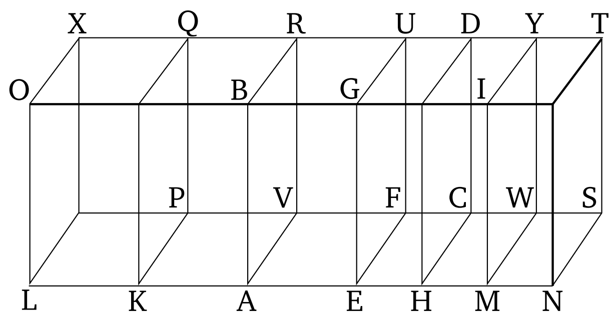
Book 11
Proposition 25

If a parallelipiped solid is cut by a plane which is parallel to the opposite planes (of the parallelipiped) then as the base (is) to the base, so the solid will be to the solid.



For let the parallelipiped solid $ABCD$ have been cut by the plane FG which is parallel to the opposite planes RA and DH. I say that as the base $AEFV$ (is) to the base $EHCF$, so the solid $ABFU$ (is) to the solid $EGCD$.

For let AH have been produced in each direction. And let any number whatsoever (of lengths), AK and KL, be made equal to AE, and any number whatsoever (of lengths), HM and MN, equal to EH. And let the parallelograms LP, KV, HW, and MS have been completed, and the solids LQ, KR, DM, and MT.

And since the straight-lines LK, KA, and AE are equal to one another, the parallelograms LP, KV, and AF are also equal to one another, and KO, KB, and AG (are equal) to one another, and, further, LX, KQ, and AR (are equal) to one another. For (they are) opposite [Prop. 11.24]. So, for the same (reasons), the parallelograms EC, HW, and MS are also equal to one another, and HG, HI, and IN are equal to one another, and, further, DH, MY, and NT (are equal to one another). Thus, three planes of (one of) the solids LQ, KR, and AU are equal to the (corresponding) three planes (of the others). But, the three planes (in one of the soilds) are equal to the three opposite planes [Prop. 11.24]. Thus, the three solids LQ, KR, and AU are equal to one another [Def. 11.10]. So, for the same (reasons), the three solids ED, DM, and MT are also equal to one another. Thus, as many multiples as the base LF is of the base AF, so many multiples is the solid LU also of the the solid AU. So, for the same (reasons), as many multiples as the base NF is of the base FH, so many multiples is the solid NU also of the solid HU. And if the base LF is equal to the base NF then the solid LU is also equal to the solid NU.† And if the base LF exceeds the base NF then the solid LU also exceeds the solid NU. And if (LF) is less than (NF) then (LU) is (also) less than (NU). So, there are four magnitudes, the two bases AF and FH, and the two solids AU and UH, and equal multiples have been taken of the base AF and the solid AU— (namely), the base LF and the solid LU—and of the base HF and the solid HU—(namely), the base NF and the solid NU. And it has been shown that if the base LF exceeds the base FN then the solid LU also exceeds the [solid] NU, and if (LF is) equal (to FN) then (LU is) equal (to NU), and if (LF is) less than (FN) then (LU is) less than (NU). Thus, as the base AF is to the base FH, so the solid AU (is) to the solid UH [Def. 5.5]. (Which is) the very thing it was required to show.