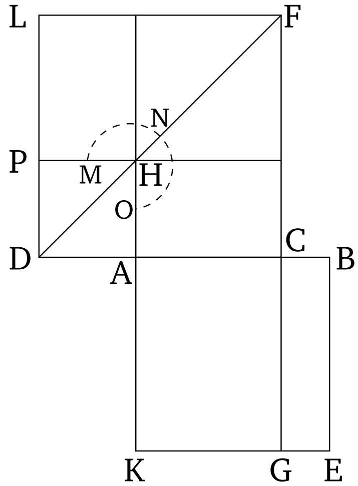
Book 13
Proposition 1

If a straight-line is cut in extreme and mean ratio then the square on the greater piece, added to half of the whole, is five times the square on the half.



For let the straight-line AB have been cut in extreme and mean ratio at point C, and let AC be the greater piece. And let the straight-line AD have been produced in a straight-line with CA. And let AD be made (equal to) half of AB. I say that the (square) on CD is five times the (square) on DA.

For let the squares AE and DF have been described on AB and DC (respectively). And let the figure in DF have been drawn. And let FC have been drawn across to G. And since AB has been cut in extreme and mean ratio at C, the (rectangle contained) by ABC is thus equal to the (square) on AC [Def. 6.3, Prop. 6.17]. And CE is the (rectangle contained) by ABC, and FH the (square) on AC. Thus, CE (is) equal to FH. And since BA is double AD, and BA (is) equal to KA, and AD to AH, KA (is) thus also double AH. And as KA (is) to AH, so CK (is) to CH [Prop. 6.1]. Thus, CK (is) double CH. And LH plus HC is also double CH [Prop. 1.43]. Thus, KC (is) equal to LH plus HC. And CE was also shown (to be) equal to HF. Thus, the whole square AE is equal to the gnomon MNO. And since BA is double AD, the (square) on BA is four times the (square) on AD—that is to say, AE (is four times) DH. And AE (is) equal to gnomon MNO. And, thus, gnomon MNO is also four times AP. Thus, the whole of DF is five times AP. And DF is the (square) on DC, and AP the (square) on DA. Thus, the (square) on CD is five times the (square) on DA.

Thus, if a straight-line is cut in extreme and mean ratio then the square on the greater piece, added to half of the whole, is five times the square on the half. (Which is) the very thing it was required to show.