

Book 11

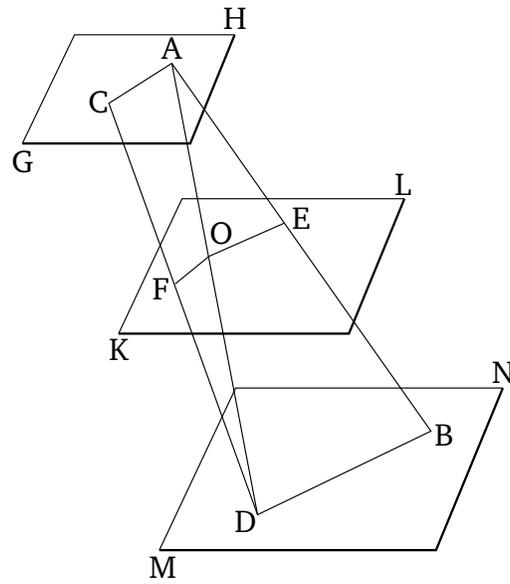
Proposition 17

If two straight-lines are cut by parallel planes then they will be cut in the same ratios.

For let the two straight-lines AB and CD be cut by the parallel planes GH , KL , and MN at the points A , E , B , and C , F , D (respectively). I say that as the straight-line AE is to EB , so CF (is) to FD .

For let AC , BD , and AD have been joined, and let AD meet the plane KL at point O , and let EO and OF have been joined.

And since two parallel planes KL and MN are cut by the plane $EBDO$, their common sections EO and BD are parallel [Prop. 11.16]. So, for the same (reasons), since two parallel planes GH and KL are cut by the plane $AOFC$, their common sections AC and OF are parallel [Prop. 11.16]. And since the straight-line EO has been drawn parallel to one of the sides BD of triangle ABD , thus, proportionally, as AE is to EB , so AO (is) to OD [Prop. 6.2]. Again, since the straight-line OF has been drawn parallel to one of the sides AC of triangle ADC , proportionally, as AO is to OD , so CF (is) to FD [Prop. 6.2]. And it was also shown that as AO (is) to OD , so AE (is) to EB . And thus as AE (is) to EB , so CF (is) to FD [Prop. 5.11].



Thus, if two straight-lines are cut by parallel planes then they will be cut in the same ratios. (Which is) the very thing it was required to show.