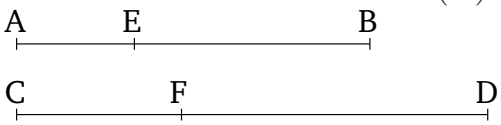
Book 5
Proposition 19

If as the whole is to the whole so the (part) taken away is to the (part) taken away then the remainder to the remainder will also be as the whole (is) to the whole.



For let the whole AB be to the whole CD as the (part) taken away AE (is) to the (part) taken away CF. I say that the remainder EB to the remainder FD will also be as the whole AB (is) to the whole CD.

For since as AB is to CD, so AE (is) to CF, (it is) also (the case), alternately, (that) as BA (is) to AE, so DC (is) to CF [Prop. 5.16]. And since composed magnitudes are proportional then they will also be proportional (when) separated, (so that) as BE (is) to EA, so DF (is) to CF [Prop. 5.17]. Also, alternately, as BE (is) to DF, so EA (is) to FC [Prop. 5.16]. And it was assumed that as AE (is) to CF, so the whole AB (is) to the whole CD. And, thus, as the remainder EB (is) to the remainder FD, so the whole AB will be to the whole CD.

Thus, if as the whole is to the whole so the (part) taken away is to the (part) taken away then the remainder to the remainder will also be as the whole (is) to the whole. [(Which is) the very thing it was required to show.]

[And since it was shown (that) as AB (is) to CD, so EB (is) to FD, (it is) also (the case), alternately, (that) as AB (is) to BE, so CD (is) to FD. Thus, composed

magnitudes are proportional. And it was shown (that) as BA (is) to AE, so DC (is) to CF. And (the latter) is converted (from the former).]

Corollary‡

So (it is) clear, from this, that if composed magnitudes are proportional then they will also be proportional (when) converted. (Which is) the very thing it was required to show.