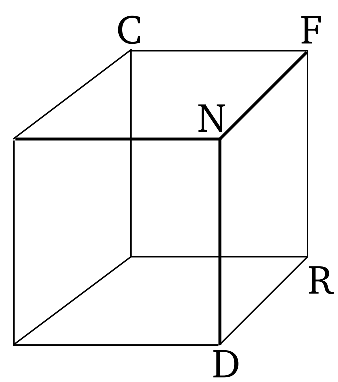
Book 11
Proposition 33

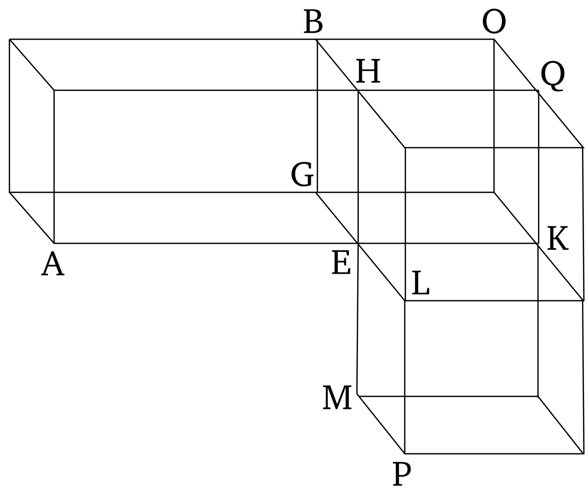
Similar parallelepiped solids are to one another as the cubed ratio of their corresponding sides.

Let AB and CD be similar parallelepiped solids, and let AE correspond to CF. I say that solid AB has to solid CD the cubed ratio that AE (has) to CF.

For let EK, EL, and EM have been produced in a straight-line with AE, GE, and HE (respectively). And let EK be made equal to CF, and EL equal to FN, and, further, EM equal to FR. And let the parallelogram KL have been completed, and the solid KP.



And since the two (straight-lines) KE and EL are equal to the two (straight-lines) CF and FN, but angle KEL is also equal to angle CFN, inasmuch as AEG is also equal to CFN, on account of the similarity of the solids AB and CD, parallelogram KL is thus equal [and similar] to parallelogram CN. So, for the same (reasons), parallelogram KM is also equal and similar to [parallelogram] CR, and, further, EP to DF. Thus, three parallelograms of solid KP are equal and similar to three parallelograms of solid CD. But the three (former parallelograms) are equal and similar to the three opposite (parallelograms), and the three (latter parallelograms) are equal and similar to the three opposite (parallelograms) [Prop. 11.24]. Thus, the whole of solid KP is equal and similar to the whole of solid CD [Def. 11.10]. Let parallelogram GK have been completed. And let the the solids EO and LQ, with bases the parallelograms GK and KL (respectively), and with the same height as AB, have been completed. And since, on account of the similarity of solids AB and CD, as AE is to CF, so EG (is) to FN, and EH to FR [Defs. 6.1, 11.9], and CF (is) equal to EK, and FN to EL, and FR to EM, thus as AE is to EK, so GE (is) to EL, and HE to EM. But, as AE (is) to EK, so [parallelogram] AG (is) to parallelogram GK, and as GE (is) to EL, so GK (is) to KL, and as HE (is) to EM, so QE (is) to KM [Prop. 6.1]. And thus as parallelogram AG (is) to GK, so GK (is) to KL, and QE (is) to KM. But, as AG (is) to GK, so solid AB (is) to solid EO, and as GK (is) to KL, so solid OE (is) to solid QL, and as QE (is) to KM, so solid QL (is) to solid KP [Prop. 11.32]. And, thus, as solid AB is to EO, so EO (is) to QL, and QL to KP. And if four magnitudes are continuously proportional then the first has to the fourth the cubed ratio that (it has) to the second [Def. 5.10]. Thus, solid AB has to KP the cubed ratio which AB (has) to EO. But, as AB (is) to EO, so parallelogram AG (is) to GK, and the straight-line AE to EK [Prop. 6.1]. Hence, solid AB also has to KP the cubed ratio that AE (has) to EK. And solid KP (is) equal to solid CD, and straight-line EK to CF. Thus, solid AB also has to solid CD the cubed ratio which its corresponding side AE (has) to the corresponding side CF.



Thus, similar parallelepiped solids are to one another as the cubed ratio of their corresponding sides. (Which is) the very thing it was required to show.

Corollary

So, (it is) clear, from this, that if four straight-lines are (continuously) proportional then as the first is to the fourth, so the parallelepiped solid on the first will be to the similar, and similarly described, parallelepiped solid on the second, since the first also has to the fourth the cubed ratio that (it has) to the second.