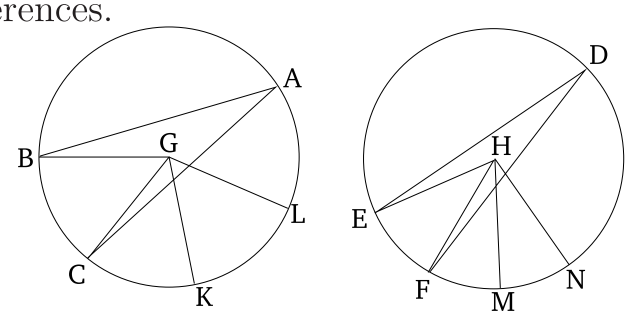
# Book 6
# Proposition 33

In equal circles, angles have the same ratio as the (ratio of the) circumferences on which they stand, whether they are standing at the centers (of the circles) or at the circumferences.




Let $ABC$ and $DEF$ be equal circles, and let $BGC$ and $EHF$ be angles at their centers, $G$ and $H$ (respectively), and $BAC$ and $EDF$ (angles) at their circumferences. I say that as circumference $BC$ is to circumference $EF$, so angle $BGC$ (is) to $EHF$, and (angle) $BAC$ to $EDF$.

For let any number whatsoever of consecutive (circumferences), $CK$ and $KL$, be made equal to circumference $BC$, and any number whatsoever, $FM$ and $MN$, to circumference $EF$. And let $GK$, $GL$, $HM$, and $HN$ have been joined.

Therefore, since circumferences $BC$, $CK$, and $KL$ are equal to one another, angles $BGC$, $CGK$, and $KGL$ are also equal to one another [Prop. 3.27]. Thus, as many times as circumference $BL$ is (divisible) by $BC$, so many times is angle $BGL$ also (divisible) by $BGC$. And so, for the same (reasons), as many times as circumference $NE$ is (divisible) by $EF$, so many times is angle $NHE$ also (divisible) by $EHF$. Thus, if circumference $BL$ is equal to circumference $EN$ then angle $BGL$ is also equal to $EHN$ [Prop. 3.27], and if circumference $BL$ is greater than circumference $EN$ then angle $BGL$ is also greater than $EHN$, and if ($BL$ is) less (than $EN$ then $BGL$ is also) less (than $EHN$). So there are four magnitudes, two circumferences $BC$ and $EF$, and two angles $BGC$ and $EHF$. And equal multiples have been taken of circumference $BC$ and angle $BGC$, (namely) circumference $BL$ and angle $BGL$, and of circumference $EF$ and angle $EHF$, (namely) circumference $EN$ and angle $EHN$. And it has been shown that if circumference $BL$ exceeds circumference $EN$ then angle $BGL$ also exceeds angle $EHN$, and if ($BL$ is) equal (to $EN$ then $BGL$ is also) equal (to $EHN$), and if ($BL$ is) less (than $EN$ then $BGL$ is also) less (than $EHN$). Thus, as circumference $BC$ (is) to $EF$, so angle $BGC$ (is) to $EHF$ [Def. 5.5]. But as angle $BGC$ (is) to $EHF$, so (angle) $BAC$ (is) to $EDF$ [Prop. 5.15]. For the former (are) double the latter (respectively) [Prop. 3.20]. Thus, also, as circumference $BC$ (is) to circumference $EF$, so angle $BGC$ (is) to $EHF$, and $BAC$ to $EDF$.

Thus, in equal circles, angles have the same ratio as the (ratio of the) circumferences on which they stand, whether they are standing at the centers (of the circles) or at the circumferences. (Which is) the very thing it was required to show.