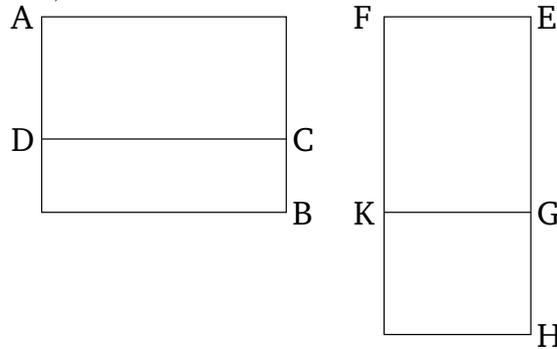


# Book 10

## Proposition 26

A medial (area) does not exceed a medial (area) by a rational (area).



For, if possible, let the medial (area)  $AB$  exceed the medial (area)  $AC$  by the rational (area)  $DB$ . And let the rational (straight-line)  $EF$  be laid down. And let the rectangular parallelogram  $FH$ , equal to  $AB$ , have been applied to to  $EF$ , producing  $EH$  as breadth. And let  $FG$ , equal to  $AC$ , have been cut off (from  $FH$ ). Thus, the remainder  $BD$  is equal to the remainder  $KH$ . And  $DB$  is rational. Thus,  $KH$  is also rational. Therefore, since  $AB$  and  $AC$  are each medial, and  $AB$  is equal to  $FH$ , and  $AC$  to  $FG$ ,  $FH$  and  $FG$  are thus each also medial. And they are applied to the rational (straight-line)  $EF$ . Thus,  $HE$  and  $EG$  are each rational, and incommensurable in length with  $EF$  [Prop. 10.22]. And since  $DB$  is rational, and is equal to  $KH$ ,  $KH$  is thus also rational. And  $(KH)$  is applied to the rational (straight-line)  $EF$ .  $GH$  is thus rational, and commensurable in length with  $EF$  [Prop. 10.20]. But,  $EG$  is also rational, and incommensurable in length with

$EF$ . Thus,  $EG$  is incommensurable in length with  $GH$  [Prop. 10.13]. And as  $EG$  is to  $GH$ , so the (square) on  $EG$  (is) to the (rectangle contained) by  $EG$  and  $GH$  [Prop. 10.13 lem.]. Thus, the (square) on  $EG$  is incommensurable with the (rectangle contained) by  $EG$  and  $GH$  [Prop. 10.11]. But, the (sum of the) squares on  $EG$  and  $GH$  is commensurable with the (square) on  $EG$ . For ( $EG$  and  $GH$  are) both rational. And twice the (rectangle contained) by  $EG$  and  $GH$  is commensurable with the (rectangle contained) by  $EG$  and  $GH$  [Prop. 10.6]. For (the former) is double the latter. Thus, the (sum of the squares) on  $EG$  and  $GH$  is incommensurable with twice the (rectangle contained) by  $EG$  and  $GH$  [Prop. 10.13]. And thus the sum of the (squares) on  $EG$  and  $GH$  plus twice the (rectangle contained) by  $EG$  and  $GH$ , that is the (square) on  $EH$  [Prop. 2.4], is incommensurable with the (sum of the squares) on  $EG$  and  $GH$  [Prop. 10.16]. And the (sum of the squares) on  $EG$  and  $GH$  (is) rational. Thus, the (square) on  $EH$  is irrational [Def. 10.4]. Thus,  $EH$  is irrational [Def. 10.4]. But, (it is) also rational. The very thing is impossible.

Thus, a medial (area) does not exceed a medial (area) by a rational (area). (Which is) the very thing it was required to show.