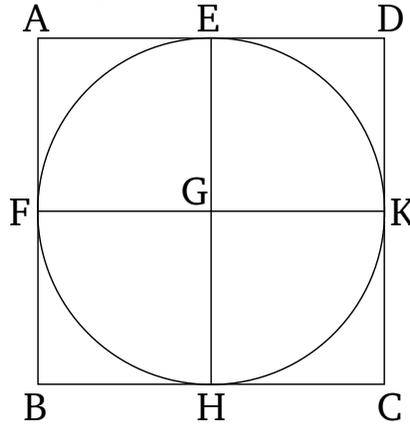


## Book 4 Proposition 8

To inscribe a circle in a given square.

Let the given square be  $ABCD$ . So it is required to inscribe a circle in square  $ABCD$ .



Let  $AD$  and  $AB$  each have been cut in half at points  $E$  and  $F$  (respectively) [Prop. 1.10]. And let  $EH$  have been drawn through  $E$ , parallel to either of  $AB$  or  $CD$ , and let  $FK$  have been drawn through  $F$ , parallel to either of  $AD$  or  $BC$  [Prop. 1.31]. Thus,  $AK$ ,  $KB$ ,  $AH$ ,  $HD$ ,  $AG$ ,  $GC$ ,  $BG$ , and  $GD$  are each parallelograms, and their opposite sides [are] manifestly equal [Prop. 1.34]. And since  $AD$  is equal to  $AB$ , and  $AE$  is half of  $AD$ , and  $AF$  half of  $AB$ ,  $AE$  (is) thus also equal to  $AF$ . So that the opposite (sides are) also (equal). Thus,  $FG$  (is) also equal to  $GE$ . So, similarly, we can also show that each of  $GH$  and  $GK$  is equal to each of  $FG$  and  $GE$ . Thus, the four (straight-lines)  $GE$ ,  $GF$ ,  $GH$ , and  $GK$  [are] equal to one another. Thus, the circle drawn with center  $G$ , and radius one of  $E$ ,  $F$ ,  $H$ , or  $K$ , will also go through the remaining points. And it will touch the

straight-lines  $AB$ ,  $BC$ ,  $CD$ , and  $DA$ , on account of the angles at  $E$ ,  $F$ ,  $H$ , and  $K$  being right-angles. For if the circle cuts  $AB$ ,  $BC$ ,  $CD$ , or  $DA$ , then a (straight-line) drawn at right-angles to a diameter of the circle, from its extremity, will fall inside the circle. The very thing was shown (to be) absurd [Prop. 3.16]. Thus, the circle drawn with center  $G$ , and radius one of  $E$ ,  $F$ ,  $H$ , or  $K$ , does not cut the straight-lines  $AB$ ,  $BC$ ,  $CD$ , or  $DA$ . Thus, it will touch them, and will have been inscribed in the square  $ABCD$ .

Thus, a circle has been inscribed in the given square. (Which is) the very thing it was required to do.