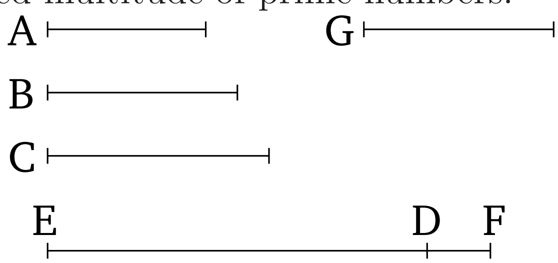
Book 9
Proposition 20

The (set of all) prime numbers is more numerous than any assigned multitude of prime numbers.



Let A, B, C be the assigned prime numbers. I say that the (set of all) primes numbers is more numerous than A, B, C.

For let the least number measured by A, B, C have been taken, and let it be DE [Prop. 7.36]. And let the unit DF have been added to DE. So EF is either prime, or not. Let it, first of all, be prime. Thus, the (set of) prime numbers A, B, C, EF, (which is) more numerous than A, B, C, has been found.

And so let EF not be prime. Thus, it is measured by some prime number [Prop. 7.31]. Let it be measured by the prime (number) G. I say that G is not the same as any of A, B, C. For, if possible, let it be (the same). And A, B, C (all) measure DE. Thus, G will also measure DE. And it also measures EF. (So) G will also measure the remainder, unit DF, (despite) being a number [Prop. 7.28]. The very thing (is) absurd. Thus, G is not the same as one of A, B, C. And it was assumed (to be) prime. Thus, the (set of) prime numbers A, B, C, G, (which is) more numerous than the assigned multitude (of prime numbers), A, B, C, has been found. (Which is) the very thing it was required to show.