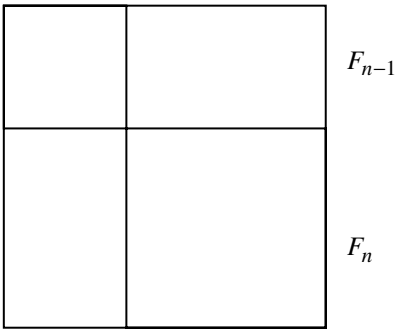
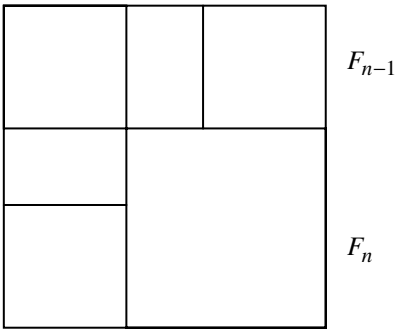
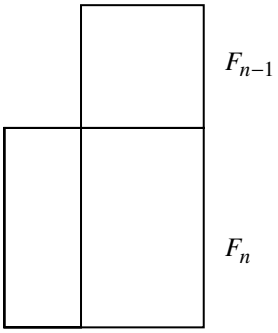
Proof Without Words: Fibonacci Tiles



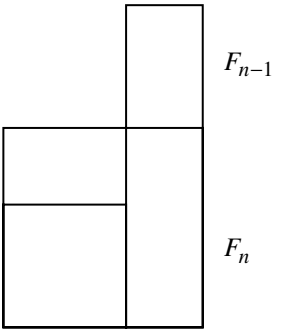
$$\begin{aligned} F_{n+1}^2 &= 2F_{n+1}F_n - F_n^2 + F_{n-1}^2 \\ &= 2F_{n+1}F_{n-1} + F_n^2 - F_{n-1}^2 \\ &= 2F_n F_{n-1} + F_n^2 + F_{n-1}^2 \\ &= F_{n+1}F_n + F_n F_{n-1} + F_{n-1}^2 \\ &= F_{n+1}F_{n-1} + F_n^2 + F_n F_{n-1} \end{aligned}$$



$$F_{n+1}^2 = F_n^2 + 3F_{n-1}^2 + 2F_{n-1}F_{n-2}$$



$$F_n^2 = F_{n+1}F_{n-1} + F_n F_{n-2} - F_{n-1}^2$$



$$F_n = F_{n+1}F_{n-2} + F_{n-1}^2$$

F_n denotes the nth Fibonacci number, where $F_{n+1} = F_n + F_{n-1}$, $F_0 = 0$, $F_1 = 1$. Obvious assumptions concerning the least value of n in each identity should be made as required. Further visual proofs of Fibonacci identities may be found in:

A. Brousseau, Fibonacci numbers and geometry, *Fibonacci Quarterly* **10** (1972) 308–318.

M. Bicknell-Johnson and D. DeTemple, Visualizing golden ratio sums with tiling patterns, *Fibonacci Quarterly* **33** (1995) 298–303.

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