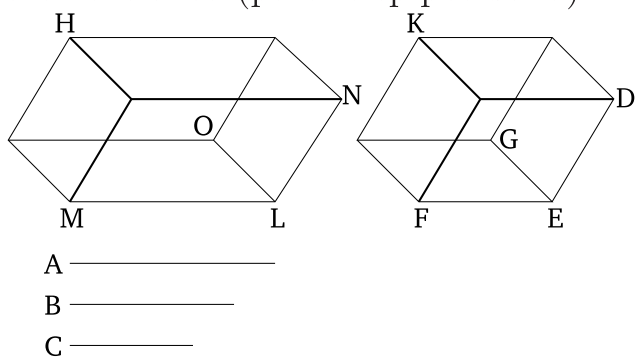
Book 11
Proposition 36

If three straight-lines are (continuously) proportional then the parallelepiped solid (formed) from the three (straight-lines) is equal to the equilateral parallelepiped solid on the middle (straight-line which is) equiangular to the aforementioned (parallelepiped solid).



Let A, B, and C be three (continuously) proportional straight-lines, (such that) as A (is) to B, so B (is) to C. I say that the (parallelepiped) solid (formed) from A, B, and C is equal to the equilateral solid on B (which is) equiangular with the aforementioned (solid).

Let the solid angle at E, contained by DEG, GEF, and FED, be set out. And let DE, GE, and EF each be made equal to B. And let the parallelepiped solid EK have been completed. And (let) LM (be made) equal to A. And let the solid angle contained by NLO, OLM, and MLN have been constructed on the straight-line LM, and at the point L on it, (so as to be) equal to the solid angle E [Prop. 11.23]. And let LO be made equal to B, and LN equal to C. And since as A (is) to B, so B (is) to C, and A (is) equal to LM, and B to each of LO and ED, and C to LN, thus as LM (is) to EF, so DE (is) to LN. And (so) the sides around the equal angles NLM and DEF are reciprocally proportional. Thus, parallelogram MN is equal to parallelogram DF [Prop. 6.14]. And since the two plane rectilinear angles DEF and NLM are equal, and the raised straight-lines stood on them (at their apexes), LO and EG, are equal to one another, and contain equal angles respectively with the original straight-lines (forming the angles), the perpendiculars drawn from points G and O to the planes through NLM and DEF (respectively) are thus equal to one another [Prop. 11.35 corr.]. Thus, the solids LH and EK (have) the same height. And parallelepiped solids on equal bases, and with the same height, are equal to one another [Prop. 11.31]. Thus, solid HL is equal to solid EK. And LH is the solid (formed) from A, B, and C, and EK the solid on B. Thus, the parallelepiped solid (formed) from A, B, and C is equal to the equilateral solid on B (which is) equiangular with the aforementioned (solid). (Which is) the very thing it was required to show.