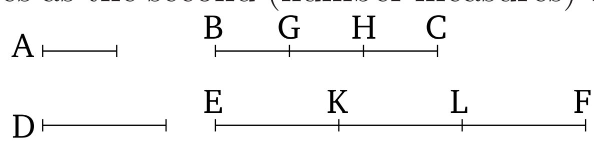
Book 7
Proposition 15

If a unit measures some number, and another number measures some other number as many times, then, also, alternately, the unit will measure the third number as many times as the second (number measures) the fourth.



For let a unit A measure some number BC, and let another number D measure some other number EF as many times. I say that, also, alternately, the unit A also measures the number D as many times as BC (measures) EF.

For since the unit A measures the number BC as many times as D (measures) EF, thus as many units as are in BC, so many numbers are also in EF equal to D. Let BC have been divided into its constituent units, BG, GH, and HC, and EF into the (divisions) EK, KL, and LF, equal to D. So the multitude of (units) BG, GH, HC will be equal to the multitude of (divisions) EK, KL, LF. And since the units BG, GH, and HC are equal to one another, and the numbers EK, KL, and LF are also equal to one another, and the multitude of the (units) BG, GH, HC is equal to the multitude of the numbers EK, KL, LF, thus as the unit BG (is) to the number EK, so the unit GH will be to the number KL, and the unit HC to the number LF. And thus, as one of the leading (numbers is) to one of the following, so (the sum of) all of the leading will be to (the sum of) all of the following [Prop. 7.12]. Thus, as the unit BG (is) to the number EK, so BC (is) to EF. And the unit BG (is) equal to the unit A, and the number EK to the number D. Thus, as the unit A is to the number D, so BC (is) to EF. Thus, the unit A measures the number D as many times as BC (measures) EF [Def. 7.20]. (Which is) the very thing it was required to show.