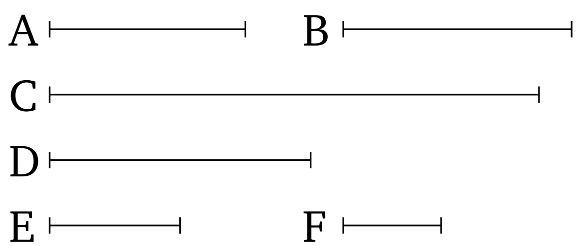
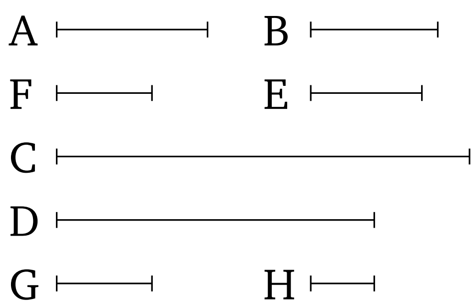
Book 7
Proposition 34

To find the least number which two given numbers (both) measure.

Let A and B be the two given numbers. So it is required to find the least number which they (both) measure.



For A and B are either prime to one another, or not. Let them, first of all, be prime to one another. And let A make C (by) multiplying B. Thus, B has also made C (by) multiplying A [Prop. 7.16]. Thus, A and B (both) measure C. So I say that (C) is also the least (number which they both measure). For if not, A and B will (both) measure some (other) number which is less than C. Let them (both) measure D (which is less than C). And as many times as A measures D, so many units let there be in E. And as many times as B measures D, so many units let there be in F. Thus, A has made D (by) multiplying E, and B has made D (by) multiplying F. Thus, the (number created) from (multiplying) A and E is equal to the (number created) from (multiplying) B and F. Thus, as A (is) to B, so F (is) to E [Prop. 7.19]. And A and B are prime (to one another), and prime (numbers) are the least (of those numbers having the same ratio) [Prop. 7.21], and the least (numbers) measure those (numbers) having the same ratio (as them) an equal number of times, the greater (measuring) the greater, and the lesser the lesser [Prop. 7.20]. Thus, B measures E, as the following (number measuring) the following. And since A has made C and D (by) multiplying B and E (respectively), thus as B is to E, so C (is) to D [Prop. 7.17]. And B measures E. Thus, C also measures D, the greater (measuring) the lesser. The very thing is impossible. Thus, A and B do not (both) measure some number which is less than C. Thus, C is the least (number) which is measured by (both) A and B.



So let A and B be not prime to one another. And let the least numbers, F and E, have been taken having the same ratio as A and B (respectively) [Prop. 7.33]. Thus, the (number created) from (multiplying) A and E is equal to the (number created) from (multiplying) B and F [Prop. 7.19]. And let A make C (by) multiplying E. Thus, B has also made C (by) multiplying F. Thus, A and B (both) measure C. So I say that (C) is also the least (number which they both measure). For if not, A and B will (both) measure some number which is less than C. Let them (both) measure D (which is less than C). And as many times as A measures D, so many units let there be in G. And as many times as B measures D, so many units let there be in H. Thus, A has made D

(by) multiplying G, and B has made D (by) multiplying H. Thus, the (number created) from (multiplying) A and G is equal to the (number created) from (multiplying) B and H. Thus, as A is to B, so H (is) to G [Prop. 7.19]. And as A (is) to B, so F (is) to E. Thus, also, as F (is) to E, so H (is) to G. And F and E are the least (numbers having the same ratio as A and B), and the least (numbers) measure those (numbers) having the same ratio an equal number of times, the greater (measuring) the greater, and the lesser the lesser [Prop. 7.20]. Thus, E measures G. And since A has made C and D (by) multiplying E and G (respectively), thus as E is to G, so C (is) to D [Prop. 7.17]. And E measures G. Thus, C also measures D, the greater (measuring) the lesser. The very thing is impossible. Thus, A and B do not (both) measure some (number) which is less than C. Thus, C (is) the least (number) which is measured by (both) A and B. (Which is) the very thing it was required to show.