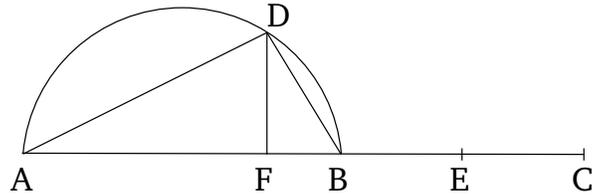


# Book 10

## Proposition 34

To find two straight-lines (which are) incommensurable in square, making the sum of the squares on them medial, and the (rectangle contained) by them rational.



Let the two medial (straight-lines)  $AB$  and  $BC$ , (which are) commensurable in square only, be laid out having the (rectangle contained) by them rational, (and) such that the square on  $AB$  is greater than (the square on)  $BC$  by the (square) on (some straight-line) incommensurable (in length) with ( $AB$ ) [Prop. 10.31]. And let the semi-circle  $ADB$  have been drawn on  $AB$ . And let  $BC$  have been cut in half at  $E$ . And let a (rectangular) parallelogram equal to the (square) on  $BE$ , (and) falling short by a square figure, have been applied to  $AB$ , (and let it be) the (rectangle contained by)  $AFB$  [Prop. 6.28]. Thus,  $AF$  [is] incommensurable in length with  $FB$  [Prop. 10.18]. And let  $FD$  have been drawn from  $F$  at right-angles to  $AB$ . And let  $AD$  and  $DB$  have been joined.

Since  $AF$  is incommensurable (in length) with  $FB$ , the (rectangle contained) by  $BA$  and  $AF$  is thus also incommensurable with the (rectangle contained) by  $AB$  and  $BF$  [Prop. 10.11]. And the (rectangle contained) by  $BA$  and  $AF$  (is) equal to the (square) on  $AD$ , and the (rect-

angle contained) by  $AB$  and  $BF$  to the (square) on  $DB$  [Prop. 10.32 lem.]. Thus, the (square) on  $AD$  is also incommensurable with the (square) on  $DB$ . And since the (square) on  $AB$  is medial, the sum of the (squares) on  $AD$  and  $DB$  (is) thus also medial [Props. 3.31, 1.47]. And since  $BC$  is double the (rectangle contained) by  $AB$  and  $BC$  (is) thus also double the (rectangle contained) by  $AB$  and  $FD$ . And the (rectangle contained) by  $AB$  and  $BC$  (is) rational. Thus, the (rectangle contained) by  $AB$  and  $FD$  (is) also rational [Prop. 10.6, Def. 10.4]. And the (rectangle contained) by  $AB$  and  $FD$  (is) equal to the (rectangle contained) by  $AD$  and  $DB$  [Prop. 10.32 lem.]. And hence the (rectangle contained) by  $AD$  and  $DB$  is rational.

Thus, two straight-lines,  $AD$  and  $DB$ , (which are) incommensurable in square, have been found, making the sum of the squares on them medial, and the (rectangle contained) by them rational.<sup>†</sup> (Which is) the very thing it was required to show.