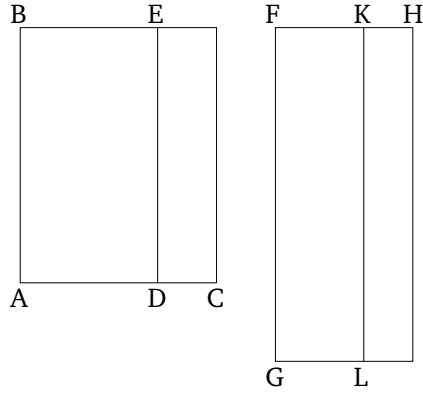# Book 10
Proposition 109

A rational (area) being subtracted from a medial (area), two other irrational (straight-lines) arise (as the square-root of the remaining area)—either a first apotome of a medial (straight-line), or that (straight-line) which with a rational (area) makes a medial whole.

For let the rational (area) BD have been subtracted from the medial (area) BC. I say that one of two irrational (straight-lines) arise (as) the square-root of the remaining (area), EC—either a first apotome of a medial (straight-line), or that (straight-line) which with a rational (area) makes a medial whole.

For let the rational (straight-line) FG be laid down, and let similar areas (to the preceding proposition) have been applied (to it). So, accordingly, FH is rational, and incommensurable in length with FG, and KF (is) also rational, and commensurable in length with FG. Thus, FH and FK are rational (straight-lines which are) commensurable in square only [Prop. 10.13]. KH is thus an apotome [Prop. 10.73], and FK an attachment to it. So, the square on HF is greater than (the square on) FK either by the (square) on (some straight-line) commensurable (in length) with (HF), or by the (square) on (some straight-line) incommensurable (in length with HF).

Therefore, if the square on HF is greater than (the square on) FK by the (square) on (some straight-line) commensurable (in length) with (HF), and (since) the attachment FK is commensurable in length with the (previously) laid down rational (straight-line) FG, KH is a second apotome [Def. 10.12]. And FG (is) rational. Hence, the square-root of LH—that is to say, (of) EC—is a first apotome of a medial (straight-line) [Prop. 10.92].

And if the square on HF is greater than (the square on) FK by the (square) on (some straight-line) incommensurable (in length with HF), and (since) the attachment FK is commensurable in length with the (previously) laid down rational (straight-line) FG, KH is a fifth apotome [Def. 10.15]. Hence, the square-root of EC is that (straight-line) which with a rational (area) makes a medial whole [Prop. 10.95]. (Which is) the very thing it was required to show.