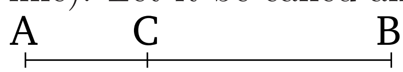
Book 10
Proposition 73

If a rational (straight-line), which is commensurable in square only with the whole, is subtracted from a(nother) rational (straight-line) then the remainder is an irrational (straight-line). Let it be called an apotome.



For let the rational (straight-line) BC, which commensurable in square only with the whole, have been subtracted from the rational (straight-line) AB. I say that the remainder AC is that irrational (straight-line) called an apotome.

For since AB is incommensurable in length with BC, and as AB is to BC, so the (square) on AB (is) to the (rectangle contained) by AB and BC [Prop. 10.21 lem.], the (square) on AB is thus incommensurable with the (rectangle contained) by AB and BC [Prop. 10.11]. But, the (sum of the) squares on AB and BC is commensurable with the (square) on AB [Prop. 10.15], and twice the (rectangle contained) by AB and BC is commensurable with the (rectangle contained) by AB and BC [Prop. 10.6]. And, inasmuch as the (sum of the squares) on AB and BC is equal to twice the (rectangle contained) by AB and BC plus the (square) on CA [Prop. 2.7], the (sum of the squares) on AB and BC is thus also incommensurable with the remaining (square) on AC [Props. 10.13, 10.16]. And the (sum of the squares) on AB and BC is rational. AC is thus an irrational (straight-line) [Def. 10.4]. And let it be called an apotome. (Which is) the very thing it was required to show.