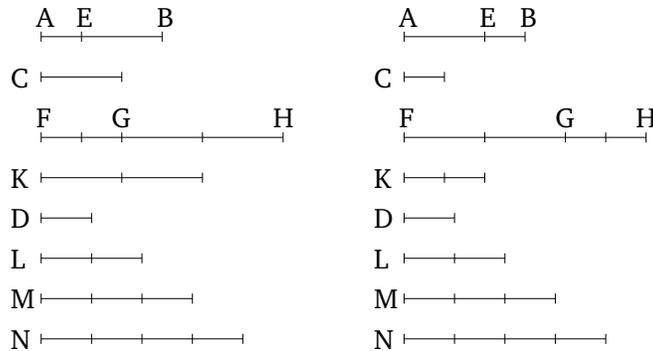


Book 5

Proposition 8

For unequal magnitudes, the greater (magnitude) has a greater ratio than the lesser to the same (magnitude). And the latter (magnitude) has a greater ratio to the lesser (magnitude) than to the greater.

Let AB and C be unequal magnitudes, and let AB be the greater (of the two), and D another random magnitude. I say that AB has a greater ratio to D than C (has) to D , and (that) D has a greater ratio to C than (it has) to AB .



For since AB is greater than C , let BE be made equal to C . So, the lesser of AE and EB , being multiplied, will sometimes be greater than D [Def. 5.4]. First of all, let AE be less than EB , and let AE have been multiplied, and let FG be a multiple of it which (is) greater than D . And as many times as FG is (divisible) by AE , so many times let GH also have become (divisible) by EB , and K by C . And let the double multiple L of D have been taken, and the triple multiple M , and several more, (each increasing) in order by one, until the (multiple) taken becomes the first multiple of D (which is) greater

than K . Let it have been taken, and let it also be the quadruple multiple N of D —the first (multiple) greater than K .

Therefore, since K is less than N first, K is thus not less than M . And since FG and GH are equal multiples of AE and EB (respectively), FG and FH are thus equal multiples of AE and AB (respectively) [Prop. 5.1]. And FG and K are equal multiples of AE and C (respectively). Thus, FH and K are equal multiples of AB and C (respectively). Thus, FH, K are equal multiples of AB, C . Again, since GH and K are equal multiples of EB and C , and EB (is) equal to C , GH (is) thus also equal to K . And K is not less than M . Thus, GH not less than M either. And FG (is) greater than D . Thus, the whole of FH is greater than D and M (added) together. But, D and M (added) together is equal to N , inasmuch as M is three times D , and M and D (added) together is four times D , and N is also four times D . Thus, M and D (added) together is equal to N . But, FH is greater than M and D . Thus, FH exceeds N . And K does not exceed N . And FH, K are equal multiples of AB, C , and N another random multiple of D . Thus, AB has a greater ratio to D than C (has) to D [Def. 5.7].

So, I say that D also has a greater ratio to C than D (has) to AB .

For, similarly, by the same construction, we can show that N exceeds K , and N does not exceed FH . And N is a multiple of D , and FH, K other random equal multiples of AB, C (respectively). Thus, D has a greater

ratio to C than D (has) to AB [Def. 5.5].

And so let AE be greater than EB . So, the lesser, EB , being multiplied, will sometimes be greater than D . Let it have been multiplied, and let GH be a multiple of EB (which is) greater than D . And as many times as GH is (divisible) by EB , so many times let FG also have become (divisible) by AE , and K by C . So, similarly (to the above), we can show that FH and K are equal multiples of AB and C (respectively). And, similarly (to the above), let the multiple N of D , (which is) the first (multiple) greater than FG , have been taken. So, FG is again not less than M . And GH (is) greater than D . Thus, the whole of FH exceeds D and M , that is to say N . And K does not exceed N , inasmuch as FG , which (is) greater than GH —that is to say, K —also does not exceed N . And, following the above (arguments), we (can) complete the proof in the same manner.

Thus, for unequal magnitudes, the greater (magnitude) has a greater ratio than the lesser to the same (magnitude). And the latter (magnitude) has a greater ratio to the lesser (magnitude) than to the greater. (Which is) the very thing it was required to show.