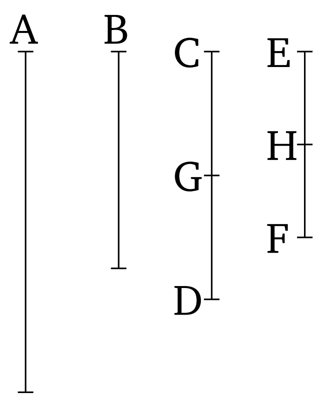
Book 7
Proposition 20

The least numbers of those (numbers) having the same ratio measure those (numbers) having the same ratio as them an equal number of times, the greater (measuring) the greater, and the lesser the lesser.

For let CD and EF be the least numbers having the same ratio as A and B (respectively). I say that CD measures A the same number of times as EF (measures) B.



For CD is not parts of A. For, if possible, let it be (parts of A). Thus, EF is also the same parts of B that CD (is) of A [Def. 7.20, Prop. 7.13]. Thus, as many parts of A as are in CD, so many parts of B are also in EF. Let CD have been divided into the parts of A, CG and GD, and EF into the parts of B, EH and HF. So the multitude of (divisions) CG, GD will be equal to the multitude of (divisions) EH, HF. And since the numbers CG and GD are equal to one another, and the numbers EH and HF are also equal to one another, and the multitude of (divisions) CG, GD is equal to the multitude of (divisions) EH, HF, thus as CG is to EH, so GD (is) to HF. Thus, as one of the leading (numbers is) to one of the following, so will (the sum of) all of the leading (numbers) be to (the sum of) all of the following [Prop. 7.12]. Thus, as CG is to EH, so CD (is) to EF. Thus, CG and EH are in the same ratio as CD and EF, being less than them. The very thing is impossible. For CD and EF were assumed (to be) the least of those (numbers) having the same ratio as them. Thus, CD is not parts of A. Thus, (it is) a part (of A) [Prop. 7.4]. And EF is the same part of B that CD (is) of A [Def. 7.20, Pr

Thus, CD measures A the same number of times that EF (measures) B. (Which is) the very thing it was required to show.