

Book 7

Proposition 31

Every composite number is measured by some prime number.

Let A be a composite number. I say that A is measured by some prime number.

For since A is composite, some number will measure it. Let it (so) measure (A), and let it be B . And if B is prime then that which was prescribed has happened. And if (B is) composite then some number will measure it. Let it (so) measure (B), and let it be C . And since C measures B , and B measures A , C thus also measures A . And if C is prime then that which was prescribed has happened. And if (C is) composite then some number will measure it. So, in this manner of continued investigation, some prime number will be found which will measure (the number preceding it, which will also measure A). And if (such a number) cannot be found then an infinite (series of) numbers, each of which is less than the preceding, will measure the number A . The very thing is impossible for numbers. Thus, some prime number will (eventually) be found which will measure the (number) preceding it, which will also measure A .

A ───────────┘

B ───────────┘

C ───────────┘

Thus, every composite number is measured by some prime number. (Which is) the very thing it was required to show.