

Book 1

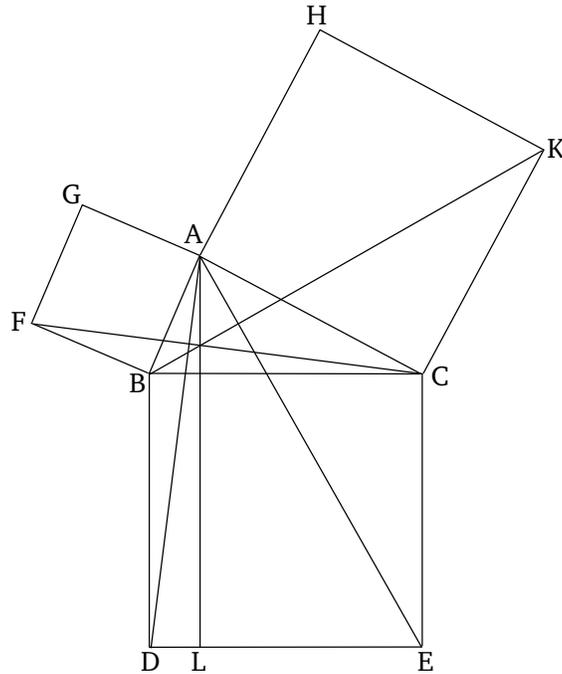
Proposition 47

In right-angled triangles, the square on the side subtending the right-angle is equal to the (sum of the) squares on the sides containing the right-angle.

Let ABC be a right-angled triangle having the angle BAC a right-angle. I say that the square on BC is equal to the (sum of the) squares on BA and AC .

For let the square $BDEC$ have been described on BC , and (the squares) GB and HC on AB and AC (respectively) [Prop. 1.46]. And let AL have been drawn through point A parallel to either of BD or CE [Prop. 1.31]. And let AD and FC have been joined. And since angles BAC and BAG are each right-angles, then two straight-lines AC and AG , not lying on the same side, make the adjacent angles with some straight-line BA , at the point A on it, (whose sum is) equal to two right-angles. Thus, CA is straight-on to AG [Prop. 1.14]. So, for the same (reasons), BA is also straight-on to AH . And since angle DBC is equal to FBA , for (they are) both right-angles, let ABC have been added to both. Thus, the whole (angle) DBA is equal to the whole (angle) FBC . And since DB is equal to BC , and FB to BA , the two (straight-lines) DB , BA are equal to the two (straight-lines) CB , BF ,[†] respectively. And angle DBA (is) equal to angle FBC . Thus, the base AD [is] equal to the base FC , and the triangle ABD is equal to the triangle FBC [Prop. 1.4]. And parallelogram BL [is] double (the area) of triangle ABD . For they have the same base, BD , and are between the same parallels, BD

and AL [Prop. 1.41]. And square GB is double (the area) of triangle FBC . For again they have the same base, FB , and are between the same parallels, FB and GC [Prop. 1.41]. [And the doubles of equal things are equal to one another.][‡] Thus, the parallelogram BL is also equal to the square GB . So, similarly, AE and BK being joined, the parallelogram CL can be shown (to be) equal to the square HC . Thus, the whole square $BDEC$ is equal to the (sum of the) two squares GB and HC . And the square $BDEC$ is described on BC , and the (squares) GB and HC on BA and AC (respectively). Thus, the square on the side BC is equal to the (sum of the) squares on the sides BA and AC .



Thus, in right-angled triangles, the square on the side subtending the right-angle is equal to the (sum of the) squares on the sides surrounding the right-[angle]. (Which is) the very thing it was required to show.