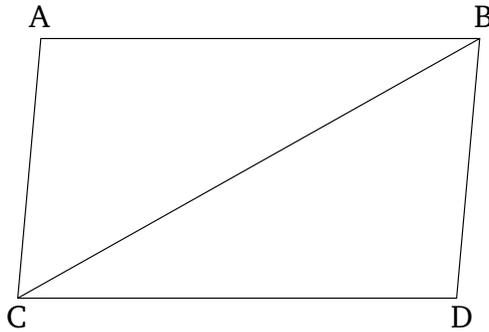


Book 1

Proposition 34

In parallelogrammic figures the opposite sides and angles are equal to one another, and a diagonal cuts them in half.



Let $ACDB$ be a parallelogrammic figure, and BC its diagonal. I say that for parallelogram $ACDB$, the opposite sides and angles are equal to one another, and the diagonal BC cuts it in half.

For since AB is parallel to CD , and the straight-line BC has fallen across them, the alternate angles ABC and BCD are equal to one another [Prop. 1.29]. Again, since AC is parallel to BD , and BC has fallen across them, the alternate angles ACB and CBD are equal to one another [Prop. 1.29]. So ABC and BCD are two triangles having the two angles ABC and BCA equal to the two (angles) BCD and CBD , respectively, and one side equal to one side—the (one) by the equal angles and common to them, (namely) BC . Thus, they will also have the remaining sides equal to the corresponding remaining (sides), and the remaining angle (equal) to the

remaining angle [Prop. 1.26]. Thus, side AB is equal to CD , and AC to BD . Furthermore, angle BAC is equal to CDB . And since angle ABC is equal to BCD , and CBD to ACB , the whole (angle) ABD is thus equal to the whole (angle) ACD . And BAC was also shown (to be) equal to CDB .

Thus, in parallelogrammic figures the opposite sides and angles are equal to one another.

And, I also say that a diagonal cuts them in half. For since AB is equal to CD , and BC (is) common, the two (straight-lines) AB, BC are equal to the two (straight-lines) DC, CB , respectively. And angle ABC is equal to angle BCD . Thus, the base AC (is) also equal to DB , and triangle ABC is equal to triangle BCD [Prop. 1.4].

Thus, the diagonal BC cuts the parallelogram $ACDB$ in half. (Which is) the very thing it was required to show.