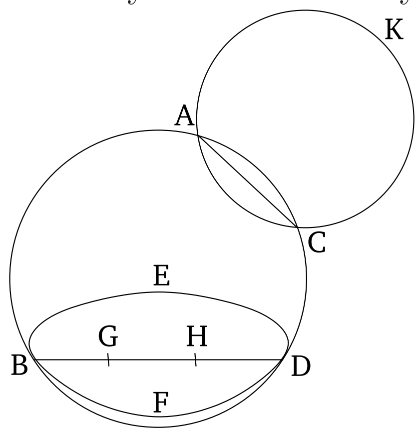
Book 3
Proposition 13

A circle does not touch a(nother) circle at more than one point, whether they touch internally or externally.



For, if possible, let circle $ABDC$ touch circle $EBFD$—first of all, internally—at more than one point, D and B.

And let the center G of circle $ABDC$ have been found [Prop. 3.1], and (the center) H of $EBFD$ [Prop. 3.1].

Thus, the (straight-line) joining G and H will fall on B and D [Prop. 3.11]. Let it fall like $BGHD$ (in the figure). And since point G is the center of circle $ABDC$, BG is equal to GD. Thus, BG (is) greater than HD. Thus, BH (is) much greater than HD. Again, since point H is the center of circle $EBFD$, BH is equal to HD. But it was also shown (to be) much greater than it. The very thing (is) impossible. Thus, a circle does not touch a(nother) circle internally at more than one point.

So, I say that neither (does it touch) externally (at more than one point).

For, if possible, let circle ACK touch circle $ABDC$

externally at more than one point, A and C. And let AC have been joined.

Therefore, since two points, A and C, have been taken at random on the circumference of each of the circles $ABDC$ and ACK, the straight-line joining the points will fall inside each (circle) [Prop. 3.2]. But, it fell inside $ABDC$, and outside ACK [Def. 3.3]. The very thing (is) absurd. Thus, a circle does not touch a(nother) circle externally at more than one point. And it was shown that neither (does it) internally.

Thus, a circle does not touch a(nother) circle at more than one point, whether they touch internally or externally. (Which is) the very thing it was required to shown.