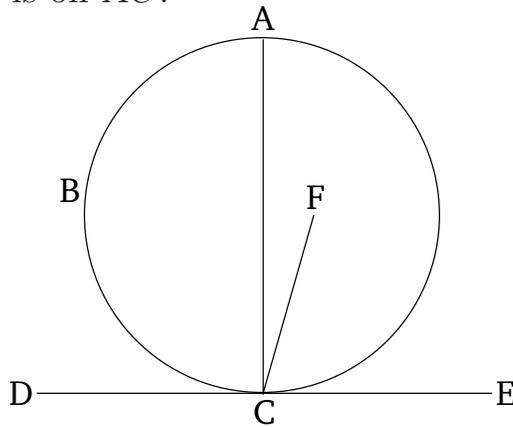


# Book 3

## Proposition 19

If some straight-line touches a circle, and a straight-line is drawn from the point of contact, at right-[angles] to the tangent, then the center (of the circle) will be on the (straight-line) so drawn.

For let some straight-line  $DE$  touch the circle  $ABC$  at point  $C$ . And let  $CA$  have been drawn from  $C$ , at right-angles to  $DE$  [Prop. 1.11]. I say that the center of the circle is on  $AC$ .



For (if) not, if possible, let  $F$  be (the center of the circle), and let  $CF$  have been joined.

[Therefore], since some straight-line  $DE$  touches the circle  $ABC$ , and  $FC$  has been joined from the center to the point of contact,  $FC$  is thus perpendicular to  $DE$  [Prop. 3.18]. Thus,  $FCE$  is a right-angle. And  $ACE$  is also a right-angle. Thus,  $FCE$  is equal to  $ACE$ , the lesser to the greater. The very thing is impossible. Thus,  $F$  is not the center of circle  $ABC$ . So, similarly, we can show that neither is any (point) other (than one) on  $AC$ . Thus, if some straight-line touches a circle, and a

straight-line is drawn from the point of contact, at right-angles to the tangent, then the center (of the circle) will be on the (straight-line) so drawn. (Which is) the very thing it was required to show.