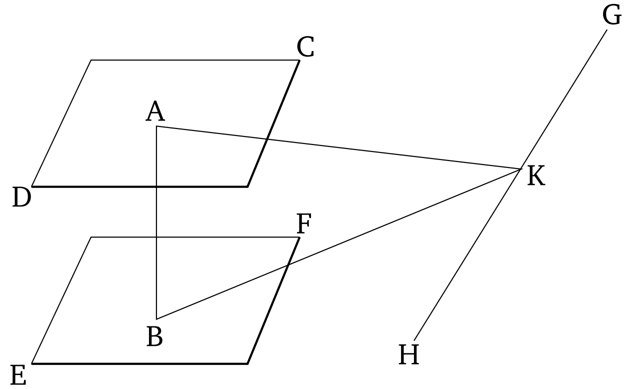
Book 11
Proposition 14

Planes to which the same straight-line is at right-angles will be parallel planes.

For let some straight-line AB be at right-angles to each of the planes CD and EF. I say that the planes are parallel.



For, if not, being produced, they will meet. Let them have met. So they will make a straight-line as a common section [Prop. 11.3]. Let them have made GH. And let some random point K have been taken on GH. And let AK and BK have been joined.

And since AB is at right-angles to the plane EF, AB is thus also at right-angles to BK, which is a straight-line in the produced plane EF [Def. 11.3]. Thus, angle ABK is a right-angle. So, for the same (reasons), BAK is also a right-angle. So the (sum of the) two angles ABK and BAK in the triangle ABK is equal to two right-angles. The very thing is impossible [Prop. 1.17]. Thus, planes CD and EF, being produced, will not meet. Planes CD and EF are thus parallel [Def. 11.8].

Thus, planes to which the same straight-line is at right-angles are parallel planes. (Which is) the very thing it was required to show.