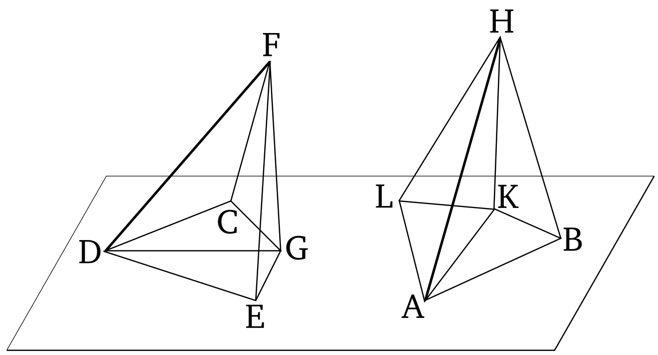
Book 11
Proposition 26

To construct a solid angle equal to a given solid angle on a given straight-line, and at a given point on it.

Let AB be the given straight-line, and A the given point on it, and D the given solid angle, contained by the plane angles EDC, EDF, and FDC. So, it is necessary to construct a solid angle equal to the solid angle D on the straight-line AB, and at the point A on it.



For let some random point F have been taken on DF, and let FG have been drawn from F perpendicular to the plane through ED and DC [Prop. 11.11], and let it meet the plane at G, and let DG have been joined. And let BAL, equal to the angle EDC, and BAK, equal to EDG, have been constructed on the straight-line AB at the point A on it [Prop. 1.23]. And let AK be made equal to DG. And let KH have been set up at the point K at right-angles to the plane through BAL [Prop. 11.12]. And let KH be made equal to GF. And let HA have been joined. I say that the solid angle at A, contained by the (plane) angles BAL, BAH, and

HAL, is equal to the solid angle at D, contained by the (plane) angles EDC, EDF, and FDC.

For let AB and DE have been cut off (so as to be) equal, and let HB, KB, FE, and GE have been joined. And since FG is at right-angles to the reference plane (EDC), it will also make right-angles with all of the straight-lines joined to it which are also in the reference plane [Def. 11.3]. Thus, the angles FGD and FGE are right-angles. So, for the same (reasons), the angles HKA and HKB are also right-angles. And since the two (straight-lines) KA and AB are equal to the two (straight-lines) GD and DE, respectively, and they contain equal angles, the base KB is thus equal to the base GE [Prop. 1.4]. And KH is also equal to GF. And they contain right-angles (with the respective bases). Thus, HB (is) also equal to FE [Prop. 1.4]. Again, since the two (straight-lines) AK and KH are equal to the two (straight-lines) DG and GF (respectively), and they contain right-angles, the base AH is thus equal to the base FD [Prop. 1.4]. And AB (is) also equal to DE. So, the two (straight-lines) HA and AB are equal to the two (straight-lines) DF and DE (respectively). And the base HB (is) equal to the base FE. Thus, the angle BAH is equal to the angle EDF [Prop. 1.8]. So, for the same (reasons), HAL is also equal to FDC. And BAL is also equal to EDC.

Thus, (a solid angle) has been constructed, equal to the given solid angle at D, on the given straight-line AB, at the given point A on it. (Which is) the very thing it was required to do.