

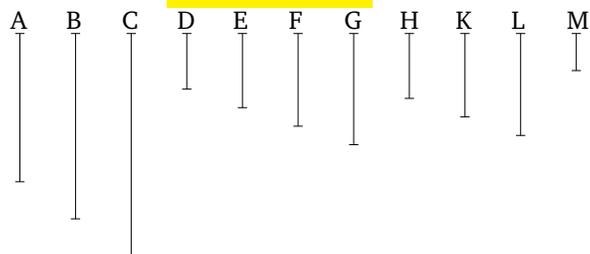
Book 7

Proposition 33

To find the least of those (numbers) having the same ratio as any given multitude of numbers.

Let A , B , and C be any given multitude of numbers. So it is required to find the least of those (numbers) having the same ratio as A , B , and C .

For A , B , and C are either prime to one another, or not. In fact, if A , B , and C are prime to one another then they are the least of those (numbers) having the same ratio as them [Prop. 7.22].



And if not, let the greatest common measure, D , of A , B , and C have been taken [Prop. 7.3]. And as many times as D measures A , B , C , so many units let there be in E , F , G , respectively. And thus E , F , G measure A , B , C , respectively, according to the units in D [Prop. 7.15]. Thus, E , F , G measure A , B , C (respectively) an equal number of times. Thus, E , F , G are in the same ratio as A , B , C (respectively) [Def. 7.20]. So I say that (they are) also the least (of those numbers having the same ratio as A , B , C). For if E , F , G are not the least of those (numbers) having the same ratio as A , B , C (respectively), then there will be [some] numbers less than E , F , G which are in the same ratio as A , B , C (respectively). Let them be H , K , L . Thus, H measures A the same number of times that K , L also

measure B , C , respectively. And as many times as H measures A , so many units let there be in M . Thus, K , L measure B , C , respectively, according to the units in M . And since H measures A according to the units in M , M thus also measures A according to the units in H [Prop. 7.15]. So, for the same (reasons), M also measures B , C according to the units in K , L , respectively. Thus, M measures A , B , and C . And since H measures A according to the units in M , H has thus made A (by) multiplying M . So, for the same (reasons), E has also made A (by) multiplying D . Thus, the (number created) from (multiplying) E and D is equal to the (number created) from (multiplying) H and M . Thus, as E (is) to H , so M (is) to D [Prop. 7.19]. And E (is) greater than H . Thus, M (is) also greater than D [Prop. 5.13]. And (M) measures A , B , and C . The very thing is impossible. For D was assumed (to be) the greatest common measure of A , B , and C . Thus, there cannot be any numbers less than E , F , G which are in the same ratio as A , B , C (respectively). Thus, E , F , G are the least of (those numbers) having the same ratio as A , B , C (respectively). (Which is) the very thing it was required to show.