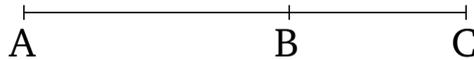


# Book 10

## Proposition 40

If two straight-lines (which are) incommensurable in square, making the sum of the squares on them medial, and the (rectangle contained) by them rational, are added together then the whole straight-line is irrational—let it be called the square-root of a rational plus a medial (area).



For let the two straight-lines,  $AB$  and  $BC$ , incommensurable in square, (and) fulfilling the prescribed (conditions), be laid down together [Prop. 10.34]. I say that  $AC$  is irrational.

For since the sum of the (squares) on  $AB$  and  $BC$  is medial, and twice the (rectangle contained) by  $AB$  and  $BC$  (is) rational, the sum of the (squares) on  $AB$  and  $BC$  is thus incommensurable with twice the (rectangle contained) by  $AB$  and  $BC$ . Hence, the (square) on  $AC$  is also incommensurable with twice the (rectangle contained) by  $AB$  and  $BC$  [Prop. 10.16]. And twice the (rectangle contained) by  $AB$  and  $BC$  (is) rational. The (square) on  $AC$  (is) thus irrational. Thus,  $AC$  (is) irrational [Def. 10.4]—let it be called the square-root of a rational plus a medial (area).<sup>†</sup> (Which is) the very thing it was required to show.