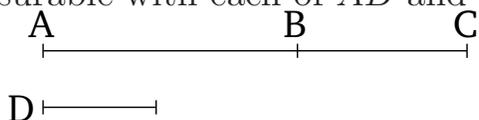


# Book 10

## Proposition 15

If two commensurable magnitudes are added together then the whole will also be commensurable with each of them. And if the whole is commensurable with one of them then the original magnitudes will also be commensurable (with one another).

For let the two commensurable magnitudes  $AB$  and  $BC$  be laid down together. I say that the whole  $AC$  is also commensurable with each of  $AB$  and  $BC$ .



For since  $AB$  and  $BC$  are commensurable, some magnitude will measure them. Let it (so) measure (them), and let it be  $D$ . Therefore, since  $D$  measures (both)  $AB$  and  $BC$ , it will also measure the whole  $AC$ . And it also measures  $AB$  and  $BC$ . Thus,  $D$  measures  $AB$ ,  $BC$ , and  $AC$ . Thus,  $AC$  is commensurable with each of  $AB$  and  $BC$  [Def. 10.1].

And so let  $AC$  be commensurable with  $AB$ . I say that  $AB$  and  $BC$  are also commensurable.

For since  $AC$  and  $AB$  are commensurable, some magnitude will measure them. Let it (so) measure (them), and let it be  $D$ . Therefore, since  $D$  measures (both)  $CA$  and  $AB$ , it will thus also measure the remainder  $BC$ . And it also measures  $AB$ . Thus,  $D$  will measure (both)  $AB$  and  $BC$ . Thus,  $AB$  and  $BC$  are commensurable [Def. 10.1].

Thus, if two magnitudes, and so on . . . .