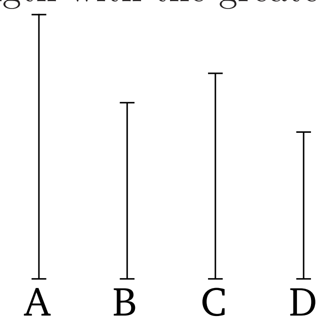
Book 10
Proposition 31

To find two medial (straight-lines), commensurable in square only, (and) containing a rational (area), such that the square on the greater is larger than the (square on the) lesser by the (square) on (some straight-line) commensurable in length with the greater.



Let two rational (straight-lines), A and B, commensurable in square only, be laid out, such that the square on the greater A is larger than the (square on the) lesser B by the (square) on (some straight-line) commensurable in length with (A) [Prop. 10.29]. And let the (square) on C be equal to the (rectangle contained) by A and B. And the (rectangle contained by) A and B (is) medial [Prop. 10.21]. Thus, the (square) on C (is) also medial. Thus, C (is) also medial [Prop. 10.21]. And let the (rectangle contained) by C and D be equal to the (square) on B. And the (square) on B (is) rational. Thus, the (rectangle contained) by C and D (is) also rational. And since as A is to B, so the (rectangle contained) by A and B (is) to the (square) on B [Prop. 10.21 lem.], but the (square) on C is equal to the (rectangle contained) by A and B, and the (rectangle contained) by C and D to the (square) on B, thus as A (is) to B, so the (square) on C (is) to the (rectangle contained) by C and D. And as the (square) on C (is) to the (rectangle contained) by C and D, so C (is) to D [Prop. 10.21 lem.]. And thus as A (is) to B, so C (is) to D. And A is commensurable in square only with B. Thus, C (is) also commensurable in square only with D [Prop. 10.11]. And C is medial. Thus, D (is) also medial [Prop. 10.23]. And since as A is to B, (so) C (is) to D, and the square on A is greater than (the square on) B by the (square) on (some straight-line) commensurable (in length) with (A), the square on C is thus also greater than (the square on) D by the (square) on (some straight-line) commensurable (in length) with (C) [Prop. 10.14].

Thus, two medial (straight-lines), C and D, commensurable in square only, (and) containing a rational (area), have been found. And the square on C is greater than (the square on) D by the (square) on (some straight-line) commensurable in length with (C).

So, similarly, (the proposition) can also be demonstrated for (some straight-line) incommensurable (in length with C), provided that the square on A is greater than (the square on B) by the (square) on (some straight-line) incommensurable (in length) with (A) [Prop. 10.30].