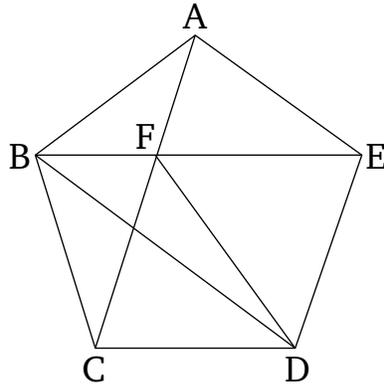


Book 13

Proposition 7

If three angles, either consecutive or not consecutive, of an equilateral pentagon are equal then the pentagon will be equiangular.



For let three angles of the equilateral pentagon $ABCDE$ —first of all, the consecutive (angles) at A , B , and C —be equal to one another. I say that pentagon $ABCDE$ is equiangular.

For let AC , BE , and FD have been joined. And since the two (straight-lines) CB and BA are equal to the two (straight-lines) BA and AE , respectively, and angle CBA is equal to angle BAE , base AC is thus equal to base BE , and triangle ABC equal to triangle ABE , and the remaining angles will be equal to the remaining angles which the equal sides subtend [Prop. 1.4], (that is), BCA (equal) to BEA . And the whole of AC was also shown (to be) equal to the whole of BE . Thus, the remainder FC is also equal to the remainder FE . And CD is also equal to DE . So, the two (straight-lines) FC and CD are equal to the two (straight-lines) FE and ED (respectively). And FD is their common base. Thus, angle FCD is equal to an-

gle FED [Prop. 1.8]. And BCA was also shown (to be) equal to AEB . And thus the whole of BCD (is) equal to the whole of AED . But, (angle) BCD was assumed (to be) equal to the angles at A and B . Thus, (angle) AED is also equal to the angles at A and B . So, similarly, we can show that angle CDE is also equal to the angles at A , B , C . Thus, pentagon $ABCDE$ is equiangular.

And so let consecutive angles not be equal, but let the (angles) at points A , C , and D be equal. I say that pentagon $ABCDE$ is also equiangular in this case.

For let BD have been joined. And since the two (straight-lines) BA and AE are equal to the (straight-lines) BC and CD , and they contain equal angles, base BE is thus equal to base BD , and triangle ABE is equal to triangle BCD , and the remaining angles will be equal to the remaining angles which the equal sides subtend [Prop. 1.4]. Thus, angle AEB is equal to (angle) CDB . And angle BED is also equal to (angle) BDE , since side BE is also equal to side BD [Prop. 1.5]. Thus, the whole angle AED is also equal to the whole (angle) CDE . But, (angle) CDE was assumed (to be) equal to the angles at A and C . Thus, angle AED is also equal to the (angles) at A and C . So, for the same (reasons), (angle) ABC is also equal to the angles at A , C , and D . Thus, pentagon $ABCDE$ is equiangular. (Which is) the very thing it was required to show.