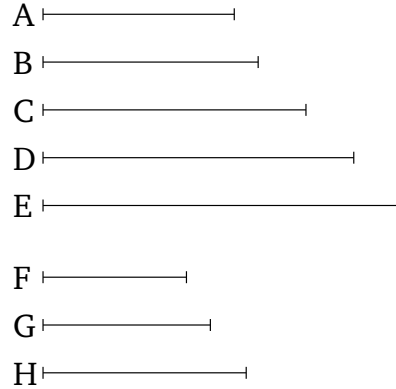


Book 8

Proposition 6

If there are any multitude whatsoever of continuously proportional numbers, and the first does not measure the second, then no other (number) will measure any other (number) either.



Let A , B , C , D , E be any multitude whatsoever of continuously proportional numbers, and let A not measure B . I say that no other (number) will measure any other (number) either.

Now, (it is) clear that A , B , C , D , E do not successively measure one another. For A does not even measure B . So I say that no other (number) will measure any other (number) either. For, if possible, let A measure C . And as many (numbers) as are A , B , C , let so many of the least numbers, F , G , H , have been taken of those (numbers) having the same ratio as A , B , C [Prop. 7.33]. And since F , G , H are in the same ratio as A , B , C , and the multitude of A , B , C is equal to the multitude of F , G , H , thus, via equality, as A is to C , so F (is) to H [Prop. 7.14]. And since as A is to B , so F (is) to G , and A does not measure B , F does not measure G either [Def. 7.20]. Thus, F is not a unit. For a unit

measures all numbers. And F and H are prime to one another [Prop. 8.3] [and thus F does not measure H]. And as F is to H , so A (is) to C . And thus A does not measure C either [Def. 7.20]. So, similarly, we can show that no other (number) can measure any other (number) either. (Which is) the very thing it was required to show.