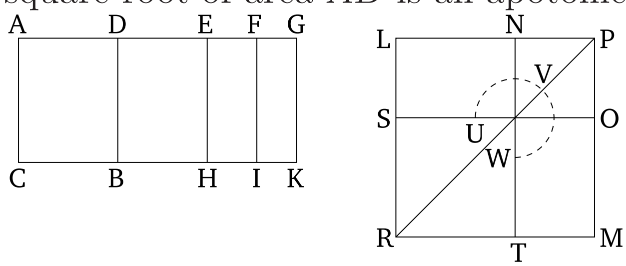
Book 10
Proposition 91

If an area is contained by a rational (straight-line) and a first apotome then the square-root of the area is an apotome.

For let the area AB have been contained by the rational (straight-line) AC and the first apotome AD. I say that the square-root of area AB is an apotome.



For since AD is a first apotome, let DG be its attachment. Thus, AG and DG are rational (straight-lines which are) commensurable in square only [Prop. 10.73]. And the whole, AG, is commensurable (in length) with the (previously) laid down rational (straight-line) AC, and the square on AG is greater than (the square on) GD by the (square) on (some straight-line) commensurable in length with (AG) [Def. 10.11]. Thus, if (an area) equal to the fourth part of the (square) on DG is applied to AG, falling short by a square figure, then it divides (AG) into (parts which are) commensurable (in length) [Prop. 10.17]. Let DG have been cut in half at E. And let (an area) equal to the (square) on EG have been applied to AG, falling short by a square figure. And let it be the (rectangle contained) by AF and FG. AF is thus commensurable (in length) with FG. And let EH,

FI, and GK have been drawn through points E, F, and G (respectively), parallel to AC.

And since AF is commensurable in length with FG, AG is thus also commensurable in length with each of AF and FG [Prop. 10.15]. But AG is commensurable (in length) with AC. Thus, each of AF and FG is also commensurable in length with AC [Prop. 10.12]. And AC is a rational (straight-line). Thus, AF and FG (are) each also rational (straight-lines). Hence, AI and FK are also each rational (areas) [Prop. 10.19]. And since DE is commensurable in length with EG, DG is thus also commensurable in length with each of DE and EG [Prop. 10.15]. And DG (is) rational, and incommensurable in length with AC. DE and EG (are) thus each rational, and incommensurable in length with AC [Prop. 10.13]. Thus, DH and EK are each medial (areas) [Prop. 10.21].

So let the square LM, equal to AI, be laid down. And let the square NO, equal to FK, have been subtracted (from LM), having with it the common angle LPM. Thus, the squares LM and NO are about the same diagonal [Prop. 6.26]. Let PR be their (common) diagonal, and let the (rest of the) figure have been drawn. Therefore, since the rectangle contained by AF and FG is equal to the square EG, thus as AF is to EG, so EG (is) to FG [Prop. 6.17]. But, as AF (is) to EG, so AI (is) to EK, and as EG (is) to FG, so EK is to KF [Prop. 6.1]. Thus, EK is the mean proportional to AI and KF [Prop. 5.11]. And MN is also the mean proportional to LM and NO, as shown before

[Prop. 10.53 lem.]. And AI is equal to the square LM, and KF to NO. Thus, MN is also equal to EK. But, EK is equal to DH, and MN to LO [Prop. 1.43]. Thus, DK is equal to the gnomon UVW and NO. And AK is also equal to (the sum of) the squares LM and NO. Thus, the remainder AB is equal to ST. And ST is the square on LN. Thus, the square on LN is equal to AB. Thus, LN is the square-root of AB. So, I say that LN is an apotome.

For since AI and FK are each rational (areas), and are equal to LM and NO (respectively), thus LM and NO—that is to say, the (squares) on each of LP and PN (respectively)—are also each rational (areas). Thus, LP and PN are also each rational (straight-lines). Again, since DH is a medial (area), and is equal to LO, LO is thus also a medial (area). Therefore, since LO is medial, and NO rational, LO is thus incommensurable with NO. And as LO (is) to NO, so LP is to PN [Prop. 6.1]. LP is thus incommensurable in length with PN [Prop. 10.11]. And they are both rational (straight-lines). Thus, LP and PN are rational (straight-lines which are) commensurable in square only. Thus, LN is an apotome [Prop. 10.73]. And it is the square-root of area AB. Thus, the square-root of area AB is an apotome.

Thus, if an area is contained by a rational (straight-line), and so on ....