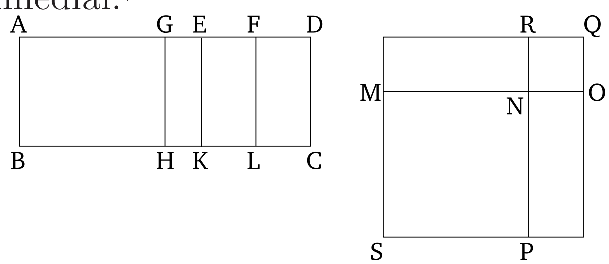
Book 10
Proposition 56

If an area is contained by a rational (straight-line) and a third binomial (straight-line) then the square-root of the area is the irrational (straight-line which is) called second bimedial.†



For let the area $ABCD$ be contained by the rational (straight-line) AB and by the third binomial (straight-line) AD, which has been divided into its (component) terms at E, of which AE is the greater. I say that the square-root of area AC is the irrational (straight-line which is) called second bimedial.

For let the same construction be made as previously. And since AD is a third binomial (straight-line), AE and ED are thus rational (straight-lines which are) commensurable in square only, and the square on AE is greater than (the square on) ED by the (square) on (some straight-line) commensurable (in length) with (AE), and neither of AE and ED [is] commensurable in length with AB [Def. 10.7]. So, similarly to that which has been previously demonstrated, we can show that MO is the square-root of area AC, and MN and NO are medial (straight-lines which are) commensurable in square only. Hence, MO is bimedial. So, we must show that (it is) also second (bimedial).

[And] since DE is incommensurable in length with AB—that is to say, with EK—and DE (is) commensurable (in length) with EF, EF is thus incommensurable in length with EK [Prop. 10.13]. And they are (both) rational (straight-lines). Thus, FE and EK are rational (straight-lines which are) commensurable in square only. EL—that is to say, MR—[is] thus medial [Prop. 10.21]. And it is contained by MNO. Thus, the (rectangle contained) by MNO is medial.

Thus, MO is a second bimedial (straight-line) [Prop. 10.38]. (Which is) the very thing it was required to show.