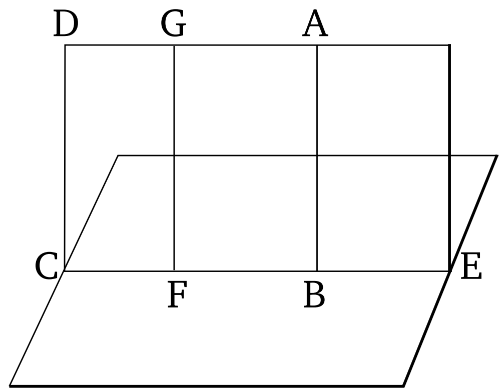
Book 11
Proposition 18

If a straight-line is at right-angles to some plane then all of the planes (passing) through it will also be at right-angles to the same plane.

For let some straight-line AB be at right-angles to a reference plane. I say that all of the planes (passing) through AB are also at right-angles to the reference plane.

For let the plane DE have been produced through AB. And let CE be the common section of the plane DE and the reference (plane). And let some random point F have been taken on CE. And let FG have been drawn from F, at right-angles to CE, in the plane DE [Prop. 1.11].

And since AB is at right-angles to the reference plane, AB is thus also at right-angles to all of the straight-lines joined to it which are also in the reference plane [Def. 11.3]. Hence, it is also at right-angles to CE. Thus, angle ABF is a right-angle. And GFB is also a right-angle. Thus, AB is parallel to FG [Prop. 1.28]. And AB is at right-angles to the reference plane. Thus, FG is also at right-angles to the reference plane [Prop. 11.8]. And a plane is at right-angles to a(nother) plane when the straight-lines drawn at right-angles to the common section of the planes, (and lying) in one of the planes, are at right-angles to the remaining plane [Def. 11.4]. And FG, (which was) drawn at right-angles to the common section of the planes, CE, in one of the planes, DE, was shown to be at right-angles to the reference plane. Thus, plane DE is at right-angles to the reference (plane). So, similarly, it can be shown that all of the planes (passing) at random through AB (are) at right-angles to the reference plane.



Thus, if a straight-line is at right-angles to some plane then all of the planes (passing) through it will also be at right-angles to the same plane. (Which is) the very thing it was required to show.