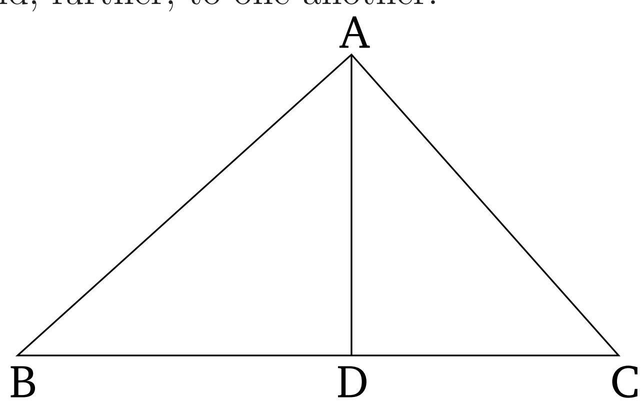
# Book 6
# Proposition 8

If, in a right-angled triangle, a (straight-line) is drawn from the right-angle perpendicular to the base then the triangles around the perpendicular are similar to the whole (triangle), and to one another.

Let $ABC$ be a right-angled triangle having the angle $BAC$ a right-angle, and let $AD$ have been drawn from $A$, perpendicular to $BC$ [Prop. 1.12]. I say that triangles $ABD$ and $ADC$ are each similar to the whole (triangle) $ABC$ and, further, to one another.




For since (angle) $BAC$ is equal to $ADB$—for each (are) right-angles—and the (angle) at $B$ (is) common to the two triangles $ABC$ and $ABD$, the remaining (angle) $ACB$ is thus equal to the remaining (angle) $BAD$ [Prop. 1.32]. Thus, triangle $ABC$ is equiangular to triangle $ABD$. Thus, as $BC$, subtending the right-angle in triangle $ABC$, is to $BA$, subtending the right-angle in triangle $ABD$, so the same $AB$, subtending the angle at $C$ in triangle $ABC$, (is) to $BD$, subtending the equal (angle) $BAD$ in triangle $ABD$, and, further, (so is) $AC$ to $AD$, (both) subtending the angle at $B$ common to the two triangles [Prop. 6.4]. Thus, triangle $ABC$ is equiangular to triangle $ABD$, and has the sides about the equal angles proportional. Thus, triangle $ABC$ [is] similar to triangle $ABD$ [Def. 6.1]. So, similarly, we can show that triangle $ABC$ is also similar to triangle $ADC$. Thus, [triangles] $ABD$ and $ADC$ are each similar to the whole (triangle) $ABC$.

So I say that triangles $ABD$ and $ADC$ are also similar to one another.

For since the right-angle $BDA$ is equal to the right-angle $ADC$, and, indeed, (angle) $BAD$ was also shown (to be) equal to the (angle) at $C$, thus the remaining (angle) at $B$ is also equal to the remaining (angle) $DAC$ [Prop. 1.32]. Thus, triangle $ABD$ is equiangular to triangle $ADC$. Thus, as $BD$, subtending (angle) $BAD$ in triangle $ABD$, is to $DA$, subtending the (angle) at $C$ in triangle $ADC$, (which is) equal to (angle) $BAD$, so (is) the same $AD$, subtending the angle at $B$ in triangle $ABD$, to $DC$, subtending (angle) $DAC$ in triangle $ADC$, (which is) equal to the (angle) at $B$, and, further, (so is) $BA$ to $AC$, (each) subtending right-angles [Prop. 6.4]. Thus, triangle $ABD$ is similar to triangle $ADC$ [Def. 6.1].

Thus, if, in a right-angled triangle, a (straight-line) is drawn from the right-angle perpendicular to the base then the triangles around the perpendicular are similar to the whole (triangle), and to one another. [(Which is) the very thing it was required to show.]

## Corollary

So (it is) clear, from this, that if, in a right-angled triangle, a (straight-line) is drawn from the right-angle perpendicular to the base then the (straight-line so) drawn is in mean proportion to the pieces of the base.† (Which is) the very thing it was required to show.