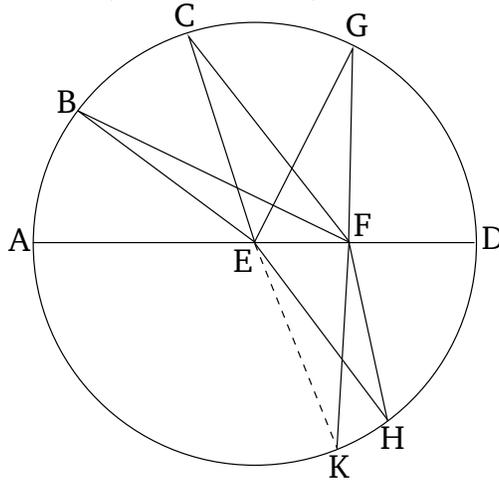


Book 3 Proposition 7

If some point, which is not the center of the circle, is taken on the diameter of a circle, and some straight-lines radiate from the point towards the (circumference of the) circle, then the greatest (straight-line) will be that on which the center (lies), and the least the remainder (of the same diameter). And for the others, a (straight-line) nearer[†] to the (straight-line) through the center is always greater than a (straight-line) further away. And only two equal (straight-lines) will radiate from the point towards the (circumference of the) circle, (one) on each (side) of the least (straight-line).



Let $ABCD$ be a circle, and let AD be its diameter, and let some point F , which is not the center of the circle, have been taken on AD . Let E be the center of the circle. And let some straight-lines, FB , FC , and FG , radiate from F towards (the circumference of) circle $ABCD$. I say that FA is the greatest (straight-line), FD the least,

and of the others, FB (is) greater than FC , and FC than FG .

For let BE , CE , and GE have been joined. And since for every triangle (any) two sides are greater than the remaining (side) [Prop. 1.20], EB and EF is thus greater than BF . And AE (is) equal to BE [thus, BE and EF is equal to AF]. Thus, AF (is) greater than BF . Again, since BE is equal to CE , and FE (is) common, the two (straight-lines) BE , EF are equal to the two (straight-lines) CE , EF (respectively). But, angle BEF (is) also greater than angle CEF .[‡] Thus, the base BF is greater than the base CF . Thus, the base BF is greater than the base CF [Prop. 1.24]. So, for the same (reasons), CF is also greater than FG .

Again, since GF and FE are greater than EG [Prop. 1.20], and EG (is) equal to ED , GF and FE are thus greater than ED . Let EF have been taken from both. Thus, the remainder GF is greater than the remainder FD . Thus, FA (is) the greatest (straight-line), FD the least, and FB (is) greater than FC , and FC than FG .

I also say that from point F only two equal (straight-lines) will radiate towards (the circumference of) circle $ABCD$, (one) on each (side) of the least (straight-line) FD . For let the (angle) FEH , equal to angle GEF , have been constructed on the straight-line EF , at the point E on it [Prop. 1.23], and let FH have been joined. Therefore, since GE is equal to EH , and EF (is) common, the two (straight-lines) GE , EF are equal to the two (straight-lines) HE , EF (respectively). And angle GEF (is) equal to angle HEF . Thus, the base FG is

equal to the base FH [Prop. 1.4]. So I say that another (straight-line) equal to FG will not radiate towards (the circumference of) the circle from point F . For, if possible, let FK (so) radiate. And since FK is equal to FG , but FH [is equal] to FG , FK is thus also equal to FH , the nearer to the (straight-line) through the center equal to the further away. The very thing (is) impossible. Thus, another (straight-line) equal to GF will not radiate from the point F towards (the circumference of) the circle. Thus, (there is) only one (such straight-line).

Thus, if some point, which is not the center of the circle, is taken on the diameter of a circle, and some straight-lines radiate from the point towards the (circumference of the) circle, then the greatest (straight-line) will be that on which the center (lies), and the least the remainder (of the same diameter). And for the others, a (straight-line) nearer to the (straight-line) through the center is always greater than a (straight-line) further away. And only two equal (straight-lines) will radiate from the same point towards the (circumference of the) circle, (one) on each (side) of the least (straight-line). (Which is) the very thing it was required to show.