

Book 3

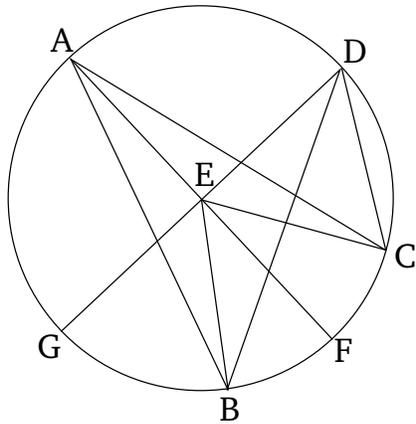
Proposition 20

In a circle, the angle at the center is double that at the circumference, when the angles have the same circumference base.

Let ABC be a circle, and let BEC be an angle at its center, and BAC (one) at (its) circumference. And let them have the same circumference base BC . I say that angle BEC is double (angle) BAC .

For being joined, let AE have been drawn through to F .

Therefore, since EA is equal to EB , angle EAB (is) also equal to EBA [Prop. 1.5]. Thus, angle EAB and EBA is double (angle) EAB . And BEF (is) equal to EAB and EBA [Prop. 1.32]. Thus, BEF is also double EAB . So, for the same (reasons), FEC is also double EAC . Thus, the whole (angle) BEC is double the whole (angle) BAC .



So let another (straight-line) have been inflected, and let there be another angle, BDC . And DE being joined,

let it have been produced to G . So, similarly, we can show that angle GEC is double EDC , of which GEB is double EDB . Thus, the remaining (angle) BEC is double the (remaining angle) BDC .

Thus, in a circle, the angle at the center is double that at the circumference, when [the angles] have the same circumference base. (Which is) the very thing it was required to show.