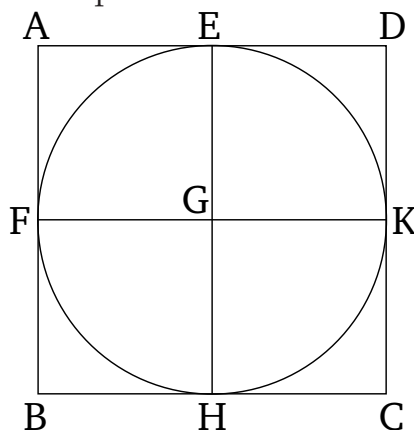


Book 4

Proposition 8

To inscribe a circle in a given square.

Let the given square be $ABCD$. So it is required to inscribe a circle in square $ABCD$.



Let AD and AB each have been cut in half at points E and F (respectively) [Prop. 1.10]. And let EH have been drawn through E , parallel to either of AB or CD , and let FK have been drawn through F , parallel to either of AD or BC [Prop. 1.31]. Thus, AK , KB , AH , HD , AG , GC , BG , and GD are each parallelograms, and their opposite sides [are] manifestly equal [Prop. 1.34]. And since AD is equal to AB , and AE is half of AD , and AF half of AB , AE (is) thus also equal to AF . So that the opposite (sides are) also (equal). Thus, FG (is) also equal to GE . So, similarly, we can also show that each of GH and GK is equal to each of FG and GE . Thus, the four (straight-lines) GE , GF , GH , and GK [are] equal to one another. Thus, the circle drawn with center G , and radius one of E , F , H , or K , will also go through the remaining points. And it will touch the

straight-lines AB , BC , CD , and DA , on account of the angles at E , F , H , and K being right-angles. For if the circle cuts AB , BC , CD , or DA , then a (straight-line) drawn at right-angles to a diameter of the circle, from its extremity, will fall inside the circle. The very thing was shown (to be) absurd [Prop. 3.16]. Thus, the circle drawn with center G , and radius one of E , F , H , or K , does not cut the straight-lines AB , BC , CD , or DA . Thus, it will touch them, and will have been inscribed in the square $ABCD$.

Thus, a circle has been inscribed in the given square. (Which is) the very thing it was required to do.