

Book 10

Proposition 36

If two rational (straight-lines which are) commensurable in square only are added together then the whole (straight-line) is irrational—let it be called a binomial (straight-line).[†]



For let the two rational (straight-lines), AB and BC , (which are) commensurable in square only, be laid down together. I say that the whole (straight-line), AC , is irrational. For since AB is incommensurable in length with BC —for they are commensurable in square only—and as AB (is) to BC , so the (rectangle contained) by ABC (is) to the (square) on BC , the (rectangle contained) by AB and BC is thus incommensurable with the (square) on BC [Prop. 10.11]. But, twice the (rectangle contained) by AB and BC is commensurable with the (rectangle contained) by AB and BC [Prop. 10.6]. And (the sum of) the (squares) on AB and BC is commensurable with the (square) on BC —for the rational (straight-lines) AB and BC are commensurable in square only [Prop. 10.15]. Thus, twice the (rectangle contained) by AB and BC is incommensurable with (the sum of) the (squares) on AB and BC [Prop. 10.13]. And, via composition, twice the (rectangle contained) by AB and BC , plus (the sum of) the (squares) on AB and BC —that is to say, the (square) on AC [Prop. 2.4]—is incommensurable with the sum of the (squares) on AB and BC [Prop. 10.16]. And the sum of the (squares) on AB and BC (is) rational. The irrational [Def. 10.4]. Hence, AC is also irrational [Def. 10.4]—let it be called a binomial (straight-line).[‡] (Which is) the

very thing it was required to show.