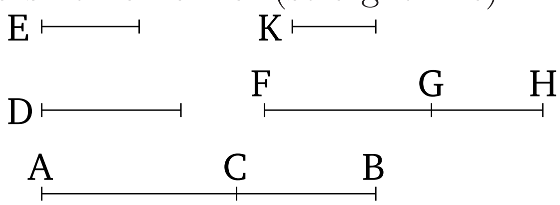
Book 10
Proposition 53

To find a sixth binomial (straight-line).



Let the two numbers AC and CB be laid down such that AB does not have to each of them the ratio which (some) square number (has) to (some) square number. And let D also be another number, which is not square, and does not have to each of BA and AC the ratio which (some) square number (has) to (some) square number either [Prop. 10.28 lem. I]. And let some rational straight-line E be laid down. And let it have been contrived that as D (is) to AB, so the (square) on E (is) to the (square) on FG [Prop. 10.6 corr.]. Thus, the (square) on E (is) commensurable with the (square) on FG [Prop. 10.6]. And E is rational. Thus, FG (is) also rational. And since D does not have to AB the ratio which (some) square number (has) to (some) square number, the (square) on E thus does not have to the (square) on FG the ratio which (some) square number (has) to (some) square number either. Thus, E (is) incommensurable in length with FG [Prop. 10.9]. So, again, let it have be contrived that as BA (is) to AC, so the (square) on FG (is) to the (square) on GH [Prop. 10.6 corr.]. The (square) on FG (is) thus commensurable with the (square) on HG [Prop. 10.6]. The (square) on HG (is) thus rational.

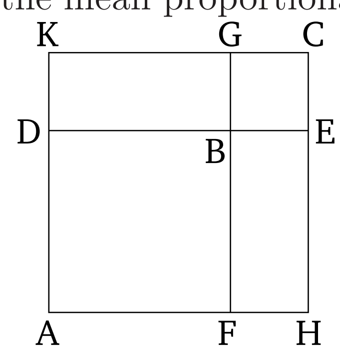
Thus, HG (is) rational. And since BA does not have to AC the ratio which (some) square number (has) to (some) square number, the (square) on FG does not have to the (square) on GH the ratio which (some) square number (has) to (some) square number either. Thus, FG is incommensurable in length with GH [Prop. 10.9]. Thus, FG and GH are rational (straight-lines which are) commensurable in square only. Thus, FH is a binomial (straight-line) [Prop. 10.36]. So, we must show that (it is) also a sixth (binomial straight-line).

For since as D is to AB, so the (square) on E (is) to the (square) on FG, and also as BA is to AC, so the (square) on FG (is) to the (square) on GH, thus, via equality, as D is to AC, so the (square) on E (is) to the (square) on GH [Prop. 5.22]. And D does not have to AC the ratio which (some) square number (has) to (some) square number. Thus, the (square) on E does not have to the (square) on GH the ratio which (some) square number (has) to (some) square number either. E is thus incommensurable in length with GH [Prop. 10.9]. And (E) was also shown (to be) incommensurable (in length) with FG. Thus, FG and GH are each incommensurable in length with E. And since as BA is to AC, so the (square) on FG (is) to the (square) on GH, the (square) on FG (is) thus greater than the (square) on GH [Prop. 5.14]. Therefore, let (the sum of) the (squares) on GH and K be equal to the (square) on FG. Thus, via conversion, as AB (is) to BC, so the (square) on FG (is) to the (square) on K [Prop. 5.19 corr.]. And AB does not have to BC the ratio which (some) square number (has) to (some) square number. Hence, the (square) on FG does not have to the (square) on K the ratio which (some) square number (has) to (some) square number either. Thus, FG is incommensurable in length with K [Prop. 10.9]. The square on FG is thus greater than (the square on) GH by the (square) on (some straight-line which is) incommensurable (in length) with (FG). And FG and GH are rational (straight-lines which are) commensurable in square only, and neither of them is commensurable in length with the rational (straight-line) E (previously) laid down.

Thus, FH is a sixth binomial (straight-line) [Def. 10.10].† (Which is) the very thing it was required to show.

Lemma

Let AB and BC be two squares, and let them be laid down such that DB is straight-on to BE. FB is, thus, also straight-on to BG. And let the parallelogram AC have been completed. I say that AC is a square, and that DG is the mean proportional to AB and BC, and, moreover, DC is the mean proportional to AC and CB.



For since DB is equal to BF, and BE to BG, the whole of DE is thus equal to the whole of FG. But DE is equal to each of AH and KC, and FG is equal to each of AK and HC [Prop. 1.34]. Thus, AH and KC are also equal to AK and HC, respectively. Thus, the parallelogram AC is equilateral. And (it is) also right-angled. Thus, AC is a square.

And since as FB is to BG, so DB (is) to BE, but as FB (is) to BG, so AB (is) to DG, and as DB (is) to BE, so DG (is) to BC [Prop. 6.1], thus also as AB (is) to DG, so DG (is) to BC [
Thus, DG is the mean proportional to AB and BC.

So I also say that DC [is] the mean proportional to AC and CB.

For since as AD is to DK, so KG (is) to GC. For [they are] respectively equal. And, via composition, as AK (is) to KD, so KC (is) to CG [Prop. 5.18]. But as AK (is) to KD, so AC (is) to CD, and as KC (is) to CG, so DC (is) to CB [Prop. 6.1]. Thus, also, as AC (is) to DC, so DC (is) to BC [Prop. 5.11]. Thus, DC is the mean proportional to AC and CB. Which (is the very thing) it was prescribed to show.