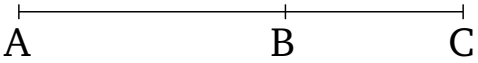
Book 10
Proposition 40

If two straight-lines (which are) incommensurable in square, making the sum of the squares on them medial, and the (rectangle contained) by them rational, are added together then the whole straight-line is irrational—let it be called the square-root of a rational plus a medial (area).



For let the two straight-lines, AB and BC, incommensurable in square, (and) fulfilling the prescribed (conditions), be laid down together [Prop. 10.34]. I say that AC is irrational.

For since the sum of the (squares) on AB and BC is medial, and twice the (rectangle contained) by AB and BC (is) rational, the sum of the (squares) on AB and BC is thus incommensurable with twice the (rectangle contained) by AB and BC. Hence, the (square) on AC is also incommensurable with twice the (rectangle contained) by AB and BC [Prop. 10.16]. And twice the (rectangle contained) by AB and BC (is) rational. The (square) on AC (is) thus irrational. Thus, AC (is) irrational [Def. 10.4]—let it be called the square-root of a rational plus a medial (area). (Which is) the very thing it was required to show.