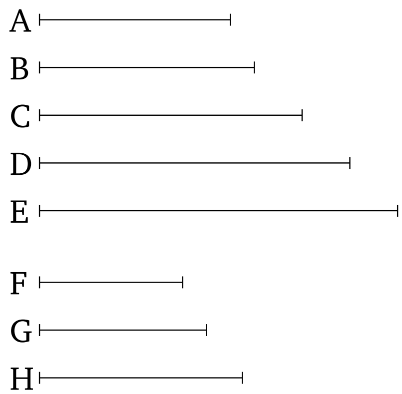
Book 8
Proposition 6

If there are any multitude whatsoever of continuously proportional numbers, and the first does not measure the second, then no other (number) will measure any other (number) either.



Let A, B, C, D, E be any multitude whatsoever of continuously proportional numbers, and let A not measure B. I say that no other (number) will measure any other (number) either.

Now, (it is) clear that A, B, C, D, E do not successively measure one another. For A does not even measure B. So I say that no other (number) will measure any other (number) either. For, if possible, let A measure C. And as many (numbers) as are A, B, C, let so many of the least numbers, F, G, H, have been taken of those (numbers) having the same ratio as A, B, C [Prop. 7.33]. And since F, G, H are in the same ratio as A, B, C, and the multitude of A, B, C is equal to the multitude of F, G, H, thus, via equality, as A is to C, so F (is) to H [Prop. 7.14]. And since as A is to B, so F (is) to G, and A does not measure B, F does not measure G either [Def. 7.20]. Thus, F is not a unit. For a unit measures all numbers. And F and H are prime to one another [Prop. 8.3] [and thus F does not measure H]. And as F is to H, so A (is) to C. And thus A does not measure C either [Def. 7.20]. So, similarly, we can show that no other (number) can measure any other (number) either. (Which is) the very thing it was required to show.