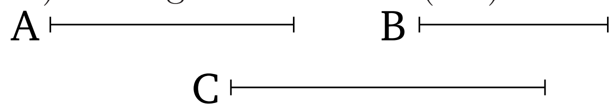
Book 5
Proposition 10

For (magnitudes) having a ratio to the same (magnitude), that (magnitude which) has the greater ratio is (the) greater. And that (magnitude) to which the latter (magnitude) has a greater ratio is (the) lesser.



For let A have a greater ratio to C than B (has) to C. I say that A is greater than B.

For if not, A is surely either equal to or less than B. In fact, A is not equal to B. For (then) A and B would each have the same ratio to C [Prop. 5.7]. But they do not. Thus, A is not equal to B. Neither, indeed, is A less than B. For (then) A would have a lesser ratio to C than B (has) to C [Prop. 5.8]. But it does not. Thus, A is not less than B. And it was shown not (to be) equal either. Thus, A is greater than B.

So, again, let C have a greater ratio to B than C (has) to A. I say that B is less than A.

For if not, (it is) surely either equal or greater. In fact, B is not equal to A. For (then) C would have the same ratio to each of A and B [Prop. 5.7]. But it does not. Thus, A is not equal to B. Neither, indeed, is B greater than A. For (then) C would have a lesser ratio to B than (it has) to A [Prop. 5.8]. But it does not. Thus, B is not greater than A. And it was shown that (it is) not equal (to A) either. Thus, B is less than A.

Thus, for (magnitudes) having a ratio to the same (magnitude), that (magnitude which) has the greater ratio is (the) greater. And that (magnitude) to which the latter (magnitude) has a greater ratio is (the) lesser. (Which is) the very thing it was required to show.