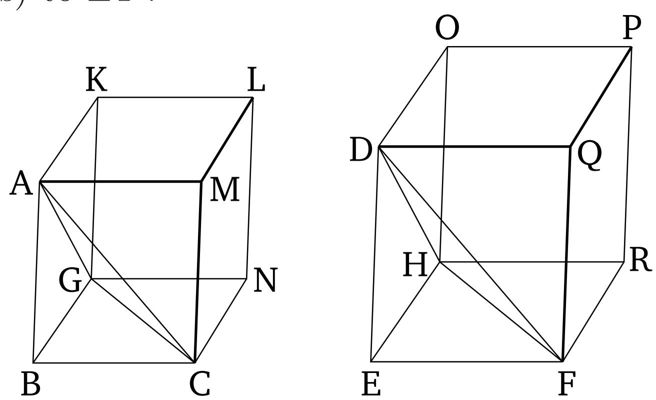
Book 12
Proposition 8

Similar pyramids which also have triangular bases are in the cubed ratio of their corresponding sides.

Let there be similar, and similarly laid out, pyramids whose bases are triangles ABC and DEF, and apexes the points G and H (respectively). I say that pyramid $ABCG$ has to pyramid $DEFH$ the cubed ratio of that BC (has) to EF.



For let the parallelepiped solids $BGML$ and $EHQP$ have been completed. And since pyramid $ABCG$ is similar to pyramid $DEFH$, angle ABC is thus equal to angle DEF, and GBC to HEF, and ABG to DEH. And as AB is to DE, so BC (is) to EF, and BG to EH [Def. 11.9]. And since as AB is to DE, so BC (is) to EF, and (so) the sides around equal angles are proportional, parallelogram BM is thus similar to paralleleogram EQ. So, for the same (reasons), BN is also similar to ER, and BK to EO. Thus, the three (parallelograms) MB, BK, and BN are similar to the three (parallelograms) EQ, EO, ER (respectively). But, the three (parallelograms) MB, BK, and BN are (both) equal and similar to the three opposite (parallelograms), and the three (parallelograms) EQ, EO, and ER are (both) equal and similar to the three opposite (parallelograms) [Prop. 11.24]. Thus, the solids $BGML$ and $EHQP$ are contained by equal numbers of similar (and similarly laid out) planes. Thus, solid $BGML$ is similar to solid $EHQP$ [Def. 11.9]. And similar parallelepiped solids are in the cubed ratio of corresponding sides [Prop. 11.33]. Thus, solid $BGML$ has to solid $EHQP$ the cubed ratio that the corresponding side BC (has) to the corresponding side EF. And as solid $BGML$ (is) to solid $EHQP$, so pyramid $ABCG$ (is) to pyramid $DEFH$, inasmuch as the pyramid is the sixth part of the solid, on account of the prism, being half of the parallelepiped solid [Prop. 11.28], also being three times the pyramid [Prop. 12.7]. Thus, pyramid $ABCG$ also has to pyramid $DEFH$ the cubed ratio that BC (has) to EF. (Which is) the very thing it was required to show.

Corollary

So, from this, (it is) also clear that similar pyramids having polygonal bases (are) to one another as the cubed ratio of their corresponding sides. For, dividing them into the pyramids (contained) within them which have triangular bases, with the similar polygons of the bases also being divided into similar triangles (which are) both equal in number, and corresponding, to the wholes [Prop. 6.20]. As one pyramid having a triangular base in the former (pyramid having a polygonal base is) to one pyramid having a triangular base in the latter (pyramid having a polygonal base), so (the sum of) all the pyramids having triangular bases in the former pyramid will also be to (the sum of) all the pyramids having triangular bases in the latter pyramid [Prop. 5.12]—that is to say, the (former) pyramid itself having a polygonal base to the (latter) pyramid having a polygonal base. And a pyramid having a triangular base is to a (pyramid) having a triangular base in the cubed ratio of corresponding sides [Prop. 12.8][1]. Thus, a (pyramid) having a polygonal base also has to to a (pyramid) having a similar base the cubed ratio of a (corresponding) side to a (corresponding) side.

[1]Since this explicitly references the Proposition which this is corollary to there is no arrow in the graph windo.