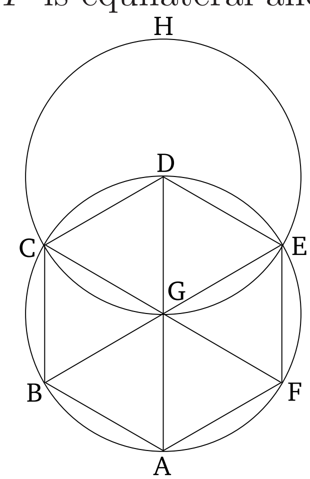
Book 4
Proposition 15

To inscribe an equilateral and equiangular hexagon in a given circle.

Let $ABCDEF$ be the given circle. So it is required to inscribe an equilateral and equiangular hexagon in circle $ABCDEF$.

Let the diameter AD of circle $ABCDEF$ have been drawn,† and let the center G of the circle have been found [Prop. 3.1]. And let the circle $EGCH$ have been drawn, with center D, and radius DG. And EG and CG being joined, let them have been drawn across (the circle) to points B and F (respectively). And let AB, BC, CD, DE, EF, and FA have been joined. I say that the hexagon $ABCDEF$ is equilateral and equiangular.



For since point G is the center of circle $ABCDEF$, GE is equal to GD. Again, since point D is the center of circle GCH, DE is equal to DG. But, GE was shown (to be) equal to GD. Thus, GE is also equal to ED. Thus, triangle EGD is equilateral. Thus, its three angles EGD, GDE, and DEG are also equal to one another, inasmuch as the angles at the base of isosceles triangles are equal to one another [Prop. 1.5]. And the three angles of the triangle are equal to two right-angles [Prop. 1.32]. Thus, angle EGD is one third of two right-angles. So, similarly, DGC can also be shown (to be) one third of two right-angles. And since the straight-line CG, standing on EB, makes adjacent angles EGC and CGB equal to two right-angles [Prop. 1.13], the remaining angle CGB is thus also one third of two right-angles. Thus, angles EGD, DGC, and CGB are equal to one another. And hence the (angles) opposite to them BGA, AGF, and FGE are also equal [to EGD, DGC, and CGB (respectively)] [Prop. 1.15]. Thus, the six angles EGD, DGC, CGB, BGA, AGF, and FGE are equal to one another. And equal angles stand on equal circumferences [Prop. 3.26]. Thus, the six circumferences AB, BC, CD, DE, EF, and FA are equal to one another. And equal circumferences are subtended by equal straight-lines [Prop. 3.29]. Thus, the six straight-lines (AB, BC, CD, DE, EF, and FA) are equal to one another. Thus, hexagon $ABCDEF$ is equilateral. So, I say that (it is) also equiangular. For since circumference FA is equal to circumference ED, let circumference $ABCD$ have been added to both. Thus, the whole of $FABCD$ is equal to the whole of $EDCBA$. And angle FED stands on circumference $FABCD$, and angle AFE on circumference $EDCBA$. Thus, angle AFE is equal to DEF [Prop. 3.27]. Similarly, it can also be shown that the remaining angles of hexagon $ABCDEF$ are individually equal to each of the angles AFE and

FED. Thus, hexagon $ABCDEF$ is equiangular. And it was also shown (to be) equilateral. And it has been inscribed in circle $ABCDE$.

Thus, an equilateral and equiangular hexagon has been inscribed in the given circle. (Which is) the very thing it was required to do.

Corollary

So, from this, (it is) manifest that a side of the hexagon is equal to the radius of the circle.

And similarly to a pentagon, if we draw tangents to the circle through the (sixfold) divisions of the (circumference of the) circle, an equilateral and equiangular hexagon can be circumscribed about the circle, analogously to the aforementioned pentagon. And, further, by (means) similar to the aforementioned pentagon, we can inscribe and circumscribe a circle in (and about) a given hexagon. (Which is) the very thing it was required to do.