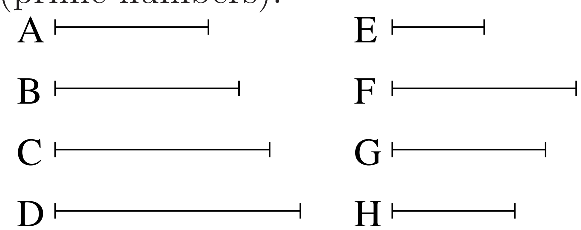
Book 9
Proposition 12

If any multitude whatsoever of numbers is continuously proportional, (starting) from a unit, then however many prime numbers the last (number) is measured by, the (number) next to the unit will also be measured by the same (prime numbers).



Let any multitude whatsoever of numbers, A, B, C, D, be (continuously) proportional, (starting) from a unit. I say that however many prime numbers D is measured by, A will also be measured by the same (prime numbers).

For let D be measured by some prime number E. I say that E measures A. For (suppose it does) not. E is prime, and every prime number is prime to every number which it does not measure [Prop. 7.29]. Thus, E and A are prime to one another. And since E measures D, let it measure it according to F. Thus, E has made D (by) multiplying F. Again, since A measures D according to the units in C [Prop. 9.11 corr.], A has thus made D (by) multiplying C. But, in fact, E has also made D (by) multiplying F. Thus, the (number created) from (multiplying) A, C is equal to the (number created) from (multiplying) E, F. Thus, as A is to E, (so) F (is) to C [Prop. 7.19]. And A and E (are) prime (to one another), and (numbers) prime (to one another are) also the least (of those numbers having the same ratio as them) [Prop. 7.21], and the least (numbers) measure those (numbers) having the same ratio as them an equal number of times, the leading (measuring) the leading, and the following the following [Prop. 7.20]. Thus, E measures C. Let it measure it according to G. Thus, E has made C (by) multiplying G. But, in fact, via the (proposition) before this, A has also made C (by) multiplying B [Prop. 9.11 corr.]. Thus, the (number created) from (multiplying) A, B is equal to the (number created) from (multiplying) E, G. Thus, as A is to E, (so) G (is) to B [Prop. 7.19]. And A and E (are) prime (to one another), and (numbers) prime (to one another are) also the least (of those numbers having the same ratio as them) [Prop. 7.21], and the least (numbers) measure those (numbers) having the same ratio as them an equal number of times, the leading (measuring) the leading, and the following the following [Prop. 7.20]. Thus, E measures B. Let it measure it according to H. Thus, E has made B (by) multiplying H. But, in fact, A has also made B (by) multiplying itself [Prop. 9.8]. Thus, the (number created) from (multiplying) E, H is equal to the (square) on A. Thus, as E is to A, (so) A (is) to H [Prop. 7.19]. And A and E are prime (to one another), and (numbers) prime (to one another are) also the least (of those numbers having the same ratio as them) [Prop. 7.21], and the least (numbers) measure those (numbers) having the same ratio as them an equal number of times, the leading (measuring) the leading, and the following the following [Prop. 7.20]. Thus, E

measures A, as the leading (measuring the) leading. But, in fact, (E) also does not measure (A). The very thing (is) impossible. Thus, E and A are not prime to one another. Thus, (they are) composite (to one another). And (numbers) composite (to one another) are (both) measured by some [prime] number [Def. 7.14]. And since E is assumed (to be) prime, and a prime (number) is not measured by another number (other) than itself [Def. 7.11], E thus measures (both) A and E. Hence, E measures A. And it also measures D. Thus, E measures (both) A and D. So, similarly, we can show that however many prime numbers D is measured by, A will also be measured by the same (prime numbers). (Which is) the very thing it was required to show.