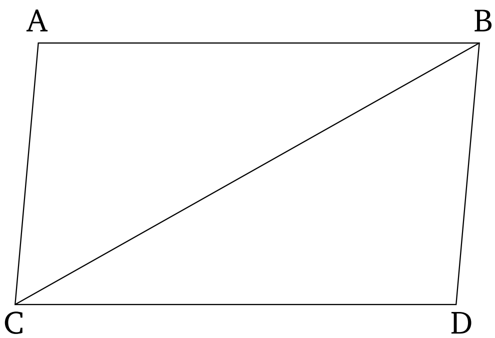
Book 1
Proposition 34

In parallelogrammic figures the opposite sides and angles are equal to one another, and a diagonal cuts them in half.



Let $ACDB$ be a parallelogrammic figure, and BC its diagonal. I say that for parallelogram $ACDB$, the opposite sides and angles are equal to one another, and the diagonal BC cuts it in half.

For since AB is parallel to CD, and the straight-line BC has fallen across them, the alternate angles ABC and BCD are equal to one another [Prop. 1.29]. Again, since AC is parallel to BD, and BC has fallen across them, the alternate angles ACB and CBD are equal to one another [Prop. 1.29]. So ABC and BCD are two triangles having the two angles ABC and BCA equal to the two (angles) BCD and CBD, respectively, and one side equal to one side—the (one) by the equal angles and common to them, (namely) BC. Thus, they will also have the remaining sides equal to the corresponding remaining (sides), and the remaining angle (equal) to the remaining angle [Prop. 1.26]. Thus, side AB is equal to

CD, and AC to BD. Furthermore, angle BAC is equal to CDB. And since angle ABC is equal to BCD, and CBD to ACB, the whole (angle) ABD is thus equal to the whole (angle) ACD. And BAC was also shown (to be) equal to CDB.

Thus, in parallelogrammic figures the opposite sides and angles are equal to one another.

And, I also say that a diagonal cuts them in half. For since AB is equal to CD, and BC (is) common, the two (straight-lines) AB, BC are equal to the two (straight-lines) DC, $CB^{\dagger}$, respectively. And angle ABC is equal to angle BCD. Thus, the base AC (is) also equal to DB, and triangle ABC is equal to triangle BCD [Prop. 1.4].

Thus, the diagonal BC cuts the parallelogram $ACDB^{\ddagger}$ in half. (Which is) the very thing it was required to show.