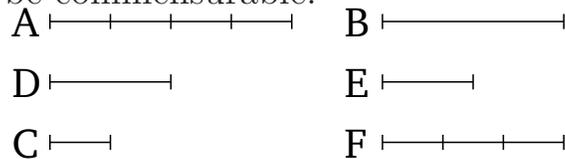


# Book 10

## Proposition 6

If two magnitudes have to one another the ratio which (some) number (has) to (some) number then the magnitudes will be commensurable.



For let the two magnitudes  $A$  and  $B$  have to one another the ratio which the number  $D$  (has) to the number  $E$ . I say that the magnitudes  $A$  and  $B$  are commensurable.

For, as many units as there are in  $D$ , let  $A$  have been divided into so many equal (divisions). And let  $C$  be equal to one of them. And as many units as there are in  $E$ , let  $F$  be the sum of so many magnitudes equal to  $C$ .

Therefore, since as many units as there are in  $D$ , so many magnitudes equal to  $C$  are also in  $A$ , therefore whichever part a unit is of  $D$ ,  $C$  is also the same part of  $A$ . Thus, as  $C$  is to  $A$ , so a unit (is) to  $D$  [Def. 7.20]. And a unit measures the number  $D$ . Thus,  $C$  also measures  $A$ . And since as  $C$  is to  $A$ , so a unit (is) to the [number]  $D$ , thus, inversely, as  $A$  (is) to  $C$ , so the number  $D$  (is) to a unit [Prop. 5.7 corr.]. Again, since as many units as there are in  $E$ , so many (magnitudes) equal to  $C$  are also in  $F$ , thus as  $C$  is to  $F$ , so a unit (is) to the [number]  $E$  [Def. 7.20]. And it was also shown that as  $A$  (is) to  $C$ , so  $D$  (is) to a unit. Thus, via equality, as  $A$  is to  $F$ , so  $D$  (is) to  $E$  [Prop. 5.22]. But, as  $D$  (is) to

$E$ , so  $A$  is to  $B$ . And thus as  $A$  (is) to  $B$ , so (it) also is to  $F$  [Prop. 5.11]. Thus,  $A$  has the same ratio to each of  $B$  and  $F$ . Thus,  $B$  is equal to  $F$  [Prop. 5.9]. And  $C$  measures  $F$ . Thus, it also measures  $B$ . But, in fact, (it) also (measures)  $A$ . Thus,  $C$  measures (both)  $A$  and  $B$ . Thus,  $A$  is commensurable with  $B$  [Def. 10.1].

Thus, if two magnitudes . . . to one another, and so on  
.....

## Corollary

So it is clear, from this, that if there are two numbers, like  $D$  and  $E$ , and a straight-line, like  $A$ , then it is possible to contrive that as the number  $D$  (is) to the number  $E$ , so the straight-line (is) to (another) straight-line (*i.e.*,  $F$ ). And if the mean proportion, (say)  $B$ , is taken of  $A$  and  $F$ , then as  $A$  is to  $F$ , so the (square) on  $A$  (will be) to the (square) on  $B$ . That is to say, as the first (is) to the third, so the (figure) on the first (is) to the similar, and similarly described, (figure) on the second [Prop. 6.19 corr.]. But, as  $A$  (is) to  $F$ , so the number  $D$  is to the number  $E$ . Thus, it has also been contrived that as the number  $D$  (is) to the number  $E$ , so the (figure) on the straight-line  $A$  (is) to the (similar figure) on the straight-line  $B$ . (Which is) the very thing it was required to show.