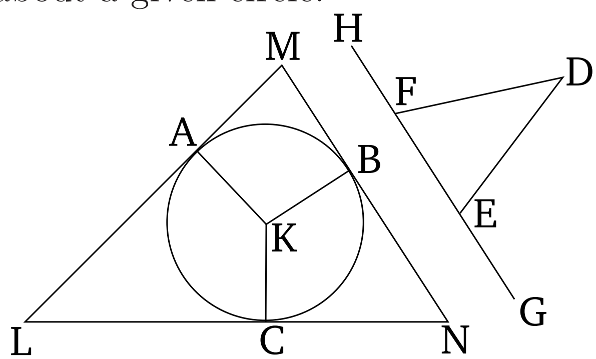
Book 4
Proposition 3

To circumscribe a triangle, equiangular with a given triangle, about a given circle.



Let ABC be the given circle, and DEF the given triangle. So it is required to circumscribe a triangle, equiangular with triangle DEF, about circle ABC.

Let EF have been produced in each direction to points G and H. And let the center K of circle ABC have been found [Prop. 3.1]. And let the straight-line KB have been drawn, at random, across (ABC). And let (angle) BKA, equal to angle DEG, have been constructed on the straight-line KB at the point K on it, and (angle) BKC, equal to DFH [Prop. 1.23]. And let the (straight-lines) LAM, MBN, and NCL have been drawn through the points A, B, and C (respectively), touching the circle ABC.

And since LM, MN, and NL touch circle ABC at points A, B, and C (respectively), and KA, KB, and KC are joined from the center K to points A, B, and C (respectively), the angles at points A, B, and C are thus right-angles [Prop. 3.18]. And since the (sum of the) four angles of quadrilateral $AMBK$ is equal to four right-angles, inasmuch as $AMBK$ (can) also (be) divided into two triangles [Prop. 1.32], and angles KAM and KBM are (both) right-angles, the (sum of the) remaining (angles), AKB and AMB, is thus equal to two right-angles. And DEG and DEF is also equal to two right-angles [Prop. 1.13]. Thus, AKB and AMB is equal to DEG and DEF, of which AKB is equal to DEG. Thus, the remainder AMB is equal to the remainder DEF. So, similarly, it can be shown that LNB is also equal to DFE. Thus, the remaining (angle) MLN is also equal to the [remaining] (angle) EDF [Prop. 1.32]. Thus, triangle LMN is equiangular with triangle DEF. And it has been drawn around circle ABC.

Thus, a triangle, equiangular with the given triangle, has been circumscribed about the given circle. (Which is) the very thing it was required to do.