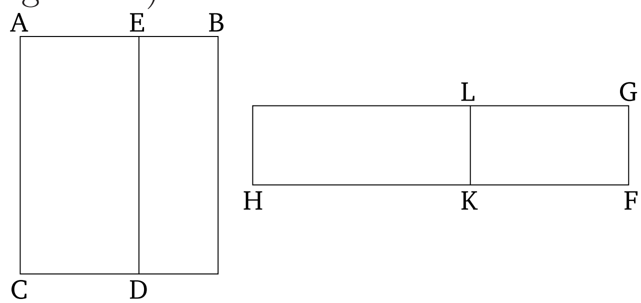
Book 10
Proposition 108

A medial (area) being subtracted from a rational (area), one of two irrational (straight-lines) arise (as) the square-root of the remaining area—either an apotome, or a minor (straight-line).



For let the medial (area) BD have been subtracted from the rational (area) BC. I say that one of two irrational (straight-lines) arise (as) the square-root of the remaining (area), EC—either an apotome, or a minor (straight-line).

For let the rational (straight-line) FG have been laid out, and let the right-angled parallelogram GH, equal to BC, have been applied to FG, and let GK, equal to DB, have been subtracted (from GH). Thus, the remainder EC is equal to LH. Therefore, since BC is a rational (area), and BD a medial (area), and BC (is) equal to GH, and BD to GK, GH is thus a rational (area), and GK a medial (area). And they are applied to the rational (straight-line) FG. Thus, FH (is) rational, and commensurable in length with FG [Prop. 10.20], and FK (is) also rational, and incommensurable in length with FG [Prop. 10.22]. Thus, FH is incommensurable in length with FK [Prop. 10.13]. FH and FK are thus rational (straight-lines which are) commensurable in square only. Thus, KH is an apotome [Prop. 10.73], and KF an attachment to it. So, the square on HF is greater than (the square on) FK by the (square) on (some straight-line which is) either commensurable, or not (commensurable), (in length with HF).

First, let the square (on it) be (greater) by the (square) on (some straight-line which is) commensurable (in length with HF). And the whole of HF is commensurable in length with the (previously) laid down rational (straight-line) FG. Thus, KH is a first apotome [Def. 10.1]. And the square-root of an (area) contained by a rational (straight-line) and a first apotome is an apotome [Prop. 10.91]. Thus, the square-root of LH—that is to say, (of) EC—is an apotome.

And if the square on HF is greater than (the square on) FK by the (square) on (some straight-line which is) incommensurable (in length) with (HF), and (since) the whole of FH is commensurable in length with the (previously) laid down rational (straight-line) FG, KH is a fourth apotome [Prop. 10.14]. And the square-root of an (area) contained by a rational (straight-line) and a fourth apotome is a minor (straight-line) [Prop. 10.94]. (Which is) the very thing it was required to show.