

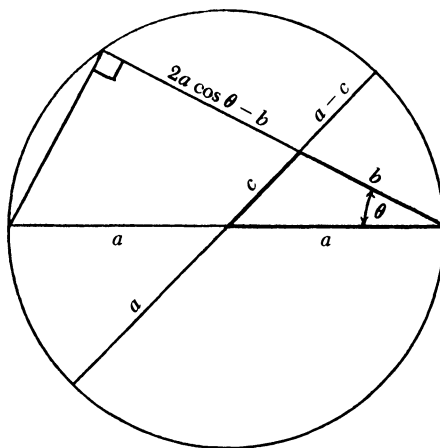
THEOREM 3. Assume the hypotheses of the previous theorem, and let \mathbf{G} be the collection of all G such that G is an algebraic complement in E of each member of \mathbf{F} . If k , the common dimension of the members of \mathbf{F} , is strictly between 0 and the dimension n of E , then \mathbf{G} is uncountable.

Proof. Since the dimensions k and $m = n - k$ of members of \mathbf{F} and \mathbf{G} , respectively, are strictly less than $n = \dim(E)$, the members of $\mathbf{F} \cup \mathbf{G}$ are proper linear subspaces of E . If \mathbf{G} is countable, then $\mathbf{F} \cup \mathbf{G}$ is countable, so there is a v in $E \setminus \bigcup(\mathbf{F} \cup \mathbf{G})$. As $\mathbf{F}_0 = \{F + \text{span}(\{v\}) : F \in \mathbf{F}\}$ satisfies the hypotheses of Theorem 2, there is a linear subspace H of E that is an algebraic complement to each member of \mathbf{F}_0 . Now $H + \text{span}(\{v\})$ is an algebraic complement to each member of \mathbf{F} , so $\text{span}(\{v\}) + H \in \mathbf{G}$. Now $v \in \bigcup \mathbf{G}$ which is a contradiction of the choice of v , and so \mathbf{G} is uncountable.

REFERENCES

1. I. N. Herstein, *Topics in Algebra*, Blaisdell, 1964.
2. J. L. Kelley, I. Namioka, et al., *Linear Topological Spaces*, Van Nostrand, 1963.
3. N. J. Lord, Simultaneous complements in finite-dimensional vector spaces, *Amer. Math. Monthly* 92 (1985), 492–493.
4. A. R. Todd and S. A. Saxon, A property of locally convex Baire spaces, *Math. Ann.* 206 (1973), 23–34.
5. A. R. Todd, Coverings of products of linear topological spaces, *J. Austral. Math. Soc. (Series A)* 29 (1980), 281–290.

Proof without Words: The Law of Cosines



$$(2a \cos \theta - b)b = (a - c)(c + a)$$

$$c^2 = a^2 + b^2 - 2ab \cos \theta$$

—SIDNEY H. KUNG
Jacksonville University
Jacksonville, FL 32211