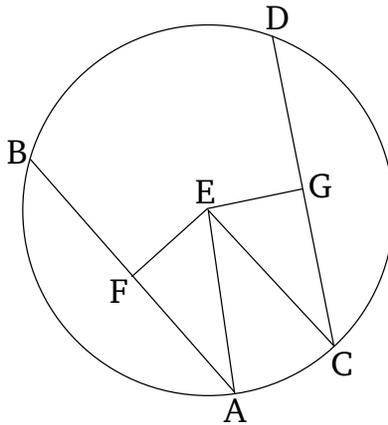


Book 3

Proposition 14

In a circle, equal straight-lines are equally far from the center, and (straight-lines) which are equally far from the center are equal to one another.



Let $ABDC$ be a circle, and let AB and CD be equal straight-lines within it. I say that AB and CD are equally far from the center.

For let the center of circle $ABDC$ have been found [Prop. 3.1], and let it be (at) E . And let EF and EG have been drawn from (point) E , perpendicular to AB and CD (respectively) [Prop. 1.12]. And let AE and EC have been joined.

Therefore, since some straight-line, EF , through the center (of the circle), cuts some (other) straight-line, AB , not through the center, at right-angles, it also cuts it in half [Prop. 3.3]. Thus, AF (is) equal to FB . Thus, AB (is) double AF . So, for the same (reasons), CD is also double CG . And AB is equal to CD . Thus, AF (is) also equal to CG . And since AE is equal to EC ,

the (square) on AE (is) also equal to the (square) on EC . But, the (sum of the squares) on AF and EF (is) equal to the (square) on AE . For the angle at F (is) a right-angle [Prop. 1.47]. And the (sum of the squares) on EG and GC (is) equal to the (square) on EC . For the angle at G (is) a right-angle [Prop. 1.47]. Thus, the (sum of the squares) on AF and FE is equal to the (sum of the squares) on CG and GE , of which the (square) on AF is equal to the (square) on CG . For AF is equal to CG . Thus, the remaining (square) on FE is equal to the (remaining square) on EG . Thus, EF (is) equal to EG . And straight-lines in a circle are said to be equally far from the center when perpendicular (straight-lines) which are drawn to them from the center are equal [Def. 3.4]. Thus, AB and CD are equally far from the center.

So, let the straight-lines AB and CD be equally far from the center. That is to say, let EF be equal to EG . I say that AB is also equal to CD .

For, with the same construction, we can, similarly, show that AB is double AF , and CD (double) CG . And since AE is equal to CE , the (square) on AE is equal to the (square) on CE . But, the (sum of the squares) on EF and FA is equal to the (square) on AE [Prop. 1.47]. And the (sum of the squares) on EG and GC (is) equal to the (square) on CE [Prop. 1.47]. Thus, the (sum of the squares) on EF and FA is equal to the (sum of the squares) on EG and GC , of which the (square) on EF is equal to the (square) on EG . For EF (is) equal to EG . Thus, the remaining (square) on AF is equal to the

(remaining square) on CG . Thus, AF (is) equal to CG . And AB is double AF , and CD double CG . Thus, AB (is) equal to CD .

Thus, in a circle, equal straight-lines are equally far from the center, and (straight-lines) which are equally far from the center are equal to one another. (Which is) the very thing it was required to show.