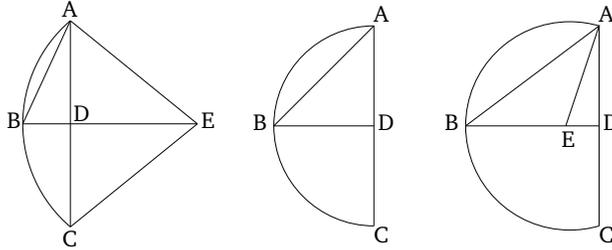


# Book 3

## Proposition 25

For a given segment of a circle, to complete the circle, the very one of which it is a segment.



Let  $ABC$  be the given segment of a circle. So it is required to complete the circle for segment  $ABC$ , the very one of which it is a segment.

For let  $AC$  have been cut in half at (point)  $D$  [Prop. 1.10], and let  $DB$  have been drawn from point  $D$ , at right-angles to  $AC$  [Prop. 1.11]. And let  $AB$  have been joined. Thus, angle  $ABD$  is surely either greater than, equal to, or less than (angle)  $BAD$ .

First of all, let it be greater. And let (angle)  $BAE$ , equal to angle  $ABD$ , have been constructed on the straight-line  $BA$ , at the point  $A$  on it [Prop. 1.23]. And let  $DB$  have been drawn through to  $E$ , and let  $EC$  have been joined. Therefore, since angle  $ABE$  is equal to  $BAE$ , the straight-line  $EB$  is thus also equal to  $EA$  [Prop. 1.6]. And since  $AD$  is equal to  $DC$ , and  $DE$  (is) common, the two (straight-lines)  $AD$ ,  $DE$  are equal to the two (straight-lines)  $CD$ ,  $DE$ , respectively. And angle  $ADE$  is equal to angle  $CDE$ . For each (is) a right-angle. Thus, the base  $AE$  is equal to the base  $CE$  [Prop. 1.4]. But,  $AE$  was shown (to be) equal to  $BE$ . Thus,  $BE$  is also

equal to  $CE$ . Thus, the three (straight-lines)  $AE$ ,  $EB$ , and  $EC$  are equal to one another. Thus, if a circle is drawn with center  $E$ , and radius one of  $AE$ ,  $EB$ , or  $EC$ , it will also go through the remaining points (of the segment), and the (associated circle) will have been completed [Prop. 3.9]. Thus, a circle has been completed from the given segment of a circle. And (it is) clear that the segment  $ABC$  is less than a semi-circle, because the center  $E$  happens to lie outside it.

[And], similarly, even if angle  $ABD$  is equal to  $BAD$ , (since)  $AD$  becomes equal to each of  $BD$  [Prop. 1.6] and  $DC$ , the three (straight-lines)  $DA$ ,  $DB$ , and  $DC$  will be equal to one another. And point  $D$  will be the center of the completed circle. And  $ABC$  will manifestly be a semi-circle.

And if  $ABD$  is less than  $BAD$ , and we construct (angle  $BAE$ ), equal to angle  $ABD$ , on the straight-line  $BA$ , at the point  $A$  on it [Prop. 1.23], then the center will fall on  $DB$ , inside the segment  $ABC$ . And segment  $ABC$  will manifestly be greater than a semi-circle.

Thus, a circle has been completed from the given segment of a circle. (Which is) the very thing it was required to do.