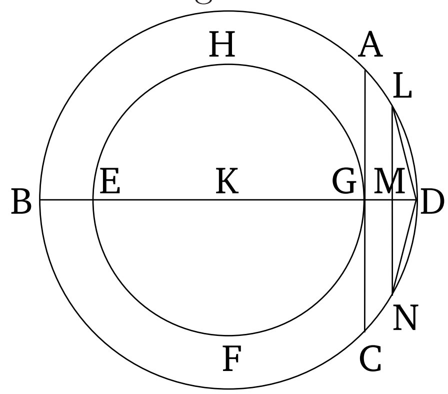
Book 12
Proposition 16

There being two circles about the same center, to inscribe an equilateral and even-sided polygon in the greater circle, not touching the lesser circle.



Let $ABCD$ and $EFGH$ be the given two circles, about the same center, K. So, it is necessary to inscribe an equilateral and even-sided polygon in the greater circle $ABCD$, not touching circle $EFGH$.

Let the straight-line BKD have been drawn through the center K. And let GA have been drawn, at right-angles to the straight-line BD, through point G, and let it have been drawn through to C. Thus, AC touches circle $EFGH$ [Prop. 3.16 corr.]. So, (by) cutting circumference BAD in half, and the half of it in half, and doing this continually, we will (eventually) leave a circumference less than AD [Prop. 10.1]. Let it have been left, and let it be LD. And let LM have been drawn, from L, perpendicular to BD, and let it have been drawn through to N. And let LD and DN have been joined. Thus, LD is equal to DN [Props. 3.3, 1.4]. And since LN is parallel to AC [Prop. 1.28], and AC touches circle $EFGH$, LN thus does not touch circle $EFGH$. Thus, even more so, LD and DN do not touch circle $EFGH$. And if we continuously insert (straight-lines) equal to straight-line LD into circle $ABCD$ [Prop. 4.1] then an equilateral and even-sided polygon, not touching the lesser circle $EFGH$, will have been inscribed in circle $ABCD$. (Which is) the very thing it was required to do.