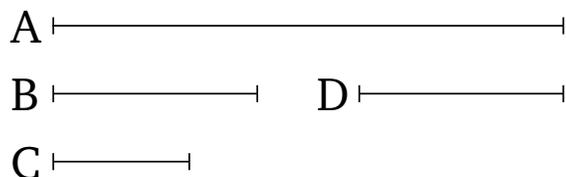


Book 6

Proposition 17

If three straight-lines are proportional then the rectangle contained by the (two) outermost is equal to the square on the middle (one). And if the rectangle contained by the (two) outermost is equal to the square on the middle (one) then the three straight-lines will be proportional.



Let A , B and C be three proportional straight-lines, (such that) as A (is) to B , so B (is) to C . I say that the rectangle contained by A and C is equal to the square on B .

Let D be made equal to B [Prop. 1.3].

And since as A is to B , so B (is) to C , and B (is) equal to D , thus as A is to B , (so) D (is) to C . And if four straight-lines are proportional then the [rectangle] contained by the (two) outermost is equal to the rectangle contained by the middle (two) [Prop. 6.16]. Thus, the (rectangle contained) by A and C is equal to the (rectangle contained) by B and D . But, the (rectangle contained) by B and D is the (square) on B . For B (is) equal to D . Thus, the rectangle contained by A and C is equal to the square on B .

And so, let the (rectangle contained) by A and C be equal to the (square) on B . I say that as A is to B , so B (is) to C .

For, with the same construction, since the (rectangle contained) by A and C is equal to the (square) on B . But, the (square) on B is the (rectangle contained) by B and D . For B (is) equal to D . The (rectangle contained) by A and C is thus equal to the (rectangle contained) by B and D . And if the (rectangle contained) by the (two) outermost is equal to the (rectangle contained) by the middle (two) then the four straight-lines are proportional [Prop. 6.16]. Thus, as A is to B , so D (is) to C . And B (is) equal to D . Thus, as A (is) to B , so B (is) to C .

Thus, if three straight-lines are proportional then the rectangle contained by the (two) outermost is equal to the square on the middle (one). And if the rectangle contained by the (two) outermost is equal to the square on the middle (one) then the three straight-lines will be proportional. (Which is) the very thing it was required to show.