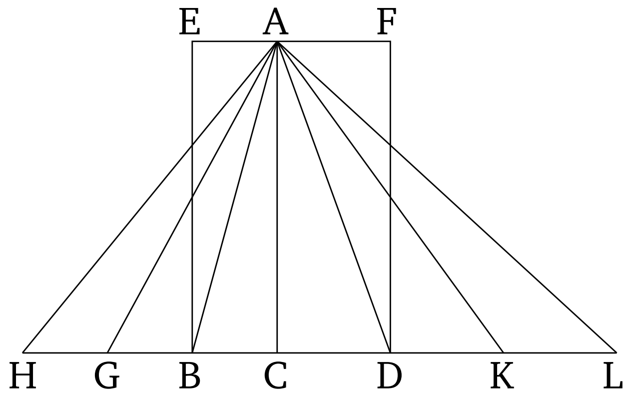
Book 6
Proposition 1

Triangles and parallelograms which are of the same height are to one another as their bases.



Let ABC and ACD be triangles, and EC and CF parallelograms, of the same height AC. I say that as base BC is to base CD, so triangle ABC (is) to triangle ACD, and parallelogram EC to parallelogram CF.

For let the (straight-line) BD have been produced in each direction to points H and L, and let [any number] (of straight-lines) BG and GH be made equal to base BC, and any number (of straight-lines) DK and KL equal to base CD. And let AG, AH, AK, and AL have been joined.

And since CB, BG, and GH are equal to one another, triangles AHG, AGB, and ABC are also equal to one another [Prop. 1.38]. Thus, as many times as base HC is (divisible by) base BC, so many times is triangle AHC also (divisible) by triangle ABC. So, for the same (reasons), as many times as base LC is (divisible) by base CD, so many times is triangle ALC also (divisible) by triangle ACD. And if base HC is equal to base

CL then triangle AHC is also equal to triangle ACL [Prop. 1.38]. And if base HC exceeds base CL then triangle AHC also exceeds triangle ACL.‡ And if (HC is) less (than CL then AHC is also) less (than ACL). So, their being four magnitudes, two bases, BC and CD, and two triangles, ABC and ACD, equal multiples have been taken of base BC and triangle ABC—(namely), base HC and triangle AHC—and other random equal multiples of base CD and triangle ADC—(namely), base LC and triangle ALC. And it has been shown that if base HC exceeds base CL then triangle AHC also exceeds triangle ALC, and if (HC is) equal (to CL then AHC is also) equal (to ALC), and if (HC is) less (than CL then AHC is also) less (than ALC). Thus, as base BC is to base CD, so triangle ABC (is) to triangle ACD [Def. 5.5]. And since parallelogram EC is double triangle ABC, and parallelogram FC is double triangle ACD [Prop. 1.34], and parts have the same ratio as similar multiples [Prop. 5.15], thus as triangle ABC is to triangle ACD, so parallelogram EC (is) to parallelogram FC. In fact, since it was shown that as base BC (is) to CD, so triangle ABC (is) to triangle ACD, and as triangle ABC (is) to triangle ACD, so parallelogram EC (is) to parallelogram CF, thus, also, as base BC (is) to base CD, so parallelogram EC (is) to parallelogram FC [Prop. 5.11].

Thus, triangles and parallelograms which are of the same height are to one another as their bases. (Which is) the very thing it was required to show.