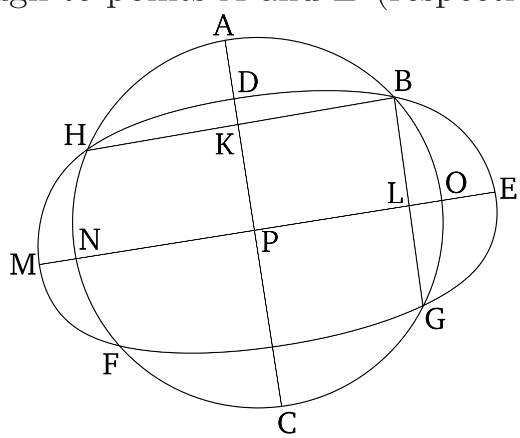
Book 3
Proposition 10

A circle does not cut a(nother) circle at more than two points.

For, if possible, let the circle ABC cut the circle DEF at more than two points, B, G, F, and H. And BH and BG being joined, let them (then) have been cut in half at points K and L (respectively). And KC and LM being drawn at right-angles to BH and BG from K and L (respectively) [Prop. 1.11], let them (then) have been drawn through to points A and E (respectively).



Therefore, since in circle ABC some straight-line AC cuts some (other) straight-line BH in half, and at right-angles, the center of circle ABC is thus on AC [Prop. 3.1 corr.]. Again, since in the same circle ABC some straight-line NO cuts some (other straight-line) BG in half, and at right-angles, the center of circle ABC is thus on NO [Prop. 3.1 corr.]. And it was also shown (to be) on AC. And the straight-lines AC and NO meet at no other (point) than P. Thus, point P is the center of circle

ABC. So, similarly, we can show that P is also the center of circle DEF. Thus, two circles cutting one another, ABC and DEF, have the same center P. The very thing is impossible [Prop. 3.5].

Thus, a circle does not cut a(nother) circle at more than two points. (Which is) the very thing it was required to show.