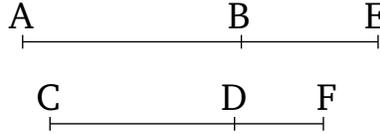


# Book 10

## Proposition 104

A (straight-line) commensurable (in length) with an apotome of a medial (straight-line) is an apotome of a medial (straight-line), and (is) the same in order.



Let  $AB$  be an apotome of a medial (straight-line), and let  $CD$  be commensurable in length with  $AB$ . I say that  $CD$  is also an apotome of a medial (straight-line), and (is) the same in order as  $AB$ .

For since  $AB$  is an apotome of a medial (straight-line), let  $EB$  be an attachment to it. Thus,  $AE$  and  $EB$  are medial (straight-lines which are) commensurable in square only [Props. 10.74, 10.75]. And let it have been contrived that as  $AB$  is to  $CD$ , so  $BE$  (is) to  $DF$  [Prop. 6.12]. Thus,  $AE$  [is] also commensurable (in length) with  $CF$ , and  $BE$  with  $DF$  [Props. 5.12, 10.11]. And  $AE$  and  $EB$  are medial (straight-lines which are) commensurable in square only.  $CF$  and  $FD$  are thus also medial (straight-lines which are) commensurable in square only [Props. 10.23, 10.13]. Thus,  $CD$  is an apotome of a medial (straight-line) [Props. 10.74, 10.75]. So, I say that it is also the same in order as  $AB$ .

[For] since as  $AE$  is to  $EB$ , so  $CF$  (is) to  $FD$  [Props. 5.12, 5.16] [but as  $AE$  (is) to  $EB$ , so the (square) on  $AE$  (is) to the (rectangle contained) by  $AE$  and  $EB$ , and as  $CF$  (is) to  $FD$ , so the (square) on  $CF$  (is) to the (rectangle con-

tained) by  $CF$  and  $FD$ ], thus as the (square) on  $AE$  is to the (rectangle contained) by  $AE$  and  $EB$ , so the (square) on  $CF$  also (is) to the (rectangle contained) by  $CF$  and  $FD$  [Prop. 10.21 lem.] [and, alternately, as the (square) on  $AE$  (is) to the (square) on  $CF$ , so the (rectangle contained) by  $AE$  and  $EB$  (is) to the (rectangle contained) by  $CF$  and  $FD$ ]. And the (square) on  $AE$  (is) commensurable with the (square) on  $CF$ . Thus, the (rectangle contained) by  $AE$  and  $EB$  is also commensurable with the (rectangle contained) by  $CF$  and  $FD$  [Props. 5.16, 10.11]. Therefore, either the (rectangle contained) by  $AE$  and  $EB$  is rational, and the (rectangle contained) by  $CF$  and  $FD$  will also be rational [Def. 10.4], or the (rectangle contained) by  $AE$  and  $EB$  [is] medial, and the (rectangle contained) by  $CF$  and  $FD$  [is] also medial [Prop. 10.23 corr.]

Therefore,  $CD$  is the apotome of a medial (straight-line), and is the same in order as  $AB$  [Props. 10.74, 10.75]. (Which is) the very thing it was required to show.