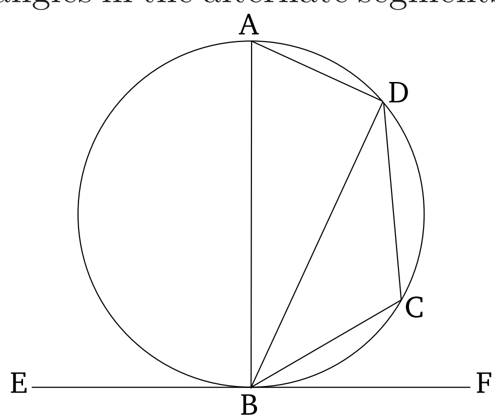
Book 3
Proposition 32

If some straight-line touches a circle, and some (other) straight-line is drawn across, from the point of contact into the circle, cutting the circle (in two), then those angles the (straight-line) makes with the tangent will be equal to the angles in the alternate segments of the circle.



For let some straight-line EF touch the circle $ABCD$ at the point B, and let some (other) straight-line BD have been drawn from point B into the circle $ABCD$, cutting it (in two). I say that the angles BD makes with the tangent EF will be equal to the angles in the alternate segments of the circle. That is to say, that angle FBD is equal to the angle constructed in segment BAD, and angle EBD is equal to the angle constructed in segment DCB.

For let BA have been drawn from B, at right-angles to EF [Prop. 1.11]. And let the point C have been taken at random on the circumference BD. And let AD, DC, and CB have been joined.

And since some straight-line EF touches the circle

$ABCD$ at point B, and BA has been drawn from the point of contact, at right-angles to the tangent, the center of circle $ABCD$ is thus on BA [Prop. 3.19]. Thus, BA is a diameter of circle $ABCD$. Thus, angle ADB, being in a semi-circle, is a right-angle [Prop. 3.31]. Thus, the remaining angles (of triangle ADB) BAD and ABD are equal to one right-angle [Prop. 1.32]. And ABF is also a right-angle. Thus, ABF is equal to BAD and ABD. Let ABD have been subtracted from both. Thus, the remaining angle DBF is equal to the angle BAD in the alternate segment of the circle. And since $ABCD$ is a quadrilateral in a circle, (the sum of) its opposite angles is equal to two right-angles [Prop. 3.22]. And DBF and DBE is also equal to two right-angles [Prop. 1.13]. Thus, DBF and DBE is equal to BAD and BCD, of which BAD was shown (to be) equal to DBF. Thus, the remaining (angle) DBE is equal to the angle DCB in the alternate segment DCB of the circle.

Thus, if some straight-line touches a circle, and some (other) straight-line is drawn across, from the point of contact into the circle, cutting the circle (in two), then those angles the (straight-line) makes with the tangent will be equal to the angles in the alternate segments of the circle. (Which is) the very thing it was required to show.