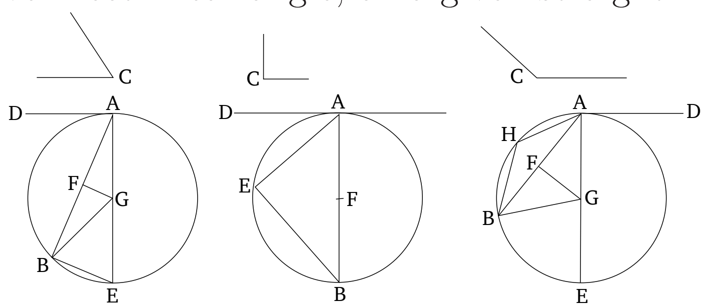
# Book 3
# Proposition 33

To draw a segment of a circle, accepting an angle equal to a given rectilinear angle, on a given straight-line.




Let $AB$ be the given straight-line, and $C$ the given rectilinear angle. So it is required to draw a segment of a circle, accepting an angle equal to $C$, on the given straight-line $AB$.

So the [angle] $C$ is surely either acute, a right-angle, or obtuse. First of all, let it be acute. And, as in the first diagram (from the left), let (angle) $BAD$, equal to angle $C$, have been constructed on the straight-line $AB$, at the point A (on it) [Prop. 1.23]. Thus, $BAD$ is also acute. Let $AE$ have been drawn, at right-angles to $DA$ [Prop. 1.11]. And let $AB$ have been cut in half at $F$ [Prop. 1.10]. And let $FG$ have been drawn from point $F$, at right-angles to $AB$ [Prop. 1.11]. And let $GB$ have been joined.

And since $AF$ is equal to $FB$, and $FG$ (is) common, the two (straight-lines) $AF$, $FG$ are equal to the two (straight-lines) $BF$, $FG$ (respectively). And angle $AFG$ (is) equal to [angle] $BFG$. Thus, the base $AG$ is equal to the base $BG$ [Prop. 1.4]. Thus, the circle drawn with center $G$, and radius $GA$, will also go through $B$ (as well as $A$). Let it have been drawn, and let it be (denoted) $ABE$. And let $EB$ have been joined. Therefore, since $AD$ is at the extremity of diameter $AE$, (namely, point) $A$, at right-angles to $AE$, the (straight-line) $AD$ thus touches the circle $ABE$ [Prop. 3.16 corr.]. Therefore, since some straight-line $AD$ touches the circle $ABE$, and some (other) straight-line $AB$ has been drawn across from the point of contact $A$ into circle $ABE$, angle $DAB$ is thus equal to the angle $AEB$ in the alternate segment of the circle [Prop. 3.32]. But, $DAB$ is equal to $C$. Thus, angle $C$ is also equal to $AEB$.

Thus, a segment $AEB$ of a circle, accepting the angle $AEB$ (which is) equal to the given (angle) $C$, has been drawn on the given straight-line $AB$.

And so let $C$ be a right-angle. And let it again be necessary to draw a segment of a circle on $AB$, accepting an angle equal to the right-[angle] $C$. Let the (angle) $BAD$ [again] have been constructed, equal to the right-angle $C$ [Prop. 1.23], as in the second diagram (from the left). And let $AB$ have been cut in half at $F$ [Prop. 1.10]. And let the circle $AEB$ have been drawn with center $F$, and radius either $FA$ or $FB$.

Thus, the straight-line $AD$ touches the circle $ABE$, on account of the angle at $A$ being a right-angle [Prop. 3.16 corr.]. And angle $BAD$ is equal to the angle in segment $AEB$. For (the latter angle), being in a semi-circle, is also a right-angle [Prop. 3.31]. But, $BAD$ is also equal to $C$. Thus, the (angle) in (segment) $AEB$ is also equal to $C$.

Thus, a segment $AEB$ of a circle, accepting an angle equal to $C$, has again been drawn on $AB$.

And so let (angle) $C$ be obtuse. And let (angle) $BAD$, equal to ($C$), have been constructed on the straight-line $AB$, at the point $A$ (on it) [Prop. 1.23], as in the third diagram (from the left). And let $AE$ have been drawn, at right-angles to $AD$ [Prop. 1.11]. And let $AB$ have again been cut in half at $F$ [Prop. 1.10]. And let $FG$ have been drawn, at right-angles to $AB$ [Prop. 1.10]. And let $GB$ have been joined.

And again, since $AF$ is equal to $FB$, and $FG$ (is) common, the two (straight-lines) $AF$, $FG$ are equal to the two (straight-lines) $BF$, $FG$ (respectively). And angle $AFG$ (is) equal to angle $BFG$. Thus, the base $AG$ is equal to the base $BG$ [Prop. 1.4]. Thus, a circle of center $G$, and radius $GA$, being drawn, will also go through $B$ (as well as $A$). Let it go like $AEB$ (in the third diagram from the left). And since $AD$ is at right-angles to the diameter $AE$, at its extremity, $AD$ thus touches circle $AEB$ [Prop. 3.16 corr.]. And $AB$ has been drawn across (the circle) from the point of contact $A$. Thus, angle $BAD$ is equal to the angle constructed in the alternate segment $AHB$ of the circle [Prop. 3.32]. But, angle $BAD$ is equal to $C$. Thus, the angle in segment $AHB$ is also equal to $C$.

Thus, a segment $AHB$ of a circle, accepting an angle equal to $C$, has been drawn on the given straight-line $AB$. (Which is) the very thing it was required to do.