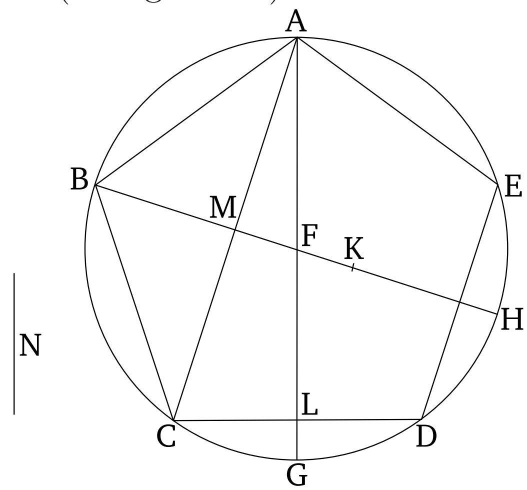
Book 13
Proposition 11

If an equilateral pentagon is inscribed in a circle which has a rational diameter then the side of the pentagon is that irrational (straight-line) called minor.



For let the equilateral pentagon $ABCDE$ have been inscribed in the circle $ABCDE$ which has a rational diameter. I say that the side of pentagon $[ABCDE]$ is that irrational (straight-line) called minor.

For let the center of the circle, point F, have been found [Prop. 3.1] . And let AF and FB have been joined. And let them have been drawn across to points G and H (respectively). And let AC have been joined. And let FK made (equal) to the fourth part of AF. And AF (is) rational. FK (is) thus also rational. And BF is also rational. Thus, the whole of BK is rational. And since circumference ACG is equal to circumference ADG, of which ABC is equal to AED, the remainder CG is thus equal to the remainder GD. And if we join AD then the angles at L are inferred (to be) right-angles, and CD (is inferred to be) double CL [Prop. 1.4]. So, for the same (reasons), the (angles) at M are also right-angles, and AC (is) double CM. Therefore, since angle ALC (is) equal to AMF, and (angle) LAC (is) common to the two triangles ACL and AMF, the remaining (angle) ACL is thus equal to the remaining (angle) MFA [Prop. 1.32]. Thus, triangle ACL is equiangular to triangle AMF. Thus, proportionally, as LC (is) to CA, so MF (is) to FA [Prop. 6.4]. And (we can take) the doubles of the leading (magnitudes). Thus, as double LC (is) to CA, so double MF (is) to FA. And as double MF (is) to FA, so MF (is) to half of FA. And, thus, as double LC (is) to CA, so MF (is) to half of FA. And (we can take) the halves of the following (magnitudes). Thus, as double LC (is) to half of CA, so MF (is) to the fourth of FA. And DC is double LC, and CM half of CA, and FK the fourth part of FA. Thus, as DC is to CM, so MF (is) to FK. Via composition, as the sum of DCM (*i.e.*, DC and CM) (is) to CM, so MK (is) to KF [Prop. 5.18]. And, thus, as the (square) on the sum of DCM (is) to the (square) on CM, so the (square) on MK (is) to the (square) on KF. And since the greater piece of a (straight-line) subtending two sides of a pentagon, such as AC, (which is) cut in extreme and mean ratio is equal to the side of the pentagon [Prop. 13.8]—that is to say, to DC—-and the square on the greater piece added to half of the whole is five times the (square) on half of the whole [Prop. 13.1], and CM (is) half of the whole, AC, thus the (square) on DCM, (taken) as one, is five times the (square) on CM. And the (square) on DCM, (taken) as one, (is) to the (square) on CM, so the (square) on MK was shown (to be) to the (square) on KF. Thus, the (square) on MK (is) five times the (square) on KF. And the square on KF (is) rational. For the diameter (is) rational. Thus, the (square) on MK (is) also rational. Thus, MK is rational [in square only]. And since BF is four times FK, BK is thus five times KF. Thus, the (square) on BK (is) twenty-five times the (square) on KF. And the (square) on MK (is) five times the square on KF. Thus, the (square) on BK (is) five times the (square) on KM. Thus, the (square) on BK does not have to the (square) on KM the ratio which a square number (has) to a square number. Thus, BK is incommensurable in length with KM [Prop. 10.9]. And each of them is a rational (straight-line). Thus, BK and KM are rational (straight-lines which are) commensurable in square only. And if from a rational (straight-line) a rational (straight-line) is subtracted, which is commensurable in square only with the whole, then the remainder is that irrational (straight-line called) an apotome [Prop. 10.73]. Thus, MB is an apotome, and MK its attachment. So, I say that (it is) also a fourth (apotome). So, let the (square) on N be (made) equal to that (magnitude) by which the (square) on BK is greater than the (square) on KM. Thus, the square on BK is greater than the (square) on KM by the (square) on N. And since KF is commensurable (in length) with FB then, via composition, KB is also commensurable (in length) with FB [Prop. 10.15]. But, BF is commensurable (in length) with BH. Thus, BK is also commensurable (in length) with BH [Prop. 10.12]. And since the (square) on BK is five times the (square) on KM, the (square) on BK thus has to the (square) on KM the ratio which 5 (has) to one. Thus, via conversion, the (square) on BK has to the (square) on N the ratio which 5 (has) to 4 [Prop. 5.19 corr.], which is not (that) of a square (number) to a square (number). BK is thus incommensurable (in length) with N [Prop. 10.9]. Thus, the square on BK is greater than the (square) on KM by the (square) on (some straight-line which is) incommensurable (in length) with (BK). Therefore, since the square on the whole, BK, is greater than the (square) on the attachment, KM, by the (square) on (some straight-line which is) incommensurable (in length) with (BK), and the whole, BK, is commensurable (in length) with the (previously) laid down rational (straight-line) BH, MB is thus a fourth apotome [Def. 10.14]. And the rectangle contained by a rational (straight-line) and a fourth apotome is irrational, and its square-root is that irrational (straight-line) called minor [Prop. 10.94]. And the square on AB is the rectangle contained by HBM, on account of joining AH, (so that) triangle ABH becomes equiangular with triangle ABM [Prop. 6.8], and (proportionally) as HB is to BA, so AB (is) to BM.

Thus, the side AB of the pentagon is that irrational (straight-line) called minor.† (Which is) the very thing it was required to show.