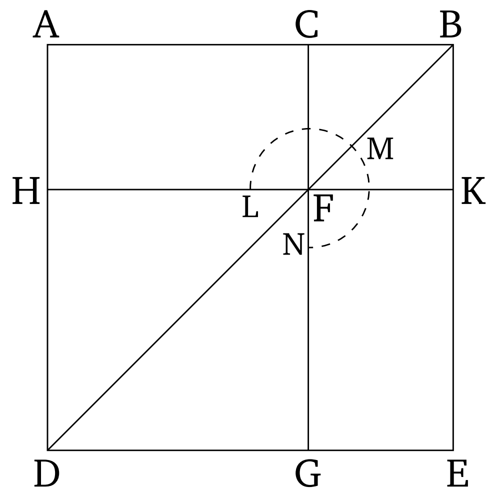
Book 13
Proposition 4

If a straight-line is cut in extreme and mean ratio then the sum of the squares on the whole and the lesser piece is three times the square on the greater piece.



Let AB be a straight-line, and let it have been cut in extreme and mean ratio at C, and let AC be the greater piece. I say that the (sum of the squares) on AB and BC is three times the (square) on CA.

For let the square $ADEB$ have been described on AB, and let the (remainder of the) figure have been drawn. Therefore, since AB has been cut in extreme and mean ratio at C, and AC is the greater piece, the (rectangle contained) by ABC is thus equal to the (square) on AC [Def. 6.3, Prop. 6.17]. And AK is the (rectangle contained) by ABC, and HG the (square) on AC. Thus, AK is equal to HG. And since AF is equal to FE [Prop. 1.43], let CK have been added to both. Thus, the whole of AK is equal to the whole of CE. Thus, AK plus CE is double AK. But, AK plus CE is the gnomon LMN plus the square CK. Thus, gnomon LMN plus square CK is double AK. But, indeed, AK was also shown (to be) equal to HG. Thus, gnomon LMN plus [square CK is double HG. Hence, gnomon LMN plus] the squares CK and HG is three times the square HG. And gnomon LMN plus the squares CK and HG is the whole of AE plus CK—which are the squares on AB and BC (respectively)—and GH (is) the square on AC. Thus, the (sum of the) squares on AB and BC is three times the square on AC. (Which is) the very thing it was required to show.