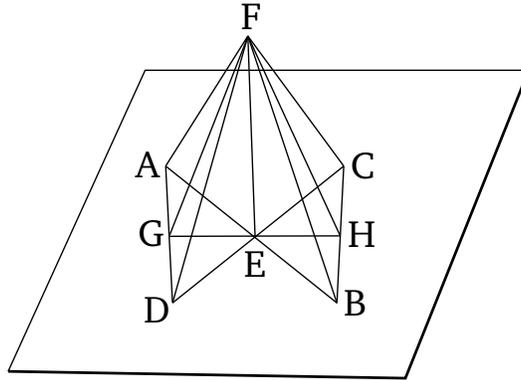


Book 11

Proposition 4

If a straight-line is set up at right-angles to two straight-lines cutting one another, at the common point of section, then it will also be at right-angles to the plane (passing) through them (both).



For let some straight-line EF have (been) set up at right-angles to two straight-lines, AB and CD , cutting one another at point E , at E . I say that EF is also at right-angles to the plane (passing) through AB and CD .

For let AE , EB , CE and ED have been cut off from (the two straight-lines so as to be) equal to one another. And let GEH have been drawn, at random, through E (in the plane passing through AB and CD). And let AD and CB have been joined. And, furthermore, let FA , FG , FD , FC , FH , and FB have been joined from the random (point) F (on EF).

For since the two (straight-lines) AE and ED are equal to the two (straight-lines) CE and EB , and they enclose equal angles [Prop. 1.15], the base AD is thus equal to the base CB , and triangle AED will be equal to

triangle CEB [Prop. 1.4]. Hence, the angle DAE [is] equal to the angle EBC . And the angle AEG (is) also equal to the angle BEH [Prop. 1.15]. So AGE and BEH are two triangles having two angles equal to two angles, respectively, and one side equal to one side—(namely), those by the equal angles, AE and EB . Thus, they will also have the remaining sides equal to the remaining sides [Prop. 1.26]. Thus, GE (is) equal to EH , and AG to BH . And since AE is equal to EB , and FE is common and at right-angles, the base FA is thus equal to the base FB [Prop. 1.4]. So, for the same (reasons), FC is also equal to FD . And since AD is equal to CB , and FA is also equal to FB , the two (straight-lines) FA and AD are equal to the two (straight-lines) FB and BC , respectively. And the base FD was shown (to be) equal to the base FC . Thus, the angle FAD is also equal to the angle FBC [Prop. 1.8]. And, again, since AG was shown (to be) equal to BH , but FA (is) also equal to FB , the two (straight-lines) FA and AG are equal to the two (straight-lines) FB and BH (respectively). And the angle FAG was shown (to be) equal to the angle FBH . Thus, the base FG is equal to the base FH [Prop. 1.4]. And, again, since GE was shown (to be) equal to EH , and EF (is) common, the two (straight-lines) GE and EF are equal to the two (straight-lines) HE and EF (respectively). And the base FG (is) equal to the base FH . Thus, the angle GEF is equal to the angle HEF [Prop. 1.8]. Each of the angles GEF and HEF (are) thus right-angles [Def. 1.10]. Thus, FE is at right-angles to GH , which was drawn at random through

FE (in the reference plane passing through AB and AC). So, similarly, we can show that FE will make right-angles with all straight-lines joined to it which are in the reference plane. And a straight-line is at right-angles to a plane when it makes right-angles with all straight-lines joined to it which are in the plane [Def. 11.3]. Thus, FE is at right-angles to the reference plane. And the reference plane is that (passing) through the straight-lines AB and CD . Thus, FE is at right-angles to the plane (passing) through AB and CD .

Thus, if a straight-line is set up at right-angles to two straight-lines cutting one another, at the common point of section, then it will also be at right-angles to the plane (passing) through them (both). (Which is) the very thing it was required to show.