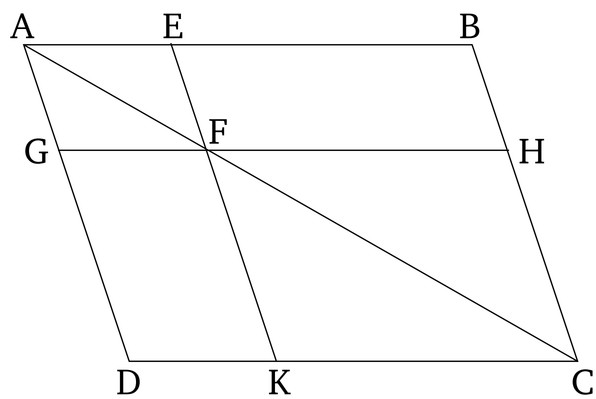
# Book 6
# Proposition 24

In any parallelogram the parallelograms about the diagonal are similar to the whole, and to one another.

Let $ABCD$ be a parallelogram, and $AC$ its diagonal. And let $EG$ and $HK$ be parallelograms about $AC$. I say that the parallelograms $EG$ and $HK$ are each similar to the whole (parallelogram) $ABCD$, and to one another.

For since $EF$ has been drawn parallel to one of the sides $BC$ of triangle $ABC$, proportionally, as $BE$ is to $EA$, so $CF$ (is) to $FA$ [Prop. 6.2]. Again, since $FG$ has been drawn parallel to one (of the sides) $CD$ of triangle $ACD$, proportionally, as $CF$ is to $FA$, so $DG$ (is) to $GA$ [Prop. 6.2]. But, as $CF$ (is) to $FA$, so it was also shown (is) $BE$ to $EA$. And thus as $BE$ (is) to $EA$, so $DG$ (is) to $GA$. And, thus, compounding, as $BA$ (is) to $AE$, so $DA$ (is) to $AG$ [Prop. 5.18]. And, alternately, as $BA$ (is) to $AD$, so $EA$ (is) to $AG$ [Prop. 5.16]. Thus, in parallelograms $ABCD$ and $EG$ the sides about the common angle $BAD$ are proportional. And since $GF$ is parallel to $DC$, angle $AFG$ is equal to $DCA$ [Prop. 1.29]. And angle $DAC$ (is) common to the two triangles $ADC$ and $AGF$. Thus, triangle $ADC$ is equiangular to triangle $AGF$ [Prop. 1.32]. So, for the same (reasons), triangle $ACB$ is equiangular to triangle $AFE$, and the whole parallelogram $ABCD$ is equiangular to parallelogram $EG$. Thus, proportionally, as $AD$ (is) to $DC$, so $AG$ (is) to $GF$, and as $DC$ (is) to $CA$, so $GF$ (is) to $FA$, and as $AC$ (is) to $CB$, so $AF$ (is) to $FE$, and, further, as $CB$ (is) to $BA$, so $FE$ (is) to $EA$ [Prop. 6.4]. And since it was shown that as $DC$ is to $CA$, so $GF$ (is) to $FA$, and as $AC$ (is) to $CB$, so $AF$ (is) to $FE$, thus, via equality, as $DC$ is to $CB$, so $GF$ (is) to $FE$ [Prop. 5.22]. Thus, in parallelograms $ABCD$ and $EG$ the sides about the equal angles are proportional. Thus, parallelogram $ABCD$ is similar to parallelogram $EG$ [Def. 6.1]. So, for the same (reasons), parallelogram $ABCD$ is also similar to parallelogram $KH$. Thus, parallelograms $EG$ and $HK$ are each similar to [parallelogram] $ABCD$. And (rectilinear figures) similar to the same rectilinear figure are also similar to one another [Prop. 6.21]. Thus, parallelogram $EG$ is also similar to parallelogram $HK$.




Thus, in any parallelogram the parallelograms about the diagonal are similar to the whole, and to one another. (Which is) the very thing it was required to show.