Book 10
Proposition 77

If a straight-line, which is incommensurable in square with the whole, and with the whole makes the sum of the squares on them medial, and twice the (rectangle contained) by them rational, is subtracted from a(nother) straight-line then the remainder is an irrational (straight-line). Let it be called that which makes with a rational (area) a medial whole.

For let the straight-line BC, which is incommensurable in square with AB, and fulfils the (other) prescribed (conditions), have been subtracted from the straight-line AB [Prop. 10.34]. I say that the remainder AC is the aforementioned irrational (straight-line).

For since the sum of the squares on AB and BC is medial, and twice the (rectangle contained) by AB and BC rational, the (sum of the squares) on AB and BC is thus incommensurable with twice the (rectangle contained) by AB and BC. Thus, the remaining (square) on AC is also incommensurable with twice the (rectangle contained) by AB and BC [Props. 2.7, 10.16]. And twice the (rectangle contained) by AB and BC is rational. Thus, the (square) on AC is irrational. Thus, AC is an irrational (straight-line) [Def. 10.4]. And let it be called that which makes with a rational (area) a medial whole. (Which is) the very thing it was required to show.