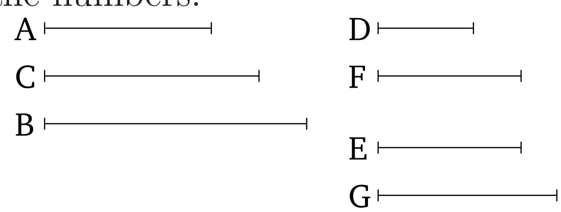
Book 8
Proposition 20

If one number falls between two numbers in mean proportion then the numbers will be similar plane (numbers).

For let one number C fall between the two numbers A and B in mean proportion. I say that A and B are similar plane numbers.



[For] let the least numbers, D and E, having the same ratio as A and C have been taken [Prop. 7.33]. Thus, D measures A as many times as E (measures) C [Prop. 7.20]. So as many times as D measures A, so many units let there be in F. Thus, F has made A (by) multiplying D [Def. 7.15]. Hence, A is plane, and D, F (are) its sides. Again, since D and E are the least of those (numbers) having the same ratio as C and B, D thus measures C as many times as E (measures) B [Prop. 7.20]. So as many times as E measures B, so many units let there be in G. Thus, E measures B according to the units in G. Thus, G has made B (by) multiplying E [Def. 7.15]. Thus, B is plane, and E, G are its sides. Thus, A and B are (both) plane numbers. So I say that (they are) also similar. For since F has made A (by) multiplying D, and has made C (by) multiplying E, thus as D is to E, so A (is) to C—that is to say, C to B [Prop. 7.17].

Again, since E has made C, B (by) multiplying F, G, respectively, thus as F is to G, so C (is) to B [Prop. 7.17]. And as C (is) to B, so D (is) to E. And thus as D (is) to E, so F (is) to G. And, alternately, as D (is) to F, so E (is) to G [Prop. 7.13]. Thus, A and B are similar plane numbers. For their sides are proportional [Def. 7.21]. (Which is) the very thing it was required to show.