

# Book 9

## Proposition 16

If two numbers are prime to one another then as the first is to the second, so the second (will) not (be) to some other (number).



For let the two numbers  $A$  and  $B$  be prime to one another. I say that as  $A$  is to  $B$ , so  $B$  is not to some other (number).

For, if possible, let it be that as  $A$  (is) to  $B$ , (so)  $B$  (is) to  $C$ . And  $A$  and  $B$  (are) prime (to one another). And (numbers) prime (to one another are) also the least (of those numbers having the same ratio as them) [Prop. 7.21]. And the least numbers measure those (numbers) having the same ratio (as them) an equal number of times, the leading (measuring) the leading, and the following the following [Prop. 7.20]. Thus,  $A$  measures  $B$ , as the leading (measuring) the leading. And ( $A$ ) also measures itself. Thus,  $A$  measures  $A$  and  $B$ , which are prime to one another. The very thing (is) absurd. Thus, as  $A$  (is) to  $B$ , so  $B$  cannot be to  $C$ . (Which is) the very thing it was required to show.