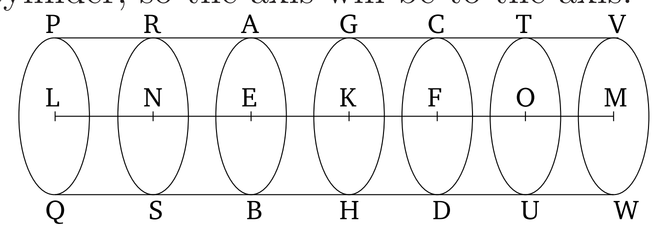
Book 12
Proposition 13

If a cylinder is cut by a plane which is parallel to the opposite planes (of the cylinder) then as the cylinder (is) to the cylinder, so the axis will be to the axis.



For let the cylinder AD have been cut by the plane GH which is parallel to the opposite planes (of the cylinder), AB and CD. And let the plane GH have met the axis at point K. I say that as cylinder BG is to cylinder GD, so axis EK (is) to axis KF.

For let axis EF have been produced in each direction to points L and M. And let any number whatsoever (of lengths), EN and NL, equal to axis EK, be set out (on the axis EL), and any number whatsoever (of lengths), FO and OM, equal to (axis) FK, (on the axis KM). And let the cylinder PW, whose bases (are) the circles PQ and VW, have been conceived on axis LM. And let planes parallel to AB, CD, and the bases of cylinder PW, have been produced through points N and O, and let them have made the circles RS and TU around the centers N and O (respectively). And since axes LN, NE, and EK are equal to one another, the cylinders QR, RB, and BG are to one another as their bases [Prop. 12.11]. But the bases are equal. Thus, the cylinders QR, RB, and BG (are) also equal to one another. Therefore, since the axes LN, NE, and EK are equal to one another, and the cylinders QR, RB, and BG are also equal to one another, and the number (of the former) is equal to the number (of the latter), thus as many multiples as axis KL is of axis EK, so many multiples is cylinder QG also of cylinder GB. And so, for the same (reasons), as many multiples as axis MK is of axis KF, so many multiples is cylinder WG also of cylinder GD. And if axis KL is equal to axis KM then cylinder QG will also be equal to cylinder GW, and if the axis (is) greater than the axis then the cylinder (will also be) greater than the cylinder, and if (the axis is) less then (the cylinder will also be) less. So, there are four magnitudes—the axes EK and KF, and the cylinders BG and GD—and equal multiples have been taken of axis EK and cylinder BG—(namely), axis LK and cylinder QG—and of axis KF and cylinder GD—(namely), axis KM and cylinder GW. And it has been shown that if axis KL exceeds axis KM then cylinder QG also exceeds cylinder GW, and if (the axes are) equal then (the cylinders are) equal, and if (KL is) less then (QG is) less. Thus, as axis EK is to axis KF, so cylinder BG (is) to cylinder GD [Def. 5.5]. (Which is) the very thing it was required to show.