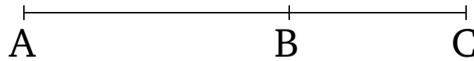


# Book 10

## Proposition 39

If two straight-lines (which are) incommensurable in square, making the sum of the squares on them rational, and the (rectangle contained) by them medial, are added together then the whole straight-line is irrational—let it be called a major (straight-line).



For let the two straight-lines,  $AB$  and  $BC$ , incommensurable in square, and fulfilling the prescribed (conditions), be laid down together [Prop. 10.33]. I say that  $AC$  is irrational.

For since the (rectangle contained) by  $AB$  and  $BC$  is medial, twice the (rectangle contained) by  $AB$  and  $BC$  is [thus] also medial [Props. 10.6, 10.23 corr.]. And the sum of the (squares) on  $AB$  and  $BC$  (is) rational. Thus, twice the (rectangle contained) by  $AB$  and  $BC$  is incommensurable with the sum of the (squares) on  $AB$  and  $BC$  [Def. 10.4]. Hence, (the sum of) the squares on  $AB$  and  $BC$ , plus twice the (rectangle contained) by  $AB$  and  $BC$ —that is, the (square) on  $AC$  [Prop. 2.4]—is also incommensurable with the sum of the (squares) on  $AB$  and  $BC$  [Prop. 10.16] [and the sum of the (squares) on  $AB$  and  $BC$  (is) rational]. Thus, the (square) on  $AC$  is irrational. Hence,  $AC$  is also irrational [Def. 10.4]—let it be called a major (straight-line). (Which is) the very thing it was required to show.