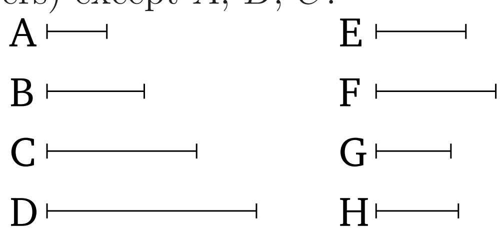
Book 9
Proposition 13

If any multitude whatsoever of numbers is continuously proportional, (starting) from a unit, and the (number) after the unit is prime, then the greatest (number) will be measured by no [other] (numbers) except (numbers) existing among the proportional numbers.

Let any multitude whatsoever of numbers, A, B, C, D, be continuously proportional, (starting) from a unit. And let the (number) after the unit, A, be prime. I say that the greatest of them, D, will be measured by no other (numbers) except A, B, C.



For, if possible, let it be measured by E, and let E not be the same as one of A, B, C. So it is clear that E is not prime. For if E is prime, and measures D, then it will also measure A, (despite A) being prime (and) not being the same as it [Prop. 9.12]. The very thing is impossible. Thus, E is not prime. Thus, (it is) composite. And every composite number is measured by some prime number [Prop. 7.31]. Thus, E is measured by some prime number. So I say that it will be measured by no other prime number than A. For if E is measured by another (prime number), and E measures D, then this (prime number) will thus also measure D. Hence, it will also measure A, (despite A) being prime (and) not being the same as it [Prop. 9.12]. The very thing is impossible. Thus, A measures E. And since E measures

D, let it measure it according to F. I say that F is not the same as one of A, B, C. For if F is the same as one of A, B, C, and measures D according to E, then one of A, B, C thus also measures D according to E. But one of A, B, C (only) measures D according to some (one) of A, B, C [Prop. 9.11]. And thus E is the same as one of A, B, C. The very opposite thing was assumed. Thus, F is not the same as one of A, B, C. Similarly, we can show that F is measured by A, (by) again showing that F is not prime. For if (F is prime), and measures D, then it will also measure A, (despite A) being prime (and) not being the same as it [Prop. 9.12]. The very thing is impossible. Thus, F is not prime. Thus, (it is) composite. And every composite number is measured by some prime number [Prop. 7.31]. Thus, F is measured by some prime number. So I say that it will be measured by no other prime number than A. For if some other prime (number) measures F, and F measures D, then this (prime number) will thus also measure D. Hence, it will also measure A, (despite A) being prime (and) not being the same as it [Prop. 9.12]. The very thing is impossible. Thus, A measures F. And since E measures D according to F, E has thus made D (by) multiplying F. But, in fact, A has also made D (by) multiplying C [Prop. 9.11 corr.]. Thus, the (number created) from (multiplying) A, C is equal to the (number created) from (multiplying) E, F. Thus, proportionally, as A is to E, so F (is) to C [Prop. 7.19]. And A measures E. Thus, F also measures C. Let it measure it according to G. So, similarly, we can show that G is not the same as one of A, B, and that it is measured by A. And since F measures C according to G, F has thus made C (by) multiplying G. But, in fact, A has also made C (by) multiplying B [Prop. 9.11 corr.]. Thus, the (number created) from (multiplying) A, B is equal to the (number created) from (multiplying) F, G. Thus, proportionally, as A (is) to F, so G (is) to B [Prop. 7.19]. And A measures F. Thus, G also measures B. Let it measure it according to H. So, similarly, we can show that H is not the same as A. And since G measures B according to H, G has thus made B (by) multiplying H. But, in fact, A has also made B (by) multiplying itself [Prop. 9.8]. Thus, the (number created) from (multiplying) H, G is equal to the square on A. Thus, as H is to A, (so) A (is) to G [Prop. 7.19]. And A measures G. Thus, H also measures A, (despite A) being prime (and) not being the same as it. The very thing (is) absurd. Thus, the greatest (number) D cannot be measured by another (number) except (one of) A, B, C. (Which is) the very thing it was required to show.