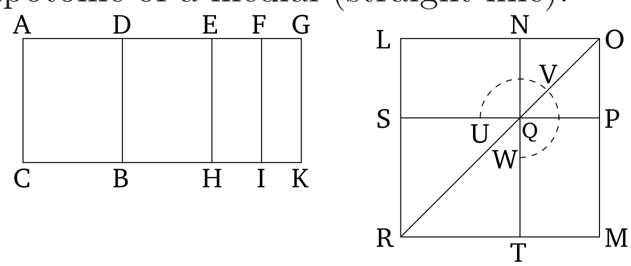
Book 10
Proposition 93

If an area is contained by a rational (straight-line) and a third apotome then the square-root of the area is a second apotome of a medial (straight-line).



For let the area AB have been contained by the rational (straight-line) AC and the third apotome AD. I say that the square-root of area AB is the second apotome of a medial (straight-line).

For let DG be an attachment to AD. Thus, AG and GD are rational (straight-lines which are) commensurable in square only [Prop. 10.73], and neither of AG and GD is commensurable in length with the (previously) laid down rational (straight-line) AC, and the square on the whole, AG, is greater than (the square on) the attachment, DG, by the (square) on (some straight-line) commensurable (in length) with (AG) [Def. 10.13]. Therefore, since the square on AG is greater than (the square on) GD by the (square) on (some straight-line) commensurable (in length) with (AG), thus if (an area) equal to the fourth part of the square on DG is applied to AG, falling short by a square figure, then it divides (AG) into (parts which are) commensurable (in length) [Prop. 10.17]. Therefore, let DG have been cut in half at E. And let (an area) equal to the (square) on EG have been applied to AG, falling short by a square figure. And let it be the (rectangle contained) by AF and FG. And let EH, FI, and GK have been drawn through points E, F, and G (respectively), parallel to AC. Thus, AF and FG are commensurable (in length). AI (is) thus also commensurable with FK [Props. 6.1, 10.11]. And since AF and FG are commensurable in length, AG is thus also commensurable in length with each of AF and FG [Prop. 10.15]. And AG (is) rational, and incommensurable in length with AC. Hence, AF and FG (are) also (rational, and incommensurable in length with AC) [Prop. 10.13]. Thus, AI and FK are each medial (areas) [Prop. 10.21]. Again, since DE is commensurable in length with EG, DG is also commensurable in length with each of DE and EG [Prop. 10.15]. And GD (is) rational, and incommensurable in length with AC. Thus, DE and EG (are) each also rational, and incommensurable in length with AC [Prop. 10.13]. DH and EK are thus each medial (areas) [Prop. 10.21]. And since AG and GD are commensurable in square only, AG is thus incommensurable in length with GD. But, AG is commensurable in length with AF, and DG with EG. Thus, AF is incommensurable in length with EG [Prop. 10.13]. And as AF (is) to EG, so AI is to EK [Prop. 6.1]. Thus, AI is incommensurable with EK [Prop. 10.11].

Therefore, let the square LM, equal to AI, have been constructed. And let NO, equal to FK, which is about the same angle as LM, have been subtracted (from LM).

Thus, LM and NO are about the same diagonal [Prop. 6.26]. Let PR be their (common) diagonal, and let the (rest of the) figure have been drawn. Therefore, since the (rectangle contained) by AF and FG is equal to the (square) on EG, thus as AF is to EG, so EG (is) to FG [Prop. 6.17]. But, as AF (is) to EG, so AI is to EK [Prop. 6.1]. And as EG (is) to FG, so EK is to FK [Prop. 6.1]. And thus as AI (is) to EK, so EK (is) to FK [Prop. 5.11]. Thus, EK is the mean proportional to AI and FK. And MN is also the mean proportional to the squares LM and NO [Prop. 10.53 lem.]. And AI is equal to LM, and FK to NO. Thus, EK is also equal to MN. But, MN is equal to LO, and EK [is] equal to DH [Prop. 1.43]. And thus the whole of DK is equal to the gnomon UVW and NO. And AK (is) also equal to LM and NO. Thus, the remainder AB is equal to ST—that is to say, to the square on LN. Thus, LN is the square-root of area AB. I say that LN is the second apotome of a medial (straight-line).

For since AI and FK were shown (to be) medial (areas), and are equal to the (squares) on LP and PN (respectively), the (squares) on each of LP and PN (are) thus also medial. Thus, LP and PN (are) each medial (straight-lines). And since AI is commensurable with FK [Props. 6.1, 10.11], the (square) on LP (is) thus also commensurable with the (square) on PN. Again, since AI was shown (to be) incommensurable with EK, LM is thus also incommensurable with MN—that is to say, the (square) on LP with the (rectangle contained) by LP and PN. Hence, LP is also incommensurable in length with PN [Props. 6.1, 10.11]. Thus, LP and PN are medial (straight-lines which are) commensurable in square only. So, I say that they also contain a medial (area).

For since EK was shown (to be) a medial (area), and is equal to the (rectangle contained) by LP and PN, the (rectangle contained) by LP and PN is thus also medial. Hence, LP and PN are medial (straight-lines which are) commensurable in square only, and which contain a medial (area). Thus, LN is the second apotome of a medial (straight-line) [Prop. 10.75]. And it is the square-root of area AB.

Thus, the square-root of area AB is the second apotome of a medial (straight-line). (Which is) the very thing it was required to show.