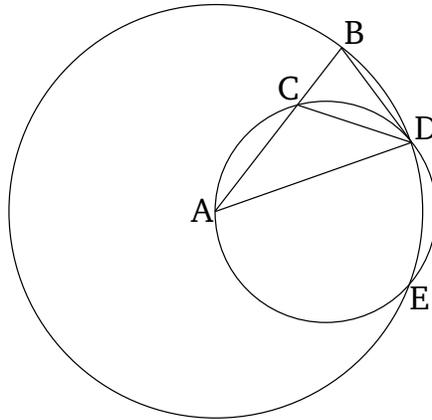


## Book 4

### Proposition 10

To construct an isosceles triangle having each of the angles at the base double the remaining (angle).

Let some straight-line  $AB$  be taken, and let it have been cut at point  $C$  so that the rectangle contained by  $AB$  and  $BC$  is equal to the square on  $CA$  [Prop. 2.11]. And let the circle  $BDE$  have been drawn with center  $A$ , and radius  $AB$ . And let the straight-line  $BD$ , equal to the straight-line  $AC$ , being not greater than the diameter of circle  $BDE$ , have been inserted into circle  $BDE$  [Prop. 4.1]. And let  $AD$  and  $DC$  have been joined. And let the circle  $ACD$  have been circumscribed about triangle  $ACD$  [Prop. 4.5].



And since the (rectangle contained) by  $AB$  and  $BC$  is equal to the (square) on  $AC$ , and  $AC$  (is) equal to  $BD$ , the (rectangle contained) by  $AB$  and  $BC$  is thus equal to the (square) on  $BD$ . And since some point  $B$  has been taken outside of circle  $ACD$ , and two straight-lines  $BA$  and  $BD$  have radiated from  $B$  towards the circle  $ACD$ , and (one) of them cuts (the circle), and (the other) meets

(the circle), and the (rectangle contained) by  $AB$  and  $BC$  is equal to the (square) on  $BD$ ,  $BD$  thus touches circle  $ACD$  [Prop. 3.37]. Therefore, since  $BD$  touches (the circle), and  $DC$  has been drawn across (the circle) from the point of contact  $D$ , the angle  $BDC$  is thus equal to the angle  $DAC$  in the alternate segment of the circle [Prop. 3.32]. Therefore, since  $BDC$  is equal to  $DAC$ , let  $CDA$  have been added to both. Thus, the whole of  $BDA$  is equal to the two (angles)  $CDA$  and  $DAC$ . But, the external (angle)  $BCD$  is equal to  $CDA$  and  $DAC$  [Prop. 1.32]. Thus,  $BDA$  is also equal to  $BCD$ . But,  $BDA$  is equal to  $CBD$ , since the side  $AD$  is also equal to  $AB$  [Prop. 1.5]. So that  $DBA$  is also equal to  $BCD$ . Thus, the three (angles)  $BDA$ ,  $DBA$ , and  $BCD$  are equal to one another. And since angle  $DBC$  is equal to  $BCD$ , side  $BD$  is also equal to side  $DC$  [Prop. 1.6]. But,  $BD$  was assumed (to be) equal to  $CA$ . Thus,  $CA$  is also equal to  $CD$ . So that angle  $CDA$  is also equal to angle  $DAC$  [Prop. 1.5]. Thus,  $CDA$  and  $DAC$  is double  $DAC$ . But  $BCD$  (is) equal to  $CDA$  and  $DAC$ . Thus,  $BCD$  is also double  $CAD$ . And  $BCD$  (is) equal to to each of  $BDA$  and  $DBA$ . Thus,  $BDA$  and  $DBA$  are each double  $DAB$ .

Thus, the isosceles triangle  $ABD$  has been constructed having each of the angles at the base  $BD$  double the remaining (angle). (Which is) the very thing it was required to do.