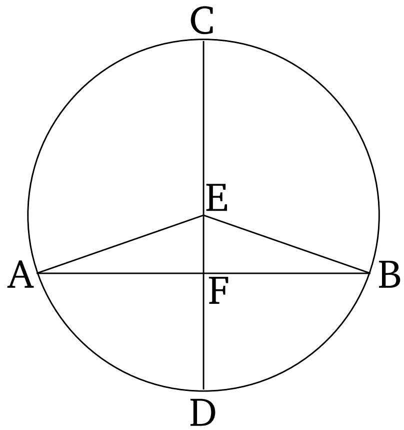
Book 3
Proposition 3

In a circle, if any straight-line through the center cuts in half any straight-line not through the center then it also cuts it at right-angles. And (conversely) if it cuts it at right-angles then it also cuts it in half.

Let ABC be a circle, and, within it, let some straight-line through the center, CD, cut in half some straight-line not through the center, AB, at the point F. I say that (CD) also cuts (AB) at right-angles.

For let the center of the circle ABC have been found [Prop. 3.1], and let it be (at point) E, and let EA and EB have been joined.

And since AF is equal to FB, and FE (is) common, two (sides of triangle AFE) [are] equal to two (sides of triangle BFE). And the base EA (is) equal to the base EB. Thus, angle AFE is equal to angle BFE [Prop. 1.8]. And when a straight-line stood upon (another) straight-line makes adjacent angles (which are) equal to one another, each of the equal angles is a right-angle [Def. 1.10]. Thus, AFE and BFE are each right-angles. Thus, the (straight-line) CD, which is through the center and cuts in half the (straight-line) AB, which is not through the center, also cuts (AB) at right-angles.



And so let CD cut AB at right-angles. I say that it also cuts (AB) in half. That is to say, that AF is equal to FB.

For, with the same construction, since EA is equal to EB, angle EAF is also equal to EBF [Prop. 1.5]. And the right-angle AFE is also equal to the right-angle BFE. Thus, EAF and EFB are two triangles having two angles equal to two angles, and one side equal to one side—(namely), their common (side) EF, subtending one of the equal angles. Thus, they will also have the remaining sides equal to the (corresponding) remaining sides [Prop. 1.26]. Thus, AF (is) equal to FB.

Thus, in a circle, if any straight-line through the center cuts in half any straight-line not through the center then it also cuts it at right-angles. And (conversely) if it cuts it at right-angles then it also cuts it in half. (Which is) the very thing it was required to show.