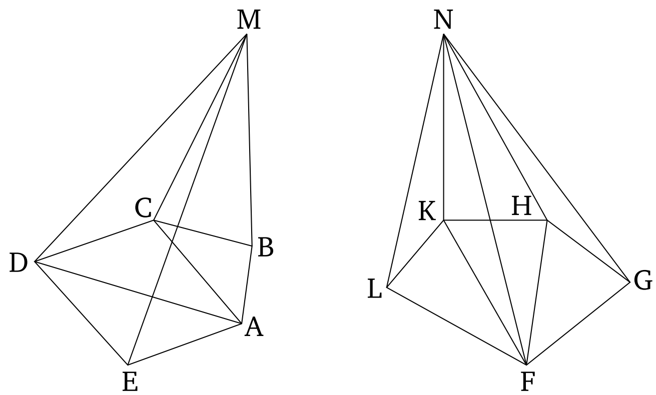
Book 12
Proposition 6

Pyramids which are of the same height, and have polygonal bases, are to one another as their bases.



Let there be pyramids of the same height whose bases (are) the polygons $ABCDE$ and $FGHKL$, and apexes the points M and N (respectively). I say that as base $ABCDE$ is to base $FGHKL$, so pyramid $ABCDEM$ (is) to pyramid $FGHKLN$.

For let AC, AD, FH, and FK have been joined. Therefore, since $ABCM$ and $ACDM$ are two pyramids having triangular bases and equal height, they are to one another as their bases [Prop. 12.5]. Thus, as base ABC is to base ACD, so pyramid $ABCM$ (is) to pyramid $ACDM$. And, via composition, as base $ABCD$ (is) to base ACD, so pyramid $ABCDM$ (is) to pyramid $ACDM$ [Prop. 5.18]. But, as base ACD (is) to base ADE, so pyramid $ACDM$ (is) also to pyramid $ADEM$ [Prop. 12.5]. Thus, via equality, as base $ABCD$ (is) to base ADE, so pyramid $ABCDM$ (is) to pyramid $ADEM$ [Prop. 5.22]. And, again, via composition, as base $ABCDE$ (is) to base ADE, so pyramid $ABCDEM$

(is) to pyramid $ADEM$ [Prop. 5.18]. So, similarly, it can also be shown that as base $FGHKL$ (is) to base FGH, so pyramid $FGHKLN$ (is) also to pyramid $FGHN$. And since $ADEM$ and $FGHN$ are two pyramids having triangular bases and equal height, thus as base ADE (is) to base FGH, so pyramid $ADEM$ (is) to pyramid $FGHN$ [Prop. 12.5]. But, as base ADE (is) to base $ABCDE$, so pyramid $ADEM$ (was) to pyramid $ABCDEM$. Thus, via equality, as base $ABCDE$ (is) to base FGH, so pyramid $ABCDEM$ (is) also to pyramid $FGHN$ [Prop. 5.22]. But, furthermore, as base FGH (is) to base $FGHKL$, so pyramid $FGHN$ was also to pyramid $FGHKLN$. Thus, via equality, as base $ABCDE$ (is) to base $FGHKL$, so pyramid $ABCDEM$ (is) also to pyramid $FGHKLN$ [Prop. 5.22]. (Which is) the very thing it was required to show.