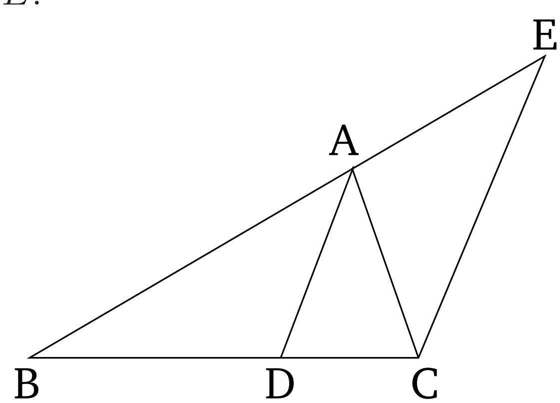
Book 6
Proposition 3

If an angle of a triangle is cut in half, and the straight-line cutting the angle also cuts the base, then the segments of the base will have the same ratio as the remaining sides of the triangle. And if the segments of the base have the same ratio as the remaining sides of the triangle, then the straight-line joining the vertex to the cutting (point) will cut the angle of the triangle in half.

Let ABC be a triangle. And let the angle BAC have been cut in half by the straight-line AD. I say that as BD is to CD, so BA (is) to AC.

For let CE have been drawn through (point) C parallel to DA. And, BA being drawn through, let it meet (CE) at (point) E.



And since the straight-line AC falls across the parallel (straight-lines) AD and EC, angle ACE is thus equal to CAD [Prop. 1.29]. But, (angle) CAD is assumed (to be) equal to BAD. Thus, (angle) BAD is also equal to ACE. Again, since the straight-line BAE falls across the parallel (straight-lines) AD and EC, the external angle

BAD is equal to the internal (angle) AEC [Prop. 1.29]. And (angle) ACE was also shown (to be) equal to BAD. Thus, angle ACE is also equal to AEC. And, hence, side AE is equal to side AC [Prop. 1.6]. And since AD has been drawn parallel to one of the sides EC of triangle BCE, thus, proportionally, as BD is to DC, so BA (is) to AE [Prop. 6.2]. And AE (is) equal to AC. Thus, as BD (is) to DC, so BA (is) to AC.

And so, let BD be to DC, as BA (is) to AC. And let AD have been joined. I say that angle BAC has been cut in half by the straight-line AD.

For, by the same construction, since as BD is to DC, so BA (is) to AC, then also as BD (is) to DC, so BA is to AE. For AD has been drawn parallel to one (of the sides) EC of triangle BCE [Prop. 6.2]. Thus, also, as BA (is) to AC, so BA (is) to AE [Prop. 5.11]. Thus, AC (is) equal to AE [Prop. 5.9]. And, hence, angle AEC is equal to ACE [Prop. 1.5]. But, AEC [is] equal to the external (angle) BAD, and ACE is equal to the alternate (angle) CAD [Prop. 1.29]. Thus, (angle) BAD is also equal to CAD. Thus, angle BAC has been cut in half by the straight-line AD.

Thus, if an angle of a triangle is cut in half, and the straight-line cutting the angle also cuts the base, then the segments of the base will have the same ratio as the remaining sides of the triangle. And if the segments of the base have the same ratio as the remaining sides of the triangle, then the straight-line joining the vertex to the cutting (point) will cut the angle of the triangle in half. (Which is) the very thing it was required to show.