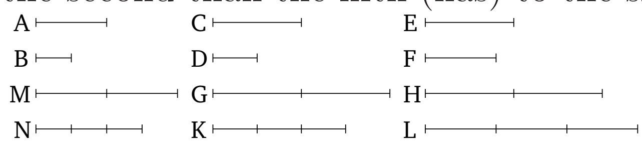
Book 5
Proposition 13

If a first (magnitude) has the same ratio to a second that a third (has) to a fourth, and the third (magnitude) has a greater ratio to the fourth than a fifth (has) to a sixth, then the first (magnitude) will also have a greater ratio to the second than the fifth (has) to the sixth.



For let a first (magnitude) A have the same ratio to a second B that a third C (has) to a fourth D, and let the third (magnitude) C have a greater ratio to the fourth D than a fifth E (has) to a sixth F. I say that the first (magnitude) A will also have a greater ratio to the second B than the fifth E (has) to the sixth F.

For since there are some equal multiples of C and E, and other random equal multiples of D and F, (for which) the multiple of C exceeds the (multiple) of D, and the multiple of E does not exceed the multiple of F [Def. 5.7], let them have been taken. And let G and H be equal multiples of C and E (respectively), and K and L other random equal multiples of D and F (respectively), such that G exceeds K, but H does not exceed L. And as many times as G is (divisible) by C, so many times let M be (divisible) by A. And as many times as K (is divisible) by D, so many times let N be (divisible) by B.

And since as A is to B, so C (is) to D, and the equal multiples M and G have been taken of A and C (respectively), and the other random equal multiples N and K of B and D (respectively), thus if M exceeds N then G exceeds K, and if (M is) equal (to N then G is also) equal (to K), and if (M is) less (than N then G is also) less (than K) [Def. 5.5]. And G exceeds K. Thus, M also exceeds N. And H does not exceeds L. And M and H are equal multiples of A and E (respectively), and N and L other random equal multiples of B and F (respectively). Thus, A has a greater ratio to B than E (has) to F [Def. 5.7].

Thus, if a first (magnitude) has the same ratio to a second that a third (has) to a fourth, and a third (magnitude) has a greater ratio to a fourth than a fifth (has) to a sixth, then the first (magnitude) will also have a greater ratio to the second than the fifth (has) to the sixth. (Which is) the very thing it was required to show.