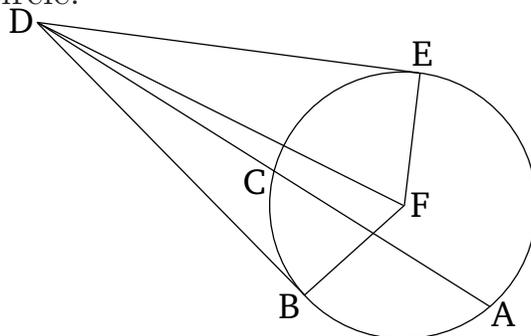


Book 3

Proposition 37

If some point is taken outside a circle, and two straight-lines radiate from the point towards the circle, and one of them cuts the circle, and the (other) meets (it), and the (rectangle contained) by the whole (straight-line) cutting (the circle), and the (part of it) cut off outside (the circle), between the point and the convex circumference, is equal to the (square) on the (straight-line) meeting (the circle), then the (straight-line) meeting (the circle) will touch the circle.



For let some point D have been taken outside circle ABC , and let two straight-lines, DCA and DB , radiate from D towards circle ABC , and let DCA cut the circle, and let DB meet (the circle). And let the (rectangle contained) by AD and DC be equal to the (square) on DB . I say that DB touches circle ABC .

For let DE have been drawn touching ABC [Prop. 3.17], and let the center of the circle ABC have been found, and let it be (at) F . And let FE , FB , and FD have been joined. (Angle) FED is thus a right-angle [Prop. 3.18]. And since DE touches circle ABC , and DCA cuts (it), the (rectangle contained) by AD and DC is thus equal

to the (square) on DE [Prop. 3.36]. And the (rectangle contained) by AD and DC was also equal to the (square) on DB . Thus, the (square) on DE is equal to the (square) on DB . Thus, DE (is) equal to DB . And FE is also equal to FB . So the two (straight-lines) DE , EF are equal to the two (straight-lines) DB , BF (respectively). And their base, FD , is common. Thus, angle DEF is equal to angle DBF [Prop. 1.8]. And DEF (is) a right-angle. Thus, DBF (is) also a right-angle. And FB produced is a diameter, And a (straight-line) drawn at right-angles to a diameter of a circle, at its extremity, touches the circle [Prop. 3.16 corr.]. Thus, DB touches circle ABC . Similarly, (the same thing) can be shown, even if the center happens to be on AC .

Thus, if some point is taken outside a circle, and two straight-lines radiate from the point towards the circle, and one of them cuts the circle, and the (other) meets (it), and the (rectangle contained) by the whole (straight-line) cutting (the circle), and the (part of it) cut off outside (the circle), between the point and the convex circumference, is equal to the (square) on the (straight-line) meeting (the circle), then the (straight-line) meeting (the circle) will touch the circle. (Which is) the very thing it was required to show.