

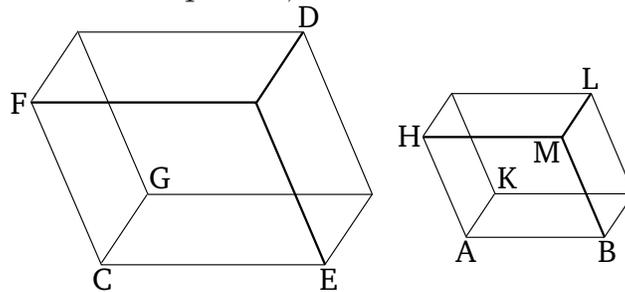
# Book 11

## Proposition 27

To describe a parallelepiped solid similar, and similarly laid out, to a given parallelepiped solid on a given straight-line.

Let the given straight-line be  $AB$ , and the given parallelepiped solid  $CD$ . So, it is necessary to describe a parallelepiped solid similar, and similarly laid out, to the given parallelepiped solid  $CD$  on the given straight-line  $AB$ .

For, let a (solid angle) contained by the (plane angles)  $BAH$ ,  $HAK$ , and  $KAB$  have been constructed, equal to solid angle at  $C$ , on the straight-line  $AB$  at the point  $A$  on it [Prop. 11.26], such that angle  $BAH$  is equal to  $ECF$ , and  $BAK$  to  $ECG$ , and  $KAH$  to  $GCF$ . And let it have been contrived that as  $EC$  (is) to  $CG$ , so  $BA$  (is) to  $AK$ , and as  $GC$  (is) to  $CF$ , so  $KA$  (is) to  $AH$  [Prop. 6.12]. And thus, via equality, as  $EC$  is to  $CF$ , so  $BA$  (is) to  $AH$  [Prop. 5.22]. And let the parallelogram  $HB$  have been completed, and the solid  $AL$ .



And since as  $EC$  is to  $CG$ , so  $BA$  (is) to  $AK$ , and the sides about the equal angles  $ECG$  and  $BAK$  are (thus) proportional, the parallelogram  $GE$  is thus similar to

the parallelogram  $KB$ . So, for the same (reasons), the parallelogram  $KH$  is also similar to the parallelogram  $GF$ , and, further,  $FE$  (is similar) to  $HB$ . Thus, three of the parallelograms of solid  $CD$  are similar to three of the parallelograms of solid  $AL$ . But, the (former) three are equal and similar to the three opposite, and the (latter) three are equal and similar to the three opposite. Thus, the whole solid  $CD$  is similar to the whole solid  $AL$  [Def. 11.9].

Thus,  $AL$ , similar, and similarly laid out, to the given parallelepiped solid  $CD$ , has been described on the given straight-lines  $AB$ . (Which is) the very thing it was required to do.