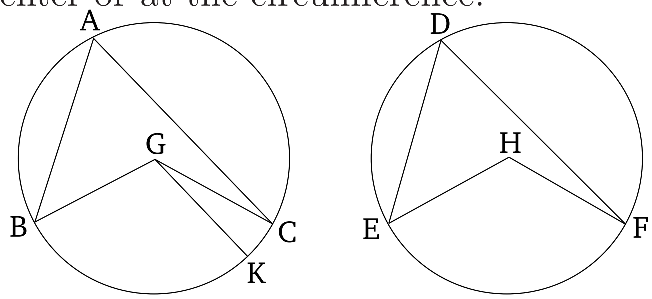
Book 3
Proposition 27

In equal circles, angles standing upon equal circumferences are equal to one another, whether they are standing at the center or at the circumference.



For let the angles BGC and EHF at the centers G and H, and the (angles) BAC and EDF at the circumferences, stand upon the equal circumferences BC and EF, in the equal circles ABC and DEF (respectively). I say that angle BGC is equal to (angle) EHF, and BAC is equal to EDF.

For if BGC is unequal to EHF, one of them is greater. Let BGC be greater, and let the (angle) BGK, equal to angle EHF, have been constructed on the straight-line BG, at the point G on it [Prop. 1.23]. But equal angles (in equal circles) stand upon equal circumferences, when they are at the centers [Prop. 3.26]. Thus, circumference BK (is) equal to circumference EF. But, EF is equal to BC. Thus, BK is also equal to BC, the lesser to the greater. The very thing is impossible. Thus, angle BGC is not unequal to EHF. Thus, (it is) equal. And the (angle) at A is half BGC, and the (angle) at D half EHF [Prop. 3.20]. Thus, the angle at A (is) also equal to the (angle) at D.

Thus, in equal circles, angles standing upon equal circumferences are equal to one another, whether they are standing at the center or at the circumference. (Which is) the very thing it was required to show.