

# Book 10

## Proposition 37

If two medial (straight-lines), commensurable in square only, which contain a rational (area), are added together then the whole (straight-line) is irrational—let it be called a first bimedial (straight-line).



For let the two medial (straight-lines),  $AB$  and  $BC$ , commensurable in square only, (and) containing a rational (area), be laid down together. I say that the whole (straight-line),  $AC$ , is irrational.

For since  $AB$  is incommensurable in length with  $BC$ , (the sum of) the (squares) on  $AB$  and  $BC$  is thus also incommensurable with twice the (rectangle contained) by  $AB$  and  $BC$  [see previous proposition]. And, via composition, (the sum of) the (squares) on  $AB$  and  $BC$ , plus twice the (rectangle contained) by  $AB$  and  $BC$ —that is, the (square) on  $AC$  [Prop. 2.4]—is incommensurable with the (rectangle contained) by  $AB$  and  $BC$  [Prop. 10.16]. And the (rectangle contained) by  $AB$  and  $BC$  (is) rational—for  $AB$  and  $BC$  were assumed to enclose a rational (area). Thus, the (square) on  $AC$  (is) irrational. Thus,  $AC$  (is) irrational [Def. 10.4]—let it be called a first bimedial (straight-line). (Which is) the very thing it was required to show.