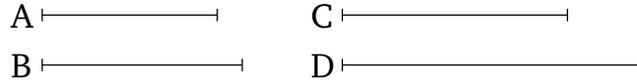


Book 9

Proposition 18

For two given numbers, to investigate whether it is possible to find a third (number) proportional to them.



Let A and B be the two given numbers. And let it be required to investigate whether it is possible to find a third (number) proportional to them.

So A and B are either prime to one another, or not. And if they are prime to one another then it has (already) been show that it is impossible to find a third (number) proportional to them [Prop. 9.16].

And so let A and B not be prime to one another. And let B make C (by) multiplying itself. So A either measures, or does not measure, C . Let it first of all measure (C) according to D . Thus, A has made C (by) multiplying D . But, in fact, B has also made C (by) multiplying itself. Thus, the (number created) from (multiplying) A , D is equal to the (square) on B . Thus, as A is to B , (so) B (is) to D [Prop. 7.19]. Thus, a third number has been found proportional to A , B , (namely) D .

And so let A not measure C . I say that it is impossible to find a third number proportional to A , B . For, if possible, let it have been found, (and let it be) D . Thus, the (number created) from (multiplying) A , D is equal to the (square) on B [Prop. 7.19]. And the (square) on B is C . Thus, the (number created) from (multiplying) A , D is equal to C . Hence, A has made C (by) multiplying D . Thus, A measures C according to D . But (A) was, in

fact, also assumed (to be) not measuring (C). The very thing (is) absurd. Thus, it is not possible to find a third number proportional to A , B when A does not measure C . (Which is) the very thing it was required to show.