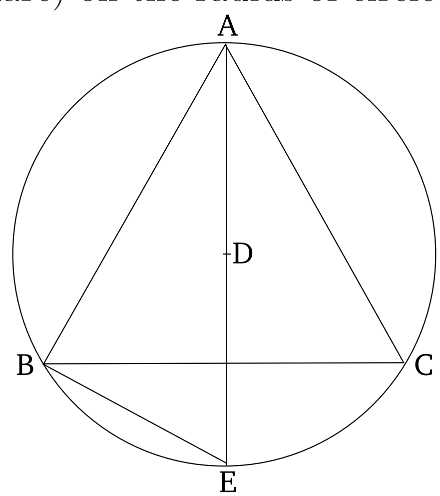
Book 13
Proposition 12

If an equilateral triangle is inscribed in a circle then the square on the side of the triangle is three times the (square) on the radius of the circle.

Let there be a circle ABC, and let the equilateral triangle ABC have been inscribed in it [Prop. 4.2]. I say that the square on one side of triangle ABC is three times the (square) on the radius of circle ABC.



For let the center, D, of circle ABC have been found [Prop. 3.1]. And AD (being) joined, let it have been drawn across to E. And let BE have been joined.

And since triangle ABC is equilateral, circumference BEC is thus the third part of the circumference of circle ABC. Thus, circumference BE is the sixth part of the circumference of the circle. Thus, straight-line BE is (the side) of a hexagon. Thus, it is equal to the radius DE [Prop. 4.15 corr.]. And since AE is double DE, the (square) on AE is four times the (square) on ED—that is to say, of the (square) on BE. And the (square) on AE (is) equal to the (sum of the squares) on AB and BE [Props. 3.31, 1.47]. Thus, the (sum of the squares) on AB and BE is four times the (square) on BE. Thus, via separation, the (square) on AB is three times the (square) on BE. And BE (is) equal to DE. Thus, the (square) on AB is three times the (square) on DE.

Thus, the square on the side of the triangle is three times the (square) on the radius [of the circle]. (Which is) the very thing it was required to show.