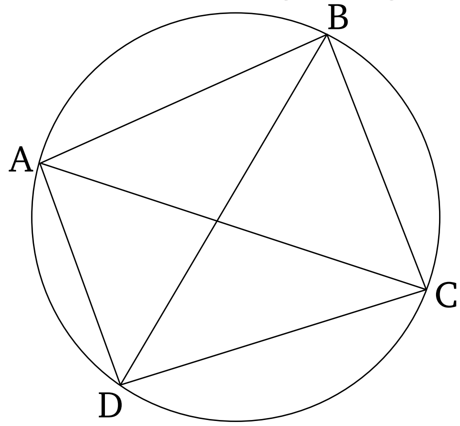
Book 3
Proposition 22

For quadrilaterals within circles, the (sum of the) opposite angles is equal to two right-angles.



Let $ABCD$ be a circle, and let $ABCD$ be a quadrilateral within it. I say that the (sum of the) opposite angles is equal to two right-angles.

Let AC and BD have been joined.

Therefore, since the three angles of any triangle are equal to two right-angles [Prop. 1.32], the three angles CAB, ABC, and BCA of triangle ABC are thus equal to two right-angles. And CAB (is) equal to BDC. For they are in the same segment $BADC$ [Prop. 3.21]. And ACB (is equal) to ADB. For they are in the same segment $ADCB$ [Prop. 3.21]. Thus, the whole of ADC is equal to BAC and ACB. Let ABC have been added to both. Thus, ABC, BAC, and ACB are equal to ABC and ADC. But, ABC, BAC, and ACB are equal to two right-angles. Thus, ABC and ADC are also equal to two right-angles. Similarly, we can show that angles BAD and DCB are also equal to two right-angles.

Thus, for quadrilaterals within circles, the (sum of the) opposite angles is equal to two right-angles. (Which is) the very thing it was required to show.