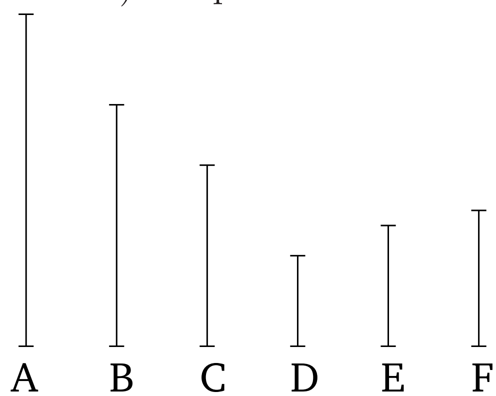
Book 7
Proposition 3

To find the greatest common measure of three given numbers (which are) not prime to one another.



Let A, B, and C be the three given numbers (which are) not prime to one another. So it is required to find the greatest common measure of A, B, and C.

For let the greatest common measure, D, of the two (numbers) A and B have been taken [Prop. 7.2]. So D either measures, or does not measure, C. First of all, let it measure (C). And it also measures A and B. Thus, D measures A, B, and C. Thus, D is a common measure of A, B, and C. So I say that (it is) also the greatest (common measure). For if D is not the greatest common measure of A, B, and C then some number greater than D will measure the numbers A, B, and C. Let it (so) measure (A, B, and C), and let it be E. Therefore, since E measures A, B, and C, it will thus also measure A and B. Thus, it will also measure the greatest common measure of A and B [Prop. 7.2 corr.]. And D is the greatest common measure of A and B. Thus, E measures

D, the greater (measuring) the lesser. The very thing is impossible. Thus, some number which is greater than D cannot measure the numbers A, B, and C. Thus, D is the greatest common measure of A, B, and C.

So let D not measure C. I say, first of all, that C and D are not prime to one another. For since A, B, C are not prime to one another, some number will measure them. So the (number) measuring A, B, and C will also measure A and B, and it will also measure the greatest common measure, D, of A and B [Prop. 7.2 corr.]. And it also measures C. Thus, some number will measure the numbers D and C. Thus, D and C are not prime to one another. Therefore, let their greatest common measure, E, have been taken [Prop. 7.2]. And since E measures D, and D measures A and B, E thus also measures A and B. And it also measures C. Thus, E measures A, B, and C. Thus, E is a common measure of A, B, and C. So I say that (it is) also the greatest (common measure). For if E is not the greatest common measure of A, B, and C then some number greater than E will measure the numbers A, B, and C. Let it (so) measure (A, B, and C), and let it be F. And since F measures A, B, and C, it also measures A and B. Thus, it will also measure the greatest common measure of A and B [Prop. 7.2 corr.]. And D is the greatest common measure of A and B. Thus, F measures D. And it also measures C. Thus, F measures D and C. Thus, it will also measure the greatest common measure of D and C [Prop. 7.2 corr.]. And E is the greatest common measure of D and C. Thus, F measures E, the greater (measuring) the lesser.

The very thing is impossible. Thus, some number which is greater than E does not measure the numbers A, B, and C. Thus, E is the greatest common measure of A, B, and C. (Which is) the very thing it was required to show.