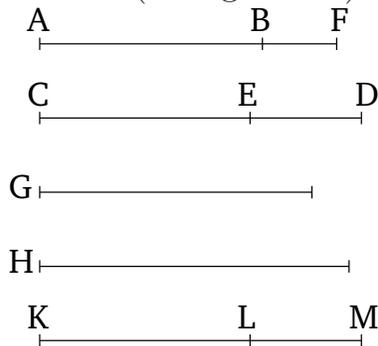


Book 10

Proposition 114

If an area is contained by an apotome, and a binomial whose terms are commensurable with, and in the same ratio as, the terms of the apotome then the square-root of the area is a rational (straight-line).



For let an area, the (rectangle contained) by AB and CD , have been contained by the apotome AB , and the binomial CD , of which let the greater term be CE . And let the terms of the binomial, CE and ED , be commensurable with the terms of the apotome, AF and FB (respectively), and in the same ratio. And let the square-root of the (rectangle contained) by AB and CD be G . I say that G is a rational (straight-line).

For let the rational (straight-line) H be laid down. And let (some rectangle), equal to the (square) on H , have been applied to CD , producing KL as breadth. Thus, KL is an apotome, of which let the terms, KM and ML , be commensurable with the terms of the binomial, CE and ED (respectively), and in the same ratio [Prop. 10.112]. But, CE and ED are also commensurable with AF and FB (respectively), and in the

same ratio. Thus, as AF is to FB , so KM (is) to ML . Thus, alternately, as AF is to KM , so BF (is) to LM [Prop. 5.16]. Thus, the remainder AB is also to the remainder KL as AF (is) to KM [Prop. 5.19]. And AF (is) commensurable with KM [Prop. 10.12]. AB is thus also commensurable with KL [Prop. 10.11]. And as AB is to KL , so the (rectangle contained) by CD and AB (is) to the (rectangle contained) by CD and KL [Prop. 6.1]. Thus, the (rectangle contained) by CD and AB is also commensurable with the (rectangle contained) by CD and KL [Prop. 10.11]. And the (rectangle contained) by CD and KL (is) equal to the (square) on H . Thus, the (rectangle contained) by CD and AB is commensurable with the (square) on H . And the (square) on G is equal to the (rectangle contained) by CD and AB . The (square) on G is thus commensurable with the (square) on H . And the (square) on H (is) rational. Thus, the (square) on G is also rational. G is thus rational. And it is the square-root of the (rectangle contained) by CD and AB .

Thus, if an area is contained by an apotome, and a binomial whose terms are commensurable with, and in the same ratio as, the terms of the apotome, then the square-root of the area is a rational (straight-line).

Corollary

And it has also been made clear to us, through this, that it is possible for a rational area to be contained by irrational straight-lines. (Which is) the very thing it was

required to show.