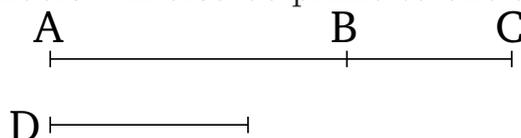


# Book 7

## Proposition 28

If two numbers are prime to one another then their sum will also be prime to each of them. And if the sum (of two numbers) is prime to any one of them then the original numbers will also be prime to one another.



For let the two numbers,  $AB$  and  $BC$ , (which are) prime to one another, be laid down together. I say that their sum  $AC$  is also prime to each of  $AB$  and  $BC$ .

For if  $CA$  and  $AB$  are not prime to one another then some number will measure  $CA$  and  $AB$ . Let it (so) measure (them), and let it be  $D$ . Therefore, since  $D$  measures  $CA$  and  $AB$ , it will thus also measure the remainder  $BC$ . And it also measures  $BA$ . Thus,  $D$  measures  $AB$  and  $BC$ , which are prime to one another. The very thing is impossible. Thus, some number cannot measure (both) the numbers  $CA$  and  $AB$ . Thus,  $CA$  and  $AB$  are prime to one another. So, for the same (reasons),  $AC$  and  $CB$  are also prime to one another. Thus,  $CA$  is prime to each of  $AB$  and  $BC$ .

So, again, let  $CA$  and  $AB$  be prime to one another. I say that  $AB$  and  $BC$  are also prime to one another.

For if  $AB$  and  $BC$  are not prime to one another then some number will measure  $AB$  and  $BC$ . Let it (so) measure (them), and let it be  $D$ . And since  $D$  measures each of  $AB$  and  $BC$ , it will thus also measure the whole of  $CA$ . And it also measures  $AB$ . Thus,  $D$  measures  $CA$  and  $AB$ , which are prime to one another. The very thing is

impossible. Thus, some number cannot measure (both) the numbers  $AB$  and  $BC$ . Thus,  $AB$  and  $BC$  are prime to one another. (Which is) the very thing it was required to show.