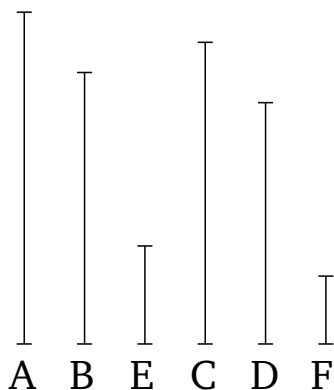# Book 10
Proposition 14

If four straight-lines are proportional, and the square on the first is greater than (the square on) the second by the (square) on (some straight-line) commensurable [in length] with the first, then the square on the third will also be greater than (the square on) the fourth by the (square) on (some straight-line) commensurable [in length] with the third. And if the square on the first is greater than (the square on) the second by the (square) on (some straight-line) incommensurable [in length] with the first, then the square on the third will also be greater than (the square on) the fourth by the (square) on (some straight-line) incommensurable [in length] with the third.

Let A, B, C, D be four proportional straight-lines, (such that) as A (is) to B, so C (is) to D. And let the square on A be greater than (the square on) B by the (square) on E, and let the square on C be greater than (the square on) D by the (square) on F. I say that A is either commensurable (in length) with E, and C is also commensurable with F, or A is incommensurable (in length) with E, and C is also incommensurable with F.

A B E C D F

For since as A is to B, so C (is) to D, thus as the (square) on A is to the (square) on B, so the (square) on C (is) to the (square) on D [Prop. 6.22]. But the (sum of the squares) on E and B is equal to the (square) on A, and the (sum of the squares) on D and F is equal to the (square) on C. Thus, as the (sum of the squares) on E and B is to the (square) on B, so the (sum of the squares) on D and F (is) to the (square) on D. Thus, via separation, as the (square) on E is to the (square) on B, so the (square) on F (is) to the (square) on D [Prop. 5.17]. Thus, also, as E is to B, so F (is) to D [Prop. 6.22]. Thus, inversely, as B is to E, so D (is) to F [Prop. 5.7 corr.]. But, as A is to B, so C also (is) to D. Thus, via equality, as A is to E, so C (is) to F [Prop. 5.22]. Therefore, A is either commensurable (in length) with E, and C is also commensurable with F, or A is incommensurable (in length) with E, and C is also incommensurable with F [Prop. 10.11].

Thus, if, and so on ....