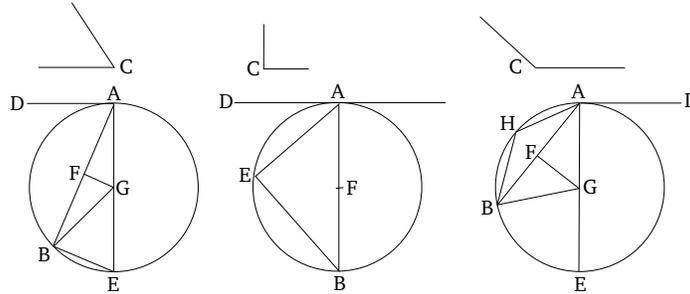


Book 3

Proposition 33

To draw a segment of a circle, accepting an angle equal to a given rectilinear angle, on a given straight-line.



Let AB be the given straight-line, and C the given rectilinear angle. So it is required to draw a segment of a circle, accepting an angle equal to C , on the given straight-line AB .

So the [angle] C is surely either acute, a right-angle, or obtuse. First of all, let it be acute. And, as in the first diagram (from the left), let (angle) BAD , equal to angle C , have been constructed on the straight-line AB , at the point A (on it) [Prop. 1.23]. Thus, BAD is also acute. Let AE have been drawn, at right-angles to DA [Prop. 1.11]. And let AB have been cut in half at F [Prop. 1.10]. And let FG have been drawn from point F , at right-angles to AB [Prop. 1.11]. And let GB have been joined.

And since AF is equal to FB , and FG (is) common, the two (straight-lines) AF , FG are equal to the two (straight-lines) BF , FG (respectively). And angle AFG (is) equal to [angle] BFG . Thus, the base AG is equal to the base BG [Prop. 1.4]. Thus, the circle drawn with

center G , and radius GA , will also go through B (as well as A). Let it have been drawn, and let it be (denoted) ABE . And let EB have been joined. Therefore, since AD is at the extremity of diameter AE , (namely, point) A , at right-angles to AE , the (straight-line) AD thus touches the circle ABE [Prop. 3.16 corr.]. Therefore, since some straight-line AD touches the circle ABE , and some (other) straight-line AB has been drawn across from the point of contact A into circle ABE , angle DAB is thus equal to the angle AEB in the alternate segment of the circle [Prop. 3.32]. But, DAB is equal to C . Thus, angle C is also equal to AEB .

Thus, a segment AEB of a circle, accepting the angle AEB (which is) equal to the given (angle) C , has been drawn on the given straight-line AB .

And so let C be a right-angle. And let it again be necessary to draw a segment of a circle on AB , accepting an angle equal to the right-[angle] C . Let the (angle) BAD [again] have been constructed, equal to the right-angle C [Prop. 1.23], as in the second diagram (from the left). And let AB have been cut in half at F [Prop. 1.10]. And let the circle AEB have been drawn with center F , and radius either FA or FB .

Thus, the straight-line AD touches the circle ABE , on account of the angle at A being a right-angle [Prop. 3.16 corr.]. And angle BAD is equal to the angle in segment AEB . For (the latter angle), being in a semi-circle, is also a right-angle [Prop. 3.31]. But, BAD is also equal to C . Thus, the (angle) in (segment) AEB is also equal to C .

Thus, a segment AEB of a circle, accepting an angle

equal to C , has again been drawn on AB .

And so let (angle) C be obtuse. And let (angle) BAD , equal to (C), have been constructed on the straight-line AB , at the point A (on it) [Prop. 1.23], as in the third diagram (from the left). And let AE have been drawn, at right-angles to AD [Prop. 1.11]. And let AB have again been cut in half at F [Prop. 1.10]. And let FG have been drawn, at right-angles to AB [Prop. 1.10]. And let GB have been joined.

And again, since AF is equal to FB , and FG (is) common, the two (straight-lines) AF , FG are equal to the two (straight-lines) BF , FG (respectively). And angle AFG (is) equal to angle BFG . Thus, the base AG is equal to the base BG [Prop. 1.4]. Thus, a circle of center G , and radius GA , being drawn, will also go through B (as well as A). Let it go like AEB (in the third diagram from the left). And since AD is at right-angles to the diameter AE , at its extremity, AD thus touches circle AEB [Prop. 3.16 corr.]. And AB has been drawn across (the circle) from the point of contact A . Thus, angle BAD is equal to the angle constructed in the alternate segment AHB of the circle [Prop. 3.32]. But, angle BAD is equal to C . Thus, the angle in segment AHB is also equal to C .

Thus, a segment AHB of a circle, accepting an angle equal to C , has been drawn on the given straight-line AB . (Which is) the very thing it was required to do.