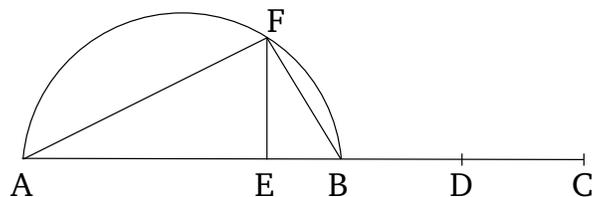


Book 10

Proposition 33

To find two straight-lines (which are) incommensurable in square, making the sum of the squares on them rational, and the (rectangle contained) by them medial.

Let the two rational (straight-lines) AB and BC , (which are) commensurable in square only, be laid out such that the square on the greater, AB , is larger than (the square on) the lesser, BC , by the (square) on (some straight-line which is) incommensurable (in length) with (AB) [Prop. 10.30]. And let BC have been cut in half at D . And let a parallelogram equal to the (square) on either of BD or DC , (and) falling short by a square figure, have been applied to AB [Prop. 6.28], and let it be the (rectangle contained) by AEB . And let the semi-circle AFB have been drawn on AB . And let EF have been drawn at right-angles to AB . And let AF and FB have been joined.



And since AB and BC are [two] unequal straight-lines, and the square on AB is greater than (the square on) BC by the (square) on (some straight-line which is) incommensurable (in length) with (AB). And a parallelogram, equal to one quarter of the (square) on BC —that is to say, (equal) to the (square) on half of it—(and) falling short by a square figure, has been applied to AB , and makes the (rectangle contained) by AEB . AE is thus in-

commensurable (in length) with EB [Prop. 10.18]. And as AE is to EB , so the (rectangle contained) by BA and AE (is) to the (rectangle contained) by AB and BE . And the (rectangle contained) by BA and AE (is) equal to the (square) on AF , and the (rectangle contained) by AB and BE to the (square) on BF [Prop. 10.32 lem.]. The (square) on AF is thus incommensurable with the (square) on FB [Prop. 10.11]. Thus, AF and FB are incommensurable in square. And since AB is rational, the (square) on AB is also rational. Hence, the sum of the (squares) on AF and FB is also rational [Prop. 1.47]. And, again, since the (rectangle contained) by AE and EB is equal to the (square) on EF , and the (rectangle contained) by AE and EB was assumed (to be) equal to the (square) on BD , FE is thus equal to BD . Thus, BC is double FE . And hence the (rectangle contained) by AB and BC is commensurable with the (rectangle contained) by AB and EF [Prop. 10.6]. And the (rectangle contained) by AB and BC (is) medial [Prop. 10.21]. Thus, the (rectangle contained) by AB and EF (is) also medial [Prop. 10.23 corr.]. And the (rectangle contained) by AB and EF (is) equal to the (rectangle contained) by AF and FB [Prop. 10.32 lem.]. Thus, the (rectangle contained) by AF and FB (is) also medial. And the sum of the squares on them was also shown (to be) rational.

Thus, the two straight-lines, AF and FB , (which are) incommensurable in square, have been found, making the sum of the squares on them rational, and the (rectangle contained) by them medial. (Which is) the very thing it was required to show.