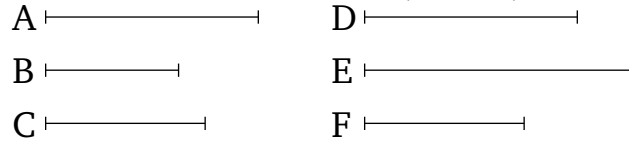


## Book 5

### Proposition 21

If there are three magnitudes, and others of equal number to them, (being) also in the same ratio taken two by two, and (if) their proportion (is) perturbed, and (if), via equality, the first is greater than the third then the fourth will also be greater than the sixth. And if (the first is) equal (to the third then the fourth will also be) equal (to the sixth). And if (the first is) less (than the third then the fourth will also be) less (than the sixth).



Let  $A$ ,  $B$ , and  $C$  be three magnitudes, and  $D$ ,  $E$ ,  $F$  other (magnitudes) of equal number to them, (being) in the same ratio taken two by two. And let their proportion be perturbed, (so that) as  $A$  (is) to  $B$ , so  $E$  (is) to  $F$ , and as  $B$  (is) to  $C$ , so  $D$  (is) to  $E$ . And let  $A$  be greater than  $C$ , via equality. I say that  $D$  will also be greater than  $F$ . And if ( $A$  is) equal (to  $C$  then  $D$  will also be) equal (to  $F$ ). And if ( $A$  is) less (than  $C$  then  $D$  will also be) less (than  $F$ ).

For since  $A$  is greater than  $C$ , and  $B$  some other (magnitude),  $A$  thus has a greater ratio to  $B$  than  $C$  (has) to  $B$  [Prop. 5.8]. But as  $A$  (is) to  $B$ , so  $E$  (is) to  $F$ . And, inversely, as  $C$  (is) to  $B$ , so  $E$  (is) to  $D$  [Prop. 5.7 corr.]. Thus,  $E$  also has a greater ratio to  $F$  than  $E$  (has) to  $D$  [Prop. 5.13]. And that (magnitude) to which the same (magnitude) has a greater ratio, that (magnitude) is less. Thus,  $F$  is less than  $D$ . Thus,  $D$  is greater than  $F$ . Similarly, we can show that even if  $A$  is equal to  $C$  then  $D$

will also be equal to  $F$ , and even if ( $A$  is) less (than  $C$  then  $D$  will also be) less (than  $F$ ).

Thus, if there are three magnitudes, and others of equal number to them, (being) also in the same ratio taken two by two, and (if) their proportion (is) perturbed, and (if), via equality, the first is greater than the third then the fourth will also be greater than the sixth. And if (the first is) equal (to the third then the fourth will also be) equal (to the sixth). And if (the first is) less (than the third then the fourth will also be) less (than the sixth). (Which is) the very thing it was required to show.