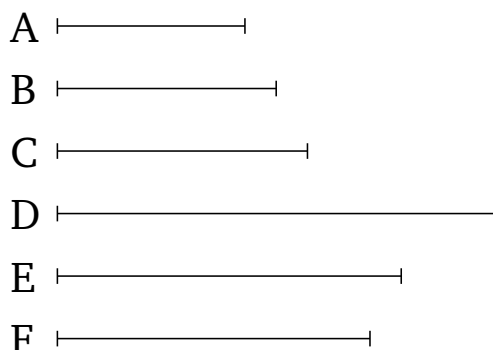# Book 7
Proposition 36

To find the least number which three given numbers (all) measure.

Let A, B, and C be the three given numbers. So it is required to find the least number which they (all) measure.

```
A  ├──────────┤
B  ├────────────┤
C  ├─────────────┤
D  ├─────────────────────┤
E  ├──────────────────┤
F  ├────────────────┤
```

For let the least (number), D, measured by the two (numbers) A and B have been taken [Prop. 7.34]. So C either measures, or does not measure, D. Let it, first of all, measure (D). And A and B also measure D. Thus, A, B, and C (all) measure D. So I say that $(D$ is) also the least (number measured by A, B, and C). For if not, A, B, and C will (all) measure [some] number which is less than D. Let them measure E (which is less than D). Since A, B, and C (all) measure E then A and B thus also measure E. Thus, the least (number) measured by A and B will also measure $[E]$ [Prop. 7.35]. And D is the least (number) measured by A and B. Thus, D will measure E, the greater (measuring) the lesser. The very thing is impossible. Thus, A, B, and C cannot (all) measure some number which is less than D. Thus, A, B,

and C (all) measure the least (number) D.

So, again, let C not measure D. And let the least number, E, measured by C and D have been taken [Prop. 7.34]. Since A and B measure D, and D measures E, A and B thus also measure E. And C also measures $[E]$. Thus, A, B, and C [also] measure E. So I say that (E is) also the least (number measured by A, B, and C). For if not, A, B, and C will (all) measure some (number) which is less than E. Let them measure F (which is less than E). Since A, B, and C (all) measure F, A and B thus also measure F. Thus, the least (number) measured by A and B will also measure F [Prop. 7.35]. And D is the least (number) measured by A and B. Thus, D measures F. And C also measures F. Thus, D and C (both) measure F. Hence, the least (number) measured by D and C will also measure F [Prop. 7.35]. And E is the least (number) measured by C and D. Thus, E measures F, the greater (measuring) the lesser. The very thing is impossible. Thus, A, B, and C cannot measure some number which is less than E. Thus, E (is) the least (number) which is measured by A, B, and C. (Which is) the very thing it was required to show.