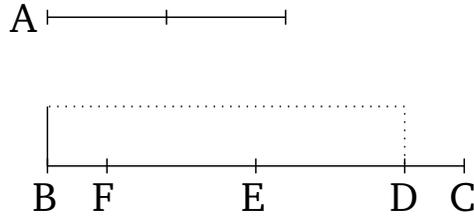


# Book 10

## Proposition 17

If there are two unequal straight-lines, and a (rectangle) equal to the fourth part of the (square) on the lesser, falling short by a square figure, is applied to the greater, and divides it into (parts which are) commensurable in length, then the square on the greater will be larger than (the square on) the lesser by the (square) on (some straight-line) commensurable [in length] with the greater. And if the square on the greater is larger than (the square on) the lesser by the (square) on (some straight-line) commensurable [in length] with the greater, and a (rectangle) equal to the fourth (part) of the (square) on the lesser, falling short by a square figure, is applied to the greater, then it divides it into (parts which are) commensurable in length.

Let  $A$  and  $BC$  be two unequal straight-lines, of which (let)  $BC$  (be) the greater. And let a (rectangle) equal to the fourth part of the (square) on the lesser,  $A$ —that is, (equal) to the (square) on half of  $A$ —falling short by a square figure, have been applied to  $BC$ . And let it be the (rectangle contained) by  $BD$  and  $DC$  [see previous lemma]. And let  $BD$  be commensurable in length with  $DC$ . I say that that the square on  $BC$  is greater than the (square on)  $A$  by (the square on some straight-line) commensurable (in length) with ( $BC$ ).



For let  $BC$  have been cut in half at the point  $E$  [Prop. 1.10]. And let  $EF$  be made equal to  $DE$  [Prop. 1.3]. Thus, the remainder  $DC$  is equal to  $BF$ . And since the straight-line  $BC$  has been cut into equal (pieces) at  $E$ , and into unequal (pieces) at  $D$ , the rectangle contained by  $BD$  and  $DC$ , plus the square on  $ED$ , is thus equal to the square on  $EC$  [Prop. 2.5]. (The same) also (for) the quadruples. Thus, four times the (rectangle contained) by  $BD$  and  $DC$ , plus the quadruple of the (square) on  $DE$ , is equal to four times the square on  $EC$ . But, the square on  $A$  is equal to the quadruple of the (rectangle contained) by  $BD$  and  $DC$ , and the square on  $DF$  is equal to the quadruple of the (square) on  $DE$ . For  $DF$  is double  $DE$ . And the square on  $BC$  is equal to the quadruple of the (square) on  $EC$ . For, again,  $BC$  is double  $CE$ . Thus, the (sum of the) squares on  $A$  and  $DF$  is equal to the square on  $BC$ . Hence, the (square) on  $BC$  is greater than the (square) on  $A$  by the (square) on  $DF$ . Thus,  $BC$  is greater in square than  $A$  by  $DF$ . It must also be shown that  $BC$  is commensurable (in length) with  $DF$ . For since  $BD$  is commensurable in length with  $DC$ ,  $BC$  is thus also commensurable in length with  $CD$  [Prop. 10.15]. But,  $CD$  is commensurable in length with  $CD$  plus  $BF$ . For  $CD$  is equal to  $BF$  [Prop. 10.6]. Thus,  $BC$  is also commensurable in length with  $BF$  plus

$CD$  [Prop. 10.12]. Hence,  $BC$  is also commensurable in length with the remainder  $FD$  [Prop. 10.15]. Thus, the square on  $BC$  is greater than (the square on)  $A$  by the (square) on (some straight-line) commensurable (in length) with  $(BC)$ .

And so let the square on  $BC$  be greater than the (square on)  $A$  by the (square) on (some straight-line) commensurable (in length) with  $(BC)$ . And let a (rectangle) equal to the fourth (part) of the (square) on  $A$ , falling short by a square figure, have been applied to  $BC$ . And let it be the (rectangle contained) by  $BD$  and  $DC$ . It must be shown that  $BD$  is commensurable in length with  $DC$ .

For, similarly, by the same construction, we can show that the square on  $BC$  is greater than the (square on)  $A$  by the (square) on  $FD$ . And the square on  $BC$  is greater than the (square on)  $A$  by the (square) on (some straight-line) commensurable (in length) with  $(BC)$ . Thus,  $BC$  is commensurable in length with  $FD$ . Hence,  $BC$  is also commensurable in length with the remaining sum of  $BF$  and  $DC$  [Prop. 10.15]. But, the sum of  $BF$  and  $DC$  is commensurable [in length] with  $DC$  [Prop. 10.6]. Hence,  $BC$  is also commensurable in length with  $CD$  [Prop. 10.12]. Thus, via separation,  $BD$  is also commensurable in length with  $DC$  [Prop. 10.15].

Thus, if there are two unequal straight-lines, and so on . . . .