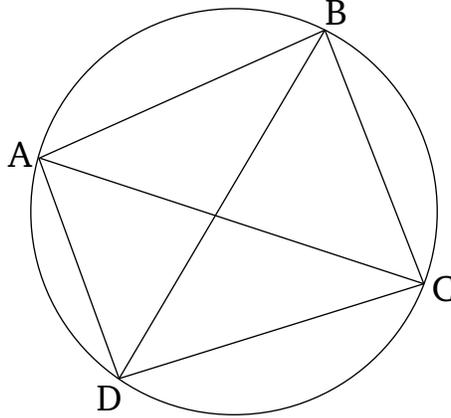


# Book 3

## Proposition 22

For quadrilaterals within circles, the (sum of the) opposite angles is equal to two right-angles.



Let  $ABCD$  be a circle, and let  $ABCD$  be a quadrilateral within it. I say that the (sum of the) opposite angles is equal to two right-angles.

Let  $AC$  and  $BD$  have been joined.

Therefore, since the three angles of any triangle are equal to two right-angles [Prop. 1.32], the three angles  $CAB$ ,  $ABC$ , and  $BCA$  of triangle  $ABC$  are thus equal to two right-angles. And  $CAB$  (is) equal to  $BDC$ . For they are in the same segment  $BADC$  [Prop. 3.21]. And  $ACB$  (is equal) to  $ADB$ . For they are in the same segment  $ADCB$  [Prop. 3.21]. Thus, the whole of  $ADC$  is equal to  $BAC$  and  $ACB$ . Let  $ABC$  have been added to both. Thus,  $ABC$ ,  $BAC$ , and  $ACB$  are equal to  $ABC$  and  $ADC$ . But,  $ABC$ ,  $BAC$ , and  $ACB$  are equal to two right-angles. Thus,  $ABC$  and  $ADC$  are also equal to two right-angles. Similarly, we can show that angles  $BAD$

and  $DCB$  are also equal to two right-angles.

Thus, for quadrilaterals within circles, the (sum of the) opposite angles is equal to two right-angles. (Which is) the very thing it was required to show.