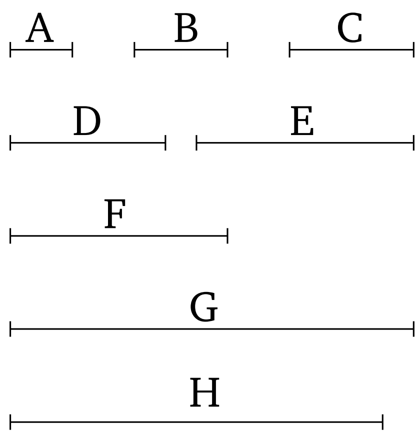
Book 7
Proposition 39

To find the least number that will have given parts.



Let A, B, and C be the given parts. So it is required to find the least number which will have the parts A, B, and C (*i.e.*, an Ath part, a Bth part, and a Cth part).

For let D, E, and F be numbers having the same names as the parts A, B, and C (respectively). And let the least number, G, measured by D, E, and F, have been taken [Prop. 7.36] .

Thus, G has parts called the same as D, E, and F [Prop. 7.37] . And A, B, and C are parts called the same as D, E, and F (respectively). Thus, G has the parts A, B, and C. So I say that (G) is also the least (number having the parts A, B, and C). For if not, there will be some number less than G which will have the parts A, B, and C. Let it be H. Since H has the parts A, B, and C, H will thus be measured by numbers called the same as the parts A, B, and C [Prop. 7.38] . And D, E, and F are numbers called the same as the parts A, B, and C (respectively). Thus, H is measured by D, E, and F.

And (H) is less than G. The very thing is impossible. Thus, there cannot be some number less than G which will have the parts A, B, and C. (Which is) the very thing it was required to show.