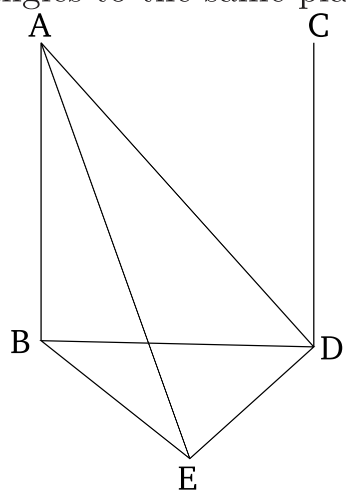
Book 11
Proposition 8

If two straight-lines are parallel, and one of them is at right-angles to some plane, then the remaining (one) will also be at right-angles to the same plane.



Let AB and CD be two parallel straight-lines, and let one of them, AB, be at right-angles to a reference plane. I say that the remaining (one), CD, will also be at right-angles to the same plane.

For let AB and CD meet the reference plane at points B and D (respectively). And let BD have been joined. AB, CD, and BD are thus in one plane [Prop. 11.7]. Let DE have been drawn at right-angles to BD in the reference plane, and let DE be made equal to AB, and let BE, AE, and AD have been joined.

And since AB is at right-angles to the reference plane, AB is thus also at right-angles to all of the straight-lines joined to it which are in the reference plane [Def. 11.3]. Thus, the angles ABD and ABE [are] each right-angles.

And since the straight-line BD has met the parallel (straight-lines) AB and CD, the (sum of the) angles ABD and CDB is thus equal to two right-angles [Prop. 1.29]. And ABD (is) a right-angle. Thus, CDB (is) also a right-angle. CD is thus at right-angles to BD. And since AB is equal to DE, and BD (is) common, the two (straight-lines) AB and BD are equal to the two (straight-lines) ED and DB (respectively). And angle ABD (is) equal to angle EDB. For each (is) a right-angle. Thus, the base AD (is) equal to the base BE [Prop. 1.4]. And since AB is equal to DE, and BE to AD, the two (sides) AB, BE are equal to the two (sides) ED, DA, respectively. And their base AE is common. Thus, angle ABE is equal to angle EDA [Prop. 1.8]. And ABE (is) a right-angle. EDA (is) thus also a right-angle. Thus, ED is at right-angles to AD. And it is also at right-angles to DB. Thus, ED is also at right-angles to the plane through BD and DA [Prop. 11.4]. And ED will thus make right-angles with all of the straight-lines joined to it which are also in the plane through BDA. And DC is in the plane through BDA, inasmuch as AB and BD are in the plane through BDA [Prop. 11.2], and in which(ever plane) AB and BD (are found), DC is also (found). Thus, ED is at right-angles to DC. Hence, CD is also at right-angles to DE. And CD is also at right-angles to BD. Thus, CD is standing at right-angles to two straight-lines, DE and DB, which meet one another, at the (point) of section, D. Hence, CD is also at right-angles to the plane through DE and DB [Prop. 11.4]. And the plane through DE and DB

is the reference (plane). CD is thus at right-angles to the reference plane.

Thus, if two straight-lines are parallel, and one of them is at right-angles to some plane, then the remaining (one) will also be at right-angles to the same plane. (Which is) the very thing it was required to show.