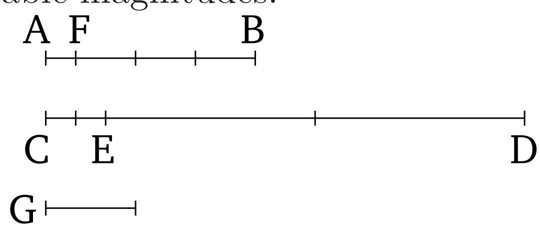
# Book 10
# Proposition 3

To find the greatest common measure of two given commensurable magnitudes.




Let $AB$ and $CD$ be the two given magnitudes, of which (let) $AB$ (be) the lesser. So, it is required to find the greatest common measure of $AB$ and $CD$.

For the magnitude $AB$ either measures, or (does) not (measure), $CD$. Therefore, if it measures ($CD$), and (since) it also measures itself, $AB$ is thus a common measure of $AB$ and $CD$. And (it is) clear that (it is) also (the) greatest. For a (magnitude) greater than magnitude $AB$ cannot measure $AB$.

So let $AB$ not measure $CD$. And continually subtracting in turn the lesser (magnitude) from the greater, the remaining (magnitude) will (at) some time measure the (magnitude) before it, on account of $AB$ and $CD$ not being incommensurable [Prop. 10.2]. And let $AB$ leave $EC$ less than itself (in) measuring $ED$, and let $EC$ leave $AF$ less than itself (in) measuring $FB$, and let $AF$ measure $CE$.

Therefore, since $AF$ measures $CE$, but $CE$ measures $FB$, $AF$ will thus also measure $FB$. And it also measures itself. Thus, $AF$ will also measure the whole (of) $AB$. But, $AB$ measures $DE$. Thus, $AF$ will also measure

$ED$. And it also measures $CE$. Thus, it also measures the whole of $CD$. Thus, $AF$ is a common measure of $AB$ and $CD$. So I say that (it is) also (the) greatest (common measure). For, if not, there will be some magnitude, greater than $AF$, which will measure (both) $AB$ and $CD$. Let it be $G$. Therefore, since $G$ measures $AB$, but $AB$ measures $ED$, $G$ will thus also measure $ED$. And it also measures the whole of $CD$. Thus, $G$ will also measure the remainder $CE$. But $CE$ measures $FB$. Thus, $G$ will also measure $FB$. And it also measures the whole (of) $AB$. And (so) it will measure the remainder $AF$, the greater (measuring) the lesser. The very thing is impossible. Thus, some magnitude greater than $AF$ cannot measure (both) $AB$ and $CD$. Thus, $AF$ is the greatest common measure of $AB$ and $CD$.

Thus, the greatest common measure of two given commensurable magnitudes, $AB$ and $CD$, has been found. (Which is) the very thing it was required to show.

## Corollary

So (it is) clear, from this, that if a magnitude measures two magnitudes then it will also measure their greatest common measure.