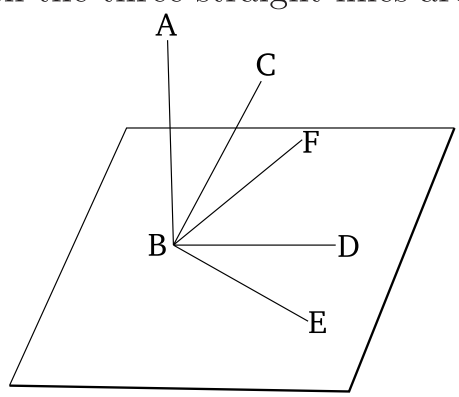
Book 11
Proposition 5

If a straight-line is set up at right-angles to three straight-lines cutting one another, at the common point of section, then the three straight-lines are in one plane.



For let some straight-line AB have been set up at right-angles to three straight-lines BC, BD, and BE, at the (common) point of section B. I say that BC, BD, and BE are in one plane.

For (if) not, and if possible, let BD and BE be in the reference plane, and BC in a more elevated (plane). And let the plane through AB and BC have been produced. So it will make a straight-line as a common section with the reference plane [Def. 11.3]. Let it make BF. Thus, the three straight-lines AB, BC, and BF are in one plane—(namely), that drawn through AB and BC. And since AB is at right-angles to each of BD and BE, AB is thus also at right-angles to the plane (passing) through BD and BE [Prop. 11.4]. And the plane (passing) through BD and BE is the reference plane. Thus, AB is at right-angles to the reference plane. Hence, AB

will also make right-angles with all straight-lines joined to it which are also in the reference plane [Def. 11.3]. And BF, which is in the reference plane, is joined to it. Thus, the angle ABF is a right-angle. And ABC was also assumed to be a right-angle. Thus, angle ABF (is) equal to ABC. And they are in one plane. The very thing is impossible. Thus, BC is not in a more elevated plane. Thus, the three straight-lines BC, BD, and BE are in one plane.

Thus, if a straight-line is set up at right-angles to three straight-lines cutting one another, at the (common) point of section, then the three straight-lines are in one plane. (Which is) the very thing it was required to show.