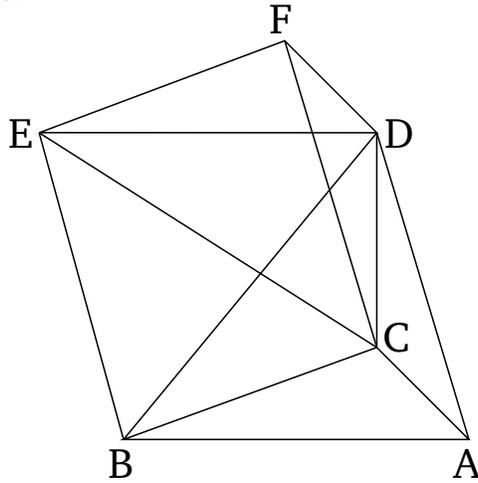


Book 12

Proposition 7

Any prism having a triangular base is divided into three pyramids having triangular bases (which are) equal to one another.



Let there be a prism whose base (is) triangle ABC , and opposite (plane) DEF . I say that prism $ABCDEF$ is divided into three pyramids having triangular bases (which are) equal to one another.

For let BD , EC , and CD have been joined. Since $ABED$ is a parallelogram, and BD is its diagonal, triangle ABD is thus equal to triangle EBD [Prop. 1.34]. And, thus, the pyramid whose base (is) triangle ABD , and apex the point C , is equal to the pyramid whose base is triangle DEB , and apex the point C [Prop. 12.5]. But, the pyramid whose base is triangle DEB , and apex the point C , is the same as the pyramid whose base is triangle EBC , and apex the point D . For they are contained by the same planes. And, thus, the pyramid

whose base is ABD , and apex the point C , is equal to the pyramid whose base is EBC and apex the point D . Again, since $FCBE$ is a parallelogram, and CE is its diagonal, triangle CEF is equal to triangle CBE [Prop. 1.34]. And, thus, the pyramid whose base is triangle BCE , and apex the point D , is equal to the pyramid whose base is triangle ECF , and apex the point D [Prop. 12.5]. And the pyramid whose base is triangle BCE , and apex the point D , was shown (to be) equal to the pyramid whose base is triangle ABD , and apex the point C . Thus, the pyramid whose base is triangle CEF , and apex the point D , is also equal to the pyramid whose base [is] triangle ABD , and apex the point C . Thus, the prism $ABCDEF$ has been divided into three pyramids having triangular bases (which are) equal to one another.

And since the pyramid whose base is triangle ABD , and apex the point C , is the same as the pyramid whose base is triangle CAB , and apex the point D . For they are contained by the same planes. And the pyramid whose base (is) triangle ABD , and apex the point C , was shown (to be) a third of the prism whose base is triangle ABC , and opposite (plane) DEF , thus the pyramid whose base is triangle ABC , and apex the point D , is also a third of the pyramid having the same base, triangle ABC , and opposite (plane) DEF .

Corollary

And, from this, (it is) clear that any pyramid is the third part of the prism having the same base as it, and an equal height. (Which is) the very thing it was required

to show.