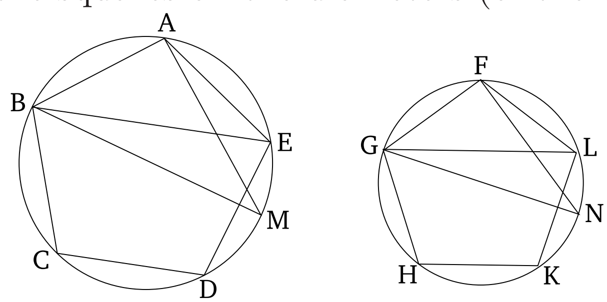
Book 12
Proposition 1

Similar polygons (inscribed) in circles are to one another as the squares on the diameters (of the circles).



Let ABC and FGH be circles, and let $ABCDE$ and $FGHKL$ be similar polygons (inscribed) in them (respectively), and let BM and GN be the diameters of the circles (respectively). I say that as the square on BM is to the square on GN, so polygon $ABCDE$ (is) to polygon $FGHKL$.

For let BE, AM, GL, and FN have been joined. And since polygon $ABCDE$ (is) similar to polygon $FGHKL$, angle BAE is also equal to (angle) GFL, and as BA is to AE, so GF (is) to FL [Def. 6.1]. So, BAE and GFL are two triangles having one angle equal to one angle, (namely), BAE (equal) to GFL, and the sides around the equal angles proportional. Triangle ABE is thus equiangular with triangle FGL [Prop. 6.6]. Thus, angle AEB is equal to (angle) FLG. But, AEB is equal to AMB, and FLG to FNG, for they stand on the same circumference [Prop. 3.27]. Thus, AMB is also equal to FNG. And the right-angle BAM is also equal to the right-angle GFN [Prop. 3.31]. Thus, the remaining (angle) is also equal to the remaining (angle) [Prop. 1.32]. Thus, triangle ABM is equiangular with triangle FGN. Thus, proportionally, as BM is to GN, so BA (is) to GF [Prop. 6.4]. But, the (ratio) of the square on BM to the square on GN is the square of the ratio of BM to GN, and the (ratio) of polygon $ABCDE$ to polygon $FGHKL$ is the square of the (ratio) of BA to GF [Prop. 6.20]. And, thus, as the square on BM (is) to the square on GN, so polygon $ABCDE$ (is) to polygon $FGHKL$.

Thus, similar polygons (inscribed) in circles are to one another as the squares on the diameters (of the circles). (Which is) the very thing it was required to show.