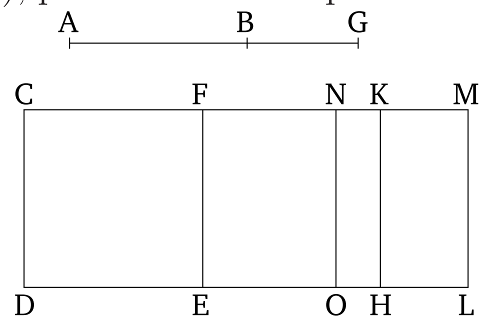
Book 10
Proposition 101

The (square) on that (straight-line) which with a rational (area) makes a medial whole, applied to a rational (straight-line), produces a fifth apotome as breadth.



Let AB be that (straight-line) which with a rational (area) makes a medial whole, and CD a rational (straight-line). And let CE, equal to the (square) on AB, have been applied to CD, producing CF as breadth. I say that CF is a fifth apotome.

Let BG be an attachment to AB. Thus, the straight-lines AG and GB are incommensurable in square, making the sum of the squares on them medial, and twice the (rectangle contained) by them rational [Prop. 10.77]. And let CH, equal to the (square) on AG, have been applied to CD, and KL, equal to the (square) on GB. The whole of CL is thus equal to the (sum of the squares) on AG and GB. And the sum of the (squares) on AG and GB together is medial. Thus, CL is medial. And it has been applied to the rational (straight-line) CD, producing CM as breadth. CM is thus rational, and incommensurable (in length) with CD [Prop. 10.22]. And since the whole of CL is equal to the (sum of the squares) on AG and GB, of which CE is equal to the (square) on AB, the remainder FL is thus equal to twice the (rectangle contained) by AG and GB [Prop. 2.7]. Therefore, let FM have been cut in half at N. And let NO have been drawn through N, parallel to either of CD or ML. Thus, FO and NL are each equal to the (rectangle contained) by AG and GB. And since twice the (rectangle contained) by AG and GB is rational, and [is] equal to FL, FL is thus rational. And it is applied to the rational (straight-line) EF, producing FM as breadth. Thus, FM is rational, and commensurable in length with CD [Prop. 10.20]. And since CL is medial, and FL rational, CL is thus incommensurable with FL. And as CL (is) to FL, so CM (is) to MF [Prop. 6.1]. CM is thus incommensurable in length with MF [Prop. 10.11]. And both are rational. Thus, CM and MF are rational (straight-lines which are) commensurable in square only. CF is thus an apotome [Prop. 10.73]. So, I say that (it is) also a fifth (apotome).

For, similarly (to the previous propositions), we can show that the (rectangle contained) by CKM is equal to the (square) on NM—that is to say, to the fourth part of the (square) on FM. And since the (square) on AG is incommensurable with the (square) on GB, and the (square) on AG (is) equal to CH, and the (square) on GB to KL, CH (is) thus incommensurable with KL. And as CH (is) to KL, so CK (is) to KM [Prop. 6.1]. Thus, CK (is) incommensurable in length with KM [Prop. 10.11]. Therefore, since CM and MF are two unequal straight-lines, and (some area), equal to the fourth part of the (square) on FM, has been applied to CM, falling short by a square figure, and divides it into incommensurable (parts), the square on CM is thus greater than (the square on) MF by the (square) on (some straight-line) incommensurable (in length) with (CM) [Prop. 10.18]. And the attachment FM is commensurable with the (previously) laid down rational (straight-line) CD. Thus, CF is a fifth apotome [Def. 10.15]. (Which is) the very thing it was required to show.