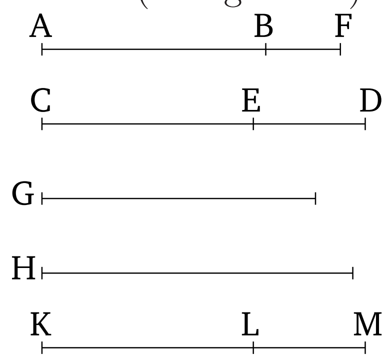
Book 10
Proposition 114

If an area is contained by an apotome, and a binomial whose terms are commensurable with, and in the same ratio as, the terms of the apotome then the square-root of the area is a rational (straight-line).



For let an area, the (rectangle contained) by AB and CD, have been contained by the apotome AB, and the binomial CD, of which let the greater term be CE. And let the terms of the binomial, CE and ED, be commensurable with the terms of the apotome, AF and FB (respectively), and in the same ratio. And let the square-root of the (rectangle contained) by AB and CD be G. I say that G is a rational (straight-line).

For let the rational (straight-line) H be laid down. And let (some rectangle), equal to the (square) on H, have been applied to CD, producing KL as breadth. Thus, KL is an apotome, of which let the terms, KM and ML, be commensurable with the terms of the binomial, CE and ED (respectively), and in the same ratio [Prop. 10.112]. But, CE and ED are also commensurable with AF and FB (respectively), and in the same ratio. Thus, as AF is to FB, so KM (is) to ML. Thus, alternately, as AF is to KM, so BF (is) to LM [Prop. 5.16]. Thus, the remainder AB is also to the remainder KL as AF (is) to KM [Prop. 5.19]. And AF (is) commensurable with KM [Prop. 10.12]. AB is thus also commensurable w And as AB is to KL, so the (rectangle contained) by CD and AB (is) to the (rectangle contained) by CD and KL [Prop. 6.1]. Thus, the (rectangle contained) by CD and AB is also commensurable with the (rectangle contained) by CD and KL [Prop. 10.11]. And the (rectangle contained) by CD and KL (is) equal to the (square) on H. Thus, the (rectangle contained) by CD and AB is commensurable with the (square) on H. And the (square) on G is equal to the (rectangle contained) by CD and AB. The (square) on G is thus commensurable with the (square) on H. And the (square) on H (is) rational. Thus, the (square) on G is also rational. G is thus rational. And it is the square-root of the (rectangle contained) by CD and AB.

Thus, if an area is contained by an apotome, and a binomial whose terms are commensurable with, and in the same ratio as, the terms of the apotome, then the square-root of the area is a rational (straight-line).

Corollary

And it has also been made clear to us, through this, that it is possible for a rational area to be contained by irrational straight-lines. (Which is) the very thing it was required to show.