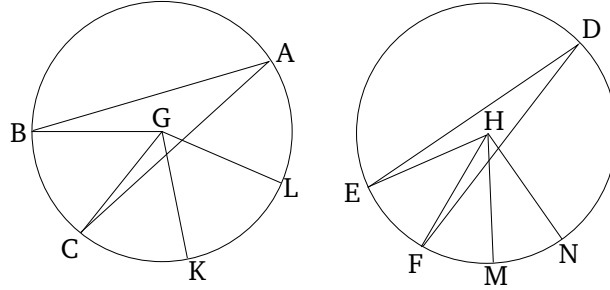


Book 6

Proposition 33

In equal circles, angles have the same ratio as the (ratio of the) circumferences on which they stand, whether they are standing at the centers (of the circles) or at the circumferences.



Let ABC and DEF be equal circles, and let BGC and EHF be angles at their centers, G and H (respectively), and BAC and EDF (angles) at their circumferences. I say that as circumference BC is to circumference EF , so angle BGC (is) to EHF , and (angle) BAC to EDF .

For let any number whatsoever of consecutive (circumferences), CK and KL , be made equal to circumference BC , and any number whatsoever, FM and MN , to circumference EF . And let GK , GL , HM , and HN have been joined.

Therefore, since circumferences BC , CK , and KL are equal to one another, angles BGC , CGK , and KGL are also equal to one another [Prop. 3.27]. Thus, as many times as circumference BL is (divisible) by BC , so many times is angle BGL also (divisible) by BGC . And so, for the same (reasons), as many times as circumference NE is (divisible) by EF , so many times is angle NHE also (divisible) by EHF . Thus, if circumference BL is equal

to circumference EN then angle BGL is also equal to EHN [Prop. 3.27], and if circumference BL is greater than circumference EN then angle BGL is also greater than EHN ,[†] and if (BL is) less (than EN then BGL is also) less (than EHN). So there are four magnitudes, two circumferences BC and EF , and two angles BGC and EHF . And equal multiples have been taken of circumference BC and angle BGC , (namely) circumference BL and angle BGL , and of circumference EF and angle EHF , (namely) circumference EN and angle EHN . And it has been shown that if circumference BL exceeds circumference EN then angle BGL also exceeds angle EHN , and if (BL is) equal (to EN then BGL is also) equal (to EHN), and if (BL is) less (than EN then BGL is also) less (than EHN). Thus, as circumference BC (is) to EF , so angle BGC (is) to EHF [Def. 5.5]. But as angle BGC (is) to EHF , so (angle) BAC (is) to EDF [Prop. 5.15]. For the former (are) double the latter (respectively) [Prop. 3.20]. Thus, also, as circumference BC (is) to circumference EF , so angle BGC (is) to EHF , and BAC to EDF .

Thus, in equal circles, angles have the same ratio as the (ratio of the) circumferences on which they stand, whether they are standing at the centers (of the circles) or at the circumferences. (Which is) the very thing it was required to show.