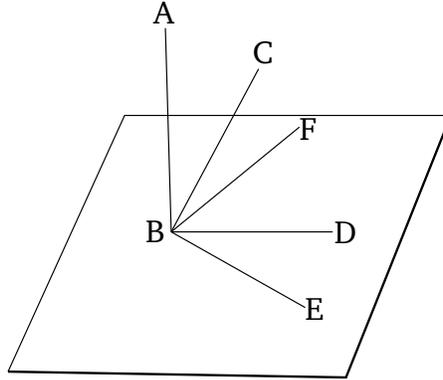


# Book 11

## Proposition 5

If a straight-line is set up at right-angles to three straight-lines cutting one another, at the common point of section, then the three straight-lines are in one plane.



For let some straight-line  $AB$  have been set up at right-angles to three straight-lines  $BC$ ,  $BD$ , and  $BE$ , at the (common) point of section  $B$ . I say that  $BC$ ,  $BD$ , and  $BE$  are in one plane.

For (if) not, and if possible, let  $BD$  and  $BE$  be in the reference plane, and  $BC$  in a more elevated (plane). And let the plane through  $AB$  and  $BC$  have been produced. So it will make a straight-line as a common section with the reference plane [Def. 11.3]. Let it make  $BF$ . Thus, the three straight-lines  $AB$ ,  $BC$ , and  $BF$  are in one plane—(namely), that drawn through  $AB$  and  $BC$ . And since  $AB$  is at right-angles to each of  $BD$  and  $BE$ ,  $AB$  is thus also at right-angles to the plane (passing) through  $BD$  and  $BE$  [Prop. 11.4]. And the plane (passing) through  $BD$  and  $BE$  is the reference plane. Thus,  $AB$  is at right-angles to the reference plane. Hence,  $AB$  will also make right-angles with all straight-lines joined

to it which are also in the reference plane [Def. 11.3]. And  $BF$ , which is in the reference plane, is joined to it. Thus, the angle  $ABF$  is a right-angle. And  $ABC$  was also assumed to be a right-angle. Thus, angle  $ABF$  (is) equal to  $ABC$ . And they are in one plane. The very thing is impossible. Thus,  $BC$  is not in a more elevated plane. Thus, the three straight-lines  $BC$ ,  $BD$ , and  $BE$  are in one plane.

Thus, if a straight-line is set up at right-angles to three straight-lines cutting one another, at the (common) point of section, then the three straight-lines are in one plane. (Which is) the very thing it was required to show.