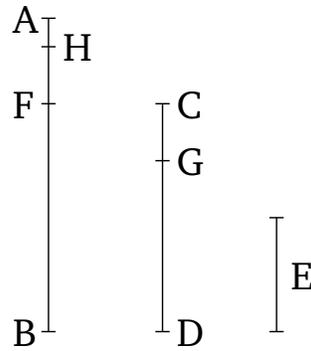


Book 7

Proposition 1

Two unequal numbers (being) laid down, and the lesser being continually subtracted, in turn, from the greater, if the remainder never measures the (number) preceding it, until a unit remains, then the original numbers will be prime to one another.



For two [unequal] numbers, AB and CD , the lesser being continually subtracted, in turn, from the greater, let the remainder never measure the (number) preceding it, until a unit remains. I say that AB and CD are prime to one another—that is to say, that a unit alone measures (both) AB and CD .

For if AB and CD are not prime to one another then some number will measure them. Let (some number) measure them, and let it be E . And let CD measuring BF leave FA less than itself, and let AF measuring DG leave GC less than itself, and let GC measuring FH leave a unit, HA .

In fact, since E measures CD , and CD measures BF , E thus also measures BF . And (E) also measures the whole of BA . Thus, (E) will also measure the remainder

AF . And AF measures DG . Thus, E also measures DG . And (E) also measures the whole of DC . Thus, (E) will also measure the remainder CG . And CG measures FH . Thus, E also measures FH . And (E) also measures the whole of FA . Thus, (E) will also measure the remaining unit AH , (despite) being a number. The very thing is impossible. Thus, some number does not measure (both) the numbers AB and CD . Thus, AB and CD are prime to one another. (Which is) the very thing it was required to show.