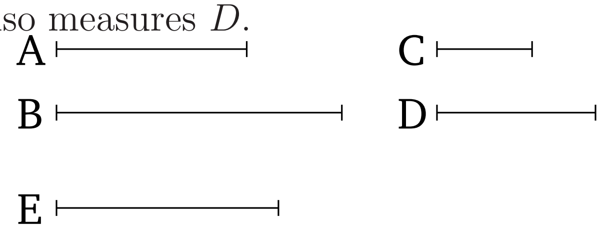
Book 8
Proposition 14

If a square (number) measures a(nother) square (number) then the side (of the former) will also measure the side (of the latter). And if the side (of a square number) measures the side (of another square number) then the (former) square (number) will also measure the (latter) square (number).

Let A and B be square numbers, and let C and D be their sides (respectively). And let A measure B. I say that C also measures D.



For let C make E (by) multiplying D. Thus, A, E, B are continuously proportional in the ratio of C to D [Prop. 8.11]. And since A, E, B are continuously proportional, and A measures B, A thus also measures E [Prop. 8.7]. And as A is to E, so C (is) to D. Thus, C also measures D [Def. 7.20].

So, again, let C measure D. I say that A also measures B.

For similarly, with the same construction, we can show that A, E, B are continuously proportional in the ratio of C to D. And since as C is to D, so A (is) to E, and C measures D, A thus also measures E [Def. 7.20]. And A, E, B are continuously proportional. Thus, A also measures B.

Thus, if a square (number) measures a(nother) square (number) then the side (of the former) will also measure the side (of the latter). And if the side (of a square number) measures the side (of another square number) then the (former) square (number) will also measure the (latter) square (number). (Which is) the very thing it was required to show.