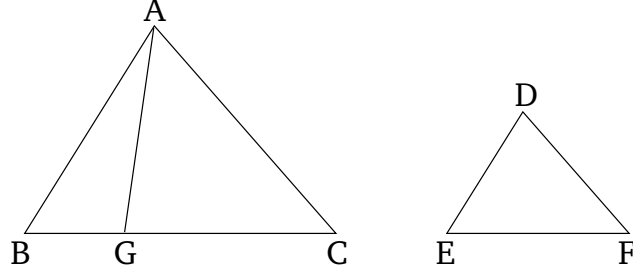


Book 6

Proposition 19

Similar triangles are to one another in the squared[†] ratio of (their) corresponding sides.



Let ABC and DEF be similar triangles having the angle at B equal to the (angle) at E , and AB to BC , as DE (is) to EF , such that BC corresponds to EF . I say that triangle ABC has a squared ratio to triangle DEF with respect to (that side) BC (has) to EF .

For let a third (straight-line), BG , have been taken (which is) proportional to BC and EF , so that as BC (is) to EF , so EF (is) to BG [Prop. 6.11]. And let AG have been joined.

Therefore, since as AB is to BC , so DE (is) to EF , thus, alternately, as AB is to DE , so BC (is) to EF [Prop. 5.16]. But, as BC (is) to EF , so EF is to BG . And, thus, as AB (is) to DE , so EF (is) to BG . Thus, for triangles ABG and DEF , the sides about the equal angles are reciprocally proportional. And those triangles having one (angle) equal to one (angle) for which the sides about the equal angles are reciprocally proportional are equal [Prop. 6.15]. Thus, triangle ABG is equal to triangle DEF . And since as BC (is) to EF , so EF (is) to BG , and if three straight-lines are proportional then

the first has a squared ratio to the third with respect to the second [Def. 5.9], BC thus has a squared ratio to BG with respect to (that) CB (has) to EF . And as CB (is) to BG , so triangle ABC (is) to triangle ABG [Prop. 6.1]. Thus, triangle ABC also has a squared ratio to (triangle) ABG with respect to (that side) BC (has) to EF . And triangle ABG (is) equal to triangle DEF . Thus, triangle ABC also has a squared ratio to triangle DEF with respect to (that side) BC (has) to EF .

Thus, similar triangles are to one another in the squared ratio of (their) corresponding sides. [(Which is) the very thing it was required to show].

Corollary

So it is clear, from this, that if three straight-lines are proportional, then as the first is to the third, so the figure (described) on the first (is) to the similar, and similarly described, (figure) on the second. (Which is) the very thing it was required to show.