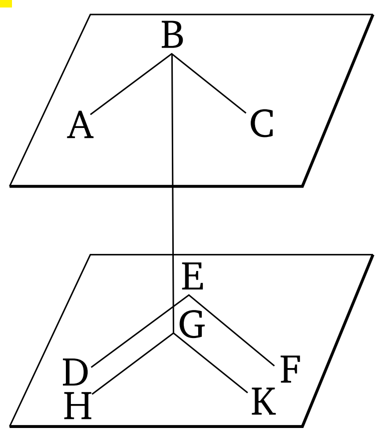
Book 11
Proposition 15

If two straight-lines joined to one another are parallel (respectively) to two straight-lines joined to one another, which are not in the same plane, then the planes through them are parallel (to one another).

For let the two straight-lines joined to one another, AB and BC, be parallel to the two straight-lines joined to one another, DE and EF (respectively), not being in the same plane. I say that the planes through AB, BC and DE, EF will not meet one another (when) produced.

For let BG have been drawn from point B perpendicular to the plane through DE and EF [Prop. 11.11], and let it meet the plane at point G. And let GH have been drawn through G parallel to ED, and GK (parallel) to EF [Prop. 1.31].



And since BG is at right-angles to the plane through DE and EF, it will thus also make right-angles with all of the straight-lines joined to it, which are also in the plane through DE and EF [Def. 11.3]. And each of GH and GK, which are in the plane through DE and

EF, are joined to it. Thus, each of the angles BGH and BGK are right-angles. And since BA is parallel to GH [Prop. 11.9], the (sum of the) angles GBA and BGH is equal to two right-angles [Prop. 1.29]. And BGH (is) a right-angle. GBA (is) thus also a right-angle. Thus, GB is at right-angles to BA. So, for the same (reasons), GB is also at right-angles to BC. Therefore, since the straight-line GB has been set up at right-angles to two straight-lines, BA and BC, cutting one another, GB is thus at right-angles to the plane through BA and BC [Prop. 11.4]. [So, for the same (reasons), BG is also at right-angles to the plane through GH and GK. And the plane through GH and GK is the (plane) through DE and EF. And it was also shown that GB is at right-angles to the plane through AB and BC.] And planes to which the same straight-line is at right-angles are parallel planes [Prop. 11.14]. Thus, the plane through AB and BC is parallel to the (plane) through DE and EF.

Thus, if two straight-lines joined to one another are parallel (respectively) to two straight-lines joined to one another, which are not in the same plane, then the planes through them are parallel (to one another). (Which is) the very thing it was required to show.