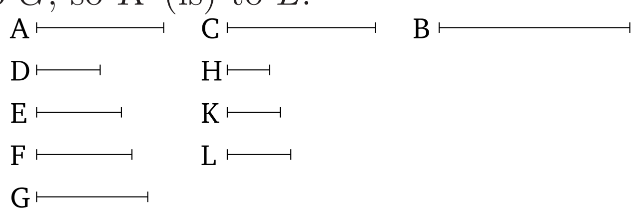
Book 10
Proposition 12

(Magnitudes) commensurable with the same magnitude are also commensurable with one another.

For let A and B each be commensurable with C. I say that A is also commensurable with B.

For since A is commensurable with C, A thus has to C the ratio which (some) number (has) to (some) number [Prop. 10.5]. Let it have (the ratio) which D (has) to E. Again, since C is commensurable with B, C thus has to B the ratio which (some) number (has) to (some) number [Prop. 10.5]. Let it have (the ratio) which F (has) to G. And for any multitude whatsoever of given ratios—(namely,) those which D has to E, and F to G—let the numbers H, K, L (which are) continuously (proportional) in the(se) given ratios have been taken [Prop. 8.4]. Hence, as D is to E, so H (is) to K, and as F (is) to G, so K (is) to L.



Therefore, since as A is to C, so D (is) to E, but as D (is) to E, so H (is) to K, thus also as A is to C, so H (is) to K [Prop. 5.11]. Again, since as C is to B, so F (is) to G, but as F (is) to G, [so] K (is) to L, thus also as C (is) to B, so K (is) to L [Prop. 5.11]. And also as A is to C, so H (is) to K. Thus, via equality, as A
Thus, A has to B the ratio which the number H (has) to the number L. Thus, A is commensurable with B

[Prop. 10.6].

Thus, (magnitudes) commensurable with the same magnitude are also commensurable with one another. (Which is) the very thing it was required to show.