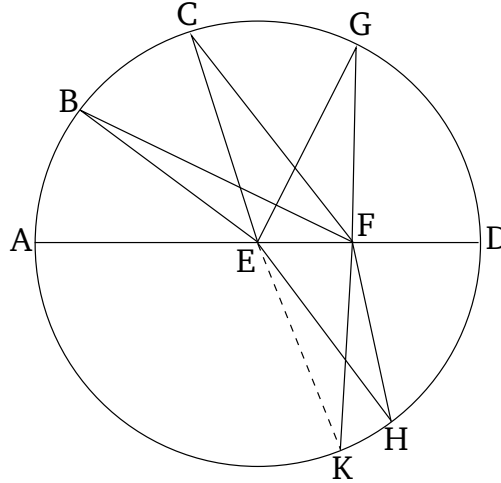


## Book 3

### Proposition 7

If some point, which is not the center of the circle, is taken on the diameter of a circle, and some straight-lines radiate from the point towards the (circumference of the) circle, then the greatest (straight-line) will be that on which the center (lies), and the least the remainder (of the same diameter). And for the others, a (straight-line) nearer<sup>†</sup> to the (straight-line) through the center is always greater than a (straight-line) further away. And only two equal (straight-lines) will radiate from the point towards the (circumference of the) circle, (one) on each (side) of the least (straight-line).



Let  $ABCD$  be a circle, and let  $AD$  be its diameter, and let some point  $F$ , which is not the center of the circle, have been taken on  $AD$ . Let  $E$  be the center of the circle. And let some straight-lines,  $FB$ ,  $FC$ , and  $FG$ , radiate from  $F$  towards (the circumference of) circle  $ABCD$ . I say that  $FA$  is the greatest (straight-line),  $FD$  the least,

and of the others,  $FB$  (is) greater than  $FC$ , and  $FC$  than  $FG$ .

For let  $BE$ ,  $CE$ , and  $GE$  have been joined. And since for every triangle (any) two sides are greater than the remaining (side) [Prop. 1.20],  $EB$  and  $EF$  is thus greater than  $BF$ . And  $AE$  (is) equal to  $BE$  [thus,  $BE$  and  $EF$  is equal to  $AF$ ]. Thus,  $AF$  (is) greater than  $BF$ . Again, since  $BE$  is equal to  $CE$ , and  $FE$  (is) common, the two (straight-lines)  $BE$ ,  $EF$  are equal to the two (straight-lines)  $CE$ ,  $EF$  (respectively). But, angle  $BEF$  (is) also greater than angle  $CEF$ .<sup>‡</sup> Thus, the base  $BF$  is greater than the base  $CF$ . Thus, the base  $BF$  is greater than the base  $CF$  [Prop. 1.24]. So, for the same (reasons),  $CF$  is also greater than  $FG$ .

Again, since  $GF$  and  $FE$  are greater than  $EG$  [Prop. 1.20], and  $EG$  (is) equal to  $ED$ ,  $GF$  and  $FE$  are thus greater than  $ED$ . Let  $EF$  have been taken from both. Thus, the remainder  $GF$  is greater than the remainder  $FD$ . Thus,  $FA$  (is) the greatest (straight-line),  $FD$  the least, and  $FB$  (is) greater than  $FC$ , and  $FC$  than  $FG$ .

I also say that from point  $F$  only two equal (straight-lines) will radiate towards (the circumference of) circle  $ABCD$ , (one) on each (side) of the least (straight-line)  $FD$ . For let the (angle)  $FEH$ , equal to angle  $GEF$ , have been constructed on the straight-line  $EF$ , at the point  $E$  on it [Prop. 1.23], and let  $FH$  have been joined. Therefore, since  $GE$  is equal to  $EH$ , and  $EF$  (is) common, the two (straight-lines)  $GE$ ,  $EF$  are equal to the two (straight-lines)  $HE$ ,  $EF$  (respectively). And angle  $GEF$  (is) equal to angle  $HEF$ . Thus, the base  $FG$  is

equal to the base  $FH$  [Prop. 1.4]. So I say that another (straight-line) equal to  $FG$  will not radiate towards (the circumference of) the circle from point  $F$ . For, if possible, let  $FK$  (so) radiate. And since  $FK$  is equal to  $FG$ , but  $FH$  [is equal] to  $FG$ ,  $FK$  is thus also equal to  $FH$ , the nearer to the (straight-line) through the center equal to the further away. The very thing (is) impossible. Thus, another (straight-line) equal to  $GF$  will not radiate from the point  $F$  towards (the circumference of) the circle. Thus, (there is) only one (such straight-line).

Thus, if some point, which is not the center of the circle, is taken on the diameter of a circle, and some straight-lines radiate from the point towards the (circumference of the) circle, then the greatest (straight-line) will be that on which the center (lies), and the least the remainder (of the same diameter). And for the others, a (straight-line) nearer to the (straight-line) through the center is always greater than a (straight-line) further away. And only two equal (straight-lines) will radiate from the same point towards the (circumference of the) circle, (one) on each (side) of the least (straight-line). (Which is) the very thing it was required to show.