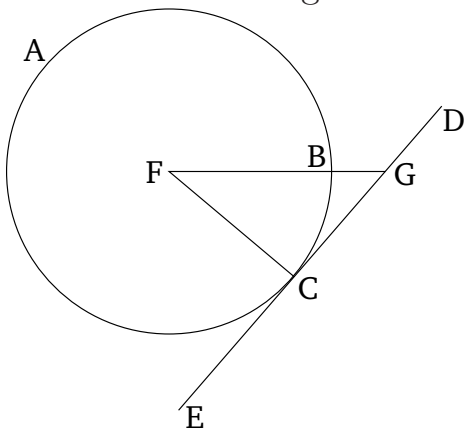
Book 3
Proposition 18

If some straight-line touches a circle, and some (other) straight-line is joined from the center (of the circle) to the point of contact, then the (straight-line) so joined will be perpendicular to the tangent.



For let some straight-line DE touch the circle ABC at point C, and let the center F of circle ABC have been found [Prop. 3.1], and let FC have been joined from F to C. I say that FC is perpendicular to DE.

For if not, let FG have been drawn from F, perpendicular to DE [Prop. 1.12].

Therefore, since angle FGC is a right-angle, (angle) FCG is thus acute [Prop. 1.17]. And the greater angle is subtended by the greater side [Prop. 1.19]. Thus, FC (is) greater than FG. And FC (is) equal to FB. Thus, FB (is) also greater than FG, the lesser than the greater. The very thing is impossible. Thus, FG is not perpendicular to DE. So, similarly, we can show that neither (is) any other (straight-line) except FC. Thus,

FC is perpendicular to DE.

Thus, if some straight-line touches a circle, and some (other) straight-line is joined from the center (of the circle) to the point of contact, then the (straight-line) so joined will be perpendicular to the tangent. (Which is) the very thing it was required to show.