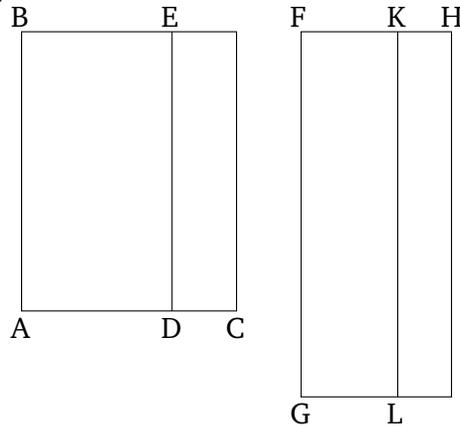


# Book 10

## Proposition 110

A medial (area), incommensurable with the whole, being subtracted from a medial (area), the two remaining irrational (straight-lines) arise (as) the (square-root of the area)—either a second apotome of a medial (straight-line), or that (straight-line) which with a medial (area) makes a medial whole.

For, as in the previous figures, let the medial (area)  $BD$ , incommensurable with the whole, have been subtracted from the medial (area)  $BC$ . I say that the square-root of  $EC$  is one of two irrational (straight-lines)—either a second apotome of a medial (straight-line), or that (straight-line) which with a medial (area) makes a medial whole.



For since  $BC$  and  $BD$  are each medial (areas), and  $BC$  (is) incommensurable with  $BD$ , accordingly,  $FH$  and  $FK$  will each be rational (straight-lines), and incommensurable in length with  $FG$  [Prop. 10.22]. And since  $BC$  is incommensurable with  $BD$ —that is to say,  $GH$  with  $GK$ — $HF$  (is) also incommensurable (in length) with

$FK$  [Props. 6.1, 10.11]. Thus,  $FH$  and  $FK$  are rational (straight-lines which are) commensurable in square only.  $KH$  is thus as apotome [Prop. 10.73], [and  $FK$  an attachment (to it)]. So, the square on  $FH$  is greater than (the square on)  $FK$  either by the (square) on (some straight-line) commensurable, or by the (square) on (some straight-line) incommensurable, (in length) with ( $FH$ ).]

So, if the square on  $FH$  is greater than (the square on)  $FK$  by the (square) on (some straight-line) commensurable (in length) with ( $FH$ ), and (since) neither of  $FH$  and  $FK$  is commensurable in length with the (previously) laid down rational (straight-line)  $FG$ ,  $KH$  is a third apotome [Def. 10.3]. And  $KL$  (is) rational. And the rectangle contained by a rational (straight-line) and a third apotome is irrational, and the square-root of it is that irrational (straight-line) called a second apotome of a medial (straight-line) [Prop. 10.93]. Hence, the square-root of  $LH$ —that is to say, (of)  $EC$ —is a second apotome of a medial (straight-line).

And if the square on  $FH$  is greater than (the square on)  $FK$  by the (square) on (some straight-line) incommensurable [in length] with ( $FH$ ), and (since) neither of  $FH$  and  $FK$  is commensurable in length with  $FG$ ,  $KH$  is a sixth apotome [Def. 10.16]. And the square-root of the (rectangle contained) by a rational (straight-line) and a sixth apotome is that (straight-line) which with a medial (area) makes a medial whole [Prop. 10.96]. Thus, the square-root of  $LH$ —that is to say, (of)  $EC$ —is that (straight-line) which with a medial (area) makes a medial whole. (Which is) the very thing it was required to

show.