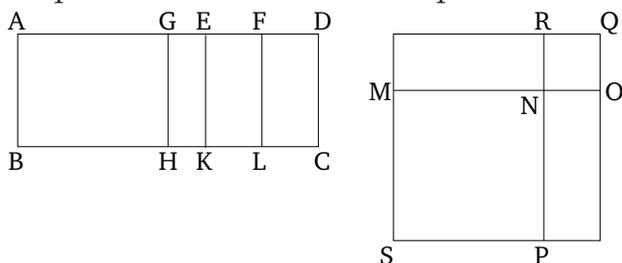


Book 10

Proposition 58

If an area is contained by a rational (straight-line) and a fifth binomial (straight-line) then the square-root of the area is the irrational (straight-line which is) called the square-root of a rational plus a medial (area).[†]

For let the area AC be contained by the rational (straight-line) AB and the fifth binomial (straight-line) AD , which has been divided into its (component) terms at E , such that AE is the greater term. [So] I say that the square-root of area AC is the irrational (straight-line which is) called the square-root of a rational plus a medial (area).



For let the same construction be made as that shown previously. So, (it is) clear that MO is the square-root of area AC . So, we must show that MO is the square-root of a rational plus a medial (area).

For since AG is incommensurable (in length) with GE [Prop. 10.18], AH is thus also incommensurable with HE —that is to say, the (square) on MN with the (square) on NO [Props. 6.1, 10.11]. Thus, MN and NO are incommensurable in square. And since AD is a fifth binomial (straight-line), and ED [is] its lesser segment, ED (is) thus commensurable in length with AB [Def. 10.9]. But, AE is incommensurable (in length)

with ED . Thus, AB is also incommensurable in length with AE [BA and AE are rational (straight-lines which are) commensurable in square only] [Prop. 10.13]. Thus, AK —that is to say, the sum of the (squares) on MN and NO —is medial [Prop. 10.21]. And since DE is commensurable in length with AB —that is to say, with EK —but, DE is commensurable (in length) with EF , EF is thus also commensurable (in length) with EK [Prop. 10.12]. And EK (is) rational. Thus, EL —that is to say, MR —that is to say, the (rectangle contained) by MNO —(is) also rational [Prop. 10.19]. MN and NO are thus (straight-lines which are) incommensurable in square, making the sum of the squares on them medial, and the (rectangle contained) by them rational.

Thus, MO is the square-root of a rational plus a medial (area) [Prop. 10.40]. And (it is) the square-root of area AC . (Which is) the very thing it was required to show.