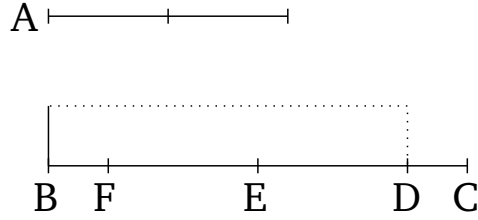# Book 10
Proposition 17

If there are two unequal straight-lines, and a (rectangle) equal to the fourth part of the (square) on the lesser, falling short by a square figure, is applied to the greater, and divides it into (parts which are) commensurable in length, then the square on the greater will be larger than (the square on) the lesser by the (square) on (some straight-line) commensurable [in length] with the greater. And if the square on the greater is larger than (the square on) the lesser by the (square) on (some straight-line) commensurable [in length] with the greater, and a (rectangle) equal to the fourth (part) of the (square) on the lesser, falling short by a square figure, is applied to the greater, then it divides it into (parts which are) commensurable in length.

Let A and BC be two unequal straight-lines, of which (let) BC (be) the greater. And let a (rectangle) equal to the fourth part of the (square) on the lesser, A—that is, (equal) to the (square) on half of A—falling short by a square figure, have been applied to BC. And let it be the (rectangle contained) by BD and DC [see previous lemma]. And let BD be commensurable in length with DC. I say that that the square on BC is greater than the (square on) A by (the square on some straight-line) commensurable (in length) with (BC).

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B   F        E          D  C
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For let BC have been cut in half at the point E [Prop. 1.10].
And let EF be made equal to DE [Prop. 1.3]. Thus, the
remainder DC is equal to BF. And since the straight-
line BC has been cut into equal (pieces) at E, and into
unequal (pieces) at D, the rectangle contained by BD
and DC, plus the square on ED, is thus equal to the
square on EC [Prop. 2.5]. (The same) also (for) the
quadruples. Thus, four times the (rectangle contained)
by BD and DC, plus the quadruple of the (square) on
DE, is equal to four times the square on EC. But, the
square on A is equal to the quadruple of the (rectan-
gle contained) by BD and DC, and the square on DF
is equal to the quadruple of the (square) on DE. For
DF is double DE. And the square on BC is equal to
the quadruple of the (square) on EC. For, again, BC is
double CE. Thus, the (sum of the) squares on A and DF
is equal to the square on BC. Hence, the (square) on BC
is greater than the (square) on A by the (square) on DF.
Thus, BC is greater in square than A by DF. It must
also be shown that BC is commensurable (in length)
with DF. For since BD is commensurable in length
with DC, BC is thus also commensurable in length with
CD [Prop. 10.15]. But, CD is commensurable in length
with CD plus BF. For CD is equal to BF [Prop. 10.6].
Thus, BC is also commensurable in length with BF plus

CD [Prop. 10.12] . Hence, BC is also commensurable in length with the remainder FD [Prop. 10.15] . Thus, the square on BC is greater than (the square on) A by the (square) on (some straight-line) commensurable (in length) with (BC).

And so let the square on BC be greater than the (square on) A by the (square) on (some straight-line) commensurable (in length) with (BC). And let a (rectangle) equal to the fourth (part) of the (square) on A, falling short by a square figure, have been applied to BC. And let it be the (rectangle contained) by BD and DC. It must be shown that BD is commensurable in length with DC.

For, similarly, by the same construction, we can show that the square on BC is greater than the (square on) A by the (square) on FD. And the square on BC is greater than the (square on) A by the (square) on (some straight-line) commensurable (in length) with (BC). Thus, BC is commensurable in length with FD. Hence, BC is also commensurable in length with the remaining sum of BF and DC [Prop. 10.15] . But, the sum of BF and DC is commensurable [in length] with DC [Prop. 10.6] . Hence, BC is also commensurable in length with CD [Prop. 10.12] . Thus, via separation, BD is also commensurable in length with DC [Prop. 10.15] .

Thus, if there are two unequal straight-lines, and so on . . . .