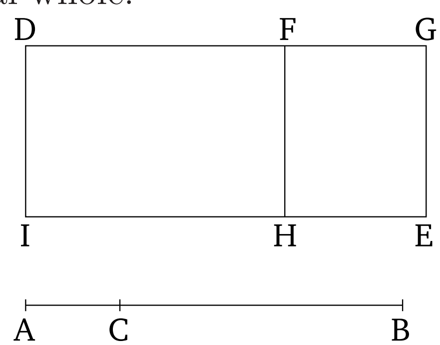
Book 10
Proposition 78

If a straight-line, which is incommensurable in square with the whole, and with the whole makes the sum of the squares on them medial, and twice the (rectangle contained) by them medial, and, moreover, the (sum of the) squares on them incommensurable with twice the (rectangle contained) by them, is subtracted from a(nother) straight-line then the remainder is an irrational (straight-line). Let it be called that which makes with a medial (area) a medial whole.



For let the straight-line BC, which is incommensurable in square AB, and fulfils the (other) prescribed (conditions), have been subtracted from the (straight-line) AB [Prop. 10.35]. I say that the remainder AC is the irrational (straight-line) called that which makes with a medial (area) a medial whole.

For let the rational (straight-line) DI be laid down. And let DE, equal to the (sum of the squares) on AB and BC, have been applied to DI, producing DG as breadth. And let DH, equal to twice the (rectangle contained) by AB and BC, have been subtracted (from DE) [producing DF as breadth]. Thus, the remainder FE is equal to the (square) on AC [Prop. 2.7]. Hence,

AC is the square-root of FE. And since the sum of the squares on AB and BC is medial, and is equal to DE, DE [is] thus medial. And it is applied to the rational (straight-line) DI, producing DG as breadth. Thus, DG is rational, and incommensurable in length with DI [Prop 10.22]. Again, since twice the (rectangle contained) by AB and BC is medial, and is equal to DH, DH is thus medial. And it is applied to the rational (straight-line) DI, producing DF as breadth. Thus, DF is also rational, and incommensurable in length with DI [Prop. 10.22]. And since the (sum of the squares) on AB and BC is incommensurable with twice the (rectangle contained) by AB and BC, DE (is) also incommensurable with DH. And as DE (is) to DH, so DG also is to DF [Prop. 6.1]. Thus, DG (is) incommensurable (in length) with DF [Prop. 10.11]. And they are both rational. Thus, GD and DF are rational (straight-lines which are) commensurable in square only. Thus, FG is an apotome [Prop. 10.73]. And FH (is) rational. And the [rectangle] contained by a rational (straight-line) and an apotome is irrational [Prop. 10.20], and its square-root is irrational. And AC is the square-root of FE. Thus, AC is irrational. Let it be called that which makes with a medial (area) a medial whole.† (Which is) the very thing it was required to show.