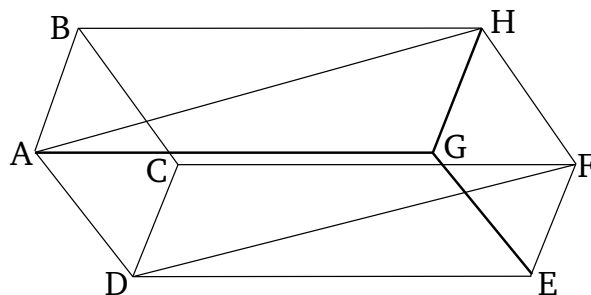


# Book 11

## Proposition 24

If a solid (figure) is contained by (six) parallel planes then its opposite planes are both equal and parallelogrammic.



For let the solid (figure)  $CDHG$  have been contained by the parallel planes  $AC$ ,  $GF$ , and  $AH$ ,  $DF$ , and  $BF$ ,  $AE$ . I say that its opposite planes are both equal and parallelogrammic.

For since the two parallel planes  $BG$  and  $CE$  are cut by the plane  $AC$ , their common sections are parallel [Prop. 11.16]. Thus,  $AB$  is parallel to  $DC$ . Again, since the two parallel planes  $BF$  and  $AE$  are cut by the plane  $AC$ , their common sections are parallel [Prop. 11.16]. Thus,  $BC$  is parallel to  $AD$ . And  $AB$  was also shown (to be) parallel to  $DC$ . Thus,  $AC$  is a parallelogram. So, similarly, we can also show that  $DF$ ,  $FG$ ,  $GB$ ,  $BF$ , and  $AE$  are each parallelograms.

Let  $AH$  and  $DF$  have been joined. And since  $AB$  is parallel to  $DC$ , and  $BH$  to  $CF$ , so the two (straight-lines) joining one another,  $AB$  and  $BH$ , are parallel to the two straight-lines joining one another,  $DC$  and  $CF$  (respectively), not (being) in the same plane. Thus,

they will contain equal angles [Prop. 11.10]. Thus, angle  $ABH$  (is) equal to (angle)  $DCF$ . And since the two (straight-lines)  $AB$  and  $BH$  are equal to the two (straight-lines)  $DC$  and  $CF$  (respectively) [Prop. 1.34], and angle  $ABH$  is equal to angle  $DCF$ , the base  $AH$  is thus equal to the base  $DF$ , and triangle  $ABH$  is equal to triangle  $DCF$  [Prop. 1.4]. And parallelogram  $BG$  is double (triangle)  $ABH$ , and parallelogram  $CE$  double (triangle)  $DCF$  [Prop. 1.34]. Thus, parallelogram  $BG$  (is) equal to parallelogram  $CE$ . So, similarly, we can show that  $AC$  is also equal to  $GF$ , and  $AE$  to  $BF$ .

Thus, if a solid (figure) is contained by (six) parallel planes then its opposite planes are both equal and parallelogrammic. (Which is) the very thing it was required to show.