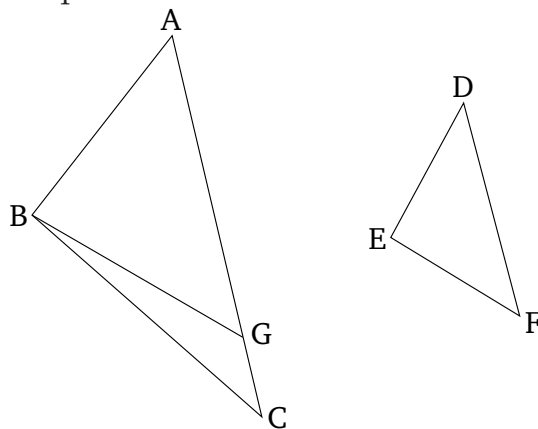


Book 6

Proposition 7

If two triangles have one angle equal to one angle, and the sides about other angles proportional, and the remaining angles either both less than, or both not less than, right-angles, then the triangles will be equiangular, and will have the angles about which the sides are proportional equal.



Let ABC and DEF be two triangles having one angle, BAC , equal to one angle, EDF (respectively), and the sides about (some) other angles, ABC and DEF (respectively), proportional, (so that) as AB (is) to BC , so DE (is) to EF , and the remaining (angles) at C and F , first of all, both less than right-angles. I say that triangle ABC is equiangular to triangle DEF , and (that) angle ABC will be equal to DEF , and (that) the remaining (angle) at C (will be) manifestly equal to the remaining (angle) at F .

For if angle ABC is not equal to (angle) DEF then one of them is greater. Let ABC be greater. And let (angle)

ABG , equal to (angle) DEF , have been constructed on the straight-line AB at the point B on it [Prop. 1.23].

And since angle A is equal to (angle) D , and (angle) ABG to DEF , the remaining (angle) AGB is thus equal to the remaining (angle) DFE [Prop. 1.32]. Thus, triangle ABG is equiangular to triangle DEF . Thus, as AB is to BG , so DE (is) to EF [Prop. 6.4]. And as DE (is) to EF , [so] it was assumed (is) AB to BC . Thus, AB has the same ratio to each of BC and BG [Prop. 5.11]. Thus, BC (is) equal to BG [Prop. 5.9]. And, hence, the angle at C is equal to angle BGC [Prop. 1.5]. And the angle at C was assumed (to be) less than a right-angle. Thus, (angle) BGC is also less than a right-angle. Hence, the adjacent angle to it, AGB , is greater than a right-angle [Prop. 1.13]. And (AGB) was shown to be equal to the (angle) at F . Thus, the (angle) at F is also greater than a right-angle. But it was assumed (to be) less than a right-angle. The very thing is absurd. Thus, angle ABC is not unequal to (angle) DEF . Thus, (it is) equal. And the (angle) at A is also equal to the (angle) at D . And thus the remaining (angle) at C is equal to the remaining (angle) at F [Prop. 1.32]. Thus, triangle ABC is equiangular to triangle DEF .

But, again, let each of the (angles) at C and F be assumed (to be) not less than a right-angle. I say, again, that triangle ABC is equiangular to triangle DEF in this case also.

For, with the same construction, we can similarly show that BC is equal to BG . Hence, also, the angle at C is equal to (angle) BGC . And the (angle) at C (is) not

less than a right-angle. Thus, BGC (is) not less than a right-angle either. So, in triangle BGC the (sum of) two angles is not less than two right-angles. The very thing is impossible [Prop. 1.17]. Thus, again, angle ABC is not unequal to DEF . Thus, (it is) equal. And the (angle) at A is also equal to the (angle) at D . Thus, the remaining (angle) at C is equal to the remaining (angle) at F [Prop. 1.32]. Thus, triangle ABC is equiangular to triangle DEF .

Thus, if two triangles have one angle equal to one angle, and the sides about other angles proportional, and the remaining angles both less than, or both not less than, right-angles, then the triangles will be equiangular, and will have the angles about which the sides (are) proportional equal. (Which is) the very thing it was required to show.