

Book 11

Proposition 18

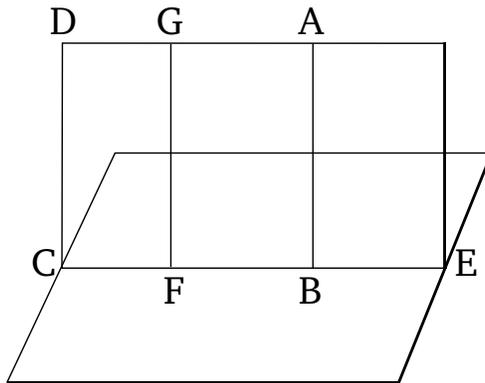
If a straight-line is at right-angles to some plane then all of the planes (passing) through it will also be at right-angles to the same plane.

For let some straight-line AB be at right-angles to a reference plane. I say that all of the planes (passing) through AB are also at right-angles to the reference plane.

For let the plane DE have been produced through AB . And let CE be the common section of the plane DE and the reference (plane). And let some random point F have been taken on CE . And let FG have been drawn from F , at right-angles to CE , in the plane DE [Prop. 1.11].

And since AB is at right-angles to the reference plane, AB is thus also at right-angles to all of the straight-lines joined to it which are also in the reference plane [Def. 11.3]. Hence, it is also at right-angles to CE . Thus, angle ABF is a right-angle. And GFB is also a right-angle. Thus, AB is parallel to FG [Prop. 1.28]. And AB is at right-angles to the reference plane. Thus, FG is also at right-angles to the reference plane [Prop. 11.8]. And a plane is at right-angles to a(nother) plane when the straight-lines drawn at right-angles to the common section of the planes, (and lying) in one of the planes, are at right-angles to the remaining plane [Def. 11.4]. And FG , (which was) drawn at right-angles to the common section of the planes, CE , in one of the planes, DE , was shown to be at right-angles to the reference plane. Thus, plane DE is at right-angles to the reference (plane). So,

similarly, it can be shown that all of the planes (passing) at random through AB (are) at right-angles to the reference plane.



Thus, if a straight-line is at right-angles to some plane then all of the planes (passing) through it will also be at right-angles to the same plane. (Which is) the very thing it was required to show.