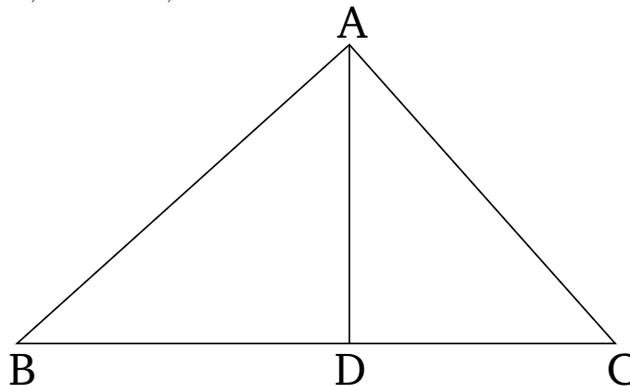


Book 6

Proposition 8

If, in a right-angled triangle, a (straight-line) is drawn from the right-angle perpendicular to the base then the triangles around the perpendicular are similar to the whole (triangle), and to one another.

Let ABC be a right-angled triangle having the angle BAC a right-angle, and let AD have been drawn from A , perpendicular to BC [Prop. 1.12]. I say that triangles ABD and ADC are each similar to the whole (triangle) ABC and, further, to one another.



For since (angle) BAC is equal to ADB —for each (are) right-angles—and the (angle) at B (is) common to the two triangles ABC and ABD , the remaining (angle) ACB is thus equal to the remaining (angle) BAD [Prop. 1.32]. Thus, triangle ABC is equiangular to triangle ABD . Thus, as BC , subtending the right-angle in triangle ABC , is to BA , subtending the right-angle in triangle ABD , so the same AB , subtending the angle at C in triangle ABC , (is) to BD , subtending the equal (angle) BAD in triangle ABD , and, further, (so is) AC

to AD , (both) subtending the angle at B common to the two triangles [Prop. 6.4]. Thus, triangle ABC is equiangular to triangle ABD , and has the sides about the equal angles proportional. Thus, triangle ABC [is] similar to triangle ABD [Def. 6.1]. So, similarly, we can show that triangle ABC is also similar to triangle ADC . Thus, [triangles] ABD and ADC are each similar to the whole (triangle) ABC .

So I say that triangles ABD and ADC are also similar to one another.

For since the right-angle BDA is equal to the right-angle ADC , and, indeed, (angle) BAD was also shown (to be) equal to the (angle) at C , thus the remaining (angle) at B is also equal to the remaining (angle) DAC [Prop. 1.32]. Thus, triangle ABD is equiangular to triangle ADC . Thus, as BD , subtending (angle) BAD in triangle ABD , is to DA , subtending the (angle) at C in triangle ADC , (which is) equal to (angle) BAD , so (is) the same AD , subtending the angle at B in triangle ABD , to DC , subtending (angle) DAC in triangle ADC , (which is) equal to the (angle) at B , and, further, (so is) BA to AC , (each) subtending right-angles [Prop. 6.4]. Thus, triangle ABD is similar to triangle ADC [Def. 6.1].

Thus, if, in a right-angled triangle, a (straight-line) is drawn from the right-angle perpendicular to the base then the triangles around the perpendicular are similar to the whole (triangle), and to one another. [(Which is) the very thing it was required to show.]

Corollary

So (it is) clear, from this, that if, in a right-angled triangle, a (straight-line) is drawn from the right-angle perpendicular to the base then the (straight-line so) drawn is in mean proportion to the pieces of the base. (Which is) the very thing it was required to show.