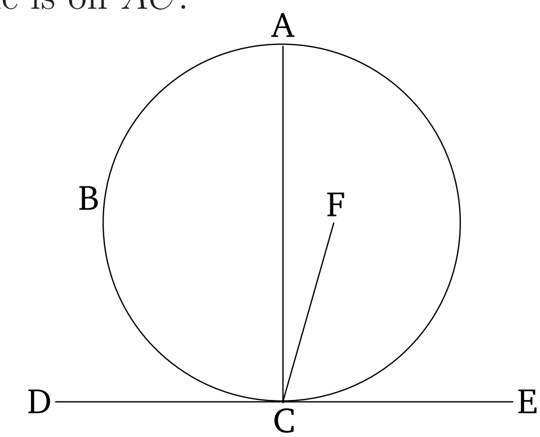
# Book 3
# Proposition 19

If some straight-line touches a circle, and a straight-line is drawn from the point of contact, at right-[angles] to the tangent, then the center (of the circle) will be on the (straight-line) so drawn.

For let some straight-line $DE$ touch the circle $ABC$ at point $C$. And let $CA$ have been drawn from $C$, at right-angles to $DE$ [Prop. 1.11]. I say that the center of the circle is on $AC$.




For (if) not, if possible, let $F$ be (the center of the circle), and let $CF$ have been joined.

[Therefore], since some straight-line $DE$ touches the circle $ABC$, and $FC$ has been joined from the center to the point of contact, $FC$ is thus perpendicular to $DE$ [Prop. 3.18]. Thus, $FCE$ is a right-angle. And $ACE$ is also a right-angle. Thus, $FCE$ is equal to $ACE$, the lesser to the greater. The very thing is impossible. Thus, $F$ is not the center of circle $ABC$. So, similarly, we can show that neither is any (point) other (than one) on $AC$. Thus, if some straight-line touches a circle, and a straight-line is drawn from the point of contact, at right-angles to the tangent, then the center (of the circle) will be on the (straight-line) so drawn. (Which is) the very thing it was required to show.