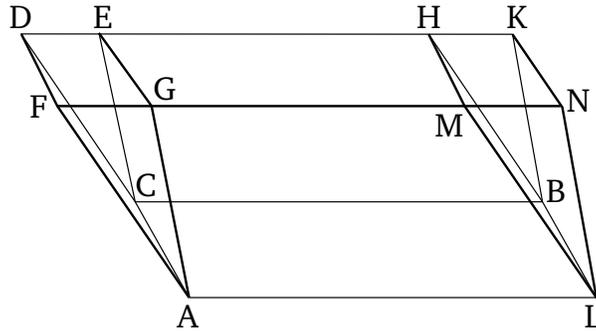


# Book 11

## Proposition 29

Parallelepiped solids which are on the same base, and (have) the same height, and in which the (ends of the straight-lines) standing up are on the same straight-lines, are equal to one another.



For let the parallelepiped solids  $CM$  and  $CN$  be on the same base  $AB$ , and (have) the same height, and let the (ends of the straight-lines) standing up in them,  $AG$ ,  $AF$ ,  $LM$ ,  $LN$ ,  $CD$ ,  $CE$ ,  $BH$ , and  $BK$ , be on the same straight-lines,  $FN$  and  $DK$ . I say that solid  $CM$  is equal to solid  $CN$ .

For since  $CH$  and  $CK$  are each parallelograms,  $CB$  is equal to each of  $DH$  and  $EK$  [Prop. 1.34]. Hence,  $DH$  is also equal to  $EK$ . Let  $EH$  have been subtracted from both. Thus, the remainder  $DE$  is equal to the remainder  $HK$ . Hence, triangle  $DCE$  is also equal to triangle  $HBK$  [Props. 1.4, 1.8], and parallelogram  $DG$  to parallelogram  $HN$  [Prop. 1.36]. So, for the same (reasons), triangle  $AFG$  is also equal to triangle  $MLN$ . And parallelogram  $CF$  is also equal to parallelogram  $BM$ , and  $CG$  to  $BN$  [Prop. 11.24]. For they are opposite.

Thus, the prism contained by the two triangles  $AFG$  and  $DCE$ , and the three parallelograms  $AD$ ,  $DG$ , and  $CG$ , is equal to the prism contained by the two triangles  $MLN$  and  $HBK$ , and the three parallelograms  $BM$ ,  $HN$ , and  $BN$ . Let the solid whose base (is) parallelogram  $AB$ , and (whose) opposite (face is)  $GEHM$ , have been added to both (prisms). Thus, the whole parallelepiped solid  $CM$  is equal to the whole parallelepiped solid  $CN$ .

Thus, parallelepiped solids which are on the same base, and (have) the same height, and in which the (ends of the straight-lines) standing up (are) on the same straight-lines, are equal to one another. (Which is) the very thing it was required to show.