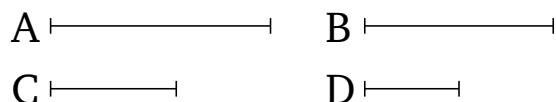# Book 10
Proposition 9

Squares on straight-lines (which are) commensurable in length have to one another the ratio which (some) square number (has) to (some) square number. And squares having to one another the ratio which (some) square number (has) to (some) square number will also have sides (which are) commensurable in length. But squares on straight-lines (which are) incommensurable in length do not have to one another the ratio which (some) square number (has) to (some) square number. And squares not having to one another the ratio which (some) square number (has) to (some) square number will not have sides (which are) commensurable in length either.

A $\longmapsto\longmapsto$ B $\longmapsto\longmapsto$

C $\longmapsto\longmapsto$ D $\longmapsto$

For let A and B be (straight-lines which are) commensurable in length. I say that the square on A has to the square on B the ratio which (some) square number (has) to (some) square number.

For since A is commensurable in length with B, A thus has to B the ratio which (some) number (has) to (some) number [Prop. 10.5]. Let it have (that) which C (has) to D. Therefore, since as A is to B, so C (is) to D. But the (ratio) of the square on A to the square on B is the square of the ratio of A to B. For similar figures are in the squared ratio of (their) corresponding sides [Prop. 6.20 corr.]. And the (ratio) of the square

on C to the square on D is the square of the ratio of the [number] C to the [number] D. For there exits one number in mean proportion to two square numbers, and (one) square (number) has to the (other) square [number] a squared ratio with respect to (that) the side (of the former has) to the side (of the latter) [Prop. 8.11]. And, thus, as the square on A is to the square on B, so the square [number] on the (number) C (is) to the square [number] on the [number] D.

And so let the square on A be to the (square) on B as the square (number) on C (is) to the [square] (number) on D. I say that A is commensurable in length with B.

For since as the square on A is to the [square] on B, so the square (number) on C (is) to the [square] (number) on D. But, the ratio of the square on A to the (square) on B is the square of the (ratio) of A to B [Prop. 6.20 corr.]. And the (ratio) of the square [number] on the [number] C to the square [number] on the [number] D is the square of the ratio of the [number] C to the [number] D [Prop. 8.11]. Thus, as A is to B, so the [number] C also (is) to the [number] D. A, thus, has to B the ratio which the number C has to the number D. Thus, A is commensurable in length with B [Prop. 10.6].

And so let A be incommensurable in length with B. I say that the square on A does not have to the [square] on B the ratio which (some) square number (has) to (some) square number.

For if the square on A has to the [square] on B the ratio which (some) square number (has) to (some) square number then A will be commensurable (in length) with

B. But it is not. Thus, the square on A does not have to the [square] on the B the ratio which (some) square number (has) to (some) square number.

So, again, let the square on A not have to the [square] on B the ratio which (some) square number (has) to (some) square number. I say that A is incommensurable in length with B.

For if A is commensurable (in length) with B then the (square) on A will have to the (square) on B the ratio which (some) square number (has) to (some) square number. But it does not have (such a ratio). Thus, A is not commensurable in length with B.

Thus, (squares) on (straight-lines which are) commensurable in length, and so on . . . .

Corollary

And it will be clear, from (what) has been demonstrated, that (straight-lines) commensurable in length (are) always also (commensurable) in square, but (straight-lines commensurable) in square (are) not always also (commensurable) in length.