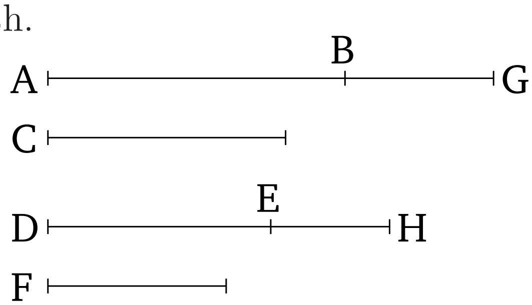
Book 5
Proposition 24

If a first (magnitude) has to a second the same ratio that third (has) to a fourth, and a fifth (magnitude) also has to the second the same ratio that a sixth (has) to the fourth, then the first (magnitude) and the fifth, added together, will also have the same ratio to the second that the third (magnitude) and sixth (added together, have) to the fourth.



For let a first (magnitude) AB have the same ratio to a second C that a third DE (has) to a fourth F. And let a fifth (magnitude) BG also have the same ratio to the second C that a sixth EH (has) to the fourth F. I say that the first (magnitude) and the fifth, added together, AG, will also have the same ratio to the second C that the third (magnitude) and the sixth, (added together), DH, (has) to the fourth F.

For since as BG is to C, so EH (is) to F, thus, inversely, as C (is) to BG, so F (is) to EH [Prop. 5.7 corr.]. Therefore, since as AB is to C, so DE (is) to F, and as C (is) to BG, so F (is) to EH, thus, via equality, as AB is to BG, so DE (is) to EH [Prop. 5.22]. And since separated magnitudes are proportional then they will also be proportional (when) composed [Prop. 5.18]. Thus, as AG is to GB, so DH (is) to HE. And, also, as BG is to C, so EH (is) to F. Thus, via equality, as AG is to C, so DH (is) to F [Prop. 5.22].

Thus, if a first (magnitude) has to a second the same ratio that a third (has) to a fourth, and a fifth (magnitude) also has to the second the same ratio that a sixth (has) to the fourth, then the first (magnitude) and the fifth, added together, will also have the same ratio to the second that the third (magnitude) and the sixth (added together, have) to the fourth. (Which is) the very thing it was required to show.