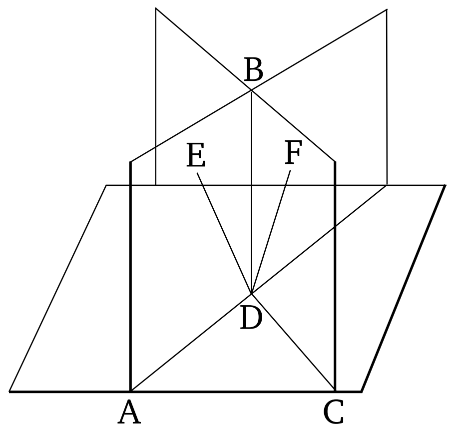
Book 11
Proposition 19

If two planes cutting one another are at right-angles to some plane then their common section will also be at right-angles to the same plane.



For let the two planes AB and BC be at right-angles to a reference plane, and let their common section be BD. I say that BD is at right-angles to the reference plane.

For (if) not, let DE also have been drawn from point D, in the plane AB, at right-angles to the straight-line AD, and DF, in the plane BC, at right-angles to CD.

And since the plane AB is at right-angles to the reference (plane), and DE has been drawn at right-angles to their common section AD, in the plane AB, DE is thus at right-angles to the reference plane [Def. 11.4]. So, similarly, we can show that DF is also at right-angles to the reference plane. Thus, two (different) straight-lines are set up, at the same point D, at right-angles to the reference plane, on the same side. The very thing is impossible [Prop. 11.13]. Thus, no (other straight-line) except the common section DB of the planes AB and BC can be set up at point D, at right-angles to the reference plane.

Thus, if two planes cutting one another are at right-angles to some plane then their common section will also be at right-angles to the same plane. (Which is) the very thing it was required to show.