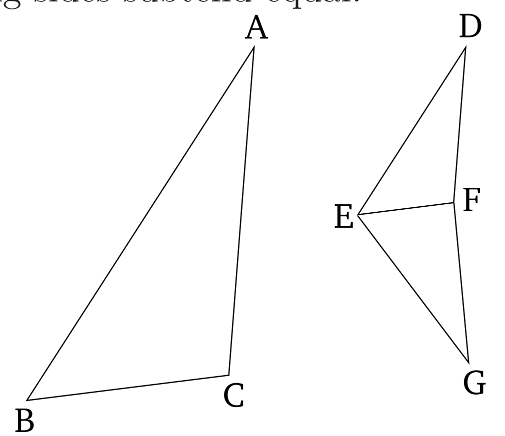
Book 6
Proposition 5

If two triangles have proportional sides then the triangles will be equiangular, and will have the angles which corresponding sides subtend equal.



Let ABC and DEF be two triangles having proportional sides, (so that) as AB (is) to BC, so DE (is) to EF, and as BC (is) to CA, so EF (is) to FD, and, further, as BA (is) to AC, so ED (is) to DF. I say that triangle ABC is equiangular to triangle DEF, and (that the triangles) will have the angles which corresponding sides subtend equal. (That is), (angle) ABC (equal) to DEF, BCA to EFD, and, further, BAC to EDF.

For let (angle) FEG, equal to angle ABC, and (angle) EFG, equal to ACB, have been constructed on the straight-line EF at the points E and F on it (respectively) [Prop. 1.23]. Thus, the remaining (angle) at A is equal to the remaining (angle) at G [Prop. 1.32].

Thus, triangle ABC is equiangular to [triangle] EGF. Thus, for triangles ABC and EGF, the sides about the equal angles are proportional, and (those) sides subtending equal angles correspond [Prop. 6.4]. Thus, as AB is to BC, [so] GE (is) to EF. But, as AB (is) to BC, so, it was assumed, (is) DE to EF. Thus, as DE (is) to EF, so GE (is) to EF [Prop. 5.11]. Thus, DE and GE each have the same ratio to EF. Thus, DE is equal to GE [Prop. 5.9]. So, for the same (reasons), DF is also equal to GF. Therefore, since DE is equal to EG, and EF (is) common, the two (sides) DE, EF are equal to the two (sides) GE, EF (respectively). And base DF [is] equal to base FG. Thus, angle DEF is equal to angle GEF [Prop. 1.8], and triangle DEF (is) equal to triangle GEF, and the remaining angles (are) equal to the remaining angles which the equal sides subtend [Prop. 1.4]. Thus, angle DFE is also equal to GFE, and (angle) EDF to EGF. And since (angle) FED is equal to GEF, and (angle) GEF to ABC, angle ABC is thus also equal to DEF. So, for the same (reasons), (angle) ACB is also equal to DFE, and, further, the (angle) at A to the (angle) at D. Thus, triangle ABC is equiangular to triangle DEF.

Thus, if two triangles have proportional sides then the triangles will be equiangular, and will have the angles which corresponding sides subtend equal. (Which is) the very thing it was required to show.