

# Book 1

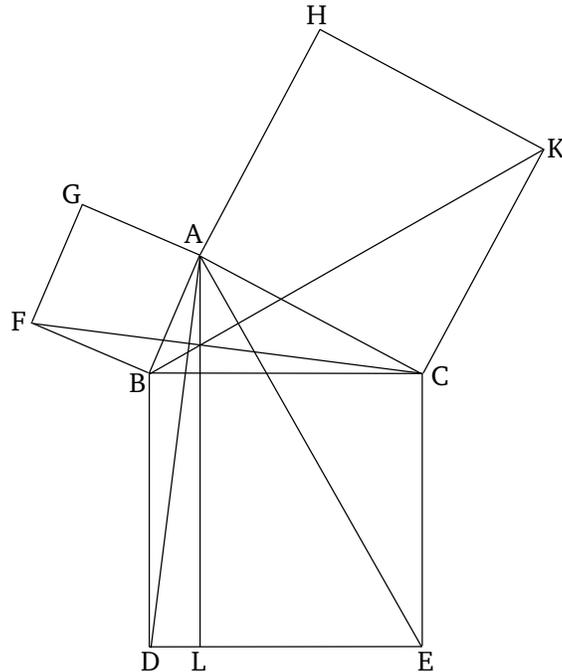
## Proposition 47

In right-angled triangles, the square on the side subtending the right-angle is equal to the (sum of the) squares on the sides containing the right-angle.

Let  $ABC$  be a right-angled triangle having the angle  $BAC$  a right-angle. I say that the square on  $BC$  is equal to the (sum of the) squares on  $BA$  and  $AC$ .

For let the square  $BDEC$  have been described on  $BC$ , and (the squares)  $GB$  and  $HC$  on  $AB$  and  $AC$  (respectively) [Prop. 1.46]. And let  $AL$  have been drawn through point  $A$  parallel to either of  $BD$  or  $CE$  [Prop. 1.31]. And let  $AD$  and  $FC$  have been joined. And since angles  $BAC$  and  $BAG$  are each right-angles, then two straight-lines  $AC$  and  $AG$ , not lying on the same side, make the adjacent angles with some straight-line  $BA$ , at the point  $A$  on it, (whose sum is) equal to two right-angles. Thus,  $CA$  is straight-on to  $AG$  [Prop. 1.14]. So, for the same (reasons),  $BA$  is also straight-on to  $AH$ . And since angle  $DBC$  is equal to  $FBA$ , for (they are) both right-angles, let  $ABC$  have been added to both. Thus, the whole (angle)  $DBA$  is equal to the whole (angle)  $FBC$ . And since  $DB$  is equal to  $BC$ , and  $FB$  to  $BA$ , the two (straight-lines)  $DB$ ,  $BA$  are equal to the two (straight-lines)  $CB$ ,  $BF$ , respectively. And angle  $DBA$  (is) equal to angle  $FBC$ . Thus, the base  $AD$  [is] equal to the base  $FC$ , and the triangle  $ABD$  is equal to the triangle  $FBC$  [Prop. 1.4]. And parallelogram  $BL$  [is] double (the area) of triangle  $ABD$ . For they have the

same base,  $BD$ , and are between the same parallels,  $BD$  and  $AL$  [Prop. 1.41]. And square  $GB$  is double (the area) of triangle  $FBC$ . For again they have the same base,  $FB$ , and are between the same parallels,  $FB$  and  $GC$  [Prop. 1.41]. [And the doubles of equal things are equal to one another.] Thus, the parallelogram  $BL$  is also equal to the square  $GB$ . So, similarly,  $AE$  and  $BK$  being joined, the parallelogram  $CL$  can be shown (to be) equal to the square  $HC$ . Thus, the whole square  $BDEC$  is equal to the (sum of the) two squares  $GB$  and  $HC$ . And the square  $BDEC$  is described on  $BC$ , and the (squares)  $GB$  and  $HC$  on  $BA$  and  $AC$  (respectively). Thus, the square on the side  $BC$  is equal to the (sum of the) squares on the sides  $BA$  and  $AC$ .



Thus, in right-angled triangles, the square on the side subtending the right-angle is equal to the (sum of the)

squares on the sides surrounding the right-[angle]. (Which is) the very thing it was required to show.