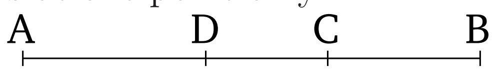
Book 10
Proposition 42

A binomial (straight-line) can be divided into its (component) terms at one point only.[†]



Let AB be a binomial (straight-line) which has been divided into its (component) terms at C. AC and CB are thus rational (straight-lines which are) commensurable in square only [Prop. 10.36]. I say that AB cannot be divided at another point into two rational (straight-lines which are) commensurable in square only.

For, if possible, let it also have been divided at D, such that AD and DB are also rational (straight-lines which are) commensurable in square only. So, (it is) clear that AC is not the same as DB. For, if possible, let it be (the same). So, AD will also be the same as CB. And as AC will be to CB, so BD (will be) to DA. And AB will (thus) also be divided at D in the same (manner) as the division at C. The very opposite was assumed. Thus, AC is not the same as DB. So, on account of this, points C and D are not equally far from the point of bisection. Thus, by whatever (amount the sum of) the (squares) on AC and CB differs from (the sum of) the (squares) on AD and DB, twice the (rectangle contained) by AD and DB also differs from twice the (rectangle contained) by AC and CB by this (same amount)—on account of both (the sum of) the (squares) on AC and CB, plus twice the (rectangle contained) by AC and CB, and (the sum of) the (squares) on AD and DB, plus twice the (rectangle contained) by AD and DB, being equal to the (square) on AB [Prop. 2.4]. But, (the sum of) the (squares) on AC and CB differs from (the sum of) the (squares) on AD and DB by a rational (area). For (they are) both rational (areas). Thus, twice the (rectangle contained) by AD and DB also differs from twice the (rectangle contained) by AC and CB by a rational (area, despite both) being medial (areas) [Prop. 10.21]. The very thing is absurd. For a medial (area) cannot exceed a medial (area) by a rational (area) [Prop. 10.26].

Thus, a binomial (straight-line) cannot be divided (into its component terms) at different points. Thus, (it can be so divided) at one point only. (Which is) the very thing it was required to show.