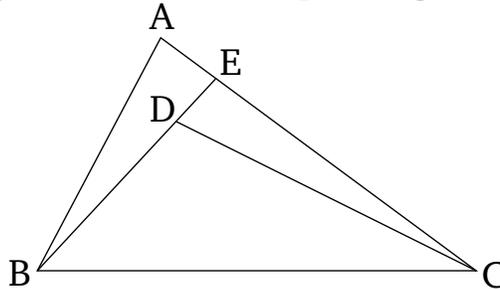


# Book 1

## Proposition 21

If two internal straight-lines are constructed on one of the sides of a triangle, from its ends, the constructed (straight-lines) will be less than the two remaining sides of the triangle, but will encompass a greater angle.



For let the two internal straight-lines  $BD$  and  $DC$  have been constructed on one of the sides  $BC$  of the triangle  $ABC$ , from its ends  $B$  and  $C$  (respectively). I say that  $BD$  and  $DC$  are less than the (sum of the) two remaining sides of the triangle  $BA$  and  $AC$ , but encompass an angle  $BDC$  greater than  $BAC$ .

For let  $BD$  have been drawn through to  $E$ . And since in any triangle (the sum of any) two sides is greater than the remaining (side) [Prop. 1.20], in triangle  $ABE$  the (sum of the) two sides  $AB$  and  $AE$  is thus greater than  $BE$ . Let  $EC$  have been added to both. Thus, (the sum of)  $BA$  and  $AC$  is greater than (the sum of)  $BE$  and  $EC$ . Again, since in triangle  $CED$  the (sum of the) two sides  $CE$  and  $ED$  is greater than  $CD$ , let  $DB$  have been added to both. Thus, (the sum of)  $CE$  and  $EB$  is greater than (the sum of)  $CD$  and  $DB$ . But, (the sum of)  $BA$  and  $AC$  was shown (to be) greater than (the sum of)

$BE$  and  $EC$ . Thus, (the sum of)  $BA$  and  $AC$  is much greater than (the sum of)  $BD$  and  $DC$ .

Again, since in any triangle the external angle is greater than the internal and opposite (angles) [Prop. 1.16], in triangle  $CDE$  the external angle  $BDC$  is thus greater than  $CED$ . Accordingly, for the same (reason), the external angle  $CEB$  of the triangle  $ABE$  is also greater than  $BAC$ . But,  $BDC$  was shown (to be) greater than  $CEB$ . Thus,  $BDC$  is much greater than  $BAC$ .

Thus, if two internal straight-lines are constructed on one of the sides of a triangle, from its ends, the constructed (straight-lines) are less than the two remaining sides of the triangle, but encompass a greater angle. (Which is) the very thing it was required to show.