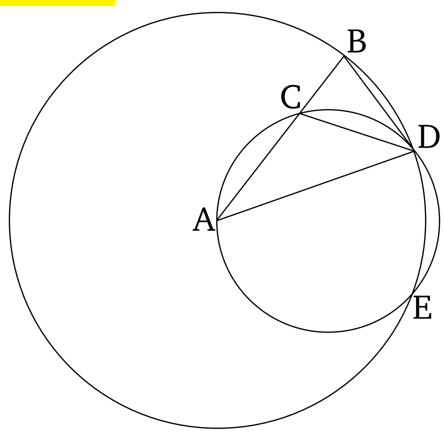
Book 4
Proposition 10

To construct an isosceles triangle having each of the angles at the base double the remaining (angle).

Let some straight-line AB be taken, and let it have been cut at point C so that the rectangle contained by AB and BC is equal to the square on CA [Prop. 2.11]. And let the circle BDE have been drawn with center A, and radius AB. And let the straight-line BD, equal to the straight-line AC, being not greater than the diameter of circle BDE, have been inserted into circle BDE [Prop. 4.1]. And let AD and DC have been joined. And let the circle ACD have been circumscribed about triangle ACD [Prop. 4.5].



And since the (rectangle contained) by AB and BC is equal to the (square) on AC, and AC (is) equal to BD, the (rectangle contained) by AB and BC is thus equal to the (square) on BD. And since some point B has been taken outside of circle ACD, and two straight-lines BA and BD have radiated from B towards the circle ACD, and (one) of them cuts (the circle), and (the other) meets (the circle), and the (rectangle contained) by AB and BC is equal to the (square) on BD, BD thus touches circle ACD [Prop. 3.37]. Therefore, since BD touches (the circle), and DC has been drawn across (the circle) from the point of contact D, the angle BDC is thus equal to the angle DAC in the alternate segment of the circle [Prop. 3.32]. Therefore, since BDC is equal to DAC, let CDA have been added to both. Thus, the whole of BDA is equal to the two (angles) CDA and DAC. But, the external (angle) BCD is equal to CDA and DAC [Prop. 1.32]. Thus, BDA is also equal to BCD. But, BDA is equal to CBD, since the side AD is also equal to AB [Prop. 1.5]. So that DBA is also equal to BCD. Thus, the three (angles) BDA, DBA, and BCD are equal to one another. And since angle DBC is equal to BCD, side BD is also equal to side DC [Prop. 1.6]. But, BD was assumed (to be) equal to CA. Thus, CA is also equal to CD. So that angle CDA is also equal to angle DAC [Prop. 1.5]y. Thus, CDA and DAC is double DAC. But BCD (is) equal to CDA and DAC. Thus, BCD is also double CAD. And BCD (is) equal to to each of BDA and DBA. Thus, BDA and DBA are each double DAB.

Thus, the isosceles triangle ABD has been constructed having each of the angles at the base BD double the remaining (angle). (Which is) the very thing it was required to do.