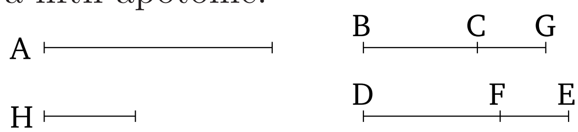
Book 10
Proposition 89

To find a fifth apotome.



Let the rational (straight-line) A be laid down, and let CG be commensurable in length with A. Thus, CG [is] a rational (straight-line). And let the two numbers DF and FE be laid down such that DE again does not have to each of DF and FE the ratio which (some) square number (has) to (some) square number. And let it have been contrived that as FE (is) to ED, so the (square) on CG (is) to the (square) on GB. Thus, the (square) on GB (is) also rational [Prop. 10.6]. Thus, BG is also rational. And since as DE is to EF, so the (square) on BG (is) to the (square) on GC. And DE does not have to EF the ratio which (some) square number (has) to (some) square number. The (square) on BG thus does not have to the (square) on GC the ratio which (some) square number (has) to (some) square number either. Thus, BG is incommensurable in length with GC [Prop. 10.9]. And they are both rational (straight-lines). BG and GC are thus rational (straight-lines which are) commensurable in square only. Thus, BC is an apotome [Prop. 10.73]. So, I say that (it is) also a fifth (apotome).

For, let the (square) on H be that (area) by which the (square) on BG is greater than the (square) on GC [Prop. 10.13 lem.]. Therefore, since as the (square) on BG (is) to the (square) on GC, so DE (is) to EF, thus, via conversion, as ED is to DF, so the (square) on BG (is) to the (square) on H [Prop. 5.19 corr.]. And ED does not have to DF the ratio which (some) square number (has) to (some) square number. Thus, the (square) on BG does not have to the (square) on H the ratio which (some) square number (has) to (some) square number either. Thus, BG is incommensurable in length with H [Prop. 10.9]. And the square on BG is greater than (the square on) GC by the (square) on H. Thus, the square on GB is greater than (the square on) GC by the (square) on (some straight-line) incommensurable in length with (GB). And the attachment CG is commensurable in length with the (previously) laid down rational (straight-line) A. Thus, BC is a fifth apotome [Def. 10.15].†

Thus, the fifth apotome BC has been found. (Which is) the very thing it was required to show.