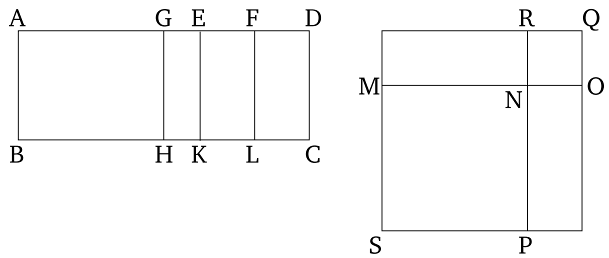
Book 10
Proposition 57

If an area is contained by a rational (straight-line) and a fourth binomial (straight-line) then the square-root of the area is the irrational (straight-line which is) called major.†



For let the area AC be contained by the rational (straight-line) AB and the fourth binomial (straight-line) AD, which has been divided into its (component) terms at E, of which let AE be the greater. I say that the square-root of AC is the irrational (straight-line which is) called major.

For since AD is a fourth binomial (straight-line), AE and ED are thus rational (straight-lines which are) commensurable in square only, and the square on AE is greater than (the square on) ED by the (square) on (some straight-line) incommensurable (in length) with (AE), and AE [is] commensurable in length with AB [Def. 10.8]. Let DE have been cut in half at F, and let the parallelogram (contained by) AG and GE, equal to the (square) on EF, (and falling short by a square figure) have been applied to AE. AG is thus incommensurable in length with GE [Prop. 10.18]. Let GH, EK, and FL have been drawn parallel to AB, and let the rest (of the construction) have been made the same as the (proposition) before this. So, it is clear that MO is the square-root of area AC. So, we must show that MO is the irrational (straight-line which is) called major.

Since AG is incommensurable in length with EG, AH is also incommensurable with GK—that is to say, SN with NQ [Props. 6.1, 10.11]. Thus, MN and NO are incommensurable in square. And since AE is commensurable in length with AB, AK is rational [Prop. 10.19]. And it is equal to the (sum of the squares) on MN and NO. Thus, the sum of the (squares) on MN and NO [is] also rational. And since DE [is] incommensurable in length with AB [Prop. 10.13]—that is to say, with EK—but DE is commensurable (in length) with EF, EF (is) thus incommensurable in length with EK [Prop. 10.13]. Thus, EK and EF are rational (straight-lines which are) commensurable in square only. LE—that is to say, MR—(is) thus medial [Prop. 10.21]. And it is contained by MN and NO. The (rectangle contained) by MN and NO is thus medial. And the [sum] of the (squares) on MN and NO (is) rational, and MN and NO are incommensurable in square. And if two straight-lines (which are) incommensurable in square, making the sum of the squares on them rational, and the (rectangle contained) by them medial, are added together, then the whole is the irrational (straight-line which is) called major [Prop. 10.39].

Thus, MO is the irrational (straight-line which is) called major. And (it is) the square-root of area AC. (Which is) the very thing it was required to show.