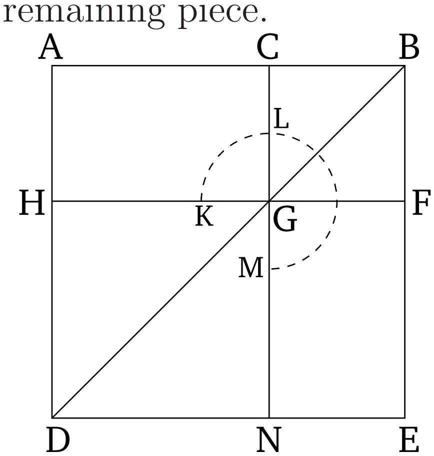
Book 2
Proposition 7

If a straight-line is cut at random then the sum of the squares on the whole (straight-line), and one of the pieces (of the straight-line), is equal to twice the rectangle contained by the whole, and the said piece, and the square on the remaining piece.



For let any straight-line AB have been cut, at random, at point C. I say that the (sum of the) squares on AB and BC is equal to twice the rectangle contained by AB and BC, and the square on CA.

For let the square $ADEB$ have been described on AB [Prop. 1.46], and let the (rest of) the figure have been drawn.

Therefore, since (rectangle) AG is equal to (rectangle) GE [Prop. 1.43], let the (square) CF have been added to both. Thus, the whole (rectangle) AF is equal to the whole (rectangle) CE. Thus, (rectangle) AF plus (rectangle) CE is double (rectangle) AF. But, (rectangle) AF plus (rectangle) CE is the gnomon KLM, and the square CF. Thus, the gnomon KLM, and the square CF, is double the (rectangle) AF. But double the (rectangle) AF is also twice the (rectangle contained) by AB and BC. For BF (is) equal to BC. Thus, the gnomon KLM, and the square CF, are equal to twice the (rectangle contained) by AB and BC. Let DG, which is the square on AC, have been added to both. Thus, the gnomon KLM, and the squares BG and GD, are equal to twice the rectangle contained by AB and BC, and the square on AC. But, the gnomon KLM and the squares BG and GD is (equivalent to) the whole of $ADEB$ and CF, which are the squares on AB and BC (respectively). Thus, the (sum of the) squares on AB and BC is equal to twice the rectangle contained by AB and BC, and the square on AC.

Thus, if a straight-line is cut at random then the sum of the squares on the whole (straight-line), and one of the pieces (of the straight-line), is equal to twice the rectangle contained by the whole, and the said piece, and the square on the remaining piece. (Which is) the very thing it was required to show.