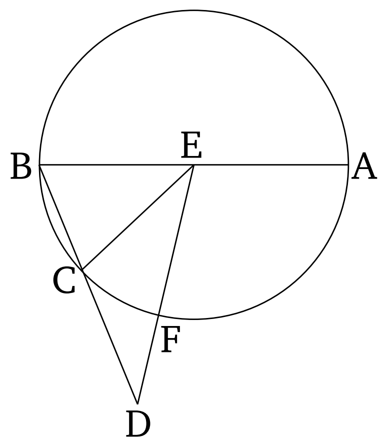
Book 13
Proposition 9

If the side of a hexagon and of a decagon inscribed in the same circle are added together then the whole straight-line has been cut in extreme and mean ratio (at the junction point), and its greater piece is the side of the hexagon.†



Let ABC be a circle. And of the figures inscribed in circle ABC, let BC be the side of a decagon, and CD (the side) of a hexagon. And let them be (laid down) straight-on (to one another). I say that the whole straight-line BD has been cut in extreme and mean ratio (at C), and that CD is its greater piece.

For let the center of the circle, point E, have been found [Prop. 3.1], and let EB, EC, and ED have been joined, and let BE have been drawn across to A. Since BC is a side on an equilateral decagon, circumference ACB (is) thus five times circumference BC. Thus, circumference AC (is) four times CB. And as circumference AC (is) to CB, so angle AEC (is) to CEB [Prop. 6.33]. Thus, (angle) AEC (is) four times CEB.

And since angle EBC (is) equal to ECB [Prop. 1.5], angle AEC is thus double
And since straight-line EC is equal to CD—for each of them is equal to the side of the hexagon [inscribed] in circle ABC [Prop. 4.15 corr.]— angle CED is also equal to angle CDE [Prop. 1.5]. Thus, angle ECB (is) double EDC [Prop. 1.32]. But, AEC was shown (to be) double ECB. Thus, AEC (is) four times EDC. And AEC was also shown (to be) four times BEC. Thus, EDC (is) equal to BEC. And angle EBD (is) common to the two triangles BEC and BED. Thus, the remaining (angle) BED is equal to the (remaining angle) ECB [Prop. 1.32]. Thus, triangle EBD is equiangular to triangle EBC. Thus, proportionally, as DB is to BE, so EB (is) to BC [Prop. 6.4]. And EB (is) equal to CD. Thus, as BD is to DC, so DC (is) to CB. And BD (is) greater than DC. Thus, DC (is) also greater than CB [Prop. 5.14]. Thus, the straight-line BD has been cut in extreme and mean ratio [at C], and DC is its greater piece. (Which is), the very thing it was required to show.