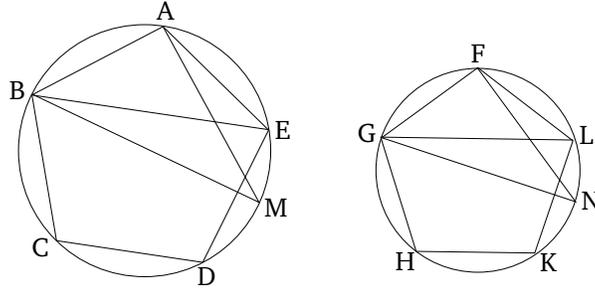


# Book 12

## Proposition 1

Similar polygons (inscribed) in circles are to one another as the squares on the diameters (of the circles).



Let  $ABC$  and  $FGH$  be circles, and let  $ABCDE$  and  $FGHKL$  be similar polygons (inscribed) in them (respectively), and let  $BM$  and  $GN$  be the diameters of the circles (respectively). I say that as the square on  $BM$  is to the square on  $GN$ , so polygon  $ABCDE$  (is) to polygon  $FGHKL$ .

For let  $BE$ ,  $AM$ ,  $GL$ , and  $FN$  have been joined. And since polygon  $ABCDE$  (is) similar to polygon  $FGHKL$ , angle  $BAE$  is also equal to (angle)  $GFL$ , and as  $BA$  is to  $AE$ , so  $GF$  (is) to  $FL$  [Def. 6.1]. So,  $BAE$  and  $GFL$  are two triangles having one angle equal to one angle, (namely),  $BAE$  (equal) to  $GFL$ , and the sides around the equal angles proportional. Triangle  $ABE$  is thus equiangular with triangle  $FGL$  [Prop. 6.6]. Thus, angle  $AEB$  is equal to (angle)  $FLG$ . But,  $AEB$  is equal to  $AMB$ , and  $FLG$  to  $FNG$ , for they stand on the same circumference [Prop. 3.27]. Thus,  $AMB$  is also equal to  $FNG$ . And the right-angle  $BAM$  is also equal to the right-angle  $GFN$  [Prop. 3.31]. Thus, the remaining (an-

gle) is also equal to the remaining (angle) [Prop. 1.32]. Thus, triangle  $ABM$  is equiangular with triangle  $FGN$ . Thus, proportionally, as  $BM$  is to  $GN$ , so  $BA$  (is) to  $GF$  [Prop. 6.4]. But, the (ratio) of the square on  $BM$  to the square on  $GN$  is the square of the ratio of  $BM$  to  $GN$ , and the (ratio) of polygon  $ABCDE$  to polygon  $FGHKL$  is the square of the (ratio) of  $BA$  to  $GF$  [Prop. 6.20]. And, thus, as the square on  $BM$  (is) to the square on  $GN$ , so polygon  $ABCDE$  (is) to polygon  $FGHKL$ .

Thus, similar polygons (inscribed) in circles are to one another as the squares on the diameters (of the circles). (Which is) the very thing it was required to show.