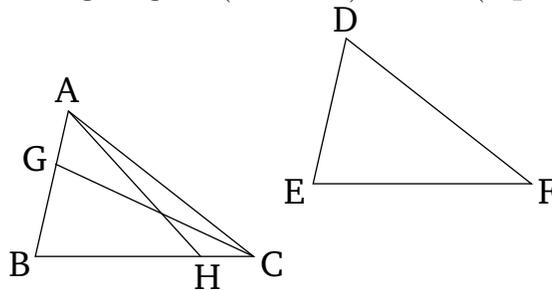


Book 1

Proposition 26

If two triangles have two angles equal to two angles, respectively, and one side equal to one side—in fact, either that by the equal angles, or that subtending one of the equal angles—then (the triangles) will also have the remaining sides equal to the [corresponding] remaining sides, and the remaining angle (equal) to the remaining angle.

Let ABC and DEF be two triangles having the two angles ABC and BCA equal to the two (angles) DEF and EFD , respectively. (That is) ABC (equal) to DEF , and BCA to EFD . And let them also have one side equal to one side. First of all, the (side) by the equal angles. (That is) BC (equal) to EF . I say that they will have the remaining sides equal to the corresponding remaining sides. (That is) AB (equal) to DE , and AC to DF . And (they will have) the remaining angle (equal) to the remaining angle. (That is) BAC (equal) to EDF .



For if AB is unequal to DE then one of them is greater. Let AB be greater, and let BG be made equal to DE [Prop. 1.3], and let GC have been joined.

Therefore, since BG is equal to DE , and BC to EF ,

the two (straight-lines) GB , BC are equal to the two (straight-lines) DE , EF , respectively. And angle GBC is equal to angle DEF . Thus, the base GC is equal to the base DF , and triangle GBC is equal to triangle DEF , and the remaining angles subtended by the equal sides will be equal to the (corresponding) remaining angles [Prop. 1.4]. Thus, GCB (is equal) to DFE . But, DFE was assumed (to be) equal to BCA . Thus, BCG is also equal to BCA , the lesser to the greater. The very thing (is) impossible. Thus, AB is not unequal to DE . Thus, (it is) equal. And BC is also equal to EF . So the two (straight-lines) AB , BC are equal to the two (straight-lines) DE , EF , respectively. And angle ABC is equal to angle DEF . Thus, the base AC is equal to the base DF , and the remaining angle BAC is equal to the remaining angle EDF [Prop. 1.4].

But, again, let the sides subtending the equal angles be equal: for instance, (let) AB (be equal) to DE . Again, I say that the remaining sides will be equal to the remaining sides. (That is) AC (equal) to DF , and BC to EF . Furthermore, the remaining angle BAC is equal to the remaining angle EDF .

For if BC is unequal to EF then one of them is greater. If possible, let BC be greater. And let BH be made equal to EF [Prop. 1.3], and let AH have been joined. And since BH is equal to EF , and AB to DE , the two (straight-lines) AB , BH are equal to the two (straight-lines) DE , EF , respectively. And the angles they encompass (are also equal). Thus, the base AH is equal to the base DF , and the triangle ABH is equal to the

triangle DEF , and the remaining angles subtended by the equal sides will be equal to the (corresponding) remaining angles [Prop. 1.4]. Thus, angle BHA is equal to EFD . But, EFD is equal to BCA . So, in triangle AHC , the external angle BHA is equal to the internal and opposite angle BCA . The very thing (is) impossible [Prop. 1.16]. Thus, BC is not unequal to EF . Thus, (it is) equal. And AB is also equal to DE . So the two (straight-lines) AB , BC are equal to the two (straight-lines) DE , EF , respectively. And they encompass equal angles. Thus, the base AC is equal to the base DF , and triangle ABC (is) equal to triangle DEF , and the remaining angle BAC (is) equal to the remaining angle EDF [Prop. 1.4].

Thus, if two triangles have two angles equal to two angles, respectively, and one side equal to one side—in fact, either that by the equal angles, or that subtending one of the equal angles—then (the triangles) will also have the remaining sides equal to the (corresponding) remaining sides, and the remaining angle (equal) to the remaining angle. (Which is) the very thing it was required to show.