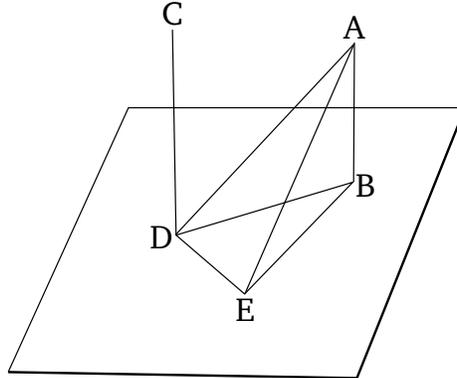


Book 11

Proposition 6

If two straight-lines are at right-angles to the same plane then the straight-lines will be parallel.



For let the two straight-lines AB and CD be at right-angles to a reference plane. I say that AB is parallel to CD .

For let them meet the reference plane at points B and D (respectively). And let the straight-line BD have been joined. And let DE have been drawn at right-angles to BD in the reference plane. And let DE be made equal to AB . And let BE , AE , and AD have been joined.

And since AB is at right-angles to the reference plane, it will [thus] also make right-angles with all straight-lines joined to it which are in the reference plane [Def. 11.3]. And BD and BE , which are in the reference plane, are each joined to AB . Thus, each of the angles ABD and ABE are right-angles. So, for the same (reasons), each of the angles CDB and CDE are also right-angles. And since AB is equal to DE , and BD (is) common, the two (straight-lines) AB and BD are equal to the two

(straight-lines) ED and DB (respectively). And they contain right-angles. Thus, the base AD is equal to the base BE [Prop. 1.4]. And since AB is equal to DE , and AD (is) also (equal) to BE , the two (straight-lines) AB and BE are thus equal to the two (straight-lines) ED and DA (respectively). And their base AE (is) common. Thus, angle ABE is equal to angle EDA [Prop. 1.8]. And ABE (is) a right-angle. Thus, EDA (is) also a right-angle. ED is thus at right-angles to DA . And it is also at right-angles to each of BD and DC . Thus, ED is standing at right-angles to the three straight-lines BD , DA , and DC at the (common) point of section. Thus, the three straight-lines BD , DA , and DC are in one plane [Prop. 11.5]. And in which(ever) plane DB and DA (are found), in that (plane) AB (will) also (be found). For every triangle is in one plane [Prop. 11.2]. And each of the angles ABD and BDC is a right-angle. Thus, AB is parallel to CD [Prop. 1.28].

Thus, if two straight-lines are at right-angles to the same plane then the straight-lines will be parallel. (Which is) the very thing it was required to show.