

Book 1

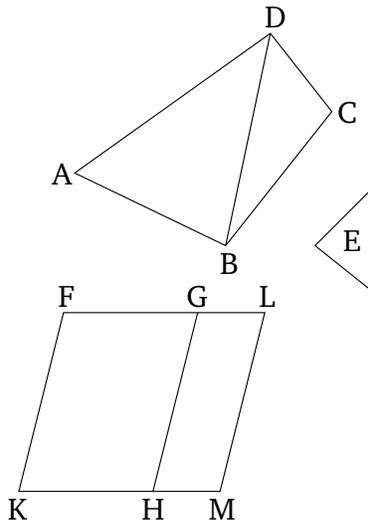
Proposition 45

To construct a parallelogram equal to a given rectilinear figure in a given rectilinear angle.

Let $ABCD$ be the given rectilinear figure, and E the given rectilinear angle. So it is required to construct a parallelogram equal to the rectilinear figure $ABCD$ in the given angle E .

Let DB have been joined, and let the parallelogram FH , equal to the triangle ABD , have been constructed in the angle HKF , which is equal to E [Prop. 1.42]. And let the parallelogram GM , equal to the triangle DBC , have been applied to the straight-line GH in the angle GHM , which is equal to E [Prop. 1.44]. And since angle E is equal to each of (angles) HKF and GHM , (angle) HKF is thus also equal to GHM . Let KHG have been added to both. Thus, (the sum of) FKH and KHG is equal to (the sum of) KHG and GHM . But, (the sum of) FKH and KHG is equal to two right-angles [Prop. 1.29]. Thus, (the sum of) KHG and GHM is also equal to two right-angles. So two straight-lines, KH and HM , not lying on the same side, make adjacent angles with some straight-line GH , at the point H on it, (whose sum is) equal to two right-angles. Thus, KH is straight-on to HM [Prop. 1.14]. And since the straight-line HG falls across the parallels KM and FG , the alternate angles MHG and HGF are equal to one another [Prop. 1.29]. Let HGL have been added to both. Thus, (the sum of) MHG and HGL

is equal to (the sum of) HGF and HGL . But, (the sum of) MHG and HGL is equal to two right-angles [Prop. 1.29]. Thus, (the sum of) HGF and HGL is also equal to two right-angles. Thus, FG is straight-on to GL [Prop. 1.14]. And since FK is equal and parallel to HG [Prop. 1.34], but also HG to ML [Prop. 1.34], KF is thus also equal and parallel to ML [Prop. 1.30]. And the straight-lines KM and FL join them. Thus, KM and FL are equal and parallel as well [Prop. 1.33]. Thus, $KFLM$ is a parallelogram. And since triangle ABD is equal to parallelogram FH , and DBC to GM , the whole rectilinear figure $ABCD$ is thus equal to the whole parallelogram $KFLM$.



Thus, the parallelogram $KFLM$, equal to the given rectilinear figure $ABCD$, has been constructed in the angle FKM , which is equal to the given (angle) E . (Which is) the very thing it was required to do.