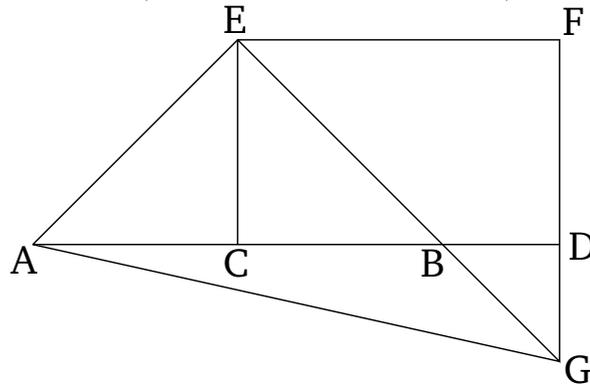


# Book 2

## Proposition 10

If a straight-line is cut in half, and any straight-line added to it straight-on, then the sum of the square on the whole (straight-line) with the (straight-line) having been added, and the (square) on the (straight-line) having been added, is double the (sum of the square) on half (the straight-line), and the square described on the sum of half (the straight-line) and (straight-line) having been added, as on one (complete straight-line).



For let any straight-line  $AB$  have been cut in half at (point)  $C$ , and let any straight-line  $BD$  have been added to it straight-on. I say that the (sum of the) squares on  $AD$  and  $DB$  is double the (sum of the) squares on  $AC$  and  $CD$ .

For let  $CE$  have been drawn from point  $C$ , at right-angles to  $AB$  [Prop. 1.11], and let it be made equal to each of  $AC$  and  $CB$  [Prop. 1.3], and let  $EA$  and  $EB$  have been joined. And let  $EF$  have been drawn through  $E$ , parallel to  $AD$  [Prop. 1.31], and let  $FD$  have been drawn through  $D$ , parallel to  $CE$  [Prop. 1.31].

And since some straight-line  $EF$  falls across the parallel straight-lines  $EC$  and  $FD$ , the (internal angles)  $CEF$  and  $EFD$  are thus equal to two right-angles [Prop. 1.29]. Thus,  $FEB$  and  $EFD$  are less than two right-angles. And (straight-lines) produced from (internal angles whose sum is) less than two right-angles meet together [Post. 5]. Thus, being produced in the direction of  $B$  and  $D$ , the (straight-lines)  $EB$  and  $FD$  will meet. Let them have been produced, and let them meet together at  $G$ , and let  $AG$  have been joined. And since  $AC$  is equal to  $CE$ , angle  $EAC$  is also equal to (angle)  $AEC$  [Prop. 1.5]. And the (angle) at  $C$  (is) a right-angle. Thus,  $EAC$  and  $AEC$  [are] each half a right-angle [Prop. 1.32]. So, for the same (reasons),  $CEB$  and  $EBC$  are also each half a right-angle. Thus, (angle)  $AEB$  is a right-angle. And since  $EBC$  is half a right-angle,  $DBG$  (is) thus also half a right-angle [Prop. 1.15]. And  $BDG$  is also a right-angle. For it is equal to  $DCE$ . For (they are) alternate (angles) [Prop. 1.29]. Thus, the remaining (angle)  $DGB$  is half a right-angle. Thus,  $DGB$  is equal to  $DBG$ . So side  $BD$  is also equal to side  $GD$  [Prop. 1.6]. Again, since  $EGF$  is half a right-angle, and the (angle) at  $F$  (is) a right-angle, for it is equal to the opposite (angle) at  $C$  [Prop. 1.34], the remaining (angle)  $FEG$  is thus half a right-angle. Thus, angle  $EGF$  (is) equal to  $FEG$ . So the side  $GF$  is also equal to the side  $EF$  [Prop. 1.6]. And since [ $EC$  is equal to  $CA$ ] the square on  $EC$  is [also] equal to the square on  $CA$ . Thus, the (sum of the) squares on  $EC$  and  $CA$  is double the square on  $CA$ . And the (square) on  $EA$  is equal to the (sum of the squares) on  $EC$  and

$CA$  [Prop. 1.47]. Thus, the square on  $EA$  is double the square on  $AC$ . Again, since  $FG$  is equal to  $EF$ , the (square) on  $FG$  is also equal to the (square) on  $FE$ . Thus, the (sum of the squares) on  $GF$  and  $FE$  is double the (square) on  $EF$ . And the (square) on  $EG$  is equal to the (sum of the squares) on  $GF$  and  $FE$  [Prop. 1.47]. Thus, the (square) on  $EG$  is double the (square) on  $EF$ . And  $EF$  (is) equal to  $CD$  [Prop. 1.34]. Thus, the square on  $EG$  is double the (square) on  $CD$ . But it was also shown that the (square) on  $EA$  (is) double the (square) on  $AC$ . Thus, the (sum of the) squares on  $AE$  and  $EG$  is double the (sum of the) squares on  $AC$  and  $CD$ . And the square on  $AG$  is equal to the (sum of the) squares on  $AE$  and  $EG$  [Prop. 1.47]. Thus, the (square) on  $AG$  is double the (sum of the squares) on  $AC$  and  $CD$ . And the (sum of the squares) on  $AD$  and  $DG$  is equal to the (square) on  $AG$  [Prop. 1.47]. Thus, the (sum of the) [squares] on  $AD$  and  $DG$  is double the (sum of the) [squares] on  $AC$  and  $CD$ . And  $DG$  (is) equal to  $DB$ . Thus, the (sum of the) [squares] on  $AD$  and  $DB$  is double the (sum of the) squares on  $AC$  and  $CD$ .

Thus, if a straight-line is cut in half, and any straight-line added to it straight-on, then the sum of the square on the whole (straight-line) with the (straight-line) having been added, and the (square) on the (straight-line) having been added, is double the (sum of the square) on half (the straight-line), and the square described on the sum of half (the straight-line) and (straight-line) having been added, as on one (complete straight-line). (Which is) the very thing it was required to show.