# Book 5
Proposition 16

If four magnitudes are proportional then they will also be proportional alternately.

Let A, B, C and D be four proportional magnitudes, (such that) as A (is) to B, so C (is) to D. I say that they will also be [proportional] alternately, (so that) as A (is) to C, so B (is) to D.

For let the equal multiples E and F have been taken of A and B (respectively), and the other random equal multiples G and H of C and D (respectively).

A ⊢——⊣   C ⊢——⊣

B ⊢—⊣   D ⊢—⊣

E ⊢——+——+——⊣   G ⊢——+——⊣

F ⊢——+——+—⊣   H ⊢—+—+—⊣
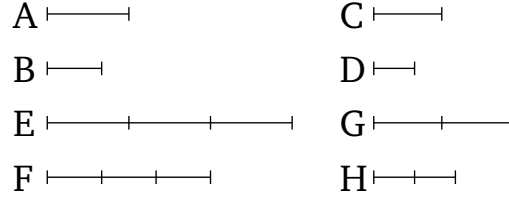
And since E and F are equal multiples of A and B (respectively), and parts have the same ratio as similar multiples [Prop. 5.15], thus as A is to B, so E (is) to F. But as A (is) to B, so C (is) to D. And, thus, as C (is) to D, so E (is) to F [Prop. 5.11]. Again, since G and H are equal multiples of C and D (respectively), thus as C is to D, so G (is) to H [Prop. 5.15]. But as C (is) to D, [so] E (is) to F. And, thus, as E (is) to F, so G (is) to H [Prop. 5.11]. And if four magnitudes are proportional, and the first is greater than the third then the second will also be greater than the fourth, and if (the first is) equal (to the third then the second will also be) equal (to the fourth), and if (the first is) less (than the third then the second will also be) less (than

the fourth) [Prop. 5.14]. Thus, if E exceeds G then F also exceeds H, and if (E is) equal (to G then F is also) equal (to H), and if (E is) less (than G then F is also) less (than H). And E and F are equal multiples of A and B (respectively), and G and H other random equal multiples of C and D (respectively). Thus, as A is to C, so B (is) to D [Def. 5.5].

Thus, if four magnitudes are proportional then they will also be proportional alternately. (Which is) the very thing it was required to show.