

Book 3

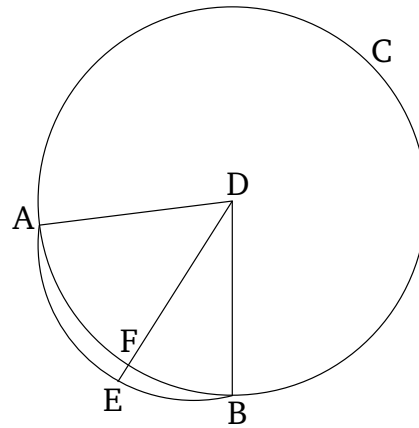
Proposition 2

If two points are taken at random on the circumference of a circle then the straight-line joining the points will fall inside the circle.

Let ABC be a circle, and let two points A and B have been taken at random on its circumference. I say that the straight-line joining A to B will fall inside the circle.

For (if) not then, if possible, let it fall outside (the circle), like AEB (in the figure). And let the center of the circle ABC have been found [Prop. 3.1], and let it be (at point) D . And let DA and DB have been joined, and let DFE have been drawn through.

Therefore, since DA is equal to DB , the angle DAE (is) thus also equal to DBE [Prop. 1.5]. And since in triangle DAE the one side, AEB , has been produced, angle DEB (is) thus greater than DAE [Prop. 1.16]. And DAE (is) equal to DBE [Prop. 1.5]. Thus, DEB (is) greater than DBE . And the greater angle is subtended by the greater side [Prop. 1.19]. Thus, DB (is) greater than DE . And DB (is) equal to DF . Thus, DF (is) greater than DE , the lesser than the greater. The very thing is impossible. Thus, the straight-line joining A to B will not fall outside the circle. So, similarly, we can show that neither (will it fall) on the circumference itself. Thus, (it will fall) inside (the circle).



Thus, if two points are taken at random on the circumference of a circle then the straight-line joining the points will fall inside the circle. (Which is) the very thing it was required to show.