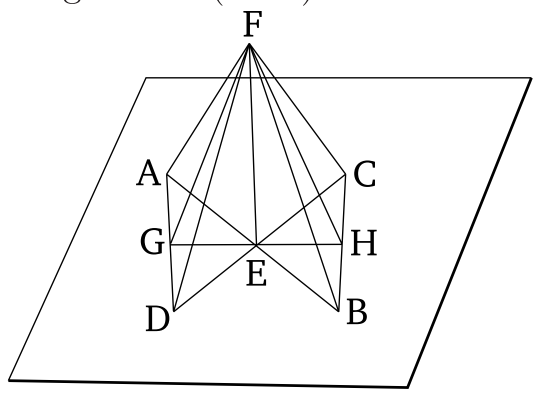
Book 11
Proposition 4

If a straight-line is set up at right-angles to two straight-lines cutting one another, at the common point of section, then it will also be at right-angles to the plane (passing) through them (both).



For let some straight-line EF have (been) set up at right-angles to two straight-lines, AB and CD, cutting one another at point E, at E. I say that EF is also at right-angles to the plane (passing) through AB and CD.

For let AE, EB, CE and ED have been cut off from (the two straight-lines so as to be) equal to one another. And let GEH have been drawn, at random, through E (in the plane passing through AB and CD). And let AD and CB have been joined. And, furthermore, let FA, FG, FD, FC, FH, and FB have been joined from the random (point) F (on EF).

For since the two (straight-lines) AE and ED are equal to the two (straight-lines) CE and EB, and they enclose equal angles [Prop. 1.15], the base AD is thus equal to the base CB, and triangle AED will be equal to triangle CEB [Prop. 1.4]. Hence, the angle DAE [is] equal to the angle EBC. And the angle AEG (is) also equal to the angle BEH [Prop. 1.15]. So AGE and BEH are two triangles having two angles equal to two angles, respectively, and one side equal to one side—(namely), those by the equal angles, AE and EB. Thus, they will also have the remaining sides equal to the remaining sides [Prop. 1.26]. Thus, GE (is) equal to EH, and AG to BH. And since AE is equal to EB, and FE is common and at right-angles, the base FA is thus equal to the base FB [Prop. 1.4]. So, for the same (reasons), FC is also equal to FD. And since AD is equal to CB, and FA is also equal to FB, the two (straight-lines) FA and AD are equal to the two (straight-lines) FB and BC, respectively. And the base FD was shown (to be) equal to the base FC. Thus, the angle FAD is also equal to the angle FBC [Prop. 1.8]. And, again, since AG was shown (to be) equal to BH, but FA (is) also equal to FB, the two (straight-lines) FA and AG are equal to the two (straight-lines) FB and BH (respectively). And the angle FAG was shown (to be) equal to the angle FBH. Thus, the base FG is equal to the base FH [Prop. 1.4]. And, again, since GE was shown (to be) equal to EH, and EF (is) common, the two (straight-lines) GE and EF are equal to the two (straight-lines) HE and EF (respectively). And the base FG (is) equal to the base FH. Thus, the angle GEF is equal to the angle HEF [Prop. 1.8]. Each of the angles GEF and HEF (are) thus right-angles [Def. 1.10]. Thus, FE is at right-angles to GH, which was drawn at random through

E (in the reference plane passing though AB and AC). So, similarly, we can show that FE will make right-angles with all straight-lines joined to it which are in the reference plane. And a straight-line is at right-angles to a plane when it makes right-angles with all straight-lines joined to it which are in the plane [Def. 11.3]. Thus, FE is at right-angles to the reference plane. And the reference plane is that (passing) through the straight-lines AB and CD. Thus, FE is at right-angles to the plane (passing) through AB and CD.

Thus, if a straight-line is set up at right-angles to two straight-lines cutting one another, at the common point of section, then it will also be at right-angles to the plane (passing) through them (both). (Which is) the very thing it was required to show.