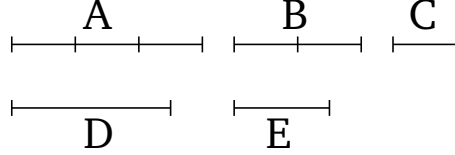


Book 10

Proposition 5

Commensurable magnitudes have to one another the ratio which (some) number (has) to (some) number.



Let A and B be commensurable magnitudes. I say that A has to B the ratio which (some) number (has) to (some) number.

For if A and B are commensurable (magnitudes) then some magnitude will measure them. Let it (so) measure (them), and let it be C . And as many times as C measures A , so many units let there be in D . And as many times as C measures B , so many units let there be in E .

Therefore, since C measures A according to the units in D , and a unit also measures D according to the units in it, a unit thus measures the number D as many times as the magnitude C (measures) A . Thus, as C is to A , so a unit (is) to D [Def. 7.20].[†] Thus, inversely, as A (is) to C , so D (is) to a unit [Prop. 5.7 corr.]. Again, since C measures B according to the units in E , and a unit also measures E according to the units in it, a unit thus measures E the same number of times that C (measures) B . Thus, as C is to B , so a unit (is) to E [Def. 7.20]. And it was also shown that as A (is) to C , so D (is) to a unit. Thus, via equality, as A is to B , so the number D (is) to the (number) E [Prop. 5.22].

Thus, the commensurable magnitudes A and B have

to one another the ratio which the number D (has) to the number E . (Which is) the very thing it was required to show.