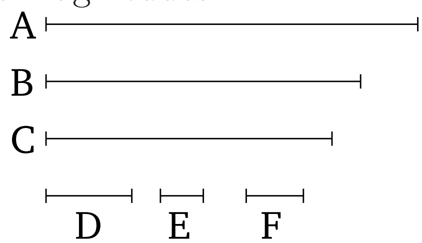
Book 10
Proposition 4

To find the greatest common measure of three given commensurable magnitudes.



Let A, B, C be the three given commensurable magnitudes. So it is required to find the greatest common measure of A, B, C.

For let the greatest common measure of the two (magnitudes) A and B have been taken [Prop. 10.3], and let it be D. So D either measures, or [does] not [measure], C. Let it, first of all, measure (C). Therefore, since D measures C, and it also measures A and B, D thus measures A, B, C. Thus, D is a common measure of A, B, C. And (it is) clear that (it is) also (the) greatest (common measure). For no magnitude larger than D measures (both) A and B.

So let D not measure C. I say, first, that C and D are commensurable. For if A, B, C are commensurable then some magnitude will measure them which will clearly also measure A and B. Hence, it will also measure D, the greatest common measure of A and B [Prop. 10.3 corr.]. And it also measures C. Hence, the aforementioned magnitude will measure (both) C and D. Thus, C and D are commensurable [Def. 10.1]. Therefore, let their greatest common measure have been taken [Prop. 10.3], and let it be E. Therefore, since E measures D, but D measures (both) A and B, E will thus also measure A and B. And it also measures C. Thus, E measures A, B, C. Thus, E is a common measure of A, B, C. So I say that (it is) also (the) greatest (common measure). For, if possible, let F be some magnitude greater than E, and let it measure A, B, C. And since F measures A, B, C, it will thus also measure A and B, and will (thus) measure the greatest common measure of A and B [Prop. 10.3 corr.]. And D is the greatest common measure of A and B. Thus, F measures D. And it also measures C. Thus, F measures (both) C and D. Thus, F will also measure the greatest common measure of C and D [Prop. 10.3 corr.]. And it is E. Thus, F will measure E, the greater (measuring) the lesser. The very thing is impossible. Thus, some [magnitude] greater than the magnitude E cannot measure A, B, C. Thus, if D does not measure C then E is the greatest common measure of A, B, C. And if it does measure (C) then D itself (is the greatest common measure).

Thus, the greatest common measure of three given commensurable magnitudes has been found. [(Which is) the very thing it was required to show.]

Corollary

So (it is) clear, from this, that if a magnitude measures three magnitudes then it will also measure their greatest common measure.

So, similarly, the greatest common measure of more (magnitudes) can also be taken, and the (above) corollary will go forward. (Which is) the very thing it was required to show.