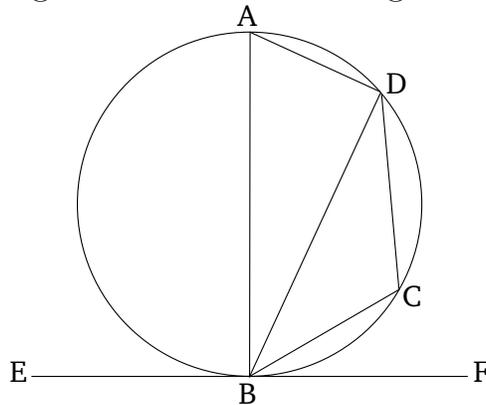


# Book 3

## Proposition 32

If some straight-line touches a circle, and some (other) straight-line is drawn across, from the point of contact into the circle, cutting the circle (in two), then those angles the (straight-line) makes with the tangent will be equal to the angles in the alternate segments of the circle.



For let some straight-line  $EF$  touch the circle  $ABCD$  at the point  $B$ , and let some (other) straight-line  $BD$  have been drawn from point  $B$  into the circle  $ABCD$ , cutting it (in two). I say that the angles  $BD$  makes with the tangent  $EF$  will be equal to the angles in the alternate segments of the circle. That is to say, that angle  $FBD$  is equal to the angle constructed in segment  $BAD$ , and angle  $EBD$  is equal to the angle constructed in segment  $DCB$ .

For let  $BA$  have been drawn from  $B$ , at right-angles to  $EF$  [Prop. 1.11]. And let the point  $C$  have been taken at random on the circumference  $BD$ . And let  $AD$ ,  $DC$ , and  $CB$  have been joined.

And since some straight-line  $EF$  touches the circle

$ABCD$  at point  $B$ , and  $BA$  has been drawn from the point of contact, at right-angles to the tangent, the center of circle  $ABCD$  is thus on  $BA$  [Prop. 3.19]. Thus,  $BA$  is a diameter of circle  $ABCD$ . Thus, angle  $ADB$ , being in a semi-circle, is a right-angle [Prop. 3.31]. Thus, the remaining angles (of triangle  $ADB$ )  $BAD$  and  $ABD$  are equal to one right-angle [Prop. 1.32]. And  $ABF$  is also a right-angle. Thus,  $ABF$  is equal to  $BAD$  and  $ABD$ . Let  $ABD$  have been subtracted from both. Thus, the remaining angle  $DBF$  is equal to the angle  $BAD$  in the alternate segment of the circle. And since  $ABCD$  is a quadrilateral in a circle, (the sum of) its opposite angles is equal to two right-angles [Prop. 3.22]. And  $DBF$  and  $DBE$  is also equal to two right-angles [Prop. 1.13]. Thus,  $DBF$  and  $DBE$  is equal to  $BAD$  and  $BCD$ , of which  $BAD$  was shown (to be) equal to  $DBF$ . Thus, the remaining (angle)  $DBE$  is equal to the angle  $DCB$  in the alternate segment  $DCB$  of the circle.

Thus, if some straight-line touches a circle, and some (other) straight-line is drawn across, from the point of contact into the circle, cutting the circle (in two), then those angles the (straight-line) makes with the tangent will be equal to the angles in the alternate segments of the circle. (Which is) the very thing it was required to show.