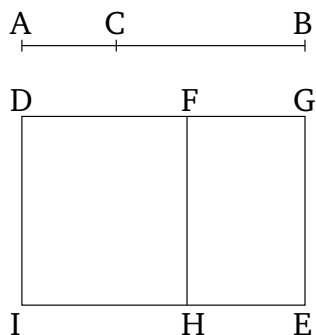


Book 10

Proposition 75

If a medial (straight-line), which is commensurable in square only with the whole, and which contains a medial (area) with the whole, is subtracted from a(nother) medial (straight-line) then the remainder is an irrational (straight-line). Let it be called a second apotome of a medial (straight-line).

For let the medial (straight-line) CB , which is commensurable in square only with the whole, AB , and which contains with the whole, AB , the medial (rectangle contained) by AB and BC , have been subtracted from the medial (straight-line) AB [Prop. 10.28]. I say that the remainder AC is an irrational (straight-line). Let it be called a second apotome of a medial (straight-line).



For let the rational (straight-line) DI be laid down. And let DE , equal to the (sum of the squares) on AB and BC , have been applied to DI , producing DG as breadth. And let DH , equal to twice the (rectangle contained) by AB and BC , have been applied to DI , producing DF as breadth. The remainder FE is thus

equal to the (square) on AC [Prop. 2.7]. And since the (squares) on AB and BC are medial and commensurable (with one another), DE (is) thus also medial [Props. 10.15, 10.23 corr.]. And it is applied to the rational (straight-line) DI , producing DG as breadth. Thus, DG is rational, and incommensurable in length with DI [Prop. 10.22]. Again, since the (rectangle contained) by AB and BC is medial, twice the (rectangle contained) by AB and BC is thus also medial [Prop. 10.23 corr.]. And it is equal to DH . Thus, DH is also medial. And it has been applied to the rational (straight-line) DI , producing DF as breadth. DF is thus rational, and incommensurable in length with DI [Prop. 10.22]. And since AB and BC are commensurable in square only, AB is thus incommensurable in length with BC . Thus, the square on AB (is) also incommensurable with the (rectangle contained) by AB and BC [Props. 10.21 lem., 10.11]. But, the (sum of the squares) on AB and BC is commensurable with the (square) on AB [Prop. 10.15], and twice the (rectangle contained) by AB and BC is commensurable with the (rectangle contained) by AB and BC [Prop. 10.6]. Thus, twice the (rectangle contained) by AB and BC is incommensurable with the (sum of the squares) on AB and BC [Prop. 10.13]. And DE is equal to the (sum of the squares) on AB and BC , and DH to twice the (rectangle contained) by AB and BC . Thus, DE [is] incommensurable with DH . And as DE (is) to DH , so GD (is) to DF [Prop. 6.1]. Thus, GD is incommensurable with DF [Prop. 10.11]. And they are both rational (straight-lines). Thus, GD and DF are ratio-

nal (straight-lines which are) commensurable in square only. Thus, FG is an apotome [Prop. 10.73]. And DI (is) rational. And the (area) contained by a rational and an irrational (straight-line) is irrational [Prop. 10.20], and its square-root is irrational. And AC is the square-root of FE . Thus, AC is an irrational (straight-line) [Def. 10.4]. And let it be called the second apotome of a medial (straight-line). (Which is) the very thing it was required to show.