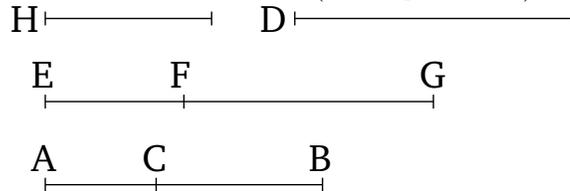


# Book 10

## Proposition 49

To find a second binomial (straight-line).



Let the two numbers  $AC$  and  $CB$  be laid down such that their sum  $AB$  has to  $BC$  the ratio which (some) square number (has) to (some) square number, and does not have to  $AC$  the ratio which (some) square number (has) to (some) square number [Prop. 10.28 lem. I]. And let the rational (straight-line)  $D$  be laid down. And let  $EF$  be commensurable in length with  $D$ .  $EF$  is thus a rational (straight-line). So, let it also have been contrived that as the number  $CA$  (is) to  $AB$ , so the (square) on  $EF$  (is) to the (square) on  $FG$  [Prop. 10.6 corr.]. Thus, the (square) on  $EF$  is commensurable with the (square) on  $FG$  [Prop. 10.6]. Thus,  $FG$  is also a rational (straight-line). And since the number  $CA$  does not have to  $AB$  the ratio which (some) square number (has) to (some) square number, the (square) on  $EF$  does not have to the (square) on  $FG$  the ratio which (some) square number (has) to (some) square number either. Thus,  $EF$  is incommensurable in length with  $FG$  [Prop. 10.9].  $EF$  and  $FG$  are thus rational (straight-lines which are) commensurable in square only. Thus,  $EG$  is a binomial (straight-line) [Prop. 10.36]. So, we must show that (it is) also a second (binomial straight-line).

For since, inversely, as the number  $BA$  is to  $AC$ , so the (square) on  $GF$  (is) to the (square) on  $FE$  [Prop. 5.7 corr.], and  $BA$  (is) greater than  $AC$ , the (square) on  $GF$  (is) thus [also] greater than the (square) on  $FE$  [Prop. 5.14]. Let (the sum of) the (squares) on  $EF$  and  $H$  be equal to the (square) on  $GF$ . Thus, via conversion, as  $AB$  is to  $BC$ , so the (square) on  $FG$  (is) to the (square) on  $H$  [Prop. 5.19 corr.]. But,  $AB$  has to  $BC$  the ratio which (some) square number (has) to (some) square number. Thus, the (square) on  $FG$  also has to the (square) on  $H$  the ratio which (some) square number (has) to (some) square number. Thus,  $FG$  is commensurable in length with  $H$  [Prop. 10.9]. Hence, the square on  $FG$  is greater than (the square on)  $FE$  by the (square) on (some straight-line) commensurable in length with ( $FG$ ). And  $FG$  and  $FE$  are rational (straight-lines which are) commensurable in square only. And the lesser term  $EF$  is commensurable in length with the rational (straight-line)  $D$  (previously) laid down.

Thus,  $EG$  is a second binomial (straight-line) [Def. 10.6]. (Which is) the very thing it was required to show.