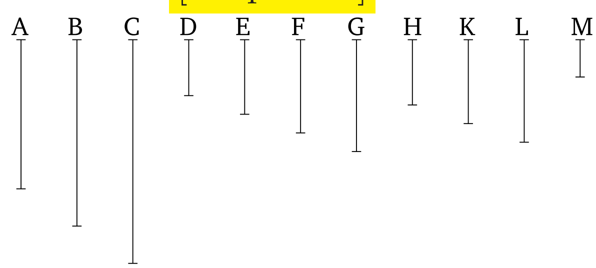
Book 7
Proposition 33

To find the least of those (numbers) having the same ratio as any given multitude of numbers.

Let A, B, and C be any given multitude of numbers. So it is required to find the least of those (numbers) having the same ratio as A, B, and C.

For A, B, and C are either prime to one another, or not. In fact, if A, B, and C are prime to one another then they are the least of those (numbers) having the same ratio as them [Prop. 7.22].



And if not, let the greatest common measure, D, of A, B, and C have be taken [Prop. 7.3]. And as many times as D measures A, B, C, so many units let there be in E, F, G, respectively. And thus E, F, G measure A, B, C, respectively, according to the units in D [Prop. 7.15]. Thus, E, F, G measure A, B, C (respectively) an equal number of times. Thus, E, F, G are in the same ratio as A, B, C (respectively) [Def. 7.20]. So I say that (they are) also the least (of those numbers having the same ratio as A, B, C). For if E, F, G are not the least of those (numbers) having the same ratio as A, B, C (respectively), then there will be [some] numbers less than E, F, G which are in the same ratio as

A, B, C (respectively). Let them be H, K, L. Thus, H measures A the same number of times that K, L also measure B, C, respectively. And as many times as H measures A, so many units let there be in M. Thus, K, L measure B, C, respectively, according to the units in M. And since H measures A according to the units in M, M thus also measures A according to the units in H [Prop. 7.15]. So, for the same (reasons), M also measures B, C according to the units in K, L, respectively. Thus, M measures A, B, and C. And since H measures A according to the units in M, H has thus made A (by) multiplying M. So, for the same (reasons), E has also made A (by) multiplying D. Thus, the (number created) from (multiplying) E and D is equal to the (number created) from (multiplying) H and M. Thus, as E (is) to H, so M (is) to D [Prop. 7.19]. And E (is) greater than H. Thus, M (is) also greater than D [Prop. 5.13]. And (M) measures A, B, and C. The very thing is impossible. For D was assumed (to be) the greatest common measure of A, B, and C. Thus, there cannot be any numbers less than E, F, G which are in the same ratio as A, B, C (respectively). Thus, E, F, G are the least of (those numbers) having the same ratio as A, B, C (respectively). (Which is) the very thing it was required to show.