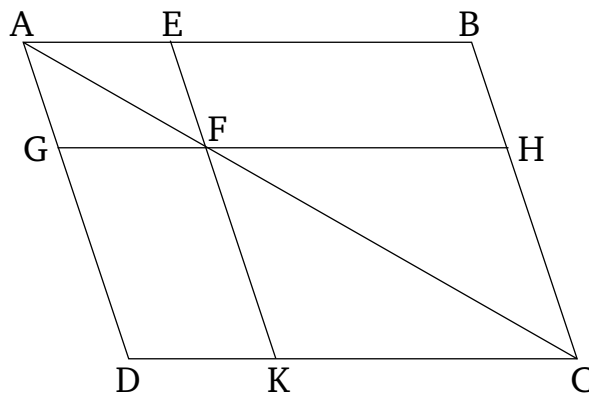# Book 6
Proposition 24

In any parallelogram the parallelograms about the diagonal are similar to the whole, and to one another.

Let $ABCD$ be a parallelogram, and AC its diagonal. And let EG and HK be parallelograms about AC. I say that the parallelograms EG and HK are each similar to the whole (parallelogram) $ABCD$, and to one another.

For since EF has been drawn parallel to one of the sides BC of triangle ABC, proportionally, as BE is to EA, so CF (is) to FA [Prop. 6.2]. Again, since FG has been drawn parallel to one (of the sides) CD of triangle ACD, proportionally, as CF is to FA, so DG (is) to GA [Prop. 6.2]. But, as CF (is) to FA, so it was also shown (is) BE to EA. And thus as BE (is) to EA, so DG (is) to GA. And, thus, compounding, as BA (is) to AE, so DA (is) to AG [Prop. 5.18]. And, alternately, as BA (is) to AD, so EA (is) to AG [Prop. 5.16]. Thus, in parallelograms $ABCD$ and EG the sides about the common angle BAD are proportional. And since GF is parallel to DC, angle AFG is equal to DCA [Prop. 1.29]. And angle DAC (is) common to the two triangles ADC and AGF. Thus, triangle ADC is equiangular to triangle AGF [Prop. 1.32]. So, for the same (reasons), triangle ACB is equiangular to triangle AFE, and the whole parallelogram $ABCD$ is equiangular to parallelogram EG. Thus, proportionally, as AD (is) to DC, so AG (is) to GF, and as DC (is) to CA, so GF (is) to FA, and as AC (is) to CB, so AF (is) to FE, and, further, as CB (is) to BA, so FE (is) to EA [Prop. 6.4]. And since it was

shown that as DC is to CA, so GF (is) to FA, and as AC (is) to CB, so AF (is) to FE, thus, via equality, as DC is to CB, so GF (is) to FE [Prop. 5.22] . Thus, in parallelograms $ABCD$ and EG the sides about the equal angles are proportional. Thus, parallelogram $ABCD$ is similar to parallelogram EG [Def. 6.1] . So, for the same (reasons), parallelogram $ABCD$ is also similar to parallelogram KH. Thus, parallelograms EG and HK are each similar to [parallelogram] $ABCD$. And (rectilinear figures) similar to the same rectilinear figure are also similar to one another [Prop. 6.21] . Thus, parallelogram EG is also similar to parallelogram HK.

Thus, in any parallelogram the parallelograms about the diagonal are similar to the whole, and to one another. (Which is) the very thing it was required to show.