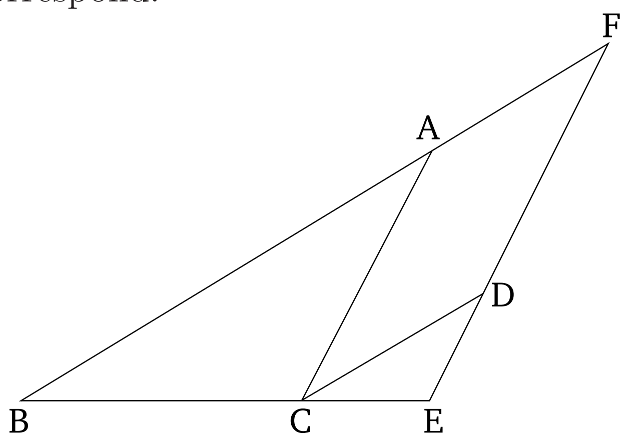
Book 6
Proposition 4

In equiangular triangles the sides about the equal angles are proportional, and those (sides) subtending equal angles correspond.



Let ABC and DCE be equiangular triangles, having angle ABC equal to DCE, and (angle) BAC to CDE, and, further, (angle) ACB to CED. I say that in triangles ABC and DCE the sides about the equal angles are proportional, and those (sides) subtending equal angles correspond.

Let BC be placed straight-on to CE. And since angles ABC and ACB are less than two right-angles [Prop 1.17], and ACB (is) equal to DEC, thus ABC and DEC are less than two right-angles. Thus, BA and ED, being produced, will meet [C.N. 5]. Let them have been produced, and let them meet at (point) F.

And since angle DCE is equal to ABC, BF is parallel to CD [Prop. 1.28]. Again, since (angle) ACB is equal to DEC, AC is parallel to FE [Prop. 1.28]. Thus,

$FACD$ is a parallelogram. Thus, FA is equal to DC, and AC to FD [Prop. 1.34]. And since AC has been drawn parallel to one (of the sides) FE of triangle FBE, thus as BA is to AF, so BC (is) to CE [Prop. 6.2]. And AF (is) equal to CD. Thus, as BA (is) to CD, so BC (is) to CE, and, alternately, as AB (is) to BC, so DC (is) to CE [Prop. 5.16]. Again, since CD is parallel to BF, thus as BC (is) to CE, so FD (is) to DE [Prop. 6.2]. And FD (is) equal to AC. Thus, as BC is to CE, so AC (is) to DE, and, alternately, as BC (is) to CA, so CE (is) to ED [Prop. 6.2]. Therefore, since it was shown that as AB (is) to BC, so DC (is) to CE, and as BC (is) to CA, so CE (is) to ED, thus, via equality, as BA (is) to AC, so CD (is) to DE [Prop. 5.22].

Thus, in equiangular triangles the sides about the equal angles are proportional, and those (sides) subtending equal angles correspond. (Which is) the very thing it was required to show.