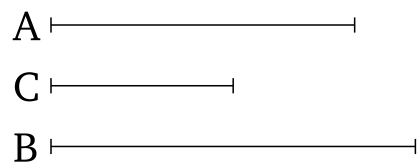
Book 10
Proposition 13

If two magnitudes are commensurable, and one of them is incommensurable with some magnitude, then the remaining (magnitude) will also be incommensurable with it.



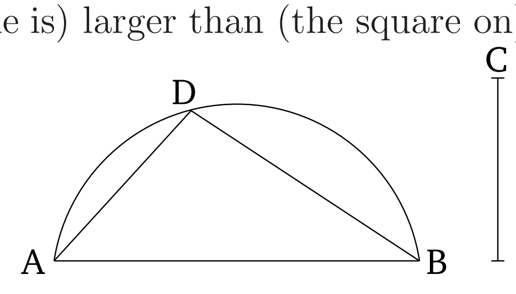
Let A and B be two commensurable magnitudes, and let one of them, A, be incommensurable with some other (magnitude), C. I say that the remaining (magnitude), B, is also incommensurable with C.

For if B is commensurable with C, but A is also commensurable with B, A is thus also commensurable with C [Prop. 10.12]. But, (it is) also incommensurable (with C). The very thing (is) impossible. Thus, B is not commensurable with C. Thus, (it is) incommensurable.

Thus, if two magnitudes are commensurable, and so on ....

Lemma

For two given unequal straight-lines, to find by (the square on) which (straight-line) the square on the greater (straight-line is) larger than (the square on) the lesser.



Let AB and C be the two given unequal straight-lines, and let AB be the greater of them. So it is required to find by (the square on) which (straight-line) the square on AB (is) greater than (the square on) C.

Let the semi-circle ADB have been described on AB. And let AD, equal to C, have been inserted into it [Prop. 4.1]. And let DB have been joined. So (it is) clear that the angle ADB is a right-angle [Prop. 3.31], and that the square on AB (is) greater than (the square on) AD—that is to say, (the square on) C—by (the square on) DB [Prop. 1.47].

And, similarly, the square-root of (the sum of the squares on) two given straight-lines is also found likeso.

Let AD and DB be the two given straight-lines. And let it be necessary to find the square-root of (the sum of the squares on) them. For let them have been laid down such as to encompass a right-angle—(namely), that (angle encompassed) by AD and DB. And let AB have been joined. (It is) again clear that AB is the square-root of (the sum of the squares on) AD and DB [Prop. 1.47]. (Which is) the very thing it was required to show.