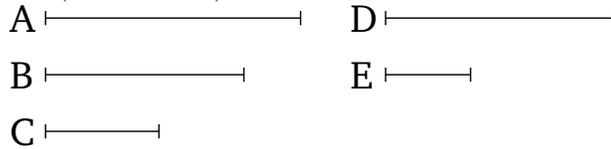


Book 10

Proposition 32

To find two medial (straight-lines), commensurable in square only, (and) containing a medial (area), such that the square on the greater is larger than the (square on the) lesser by the (square) on (some straight-line) commensurable (in length) with the greater.



Let three rational (straight-lines), A , B and C , commensurable in square only, be laid out such that the square on A is greater than (the square on C) by the (square) on (some straight-line) commensurable (in length) with (A) [Prop. 10.29]. And let the (square) on D be equal to the (rectangle contained) by A and B . Thus, the (square) on D (is) medial. Thus, D is also medial [Prop. 10.21]. And let the (rectangle contained) by D and E be equal to the (rectangle contained) by B and C . And since as the (rectangle contained) by A and B is to the (rectangle contained) by B and C , so A (is) to C [Prop. 10.21 lem.], but the (square) on D is equal to the (rectangle contained) by A and B , and the (rectangle contained) by D and E to the (rectangle contained) by B and C , thus as A is to C , so the (square) on D (is) to the (rectangle contained) by D and E . And as the (square) on D (is) to the (rectangle contained) by D and E , so D (is) to E [Prop. 10.21 lem.]. And thus as A (is) to C , so D (is) to E . And A (is) commensurable in

square [only] with C . Thus, D (is) also commensurable in square only with E [Prop. 10.11]. And D (is) medial. Thus, E (is) also medial [Prop. 10.23]. And since as A is to C , (so) D (is) to E , and the square on A is greater than (the square on) C by the (square) on (some straight-line) commensurable (in length) with (A), the square on D will thus also be greater than (the square on) E by the (square) on (some straight-line) commensurable (in length) with (D) [Prop. 10.14]. So, I also say that the (rectangle contained) by D and E is medial. For since the (rectangle contained) by B and C is equal to the (rectangle contained) by D and E , and the (rectangle contained) by B and C (is) medial [for B and C are rational (straight-lines which are) commensurable in square only] [Prop. 10.21], the (rectangle contained) by D and E (is) thus also medial.

Thus, two medial (straight-lines), D and E , commensurable in square only, (and) containing a medial (area), have been found such that the square on the greater is larger than the (square on the) lesser by the (square) on (some straight-line) commensurable (in length) with the greater..

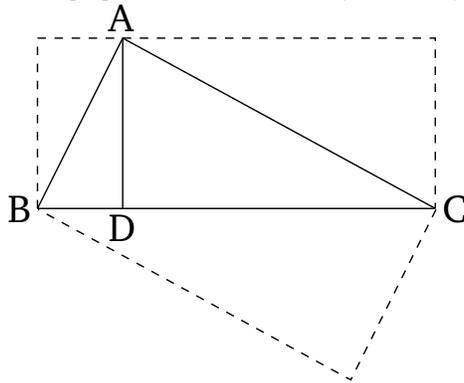
So, similarly, (the proposition) can again also be demonstrated for (some straight-line) incommensurable (in length with the greater), provided that the square on A is greater than (the square on) C by the (square) on (some straight-line) incommensurable (in length) with (A) [Prop. 10.30].

Lemma

Let ABC be a right-angled triangle having the (angle)

A a right-angle. And let the perpendicular AD have been drawn. I say that the (rectangle contained) by CBD is equal to the (square) on BA , and the (rectangle contained) by BCD (is) equal to the (square) on CA , and the (rectangle contained) by BD and DC (is) equal to the (square) on AD , and, further, the (rectangle contained) by BC and AD [is] equal to the (rectangle contained) by BA and AC .

And, first of all, (let us prove) that the (rectangle contained) by CBD [is] equal to the (square) on BA .



For since AD has been drawn from the right-angle in a right-angled triangle, perpendicular to the base, ABD and ADC are thus triangles (which are) similar to the whole, ABC , and to one another [Prop. 6.8]. And since triangle ABC is similar to triangle ABD , thus as CB is to BA , so BA (is) to BD [Prop. 6.4]. Thus, the (rectangle contained) by CBD is equal to the (square) on AB [Prop. 6.17].

So, for the same (reasons), the (rectangle contained) by BCD is also equal to the (square) on AC .

And since if a (straight-line) is drawn from the right-angle in a right-angled triangle, perpendicular to the

base, the (straight-line so) drawn is the mean proportional to the pieces of the base [Prop. 6.8 corr.], thus as BD is to DA , so AD (is) to DC . Thus, the (rectangle contained) by BD and DC is equal to the (square) on DA [Prop. 6.17].

I also say that the (rectangle contained) by BC and AD is equal to the (rectangle contained) by BA and AC . For since, as we said, ABC is similar to ABD , thus as BC is to CA , so BA (is) to AD [Prop. 6.4]. Thus, the (rectangle contained) by BC and AD is equal to the (rectangle contained) by BA and AC [Prop. 6.16]. (Which is) the very thing it was required to show.