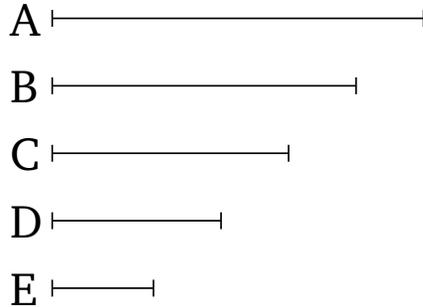


Book 7

Proposition 22

The least numbers of those (numbers) having the same ratio as them are prime to one another.



Let A and B be the least numbers of those (numbers) having the same ratio as them. I say that A and B are prime to one another.

For if they are not prime to one another then some number will measure them. Let it (so measure them), and let it be C . And as many times as C measures A , so many units let there be in D . And as many times as C measures B , so many units let there be in E .

Since C measures A according to the units in D , C has thus made A (by) multiplying D [Def. 7.15]. So, for the same (reasons), C has also made B (by) multiplying E . So the number C has made A and B (by) multiplying the two numbers D and E (respectively). Thus, as D is to E , so A (is) to B [Prop. 7.17]. Thus, D and E are in the same ratio as A and B , being less than them. The very thing is impossible. Thus, some number does not measure the numbers A and B . Thus, A and B are prime to one another. (Which is) the very thing it was required to show.