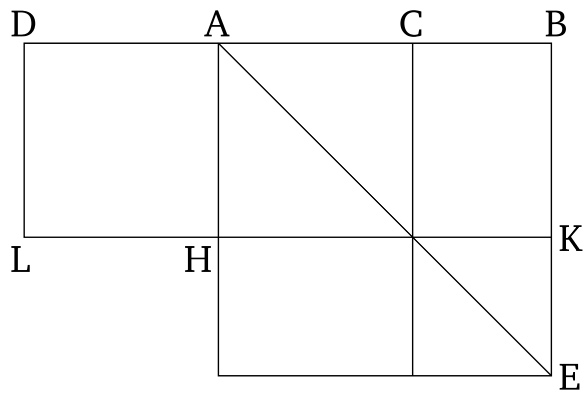
Book 13
Proposition 5

If a straight-line is cut in extreme and mean ratio, and a (straight-line) equal to the greater piece is added to it, then the whole straight-line has been cut in extreme and mean ratio, and the original straight-line is the greater piece.



For let the straight-line AB have been cut in extreme and mean ratio at point C. And let AC be the greater piece. And let AD be [made] equal to AC. I say that the straight-line DB has been cut in extreme and mean ratio at A, and that the original straight-line AB is the greater piece.

For let the square AE have been described on AB, and let the (remainder of the) figure have been drawn. And since AB has been cut in extreme and mean ratio at C, the (rectangle contained) by ABC is thus equal to the (square) on AC [Def. 6.3, Prop. 6.17]. And CE is the (rectangle contained) by ABC, and CH the (square) on AC. But, HE is equal to CE [Prop. 1.43], and DH equal to HC. Thus, DH is also equal to HE. [Let HB have been added to both.] Thus, the whole of DK is equal to the whole of AE. And DK is the (rectangle contained) by BD and DA. For AD (is) equal to DL. And AE (is) the (square) on AB. Thus, the (rectangle contained) by BDA is equal to the (square) on AB. Thus, as DB (is) to BA, so BA (is) to AD [Prop. 6.17]. And DB (is) greater than BA. Thus, BA (is) also greater than AD [Prop. 5.14].

Thus, DB has been cut in extreme and mean ratio at A, and the greater piece is AB. (Which is) the very thing it was required to show.