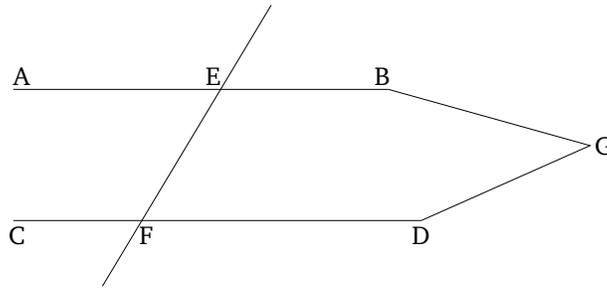


Book 1

Proposition 27

If a straight-line falling across two straight-lines makes the alternate angles equal to one another then the (two) straight-lines will be parallel to one another.



For let the straight-line EF , falling across the two straight-lines AB and CD , make the alternate angles AEF and EFD equal to one another. I say that AB and CD are parallel.

For if not, being produced, AB and CD will certainly meet together: either in the direction of B and D , or (in the direction) of A and C [Def. 1.23]. Let them have been produced, and let them meet together in the direction of B and D at (point) G . So, for the triangle GEF , the external angle AEF is equal to the interior and opposite (angle) EFG . The very thing is impossible [Prop. 1.16]. Thus, being produced, AB and CD will not meet together in the direction of B and D . Similarly, it can be shown that neither (will they meet together) in (the direction of) A and C . But (straight-lines) meeting in neither direction are parallel [Def. 1.23]. Thus, AB and CD are parallel.

Thus, if a straight-line falling across two straight-lines

makes the alternate angles equal to one another then the (two) straight-lines will be parallel (to one another). (Which is) the very thing it was required to show.