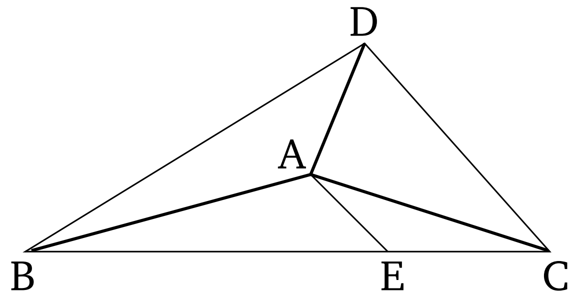
Book 11
Proposition 20

If a solid angle is contained by three plane angles then (the sum of) any two (angles) is greater than the remaining (one), (the angles) being taken up in any (possible way).



For let the solid angle A have been contained by the three plane angles BAC, CAD, and DAB. I say that (the sum of) any two of the angles BAC, CAD, and DAB is greater than the remaining (one), (the angles) being taken up in any (possible way).

For if the angles BAC, CAD, and DAB are equal to one another then (it is) clear that (the sum of) any two is greater than the remaining (one). But, if not, let BAC be greater (than CAD or DAB). And let (angle) BAE, equal to the angle DAB, have been constructed in the plane through BAC, on the straight-line AB, at the point A on it. And let AE be made equal to AD. And BEC being drawn across through point E, let it cut the straight-lines AB and AC at points B and C (respectively). And let DB and DC have been joined.

And since DA is equal to AE, and AB (is) common, the two (straight-lines AD and AB are) equal to the two (straight-lines EA and AB, respectively). And angle DAB (is) equal to angle BAE. Thus, the base DB is equal to the base BE [Prop. 1.4]. And since the (sum of the) two (straight-lines) BD and DC is greater than BC [Prop. 1.20], of which DB was shown (to be) equal to BE, the remainder DC is thus greater than the remainder EC. And since DA is equal to AE, but AC (is) common, and the base DC is greater than the base EC, the angle DAC is thus greater than the angle EAC [Prop. 1.25]. And DAB was also shown (to be) equal to BAE. Thus, (the sum of) DAB and DAC is greater than BAC. So, similarly, we can also show that the remaining (angles), being taken in pairs, are greater than the remaining (one).

Thus, if a solid angle is contained by three plane angles then (the sum of) any two (angles) is greater than the remaining (one), (the angles) being taken up in any (possible way). (Which is) the very thing it was required to show.