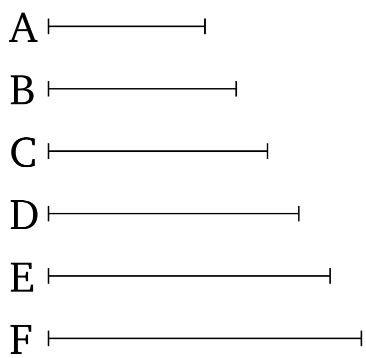
Book 9
Proposition 8

If any multitude whatsoever of numbers is continuously proportional, (starting) from a unit, then the third from the unit will be square, and (all) those (numbers after that) which leave an interval of one (number), and the fourth (will be) cube, and all those (numbers after that) which leave an interval of two (numbers), and the seventh (will be) both cube and square, and (all) those (numbers after that) which leave an interval of five (numbers).



Let any multitude whatsoever of numbers, A, B, C, D, E, F, be continuously proportional, (starting) from a unit. I say that the third from the unit, B, is square, and all those (numbers after that) which leave an interval of one (number). And the fourth (from the unit), C, (is) cube, and all those (numbers after that) which leave an interval of two (numbers). And the seventh (from the unit), F, (is) both cube and square, and all those (numbers after that) which leave an interval of five (numbers).

For since as the unit is to A, so A (is) to B, the unit thus measures the number A the same number of times as A (measures) B [Def. 7.20]. And the unit measures the number A according to the units in it. Thus, A also measures B according to the units in A. A has thus made B (by) multiplying itself [Def. 7.15]. Thus, B is square. And since B, C, D are continuously proportional, and B is square, D is thus also square [Prop. 8.22]. So, for the same (reasons), F is also square. So, similarly, we can also show that all those (numbers after that) which leave an interval of one (number) are square. So I also say that the fourth (number) from the unit, C, is cube, and all those (numbers after that) which leave an interval of two (numbers). For since as the unit is to A, so B (is) to C, the unit thus measures the number A the same number of times that B (measures) C. And the unit measures the number A according to the units in A. And thus B measures C according to the units in A. A has thus made C (by) multiplying B. Therefore, since A has made B (by) multiplying itself, and has made C (by) multiplying B, C is thus cube. And since C, D, E, F are continuously proportional, and C is cube, F is thus also cube [Prop. 8.23]. And it was also shown (to be) square. Thus, the seventh (number) from the unit is (both) cube and square. So, similarly, we can show that all those (numbers after that) which leave an interval of five (numbers) are (both) cube and square. (Which is) the very thing it was required to show.