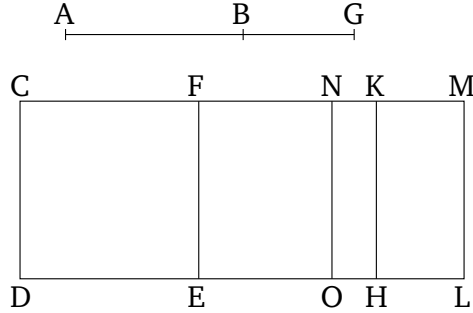


# Book 10

## Proposition 102

The (square) on that (straight-line) which with a medial (area) makes a medial whole, applied to a rational (straight-line), produces a sixth apotome as breadth.



Let  $AB$  be that (straight-line) which with a medial (area) makes a medial whole, and  $CD$  a rational (straight-line). And let  $CE$ , equal to the (square) on  $AB$ , have been applied to  $CD$ , producing  $CF$  as breadth. I say that  $CF$  is a sixth apotome.

For let  $BG$  be an attachment to  $AB$ . Thus,  $AG$  and  $GB$  are incommensurable in square, making the sum of the squares on them medial, and twice the (rectangle contained) by  $AG$  and  $GB$  medial, and the (sum of the squares) on  $AG$  and  $GB$  incommensurable with twice the (rectangle contained) by  $AG$  and  $GB$  [Prop. 10.78]. Therefore, let  $CH$ , equal to the (square) on  $AG$ , have been applied to  $CD$ , producing  $CK$  as breadth, and  $KL$ , equal to the (square) on  $BG$ . Thus, the whole of  $CL$  is equal to the (sum of the squares) on  $AG$  and  $GB$ .  $CL$  [is] thus also medial. And it is applied to the rational (straight-line)  $CD$ , producing  $CM$  as breadth. Thus,  $CM$  is rational, and incommensurable in length

with  $CD$  [Prop. 10.22]. Therefore, since  $CL$  is equal to the (sum of the squares) on  $AG$  and  $GB$ , of which  $CE$  (is) equal to the (square) on  $AB$ , the remainder  $FL$  is thus equal to twice the (rectangle contained) by  $AG$  and  $GB$  [Prop. 2.7]. And twice the (rectangle contained) by  $AG$  and  $GB$  (is) medial. Thus,  $FL$  is also medial. And it is applied to the rational (straight-line)  $FE$ , producing  $FM$  as breadth.  $FM$  is thus rational, and incommensurable in length with  $CD$  [Prop. 10.22]. And since the (sum of the squares) on  $AG$  and  $GB$  is incommensurable with twice the (rectangle contained) by  $AG$  and  $GB$ , and  $CL$  equal to the (sum of the squares) on  $AG$  and  $GB$ , and  $FL$  equal to twice the (rectangle contained) by  $AG$  and  $GB$ ,  $CL$  [is] thus incommensurable with  $FL$ . And as  $CL$  (is) to  $FL$ , so  $CM$  is to  $MF$  [Prop. 6.1]. Thus,  $CM$  is incommensurable in length with  $MF$  [Prop. 10.11]. And they are both rational. Thus,  $CM$  and  $MF$  are rational (straight-lines which are) commensurable in square only.  $CF$  is thus an apotome [Prop. 10.73]. So, I say that (it is) also a sixth (apotome).

For since  $FL$  is equal to twice the (rectangle contained) by  $AG$  and  $GB$ , let  $FM$  have been cut in half at  $N$ , and let  $NO$  have been drawn through  $N$ , parallel to  $CD$ . Thus,  $FO$  and  $NL$  are each equal to the (rectangle contained) by  $AG$  and  $GB$ . And since  $AG$  and  $GB$  are incommensurable in square, the (square) on  $AG$  is thus incommensurable with the (square) on  $GB$ . But,  $CH$  is equal to the (square) on  $AG$ , and  $KL$  is equal to the (square) on  $GB$ . Thus,  $CH$  is incommensurable

with  $KL$ . And as  $CH$  (is) to  $KL$ , so  $CK$  is to  $KM$  [Prop. 6.1]. Thus,  $CK$  is incommensurable (in length) with  $KM$  [Prop. 10.11]. And since the (rectangle contained) by  $AG$  and  $GB$  is the mean proportional to the (squares) on  $AG$  and  $GB$  [Prop. 10.21 lem.], and  $CH$  is equal to the (square) on  $AG$ , and  $KL$  equal to the (square) on  $GB$ , and  $NL$  equal to the (rectangle contained) by  $AG$  and  $GB$ ,  $NL$  is thus also the mean proportional to  $CH$  and  $KL$ . Thus, as  $CH$  is to  $NL$ , so  $NL$  (is) to  $KL$ . And for the same (reasons as the preceding propositions), the square on  $CM$  is greater than (the square on)  $MF$  by the (square) on (some straight-line) incommensurable (in length) with  $(CM)$  [Prop. 10.18]. And neither of them is commensurable with the (previously) laid down rational (straight-line)  $CD$ . Thus,  $CF$  is a sixth apotome [Def. 10.16]. (Which is) the very thing it was required to show.