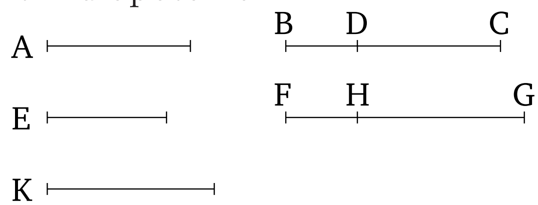
Book 10
Proposition 87

To find a third apotome.



Let the rational (straight-line) A be laid down. And let the three numbers, E, BC, and CD, not having to one another the ratio which (some) square number (has) to (some) square number, be laid down. And let CB have to BD the ratio which (some) square number (has) to (some) square number. And let it have been contrived that as E (is) to BC, so the square on A (is) to the square on FG, and as BC (is) to CD, so the square on FG (is) to the (square) on GH [Prop. 10.6 corr.]. Therefore, since as E is to BC, so the square on A (is) to the square on FG, the square on A is thus commensurable with the square on FG [Prop. 10.6]. And the square on A (is) rational. Thus, the (square) on FG (is) also rational. Thus, FG is a rational (straight-line). And since E does not have to BC the ratio which (some) square number (has) to (some) square number, the square on A thus does not have to the [square] on FG the ratio which (some) square number (has) to (some) square number either. Thus, A is incommensurable in length with FG [Prop. 10.9]. Again, since as BC is to CD, so the square on FG is to the (square) on GH, the square on FG is thus commensurable with the (square) on GH [Prop. 10.6]. And the (square) on FG (is) rational. Thus, the (square) on GH (is) also rational. Thus, GH is a rational (straight-line). And since BC does not have to CD the ratio which (some) square number (has) to (some) square number, the (square) on FG thus does not have to the (square) on GH the ratio which (some) square number (has) to (some) square number either. Thus, FG is incommensurable in length with GH [Prop. 10.9]. And both are rational (straight-lines). FG and GH are thus rational (straight-lines which are) commensurable in square only. Thus, FH is an apotome [Prop. 10.73]. So, I say that (it is) also a third (apotome).

For since as E is to BC, so the square on A (is) to the (square) on FG, and as BC (is) to CD, so the (square) on FG (is) to the (square) on HG, thus, via equality, as E is to CD, so the (square) on A (is) to the (square) on HG [Prop. 5.22]. And E does not have to CD the ratio which (some) square number (has) to (some) square number. Thus, the (square) on A does not have to the (square) on GH the ratio which (some) square number (has) to (some) square number either. A (is) thus incommensurable in length with GH [Prop. 10.9]. Thus, neither of FG and GH is commensurable in length with the (previously) laid down rational (straight-line) A. Therefore, let the (square) on K be that (area) by which the (square) on FG is greater than the (square) on GH [Prop. 10.13 lem.]. Therefore, since as BC is to CD, so the (square) on FG (is) to the (square) on GH, thus, via conversion, as BC is to BD, so the square on FG (is) to the square on K [Prop. 5.19 corr.]. And BC has to BD the ratio which (some) square number (has) to (some) square number. Thus, the (square) on FG also has to the (square) on K the ratio which (some) square number (has) to (some) square number. FG is thus commensurable in length with K [Prop. 10.9]. And the square on FG is (thus) greater than (the square on) GH by the (square) on (some straight-line) commensurable (in length) with (FG). And neither of FG and GH is commensurable in length with the (previously) laid down rational (straight-line) A. Thus, FH is a third apotome [Def. 10.13].

Thus, the third apotome FH has been found. (Which is) very thing it was required to show.