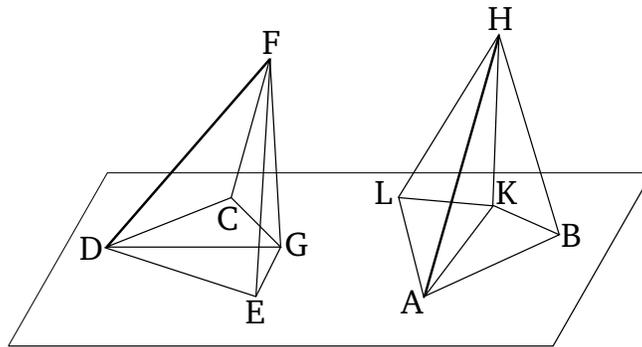


Book 11

Proposition 26

To construct a solid angle equal to a given solid angle on a given straight-line, and at a given point on it.

Let AB be the given straight-line, and A the given point on it, and D the given solid angle, contained by the plane angles EDC , EDF , and FDC . So, it is necessary to construct a solid angle equal to the solid angle D on the straight-line AB , and at the point A on it.



For let some random point F have been taken on DF , and let FG have been drawn from F perpendicular to the plane through ED and DC [Prop. 11.11], and let it meet the plane at G , and let DG have been joined. And let BAL , equal to the angle EDC , and BAK , equal to EDG , have been constructed on the straight-line AB at the point A on it [Prop. 1.23]. And let AK be made equal to DG . And let KH have been set up at the point K at right-angles to the plane through BAL [Prop. 11.12]. And let KH be made equal to GF . And let HA have been joined. I say that the solid angle at A , contained by the (plane) angles BAL , BAH , and

HAL , is equal to the solid angle at D , contained by the (plane) angles EDC , EDF , and FDC .

For let AB and DE have been cut off (so as to be) equal, and let HB , KB , FE , and GE have been joined. And since FG is at right-angles to the reference plane (EDC), it will also make right-angles with all of the straight-lines joined to it which are also in the reference plane [Def. 11.3]. Thus, the angles FGD and FGE are right-angles. So, for the same (reasons), the angles HKA and HKB are also right-angles. And since the two (straight-lines) KA and AB are equal to the two (straight-lines) GD and DE , respectively, and they contain equal angles, the base KB is thus equal to the base GE [Prop. 1.4]. And KH is also equal to GF . And they contain right-angles (with the respective bases). Thus, HB (is) also equal to FE [Prop. 1.4]. Again, since the two (straight-lines) AK and KH are equal to the two (straight-lines) DG and GF (respectively), and they contain right-angles, the base AH is thus equal to the base FD [Prop. 1.4]. And AB (is) also equal to DE . So, the two (straight-lines) HA and AB are equal to the two (straight-lines) DF and DE (respectively). And the base HB (is) equal to the base FE . Thus, the angle BAH is equal to the angle EDF [Prop. 1.8]. So, for the same (reasons), HAL is also equal to FDC . And BAL is also equal to EDC .

Thus, (a solid angle) has been constructed, equal to the given solid angle at D , on the given straight-line AB , at the given point A on it. (Which is) the very thing it was required to do.