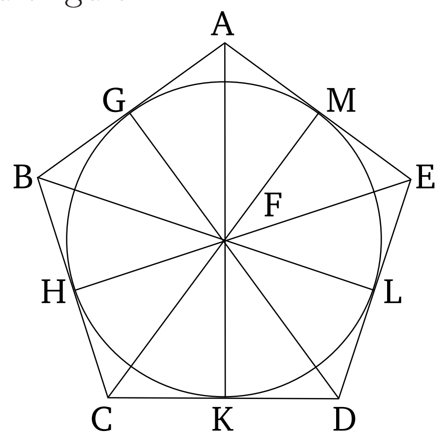
Book 4
Proposition 13

To inscribe a circle in a given pentagon, which is equilateral and equiangular.



Let $ABCDE$ be the given equilateral and equiangular pentagon. So it is required to inscribe a circle in pentagon $ABCDE$.

For let angles BCD and CDE have each been cut in half by each of the straight-lines CF and DF (respectively) [Prop. 1.9]. And from the point F, at which the straight-lines CF and DF meet one another, let the straight-lines FB, FA, and FE have been joined. And since BC is equal to CD, and CF (is) common, the two (straight-lines) BC, CF are equal to the two (straight-lines) DC, CF. And angle BCF [is] equal to angle DCF. Thus, the base BF is equal to the base DF, and triangle BCF is equal to triangle DCF, and the remaining angles will be equal to the (corresponding) remaining angles which the equal sides subtend [Prop. 1.4]. Thus, angle CBF (is) equal to CDF. And since CDE is double CDF, and CDE (is) equal to ABC, and CDF

to CBF, CBA is thus also double CBF. Thus, angle ABF is equal to FBC. Thus, angle ABC has been cut in half by the straight-line BF. So, similarly, it can be shown that BAE and AED have been cut in half by the straight-lines FA and FE, respectively. So let FG, FH, FK, FL, and FM have been drawn from point F, perpendicular to the straight-lines AB, BC, CD, DE, and EA (respectively) [Prop. 1.12]. And since angle HCF is equal to KCF, and the right-angle FHC is also equal to the [right-angle] FKC, FHC and FKC are two triangles having two angles equal to two angles, and one side equal to one side, (namely) their common (side) FC, subtending one of the equal angles. Thus, they will also have the remaining sides equal to the (corresponding) remaining sides [Prop. 1.26]. Thus, the perpendicular FH (is) equal to the perpendicular FK. So, similarly, it can be shown that FL, FM, and FG are each equal to each of FH and FK. Thus, the five straight-lines FG, FH, FK, FL, and FM are equal to one another. Thus, the circle drawn with center F, and radius one of G, H, K, L, or M, will also go through the remaining points, and will touch the straight-lines AB, BC, CD, DE, and EA, on account of the angles at points G, H, K, L, and M being right-angles. For if it does not touch them, but cuts them, it follows that a (straight-line) drawn at right-angles to the diameter of the circle, from its extremity, falls inside the circle. The very thing was shown (to be) absurd [Prop. 3.16]. Thus, the circle drawn with center F, and radius one of G, H, K, L, or M, does not cut the straight-lines AB, BC, CD, DE, or EA. Thus, it will touch them. Let it have been drawn, like $GHKLM$ (in the figure).

Thus, a circle has been inscribed in the given pentagon which is equilateral and equiangular. (Which is) the very thing it was required to do.