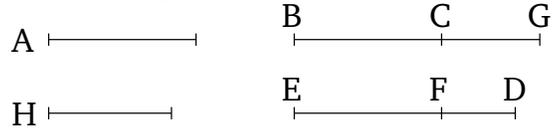


# Book 10

## Proposition 88

To find a fourth apotome.



Let the rational (straight-line)  $A$ , and  $BG$  (which is) commensurable in length with  $A$ , be laid down. Thus,  $BG$  is also a rational (straight-line). And let the two numbers  $DF$  and  $FE$  be laid down such that the whole,  $DE$ , does not have to each of  $DF$  and  $EF$  the ratio which (some) square number (has) to (some) square number. And let it have been contrived that as  $DE$  (is) to  $EF$ , so the square on  $BG$  (is) to the (square) on  $GC$  [Prop. 10.6 corr.]. The (square) on  $BG$  is thus commensurable with the (square) on  $GC$  [Prop. 10.6]. And the (square) on  $BG$  (is) rational. Thus, the (square) on  $GC$  (is) also rational. Thus,  $GC$  (is) a rational (straight-line). And since  $DE$  does not have to  $EF$  the ratio which (some) square number (has) to (some) square number, the (square) on  $BG$  thus does not have to the (square) on  $GC$  the ratio which (some) square number (has) to (some) square number either. Thus,  $BG$  is incommensurable in length with  $GC$  [Prop. 10.9]. And they are both rational (straight-lines). Thus,  $BG$  and  $GC$  are rational (straight-lines which are) commensurable in square only. Thus,  $BC$  is an apotome [Prop. 10.73]. [So, I say that (it is) also a fourth (apotome).]

Now, let the (square) on  $H$  be that (area) by which the (square) on  $BG$  is greater than the (square) on  $GC$

[Prop. 10.13 lem.]. Therefore, since as  $DE$  is to  $EF$ , so the (square) on  $BG$  (is) to the (square) on  $GC$ , thus, also, via conversion, as  $ED$  is to  $DF$ , so the (square) on  $GB$  (is) to the (square) on  $H$  [Prop. 5.19 corr.]. And  $ED$  does not have to  $DF$  the ratio which (some) square number (has) to (some) square number. Thus, the (square) on  $GB$  does not have to the (square) on  $H$  the ratio which (some) square number (has) to (some) square number either. Thus,  $BG$  is incommensurable in length with  $H$  [Prop. 10.9]. And the square on  $BG$  is greater than (the square on)  $GC$  by the (square) on  $H$ . Thus, the square on  $BG$  is greater than (the square) on  $GC$  by the (square) on (some straight-line) incommensurable (in length) with ( $BG$ ). And the whole,  $BG$ , is commensurable in length with the the (previously) laid down rational (straight-line)  $A$ . Thus,  $BC$  is a fourth apotome [Def. 10.14].

Thus, a fourth apotome has been found. (Which is) the very thing it was required to show.