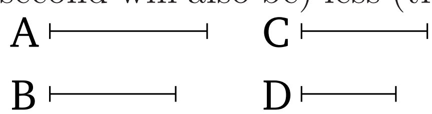
Book 5
Proposition 14

If a first (magnitude) has the same ratio to a second that a third (has) to a fourth, and the first (magnitude) is greater than the third, then the second will also be greater than the fourth. And if (the first magnitude is) equal (to the third then the second will also be) equal (to the fourth). And if (the first magnitude is) less (than the third then the second will also be) less (than the fourth).



For let a first (magnitude) A have the same ratio to a second B that a third C (has) to a fourth D. And let A be greater than C. I say that B is also greater than D.

For since A is greater than C, and B (is) another random [magnitude], A thus has a greater ratio to B than C (has) to B [Prop. 5.8]. And as A (is) to B, so C (is) to D. Thus, C also has a greater ratio to D than C (has) to B. And that (magnitude) to which the same (magnitude) has a greater ratio is the lesser [Prop. 5.10]. Thus, D (is) less than B. Hence, B is greater than D.

So, similarly, we can show that even if A is equal to C then B will also be equal to D, and even if A is less than C then B will also be less than D.

Thus, if a first (magnitude) has the same ratio to a second that a third (has) to a fourth, and the first (magnitude) is greater than the third, then the second will also be greater than the fourth. And if (the first magnitude is) equal (to the third then the second will also be) equal (to the fourth). And if (the first magnitude is) less (than the third then the second will also be) less (than the fourth). (Which is) the very thing it was required to show.