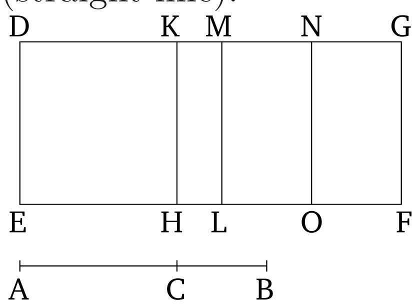
Book 10
Proposition 62

The square on a second bimedial (straight-line) applied to a rational (straight-line) produces as breadth a third binomial (straight-line).



Let AB be a second bimedial (straight-line) having been divided into its (component) medial (straight-lines) at C, such that AC is the greater segment. And let DE be some rational (straight-line). And let the parallelogram DF, equal to the (square) on AB, have been applied to DE, producing DG as breadth. I say that DG is a third binomial (straight-line).

Let the same construction be made as that shown previously. And since AB is a second bimedial (straight-line), having been divided at C, AC and CB are thus medial (straight-lines) commensurable in square only, and containing a medial (area) [Prop. 10.38]. Hence, the sum of the (squares) on AC and CB is also medial [Props. 10.15, 10.23 corr.]. And it is equal to DL. Thus, DL (is) also medial. And it is applied to the rational (straight-line) DE. MD is thus also rational, and incommensurable in length with DE [Prop. 10.22]. So, for the same (reasons), MG is also rational, and incommensurable in length with ML—that is to say, with DE. Thus, DM and MG are each rational, and incommensurable in length with DE. And since AC is incommensurable in length with CB, and as AC (is) to CB, so the (square) on AC (is) to the (rectangle contained) by ACB [Prop. 10.21 lem.], the (square) on AC (is) also incommensurable with the (rectangle contained) by ACB [Prop. 10.11]. And hence the sum of the (squares) on AC and CB is incommensurable with twice the (rectangle contained) by ACB—that is to say, DL with MF [Props. 10.12, 10.13]. Hence, DM is also incommensurable (in length) with MG [Props. 6.1, 10.11]. And they are rational. DG is thus a binomial (straight-line) [Prop. 10.36]. [So] we must show that (it is) also a third (binomial straight-line).

So, similarly to the previous (propositions), we can conclude that DM is greater than MG, and DK (is) commensurable (in length) with KM. And the (rectangle contained) by DKM is equal to the (square) on MN. Thus, the square on DM is greater than (the square on) MG by the (square) on (some straight-line) commensurable (in length) with (DM) [Prop. 10.17]. And neither of DM and MG is commensurable in length with DE.

Thus, DG is a third binomial (straight-line) [Def. 10.7]. (Which is) the very thing it was required to show.