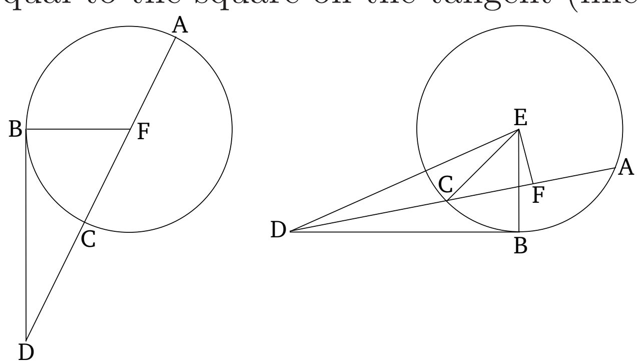
Book 3
Proposition 36

If some point is taken outside a circle, and two straight-lines radiate from it towards the circle, and (one) of them cuts the circle, and the (other) touches (it), then the (rectangle contained) by the whole (straight-line) cutting (the circle), and the (part of it) cut off outside (the circle), between the point and the convex circumference, will be equal to the square on the tangent (line).



For let some point D have been taken outside circle ABC, and let two straight-lines, $DC[A]$ and DB, radiate from D towards circle ABC. And let DCA cut circle ABC, and let BD touch (it). I say that the rectangle contained by AD and DC is equal to the square on DB.

$[D]CA$ is surely either through the center, or not. Let it first of all be through the center, and let F be the center of circle ABC, and let FB have been joined. Thus, (angle) FBD is a right-angle [Prop. 3.18]. And since straight-line AC is cut in half at F, let CD have been added to it. Thus, the (rectangle contained) by AD and DC plus the (square) on FC is equal to the (square) on FD [Prop. 2.6]. And FC (is) equal to FB. Thus, the (rectangle contained) by AD and DC plus the (square) on FB is equal to the (square) on FD. And the (square) on FD is equal to the (sum of the squares) on FB and BD [Prop. 1.47]. Thus, the (rectangle contained) by AD and DC plus the (square) on FB is equal to the (sum of the squares) on FB and BD. Let the (square) on FB have been subtracted from both. Thus, the remaining (rectangle contained) by AD and DC is equal to the (square) on the tangent DB.

And so let DCA not be through the center of circle ABC, and let the center E have been found, and let EF have been drawn from E, perpendicular to AC [Prop. 1.12]. And let EB, EC, and ED have been joined. (Angle) EBD (is) thus a right-angle [Prop. 3.18]. And since some straight-line, EF, through the center, cuts some (other) straight-line, AC, not through the center, at right-angles, it also cuts it in half [Prop. 3.3]. Thus, AF is equal to FC. And since the straight-line AC is cut in half at point F, let CD have been added to it. Thus, the (rectangle contained) by AD and DC plus the (square) on FC is equal to the (square) on FD [Prop. 2.6]. Let the (square) on FE have been added to both. Thus, the (rectangle contained) by AD and DC plus the (sum of the squares) on CF and FE is equal to the (sum of the squares) on FD and FE. But the (square) on EC is equal to the (sum of the squares) on CF and FE. For [angle] EFC [is] a right-angle [Prop. 1.47]. And the (square) on ED is equal to the (sum of the squares) on DF and FE [Prop. 1.47]. Thus, the (rectangle contained) by AD and DC plus the (square) on EC is equal to the (square) on ED. And EC (is) equal to EB. Thus, the (rectangle contained) by AD and DC plus the (square) on EB is equal to the (square) on ED. And the (sum of the squares) on EB and BD is equal to the (square) on ED. For EBD (is) a right-angle [Prop. 1.47]. Thus, the (rectangle contained) by AD and DC plus the (square) on EB is equal to the (sum of the squares) on EB and BD. Let the (square) on EB have been subtracted from both. Thus, the remaining (rectangle contained) by AD and DC is equal to the (square) on BD.

Thus, if some point is taken outside a circle, and two straight-lines radiate from it towards the circle, and (one) of them cuts the circle, and (the other) touches (it), then the (rectangle contained) by the whole (straight-line) cutting (the circle), and the (part of it) cut off outside (the circle), between the point and the convex circumference, will be equal to the square on the tangent (line). (Which is) the very thing it was required to show.