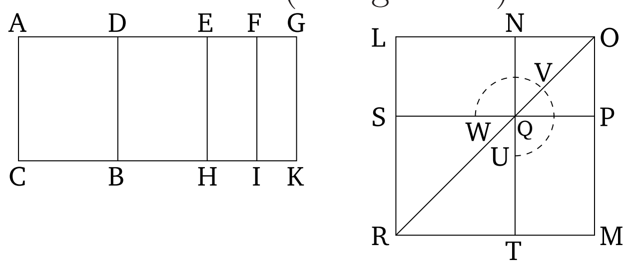
Book 10
Proposition 92

If an area is contained by a rational (straight-line) and a second apotome then the square-root of the area is a first apotome of a medial (straight-line).



For let the area AB have been contained by the rational (straight-line) AC and the second apotome AD. I say that the square-root of area AB is the first apotome of a medial (straight-line).

For let DG be an attachment to AD. Thus, AG and GD are rational (straight-lines which are) commensurable in square only [Prop. 10.73], and the attachment DG is commensurable (in length) with the (previously) laid down rational (straight-line) AC, and the square on the whole, AG, is greater than (the square on) the attachment, GD, by the (square) on (some straight-line) commensurable in length with (AG) [Def. 10.12]. Therefore, since the square on AG is greater than (the square on) GD by the (square) on (some straight-line) commensurable (in length) with (AG), thus if (an area) equal to the fourth part of the (square) on GD is applied to AG, falling short by a square figure, then it divides (AG) into (parts which are) commensurable (in length) [Prop. 10.17]. Therefore, let DG have been cut in half at E. And let (an area) equal to the (square) on EG have been applied to AG, falling short by a square figure. And let it be the (rectangle contained) by AF and FG. Thus, AF is commensurable in length with FG. AG is thus also commensurable in length with each of AF and FG [Prop. 10.15]. And AG (is) a rational (straight-line), and incommensurable in length with AC. AF and FG are thus also each rational (straight-lines), and incommensurable in length with AC [Prop. 10.13]. Thus, AI and FK are each medial (areas) [Prop. 10.21]. Again, since DE is commensurable (in length) with EG, thus DG is also commensurable (in length) with each of DE and EG [Prop. 10.15]. But, DG is commensurable in length with AC [thus, DE and EG are also each rational, and commensurable in length with AC]. Thus, DH and EK are each rational (areas) [Prop. 10.19].

Therefore, let the square LM, equal to AI, have been constructed. And let NO, equal to FK, which is about the same angle LPM as LM, have been subtracted (from LM). Thus, the squares LM and NO are about the same diagonal [Prop. 6.26]. Let PR be their (common) diagonal, and let the (rest of the) figure have been drawn. Therefore, since AI and FK are medial (areas), and are equal to the (squares) on LP and PN (respectively), [thus] the (squares) on LP and PN are also medial. Thus, LP and PN are also medial (straight-lines which are) commensurable in square only.† And since the (rectangle contained) by AF and FG is equal to the (square) on EG, thus as AF is to EG, so EG (is) to FG [Prop. 10.17]. But, as AF (is) to EG, so AI

(is) to EK. And as EG (is) to FG, so EK [is] to FK [Prop. 6.1]. Thus, EK is the mean proportional to AI and FK [Prop. 5.11]. And MN is also the mean proportional to the squares LM and NO [Prop. 10.53 lem.]. And AI is equal to LM, and FK to NO. Thus, MN is also equal to EK. But, DH [is] equal to EK, and LO equal to MN [Prop. 1.43]. Thus, the whole (of) DK is equal to the gnomon UVW and NO. Therefore, since the whole (of) AK is equal to LM and NO, of which DK is equal to the gnomon UVW and NO, the remainder AB is thus equal to TS. And TS is the (square) on LN. Thus, the (square) on LN is equal to the area AB. LN is thus the square-root of area AB. [So], I say that LN is the first apotome of a medial (straight-line).

For since EK is a rational (area), and is equal to LO, LO—that is to say, the (rectangle contained) by LP and PN—is thus a rational (area). And NO was shown (to be) a medial (area). Thus, LO is incommensurable with NO. And as LO (is) to NO, so LP is to PN [Prop. 6.1]. Thus, LP and PN are incommensurable in length [Prop. 10.11]. LP and PN are thus medial (straight-lines which are) commensurable in square only, and which contain a rational (area). Thus, LN is the first apotome of a medial (straight-line) [Prop. 10.74]. And it is the square-root of area AB.

Thus, the square root of area AB is the first apotome of a medial (straight-line). (Which is) the very thing it was required to show.