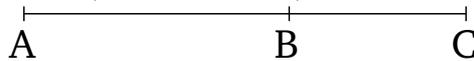


Book 10

Proposition 39

If two straight-lines (which are) incommensurable in square, making the sum of the squares on them rational, and the (rectangle contained) by them medial, are added together then the whole straight-line is irrational—let it be called a major (straight-line).



For let the two straight-lines, AB and BC , incommensurable in square, and fulfilling the prescribed (conditions), be laid down together [Prop. 10.33]. I say that AC is irrational.

For since the (rectangle contained) by AB and BC is medial, twice the (rectangle contained) by AB and BC is [thus] also medial [Props. 10.6, 10.23 corr.]. And the sum of the (squares) on AB and BC (is) rational. Thus, twice the (rectangle contained) by AB and BC is incommensurable with the sum of the (squares) on AB and BC [Def. 10.4]. Hence, (the sum of) the squares on AB and BC , plus twice the (rectangle contained) by AB and BC —that is, the (square) on AC [Prop. 2.4]—is also incommensurable with the sum of the (squares) on AB and BC [Prop. 10.16] [and the sum of the (squares) on AB and BC (is) rational]. Thus, the (square) on AC is irrational. Hence, AC is also irrational [Def. 10.4]—let it be called a major (straight-line).[†] (Which is) the very thing it was required to show.