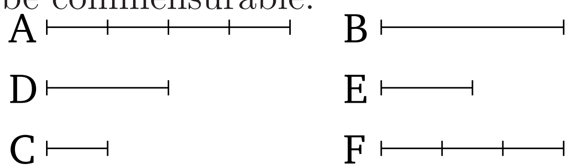
Book 10
Proposition 6

If two magnitudes have to one another the ratio which (some) number (has) to (some) number then the magnitudes will be commensurable.



For let the two magnitudes A and B have to one another the ratio which the number D (has) to the number E. I say that the magnitudes A and B are commensurable.

For, as many units as there are in D, let A have been divided into so many equal (divisions). And let C be equal to one of them. And as many units as there are in E, let F be the sum of so many magnitudes equal to C.

Therefore, since as many units as there are in D, so many magnitudes equal to C are also in A, therefore whichever part a unit is of D, C is also the same part of A. Thus, as C is to A, so a unit (is) to D [Def. 7.20]. And a unit measures the number D. Thus, C also measures A. And since as C is to A, so a unit (is) to the [number] D, thus, inversely, as A (is) to C, so the number D (is) to a unit [Prop. 5.7 corr.]. Again, since as many units as there are in E, so many (magnitudes) equal to C are also in F, thus as C is to F, so a unit (is) to the [number] E [Def. 7.20]. And it was also shown that as A (is) to C, so D (is) to a unit. Thus, via equality, as A is to F, so D (is) to E [Prop. 5.22]. But, as D (is) to E, so A is to B. And thus as A (is) to B, so (it) also is to F [Prop. 5.11]. Thus, A has the same ratio to each of B and F. Thus, B is equal to F [Prop. 5.9]. And C measures F. Thus, it also measures B. But, in fact, (it) also (measures) A. Thus, C measures (both) A and B. Thus, A is commensurable with B [Def. 10.1].

Thus, if two magnitudes ... to one another, and so on ....

Corollary

So it is clear, from this, that if there are two numbers, like D and E, and a straight-line, like A, then it is possible to contrive that as the number D (is) to the number E, so the straight-line (is) to (another) straight-line (*i.e.,* F). And if the mean proportion, (say) B, is taken of A and F, then as A is to F, so the (square) on A (will be) to the (square) on B. That is to say, as the first (is) to the third, so the (figure) on the first (is) to the similar, and similarly described, (figure) on the second [Prop. 6.19 corr.]. But, as A (is) to F, so the number D is to the number E. Thus, it has also been contrived that as the number D (is) to the number E, so the (figure) on the straight-line A (is) to the (similar figure) on the straight-line B. (Which is) the very thing it was required to show.