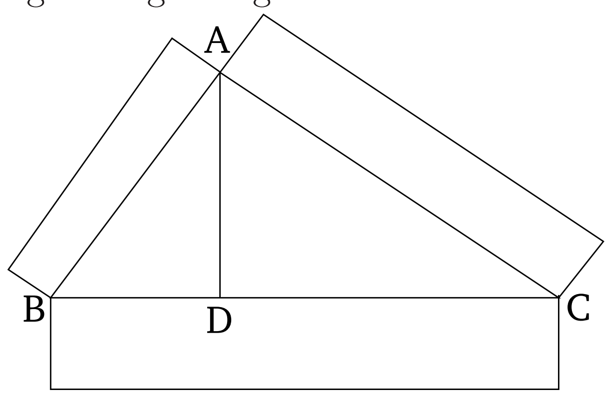
# Book 6
# Proposition 31

In right-angled triangles, the figure (drawn) on the side subtending the right-angle is equal to the (sum of the) similar, and similarly described, figures on the sides surrounding the right-angle.




Let $ABC$ be a right-angled triangle having the angle $BAC$ a right-angle. I say that the figure (drawn) on $BC$ is equal to the (sum of the) similar, and similarly described, figures on $BA$ and $AC$.

Let the perpendicular $AD$ have been drawn [Prop. 1.12].

Therefore, since, in the right-angled triangle $ABC$, the (straight-line) $AD$ has been drawn from the right-angle at $A$ perpendicular to the base $BC$, the triangles $ABD$ and $ADC$ about the perpendicular are similar to the whole (triangle) $ABC$, and to one another [Prop. 6.8]. And since $ABC$ is similar to $ABD$, thus as $CB$ is to $BA$, so $AB$ (is) to $BD$ [Def. 6.1]. And since three straight-lines are proportional, as the first is to the third, so the figure (drawn) on the first is to the similar, and similarly described, (figure) on the second [Prop. 6.19 corr.]. Thus, as $CB$ (is) to $BD$, so the figure (drawn) on $CB$ (is) to the similar, and similarly described, (figure) on $BA$. And so, for the same (reasons), as $BC$ (is) to $CD$, so the figure (drawn) on $BC$ (is) to the (figure) on $CA$. Hence, also, as $BC$ (is) to $BD$ and $DC$, so the figure (drawn) on $BC$ (is) to the (sum of the) similar, and similarly described, (figures) on $BA$ and $AC$ [Prop. 5.24]. And $BC$ is equal to $BD$ and $DC$. Thus, the figure (drawn) on $BC$ (is) also equal to the (sum of the) similar, and similarly described, figures on $BA$ and $AC$ [Prop. 5.9].

Thus, in right-angled triangles, the figure (drawn) on the side subtending the right-angle is equal to the (sum of the) similar, and similarly described, figures on the sides surrounding the right-angle. (Which is) the very thing it was required to show.