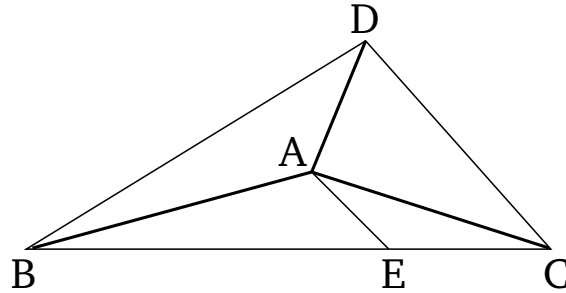


# Book 11

## Proposition 20

If a solid angle is contained by three plane angles then (the sum of) any two (angles) is greater than the remaining (one), (the angles) being taken up in any (possible way).



For let the solid angle  $A$  have been contained by the three plane angles  $BAC$ ,  $CAD$ , and  $DAB$ . I say that (the sum of) any two of the angles  $BAC$ ,  $CAD$ , and  $DAB$  is greater than the remaining (one), (the angles) being taken up in any (possible way).

For if the angles  $BAC$ ,  $CAD$ , and  $DAB$  are equal to one another then (it is) clear that (the sum of) any two is greater than the remaining (one). But, if not, let  $BAC$  be greater (than  $CAD$  or  $DAB$ ). And let (angle)  $BAE$ , equal to the angle  $DAB$ , have been constructed in the plane through  $BAC$ , on the straight-line  $AB$ , at the point  $A$  on it. And let  $AE$  be made equal to  $AD$ . And  $BE$  being drawn across through point  $E$ , let it cut the straight-lines  $AB$  and  $AC$  at points  $B$  and  $C$  (respectively). And let  $DB$  and  $DC$  have been joined.

And since  $DA$  is equal to  $AE$ , and  $AB$  (is) common, the two (straight-lines  $AD$  and  $AB$  are) equal to the two (straight-lines  $EA$  and  $AB$ , respectively). And an-

gle  $DAB$  (is) equal to angle  $BAE$ . Thus, the base  $DB$  is equal to the base  $BE$  [Prop. 1.4]. And since the (sum of the) two (straight-lines)  $BD$  and  $DC$  is greater than  $BC$  [Prop. 1.20], of which  $DB$  was shown (to be) equal to  $BE$ , the remainder  $DC$  is thus greater than the remainder  $EC$ . And since  $DA$  is equal to  $AE$ , but  $AC$  (is) common, and the base  $DC$  is greater than the base  $EC$ , the angle  $DAC$  is thus greater than the angle  $EAC$  [Prop. 1.25]. And  $DAB$  was also shown (to be) equal to  $BAE$ . Thus, (the sum of)  $DAB$  and  $DAC$  is greater than  $BAC$ . So, similarly, we can also show that the remaining (angles), being taken in pairs, are greater than the remaining (one).

Thus, if a solid angle is contained by three plane angles then (the sum of) any two (angles) is greater than the remaining (one), (the angles) being taken up in any (possible way). (Which is) the very thing it was required to show.