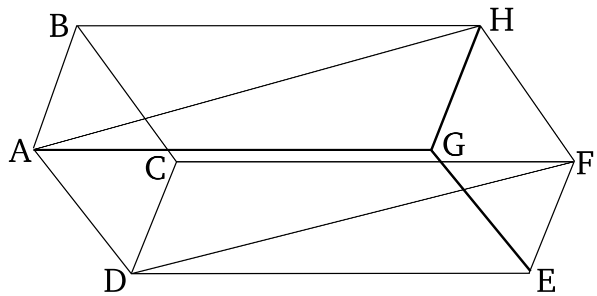
Book 11
Proposition 24

If a solid (figure) is contained by (six) parallel planes then its opposite planes are both equal and parallelogrammic.



For let the solid (figure) $CDHG$ have been contained by the parallel planes AC, GF, and AH, DF, and BF, AE. I say that its opposite planes are both equal and parallelogrammic.

For since the two parallel planes BG and CE are cut by the plane AC, their common sections are parallel [Prop. 11.16]. Thus, AB is parallel to DC. Again, since the two parallel planes BF and AE are cut by the plane AC, their common sections are parallel [Prop. 11.16]. Thus, BC is parallel to AD. And AB was also shown (to be) parallel to DC. Thus, AC is a parallelogram. So, similarly, we can also show that DF, FG, GB, BF, and AE are each parallelograms.

Let AH and DF have been joined. And since AB is parallel to DC, and BH to CF, so the two (straight-lines) joining one another, AB and BH, are parallel to the two straight-lines joining one another, DC and CF (respectively), not (being) in the same plane. Thus, they will contain equal angles [Prop. 11.10]. Thus, angle ABH (is) equal to (angle) DCF. And since the two (straight-lines) AB and BH are equal to the two (straight-lines) DC and CF (respectively) [Prop. 1.34], and angle ABH is equal to angle DCF, the base AH is thus equal to the base DF, and triangle ABH is equal to triangle DCF [Prop. 1.4]. And parallelogram BG is double (triangle) ABH, and parallelogram CE double (triangle) DCF [Prop. 1.34]. Thus, parallelogram BG (is) equal to parallelogram CE. So, similarly, we can show that AC is also equal to GF, and AE to BF.

Thus, if a solid (figure) is contained by (six) parallel planes then its opposite planes are both equal and parallelogrammic. (Which is) the very thing it was required to show.