The Case for Change in Developmental Mathematics Education

Transitioning from Prerequisites to Corequisites

Mathematical Association of America
CRAFTY: Curriculum Renewal Across the First Two Years

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Why This Paper?

The Case for Corequisites

The purpose of this paper is to provide

- supporting evidence for CRAFTY's position on corequisite support courses, and
- a starting point for mathematics departments that wish to begin conversations around implementation of corequisite courses at their institutions.

Toward that end, this paper

- describes some outcomes from corequisite coursetaking in early-implementing states and institutions,
- provides insight into the implications of corequisite courses for underprepared students, and
- surfaces important considerations for developing appropriate and effective corequisite courses in your local institution.

The paper concludes with

- an appendix on corequisites content considerations,
- a glossary that unpacks some of the terminology used in this paper (selected essential terms are also highlighted in callouts within the narrative),
- a list of further readings, including resources on corequisite courses, on mathematics pathways, and on appropriate content for statistics, for quantitative reasoning, and for the pathway to calculus, and
- a bibliography that includes the further readings and the references cited throughout the paper.

What is CRAFTY?

CRAFTY: Curriculum Renewal Across the First Two Years, is a subcommittee of CUPM: the Committee on the Undergraduate Program in Mathematics,[1] which in turn is part of the Mathematical Association of America, whose mission is to advance the mathematical sciences, especially at the collegiate level. CRAFTY's charge is to monitor ongoing developments in undergraduate education, with the intention of making general recommendations concerning the first two years of collegiate mathematics.

Toward that end, CRAFTY leads national initiatives to renew the mathematics coursework and instruction offered for students in their first two years of college. In making decisions about timely renewal efforts concerning lower level courses and programs, CRAFTY considers CUPM recommendations, including the CUPM Curriculum Guide,[2] as well as information obtained from representatives of employers and of partner disciplines. In undertaking its initiatives, CRAFTY collaborates with CUPM and CRAFTY's sister CUPM subcommittees, as well as with other MAA special interest groups (SIGs) and committees.

What are corequisites?

Corequisite,[3] as in corequisite supports or models, typically refers to the practice of placing students who are identified as being underprepared directly into college-level courses upon enrollment and supporting those students through various "corequisite" structures, such as support courses, labs, or tutoring sessions.

The guiding principle of corequisite courses is to meet students where they are academically and provide them with the content and strategies they need to succeed in their college-level courses.

Corequisites differ from *prerequisites* in that with the prerequisite coursetaking model, students are required to take developmental or remedial courses before entering into a college-level course.

Why Embrace Corequisites?

Outcomes From Corequisite Coursetaking in Early-Implementing States and Institutions

High attrition rates in long developmental mathematics sequences pose significant structural barriers for students designated as underprepared for college-level work, and disproportionately affect historically marginalized groups of students.

To begin addressing these structural barriers, many states have recently passed legislation and enacted significant policy changes to mandate the use of corequisite courses to support underprepared students in gateway-level courses—that is, introductory courses that fulfill a general mathematics or English requirement.

States including Tennessee, California, Georgia, Indiana, and West Virginia, as well as large systems such as the City University of New York (CUNY), have implemented developmental education reforms that have increased student success in gateway-level mathematics. Further, the work in these states and institutions provides a growing body of evidence suggesting that corequisite courses increase

[1]The Mathematical Association of America's Committee on the Undergraduate Program in Mathematics web page is available at https://www.maa.org/node/272

[2]Schumacher & Siegel, Co-Chairs. Edited by Paul Zorn. (2015). https://www.maa.org/node/790342

[3]For more information on corequisites, including selected resources for further reading, see the glossary and the further readings sections at the end of this paper. Selected other terms indicated with ***bold italics*** are also included in the glossary.

completion rates for students deemed as underprepared in gateway-level mathematics courses across demographic backgrounds and preparation levels.

This paper takes a deeper look at corequisite implementation in **Tennessee**, a pioneer in implementing corequisites throughout a state higher education system; in the state of **California**, whose experience offers useful insights on corequisites implemented in tandem with mathematics pathways; and in the **City University of New York**, which conducted a randomized controlled trial to examine the outcomes for students taking courses under the corequisite model as compared to the prerequisite model.

This paper also provides a brief sidebar snapshot of one additional institutional approach to corequisite implementation—the **University of Central Arkansas** corequisites for college algebra and quantitative reasoning.

What is a mathematics pathway?

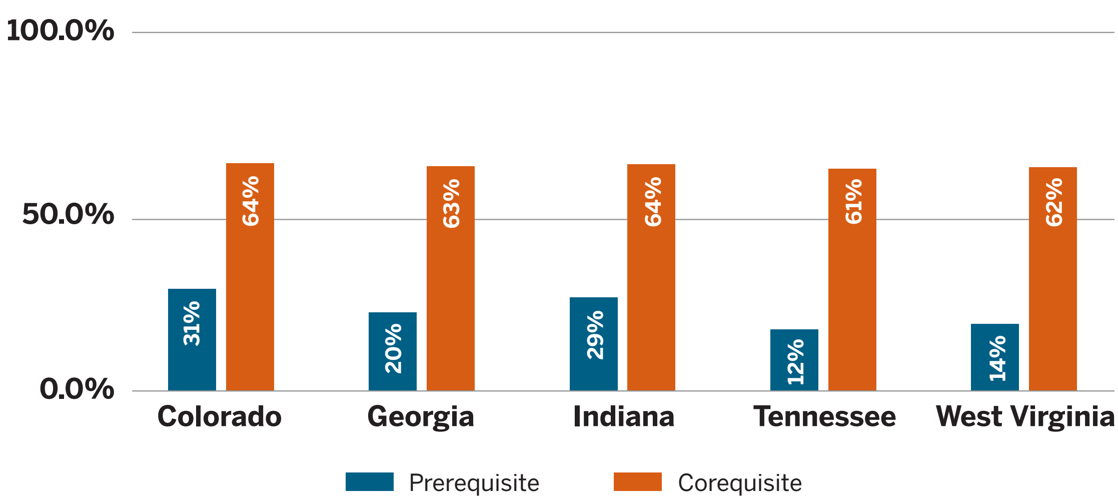
A *mathematics pathway*[4] refers to the series of mathematics courses that students take to complete requirements for an academic goal such as high school graduation or completion of a postsecondary program, certificate, or degree.

A high-quality mathematics pathway offers students a coherent and consistent learning experience that supports their development as independent mathematical learners and is aligned with their academic and career goals.

Tennessee: Implementing Corequisites Improves Student Success Rates

At two- and four-year colleges in the state of Tennessee, 61% of students taking courses under the corequisite model finished a college-level mathematics course within one year. By contrast, only 12% of students taking courses under a prerequisite model finished a college-level mathematics course within *two* years. Other states[5] report similar completion rates for students taking courses through a prerequisite versus a corequisite model.

Figure 1: Comparing student course completion rates in 5 states under prerequisite and corequisite course models in mathematics



[4] See glossary.

[5] See, for example, Complete College America. (2016).

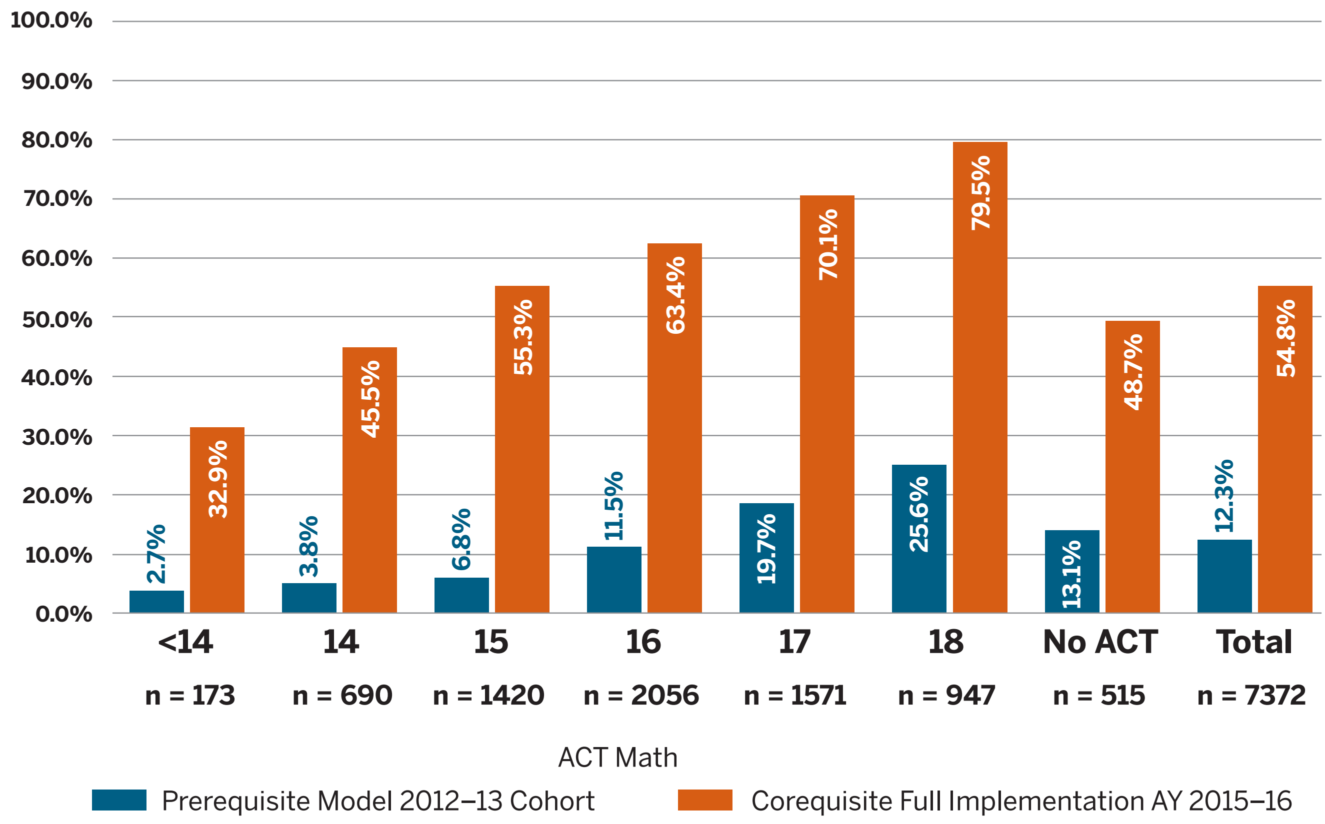
Figure 1 shows student completion rates for students taking developmental education courses under the prerequisite model compared to students taking such courses under the corequisite model, organized by the states surveyed: Colorado, Georgia, Indiana, Tennessee, and West Virginia, by developmental education type (prerequisite or corequisite) and by state. Colorado, Georgia, Indiana, Tennessee and West Virginia have greater completion rates in corequisite models.

Credit: Figure built by the Dana Center from graphics and data found in Complete College America's 2016 report *Corequisite remediation: Spanning the completion divide*. For this and all other references, please see the bibliography for full citation.

While the ways a corequisite model is implemented can vary by state and among institutions within a state, broadly speaking, deploying a corequisite course model entails placing students who have been designated as underprepared directly into college-level courses and providing required additional supports through a corequisite course or other student supports.

In 2007, only about 10% of the students who placed into developmental mathematics in Tennessee institutions of higher education completed a required college-level mathematics course. This low completion rate meant that the vast majority of students designated as underprepared were not meeting their goal of completing a college degree. To address this dire situation, Tennessee required implementation of corequisite courses at scale across all of its two- and four-year institutions beginning in the 2015–2016 academic year.

Figure 2: Comparing student success rates in gateway mathematics courses in Tennessee community colleges (2-year institutions) under prerequisite model (2012–2013) and corequisite model (2015–2016). N = 7,372



Tennessee Board of Regents Brief #3: *Co-requsite Remediation Full Implementation 2015–16*

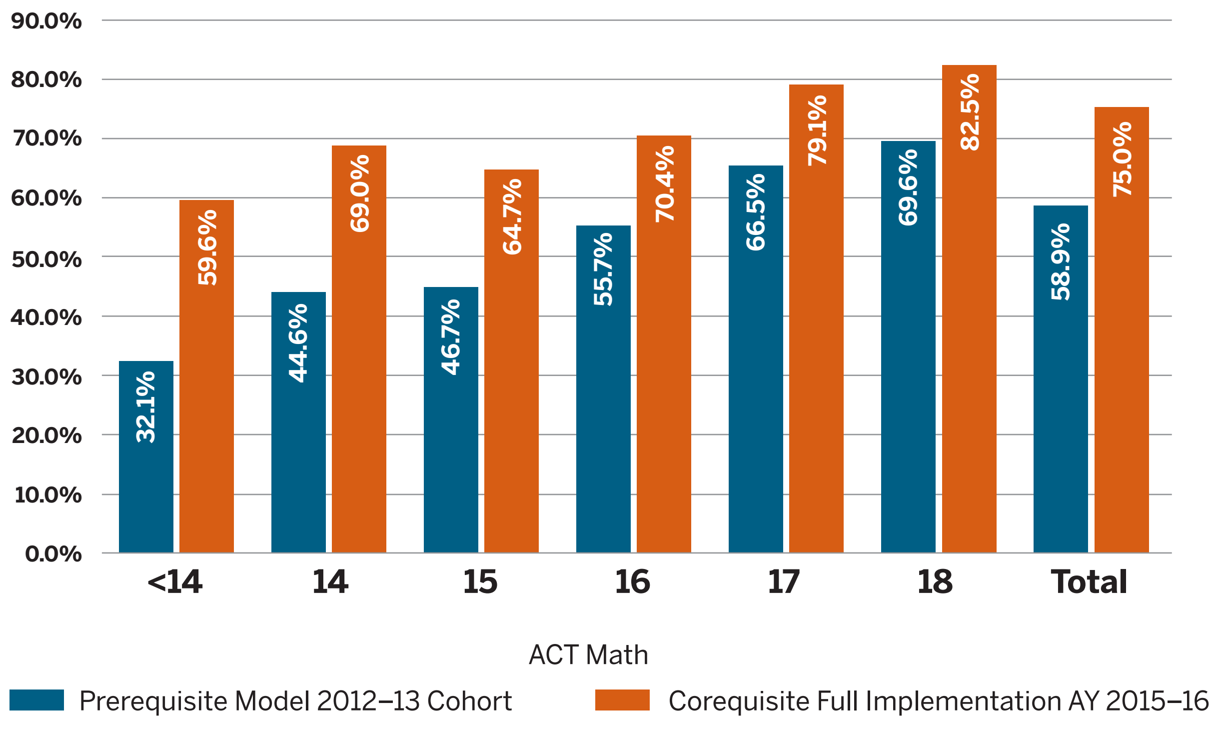
Figure 2 compares completion rates of gateway-level mathematics courses before and after full implementation of corequisite courses at Tennessee community colleges by student, using placement with ACT scores. The data show that gateway-level completion rates in corequisite courses are greater than gateway-level completion rates in prerequisite courses across all ACT scores.

Credit: Figure rebuilt by the Dana Center from graphic in Tennessee Board of Regents, Office of the Vice Chancellor for Academic Affairs. (2016). Technical Brief #3: Co-Requisite Remediation Full Implementation 2015–16.

The data in Figure 2 show that—across a wide range of preparation levels—students taking corequisite courses are passing a gateway-level course at higher proportions than were students assigned to traditional *prerequisite* developmental education. Overall, student success rates in gateway-level courses increased from 12% (of students completing a gateway-level course) within two years, to 55% (of students completing a gateway-level course) within one year. This dramatic increase in success rates represents a significant increase in the number of community college students achieving ***gateway momentum***,[6] which increases the probability that they complete a college degree.

Interestingly, increases in student success rates in gateway-level mathematics courses at universities were not as large as the increases observed at community colleges, yet the results still show that corequisite courses benefit students over a wide range of preparation levels.

Figure 3: Comparing student success rates in gateway mathematics courses in Tennessee universities (4-year institutions) under prerequisite model (2012–2013) and corequisite model (2015–2016)



Tennessee Board of Regents Brief #3: *Co-requsite Remediation Full Implementation 2015–16*

[6]See glossary.

Figure 3 compares completion rates of gateway level mathematics courses before and after full implementation of corequisite courses at Tennessee universities by student, using placement with ACT scores. The data show that gateway-level completion rates in corequisite courses are greater than gateway-level completion rates in prerequisite courses across all ACT scores.

Credit: Figure rebuilt by the Dana Center from graphic in Tennessee Board of Regents, Office of the Vice Chancellor for Academic Affairs.(2016). Technical Brief #3: Co-Requisite Remediation Full Implementation 2015–16.

Tennessee data also indicate significant increases in the success rates of historically marginalized students. As shown in Figure 4, these students also saw increases in success rates in gateway-level math courses.

Corequisites implementation in Tennessee: Completion rates were higher within one year with corequisites than within two years with prerequisites

It is important to note that completion rates before full implementation of corequisite courses measured completion of a gateway-level math course within two years, while completion rates after full implementation of corequisite courses measured completion of a gateway-level math course within *one* year.

Figure 4: Tennessee Community Colleges: Results of full implementation of Tennessee Board of Regents corequisite mathematics for historically marginalized students

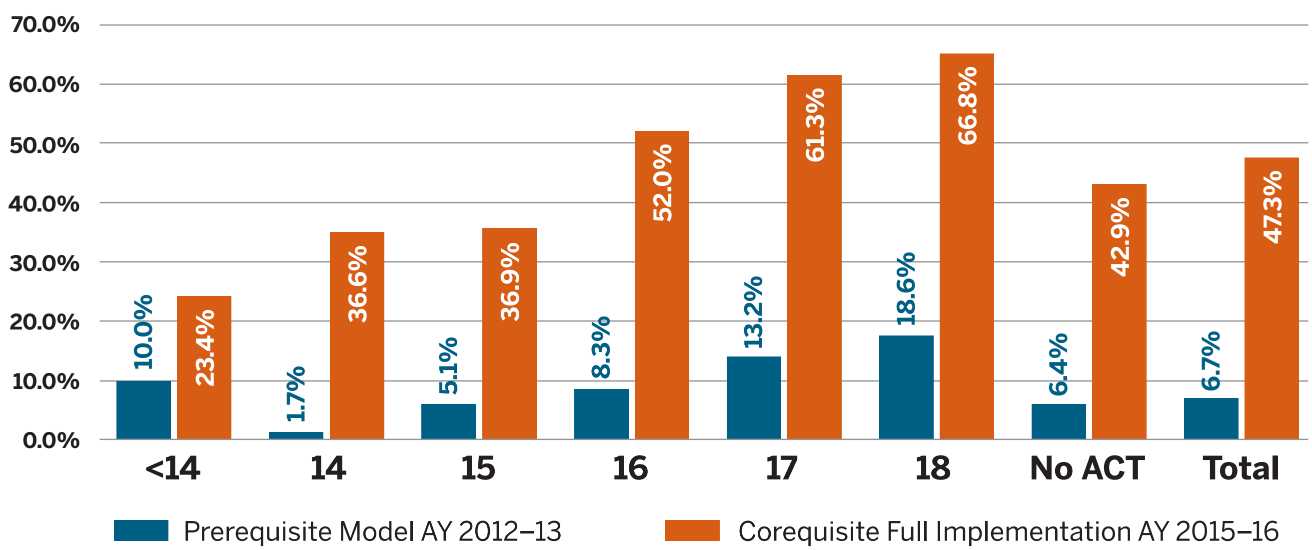


Figure 4 shows that, for historically marginalized students, gateway-level completion rates in corequisite courses are greater than gateway-level completion rates in prerequisite courses across all ACT scores.

Credit: Figure rebuilt by the Dana Center from graphic in Tennessee Board of Regents, Office of the Vice Chancellor for Academic Affairs. (2016). Technical Brief #3: Co-Requisite Remediation Full Implementation 2015–16.

These results show the potential of corequisite courses in helping two- and four-year colleges fulfill their core education mission by narrowing access gaps so that all students, including historically marginalized students, have an equitable opportunity to complete a gateway-level mathematics course, which increases the likelihood they complete a certificate or degree.[7]

[7] Jenkins & Bailey. (2017 February).

Tennessee: Digging Deeper to Better Understand Student Needs

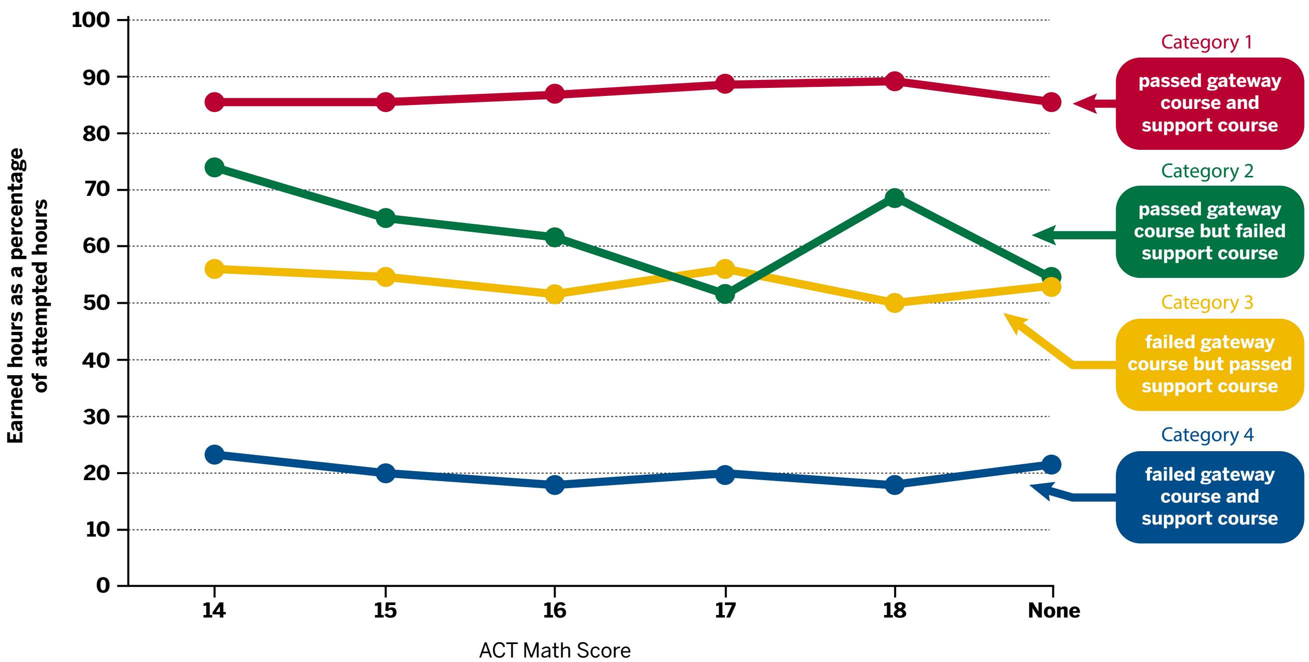
Though it is promising that these data show that implementation of corequisite models increases success rates across demographic backgrounds and preparation levels, a closer look at the Tennessee data suggests that significant work remains to be done to ensure all underprepared students are effectively served.

While student success rates in Tennessee community colleges increased from 12% (completion of a gateway mathematics course within two years) to 55% (completion within one year), a natural question arises—how can two-year colleges serve the 45% of students who did not succeed in corequisite courses?

To better understand the needs of these students, an analysis conducted by the Tennessee Board of Regents on data from 2015–2016 sorted students into four categories.

- Category 1: Students who passed the gateway-level course and the developmental support course
- Category 2: Students who passed the gateway-level course but failed the developmental support course
- Category 3: Students who failed the gateway-level course but passed the developmental support course
- Category 4: Students who failed the gateway-level course and the developmental support course

Figure 5: Percentages of hours earned by students taking corequisite courses at Tennessee Community Colleges (2015–2016)



Adapted from Tennessee Board of Regents Brief #3: *Co-requsite Remediation Full Implementation 2015–16*

Figure 5 shows that students who pass the gateway-level course and the support course earn about 85% of their attempted hours (this percentage is consistent across ACT scores), while students who fail both courses earn about 20% of their attempted hours (this percentage is consistent across ACT scores).

Credit: Figure rebuilt by the Dana Center from graphic in Tennessee Board of Regents, Office of the Vice Chancellor for Academic Affairs.(2016). Technical Brief #3: Co-Requisite Remediation Full Implementation 2015–16.

In the 2015–2016 academic year, 52% of the students in this study who were deemed to be underprepared passed both the college-level course and the developmental support course (Category 1).

On average, across ACT scores, students in Category 1 earned 85% of their attempted college hours during the same semester that they took the corequisite course. A relatively small number of underprepared students, roughly 3%, passed their college-level course but failed the developmental support course (Category 2). On average, across ACT scores, students in Category 2 earned 56% of their attempted college hours. Together, students in these two categories account for the 55% of students who passed their college-level course in Tennessee (see Figure 2, ~ 55%). Based on this analysis, students in corequisite courses who do well in their college-level math course tend to do well in most of their other courses.

The situation is different for the 36% of underprepared students who failed both the college-level course and the developmental support course (Category 4). On average, across ACT scores, students in Category 4 earned 22% of their attempted college hours in the same semester that they took the developmental support course. This outcome suggests that students in corequisite courses who fail their college-level math course tend to fail most of their other courses, and it points to a problem with college preparation in general rather than a problem with developmental mathematics. That is, students may need to develop broader "college-ready" skills, such as ***academic mindsets***[8] or gain other skills (beyond the scope of the targeted corequisite support course) that are essential for successfully navigating through college or university.

According to the 2015 joint statement on *Core Principles for Transforming Remedial Education*,[9] students for whom the default college-level course placement is not appropriate, even with additional mandatory support, should be enrolled in rigorous, streamlined developmental education options that align with the knowledge and skills required for success in gateway courses in their academic or career area of interest.

Thus, even though corequisite courses offer a significant improvement over prerequisite courses, colleges and universities must also provide advising, study skills, and college success skills for students to succeed in all courses and not just in gateway mathematics and English courses.

The Positive Impact of Math Pathways in Tennessee's Implementation of Corequisites

It is important to note that Tennessee did not implement corequisite courses as a standalone initiative. Rather, the state implemented corequisites as part of its mathematics pathways work. Math pathways enable students to take different paths through the math curriculum so that the math they learn is relevant to their course of study.[10]

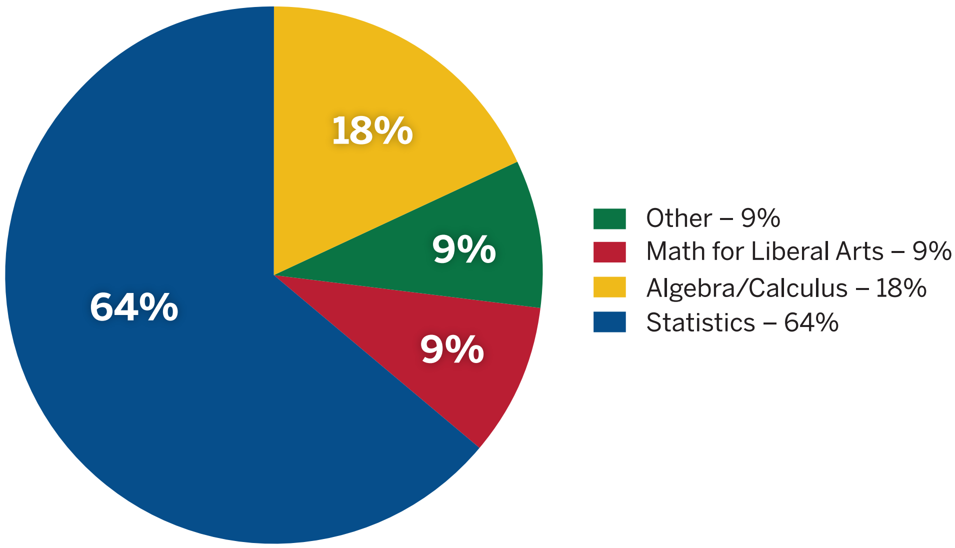
[8] See glossary.

[9] Achieving the Dream, et al. (2015 November).

[10] Ganga & Mazzariello. (2018 October).

As shown in Figure 6, during the second year (2016) of statewide corequisite implementation, Tennessee community colleges went from enrolling 20% of students in statistics to enrolling more than 60% of students in statistics.

Figure 6: Gateway mathematics courses taken by first-time students in Tennessee community colleges in Fall 2016. N = 18,956



Note. N = 18,956. "Other" includes the following courses: Finite Mathematics, Survey of Mathematics, Trigonometric Applications, and Math for Elemetary Education.

Figure 6 shows the percentages of students (N = 18,956) taking, variously, Statistics (64%), Algebra or Calculus (18%), Math for Liberal Arts (9%) or Other (9%); Other includes Finite Mathematics, Survey of Mathematics, Trigonometric Applications, and Math for Elementary Education).

Credit: Figure developed by the Dana Center from information provided by the Tennessee Board of Regents.

A recent study by the Community College Resource Center analyzed Tennessee statewide data using regression discontinuity and difference-in-regression-discontinuity designs to identify the separate effects of mathematics pathways and of corequisite courses.[11]

The study found that enrolling students in corequisite courses resulted in large positive effects, and it attributed these effects to statewide efforts to enroll students in math courses aligned with the requirements of the students' programs (that is, mathematics pathways)—for example, enrolling more students in statistics rather than placing them into algebra by default.

The study also found that corequisite courses do not compromise student performance in students' subsequent coursework. While the CCRC study found no effect in terms of enrollment or completion of a credential, it did find that math students placed into corequisite courses were more likely (by 8 percentage points) to enroll in and pass an additional college-level math course compared with students placed directly into college-level math courses.

[11]Ran & Lin. (2019 November).

Considering the conditions required to support successful corequisite models, offering viable math pathways is essential to maximizing the benefits of corequisite support courses

A significant takeaway from this study is that corequisite courses and mathematics pathways should not be viewed as competing reform efforts, but rather, as complementary systemwide strategies that lead to student success in gateway-level math courses. In fact, it could be argued that when considering the conditions required to support successful corequisite models, offering viable math pathways is essential to maximizing the benefits of corequisite support courses.

Successful Corequisites In Multiple Mathematics Pathways—California

In 2017, the California State University System revised policies for first year-student placement in English and mathematics / quantitative reasoning courses. Specifically, the CSU System made two significant changes. It banned non-credit developmental education courses, and it removed the intermediate algebra prerequisite as a requirement for transfer among CSU institutions.

In addition, in 2018 the state of California enacted legislation requiring that all community colleges offer students a path to completing a transferable course within the first year of their program, and it also required the use of high school grades as the primary means of placement into gateway-level mathematics and English courses.

The changes in placement policy were intended to address years of frustration with the placement tools used locally—whether automated software or another method. In the past, the state tried to remedy these frustrations by working on developing a common assessment tool driven by faculty, but little to no progress was made.

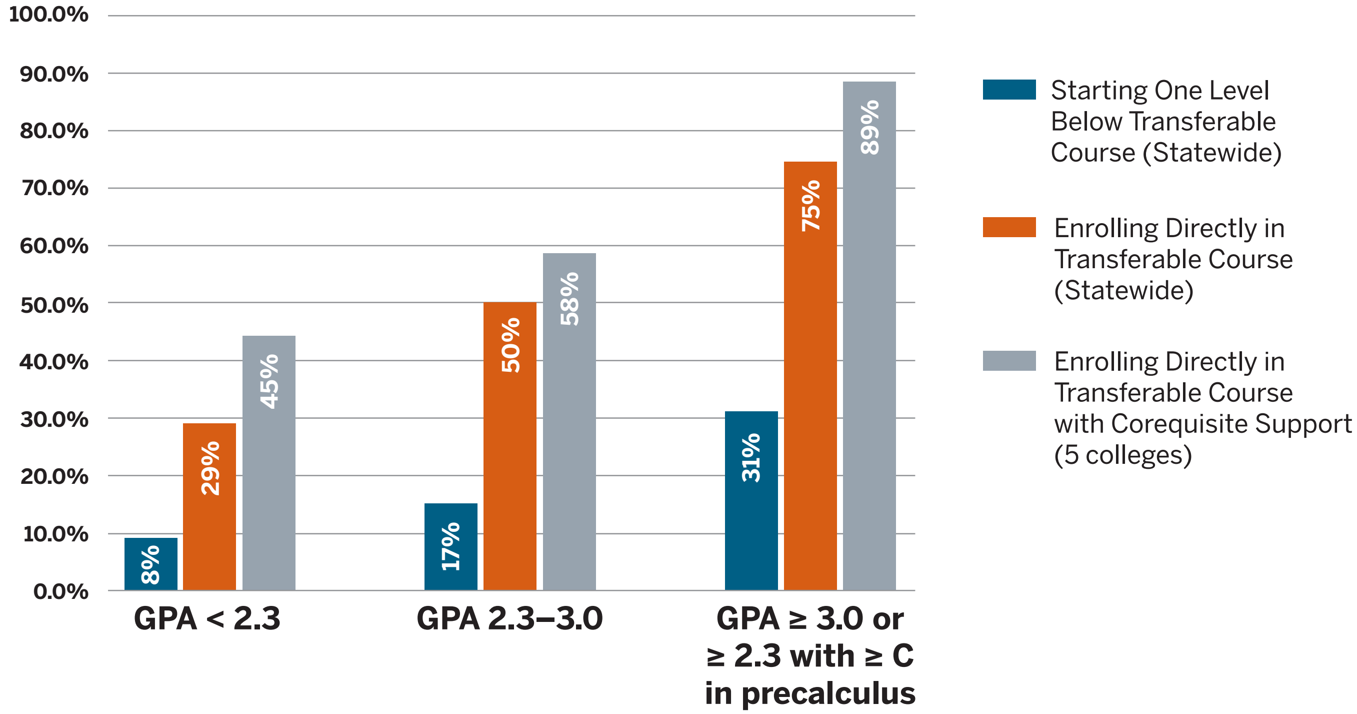
As a result of these policies and legislation, California community colleges created corequisite courses for students in mathematics pathways for statistics and for STEM (science, technology, engineering, and mathematics).

Figures 7 and 8 show an analysis by the RP Group's Multiple Measures Assessment Project[12] of statewide student data for California community colleges for 2007 to 2018, looking at completion rates for students in three different mathematics sequences for transferable ***statistics*** (Figure 7) or transferable ***STEM math*** (Figure 8). The three sequences are:

- developmental mathematics followed by college-level mathematics (e.g., a prerequisite model, in which students start one level below a transferable course),
- college-level mathematics without support courses (e.g., students enroll directly in a transferable course), and
- college-level mathematics with support courses (e.g., a corequisite model, in which students enroll directly in a transferable course, with a corequisite support).

[12] The RP Group does research, planning, and professional development for California community colleges; see, for example, https://rpgroup.org/Resources/Resources-Library/proj/118?projname=Multiple%20Measures%20Assessment%20Project%20(MMAP

Figure 7: Completion of transferable statistics: California community colleges' student completion rates for three sequences leading to a transferable statistics course; statewide data from 2007–2014 and for corequisite sequence, 2016–2018 (corequisite N = 1,888)

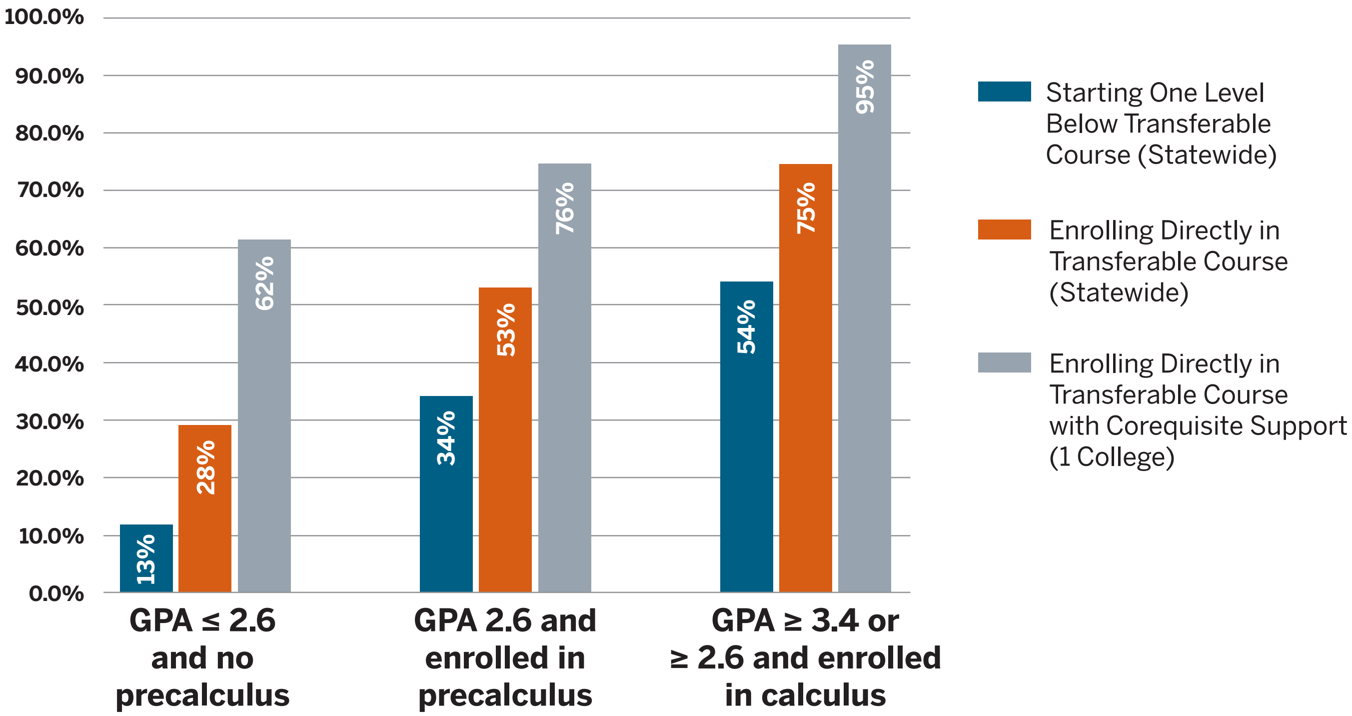


Source: Analysis by the Multiple Measures Assessment Project, Statewide Data from 2007-2014, Corequisite Data from F2016-F2018 (N = 1,888)

Figure 7 shows that students with low GPAs who are placed directly into transfer-level courses are three times more likely to complete transfer-level statistics than are their peers who are placed into a course one level below a transfer-level course (29% versus 8%), and students receiving corequisite support in a transfer-level course are almost five times more likely to complete transferable statistics than are their peers placed one level below transfer level (45% versus 8%).

Credit: Figure rebuilt by the Dana Center from graphic by Campaign for College Opportunity and the California Acceleration Project. (2019 September). *Getting There: Are California Community Colleges Maximizing Student Completion of Transfer-Level Math and English?*

Figure 8: Completion of transferable STEM Math: California community colleges' student completion rates for three sequences leading to a transferable STEM math course; statewide data from 2007–2014 and corequisite data for Precalculus and Business Calculus, 2016–2018. (N = 241) for precalculus and business calculus in a corequisite model



Source: Analysis by the Multiple Measures Assessment Project, Statewide Data from 2007-2014, Corequisite Data from Pre-Calc & Business

Figure 8 shows that when placed into transfer-level courses with corequisite support, almost two-thirds (62%) of students with low GPAs and no prior precalculus courses complete transferable STEM math. When placed one level below transfer level, just one-sixth (13%) complete transferable STEM math.

Credit: Figure rebuilt by the Dana Center from graphic by Campaign for College Opportunity and the California Acceleration Project. (2019 September). *Getting There: Are California Community Colleges Maximizing Student Completion of Transfer-Level Math and English?*

These results in California are consistent with the earlier noted results in Tennessee—corequisite courses benefited students over a wide range of preparation levels *and* across mathematics pathways, including the pathway to calculus, which leads to careers in STEM fields. As to mathematics pathways to other (not necessarily STEM) careers, such as statistics and mathematics for the liberal arts, historical data analysis strongly suggests that statistics pathways and support courses result in greater rates of gateway math completion (e.g., the Carnegie Foundation for the Advancement of Teaching's Statway program[13]).

As suggested in Figure 8, for the Business, Science, Technology, Engineering, and Mathematics (BSTEM) pathways, early indicators of success in passing transfer-level math courses such as trigonometry, college algebra, and precalculus seem promising. An important indicator to consider is what happens to these students as they progress through and take more upper-division mathematics, such as calculus and beyond.

[13] https://carnegiemathpathways.org/statway

Corequisites and Success Beyond Gateway-Level Math—City University of New York

While states implementing corequisites are seeing increases in the number of underprepared students who complete a gateway-level mathematics course in their first year, evidence is also emerging that students who complete corequisite math courses then go on to succeed in subsequent courses and are more likely to graduate. The City University of New York (CUNY) conducted a randomized control trial to understand how a corequisite coursetaking model compares with a prerequisite model.

As part of the study,[14] in 2013, 297 students at CUNY who had been assessed as needing remedial elementary algebra were assigned to a gateway-level statistics course with corequisite support (treatment group), and 297 more students in the associate's degree program who had been assessed as needing remedial elementary algebra were assigned to a prerequisite elementary algebra course to be followed by gateway-level math (control group).

After three years, researchers observed the enrollment status of all the students (treatment group and control group); data indicated that being assigned to a corequisite statistics course had a positive effect on graduation rates.

The graduation rate for students who were assigned to a *corequisite* statistics course was 25%, while the graduation rate for students assigned to a *prerequisite* elementary algebra course was 17%. The 8% difference between the two groups was statistically significant, and represents a relatively large increase in the number of graduates within the treatment group—50% more students assigned to corequisite statistics graduated within three years as compared to students assigned to the traditional prerequisite elementary algebra course.

Researchers attributed some of this effect to enrolling students in statistics rather than by default enrolling them in algebra. Researchers explained the difference in findings between this report and the CCRC report on corequisite implementation in Tennessee[15] (that there was no effect in terms of enrollment or completion of a credential) to the high degree of fidelity in implementation of the corequisite support course at CUNY, a level of fidelity that was more difficult to achieve at statewide scale.

In addition to increases in graduation rates, the CUNY study provided two additional important takeaways. Researchers found that the students in the corequisite statistics course were more likely to form study groups to support their learning than were the students in the prerequisite elementary algebra course. Researchers also found that students assigned to the corequisite statistics course were more likely to take subsequent, more advanced, math courses than were the students assigned to the prerequisite elementary algebra course.

[14] Logue, Douglas, & Watanabe-Rose. (2019).

[15] Ran & Lin. (2019 November).

Conclusion and Discussion

Based on the research surveyed in this paper, the Mathematical Association of America's Curriculum Renewal Across the First Two Years (CRAFTY) committee believes there is ample evidence that for students deemed to be underprepared, corequisite courses substantially increase success in gateway-level mathematics courses.

Large-scale implementation of corequisite models in various states and institutions suggests that corequisite support courses decrease student attrition rates by reducing the number of transition points between courses, that corequisites increase the likelihood that students enroll in more advanced mathematics courses—and that corequisites have the potential to improve overall graduation rates.

As described in this paper and in additional resources (e.g., see further readings at end of this paper), these improvements have been documented in large-scale implementation efforts in a variety of states and across institutions.

In thinking about how corequisite structures might be deployed to improve outcomes at their institutions, departments of mathematics can consider several questions.

The first and most important question is: **What are our motivations for creating a corequisite course?**

To begin answering this question, it is important to look beyond individual course success rates and study the **throughput rate**[16] for developmental sequences to understand the effects of student attrition on completion of gateway-level mathematics courses.

Once a determination is made that corequisite courses can be used to address high attrition rates, additional questions can be asked about the current state of your department's mathematics sequences and their articulation with students' potential programs of study.

- Do we offer multiple mathematics pathways aligned to students' potential programs of study? If so, what percentage of students are still on an algebra pathway, and could some of these students be better served through a different pathway?
- What work do we need to do with partner disciplines to determine if students in their programs might be better served through a statistics or a quantitative reasoning pathway?
- How effective are our current mathematics course placement procedures? Do our placement procedures reliably lead to student success? Could we consider placing students using ***multiple measures placement***?[17]

What is meant by throughput rate?

In academic contexts, the term ***throughputs*** or ***throughput rate*** refers to the number or percentage of students who complete an entire sequence of courses (starting with first developmental course through gateway-level course) in a specific period of time.

[16] See glossary.

[17] See glossary.

Effectively implementing corequisites requires a new approach to creating content for inclusion in support courses, and it also requires close alignment between support courses and their target gateway-level courses. The support content in a corequisite support course will vary depending on the course being supported—e.g., statistics, algebra, or quantitative reasoning.

Questions about corequisite content include:

- What content is necessary to prepare students for success in the college-level course?
- What resources do we need to develop appropriately aligned developmental support courses?
- Which structures (e.g., a ***cohort model*** or a ***comingling model***[18]) best support students?

For additional information on questions about corequisite course content, please see the appendix, Corequisites Content Considerations.

Corequisite courses also present a unique opportunity to reduce attrition rates along the pathway to calculus, which has traditionally underserved students who are women, Black, Indigenous, or Latinx, or who are English language learners or from low-income backgrounds.

Broadening participation in STEM fields requires deeper analysis of the pathway to calculus, which should include looking at throughput rates for success in calculus across race, gender, ethnicity, and socioeconomic status.

To leverage corequisites to broaden participation in STEM fields, math departments should ask:

- Who does our pathway to calculus serve? Are we equitably serving students (e.g., across various populations)?
- If not, what are the structural barriers—e.g., placement procedures, length of developmental course sequences, alignment of content (from developmental courses to gateway courses and to students' intended program of study)—to more equitable service, and what can we do to better support the students who are negatively affected by these barriers?
- How can we rethink the traditional content in this mathematics pathway so that we can reduce attrition rates by shortening the length of the pathway via corequisites?

The material in this paper, including the appendix Corequisites Content Considerations, should serve as a framework to help mathematics departments begin asking these important questions.

Ultimately, the CRAFTY committee hopes this paper will launch conversations that support mathematics faculty in making informed decisions about the implementation of corequisite courses at their institutions.

[18] See glossary.

Appendix

Corequisites Content Considerations

Introductory Statistics

A recommended starting point for thinking about the content for college-level introductory statistics courses is the revised *Guidelines for Assessment and Instruction in Statistics Education (GAISE) College Report* published in 2016[19] and endorsed by ASA (the American Statistical Association) and AMATYC (the American Mathematical Association of Two-Year Colleges).

The 2016 revised report takes into account the many changes in the world of statistics education and statistical practice since the original GAISE report was published in 2005, and it suggests a direction for the future of introductory statistics courses. The revised GAISE report was informed by outreach to the statistics education community and by reference to the statistics education literature, and it includes an updated list of learning objectives for students in introductory courses, along with suggested topics that might be omitted from, or deemphasized in, an introductory course. The revised GAISE report also makes six recommendations for teaching statistics:

1. Teach statistical thinking.
    a. Teach statistics as an investigative process of problem-solving and decision-making.
    b. Give students experience with multivariable thinking.
2. Focus on conceptual understanding.
3. Integrate real data with a context and purpose.
4. Foster active learning.
5. Use technology to explore concepts and analyze data.
6. Use assessments to improve and evaluate student learning.

Modernizing and updating current introductory statistics courses provide faculty with an opportunity to rethink the content required for success in introductory statistics as they develop corequisite support courses.

A recent (2019) resource, "Mathematics Foundations for Success in Introductory Statistics,"[20] provides a basis for developing an appropriate corequisite course that is aligned with the revised GAISE report by mapping relevant preparatory skills to the content knowledge those skills support in introductory statistics courses.

While some algebraic topics are needed to support students in introductory statistics courses, this paper also outlines the important non-algebraic skills that are essential for success, and it highlights the need to create coherent support courses for introductory statistics. **Creating such supports requires that institutions shift away from arbitrarily requiring content typically found in beginning or intermediate algebra.**

[19] American Statistical Association. (2016).
[20] Peck, Gould, & Utts. (2019 August).

Quantitative Reasoning

Generally speaking, quantitative reasoning courses provide students with the mathematics needed to meet the quantitative demands of everyday life. Historically, the topics covered in these courses has varied greatly. In thinking about developing appropriate corequisites for QR courses, it is important to first consider current trends in the design of college-level QR courses.

In 2015, the Mathematical Association of America and other professional societies recommended the implementation of multiple mathematics pathways aligned to fields of study, including math pathways that provide an early exposure to statistics, modeling, and computation.[21]

QR courses can play a critical role in ensuring students receive early exposure to these topics, and since 2015, there has been a significant shift towards including **probability, statistics, and modeling, as well as proportional reasoning**, in QR courses. In 2018, in an effort to bring additional consistency to quantitative reasoning courses across states and institutions, the Charles A. Dana Center at The University of Texas at Austin recommended several action items that can guide institutions seeking to implement corequisite courses.[22]

AMATYC offers additional guidance through its position on Mathematics for Liberal Arts.[23] This position statement outlines four additional recommendations describing the purpose, approach, student engagement, and audience for QR courses:

- **Purpose:** QR courses should increase students' quantitative abilities and help them realize the relevance of mathematics.
- **Approach:** Content should be useful and meaningful for students. Focus should be placed on conceptual understanding through modeling. Technology should be used to facilitate an exploration of the concepts.
- **Student Engagement:** QR courses should engage students in the learning process by incorporating active learning strategies.
- **Audience:** STEM and non-STEM students would benefit from taking a QR course.

Once institutions have developed an appropriate QR course that is in keeping with recommendations from professional associations, careful backmapping from the gateway-level course to the corequisite support can be used to determine appropriate support content.

Backmapping from the gateway-level course creates coherence between the courses, and it requires institutions to shift away from arbitrarily requiring content typically found in beginning or intermediate algebra unless students intend to take calculus.

[21] See, for example: Burdman. (2015 May); Saxe & Braddy. (2015).

[22] Gaze & Richardson. (2019).

[23] AMATYC. (2019).

Example of backmapping for a QR course: The University of Central Arkansas: Using a cohort model for corequisite support courses for college algebra and quantitative reasoning math pathways

The University of Central Arkansas developed two corequisite support courses for their College Algebra and Quantitative Reasoning pathways. The university's corequisite support courses provide just-in-time learning in a **cohort** model—all students in the college-level course enroll in the same corequisite support course. The corequisite course meets three times a week, while the college-level course meets twice a week. To determine appropriate topics for the corequisite support course, mathematics faculty used a backmapping process.

Placement into the corequisite course is determined by either ACT scores or placement testing scores. The university reports that students in general obtain a C or better in the college-level courses, and faculty report the corequisite course adequately prepares students for the corresponding credit-bearing course. For more information on success rates, see Scaling Co–Requisite Supports at the University of Central Arkansas: Perspective from a Four–Year Higher Education Institution.[24] It is important to note that the university does not admit students who score 15 or lower on the ACT.

Suggestions from faculty at the University of Central Arkansas include teaching the corequisite support course and gateway-level course in a 5-day-a-week format, and creating flexible support courses so that faculty are empowered to respond to student needs. Faculty also suggest assigning the same grade in the corequisite course and the gateway-level course.

The Pathway to Calculus

In thinking about how to create appropriate corequisite support courses for college algebra and precalculus, courses originally intended to prepare students for calculus, faculty can build on aligned math pathways to re-envision the entire pathway-to-calculus sequence. In 2004, the Mathematical Association of America acknowledged that college algebra was not an appropriate default gateway math course for mathematics.[25]

By ensuring that students are enrolled in default gateway math courses aligned to their programs of study, faculty can focus their attention on developing course sequences that can more effectively prepare calculus-intending students for success, while simultaneously making content decisions that leverage corequisite supports to shorten the pathway to calculus.

Textbooks for college algebra and precalculus usually cover a wide range of topics that may or may not be essential for success in calculus. When designing courses around these textbooks, instructors must often choose which topics to cover and which to leave out. Generally, students are introduced to functions and relations, learn properties about families of functions and identify features of their graphs, practice numerous algebraic manipulations, use theorems to locate rational roots of polynomials, and prove trigonometric identities. Often, however, the end result is an experience that emphasizes algebraic manipulations while losing sight of some of the most important conceptual features, such as deep understanding of function or grappling with change.

To identify the most appropriate content for inclusion in a re-envisioned and shortened pathway to calculus that uses corequisite supports to prepare students for calculus, it is recommended that

[24] Charles A. Dana Center at The University of Texas at Austin. (2018).

[25] The Mathematical Association of America. (2004).

faculty examine the large amount of curricular material usually associated with college algebra and precalculus, and decide which skills, concepts, and procedures best prepare students for calculus. This requires a two-step process for making content decisions: 1) backmapping and 2) selecting essential content.

Backmapping the pathway to calculus

The backmapping process begins with identifying difficult concepts in calculus that create barriers to student success. Once faculty identify these concepts, they can determine the specific content that best prepares students to learn those concepts. While this approach typically leads to a long list of mathematical topics, faculty can select the essential content for a shortened pathway to calculus by focusing on content that promotes a deep understanding of *function as a process*,[26] and that develops proficiency in *covariational reasoning*.

A deep understanding of the function as a process involves having a strong conceptual understanding of the process view of function. A curriculum that stresses the process view of a function prepares students to analyze function outputs on entire intervals of inputs, to reason about inverting functions by reversing processes, and to make stronger connections between the graph of a function and the function's relationship to generalized inputs and outputs. In such a curriculum, students also understand that a function is independent of a formula, and they are able to communicate about functions using multiple representations.

Having proficiency in covariational reasoning entails having the ability to analyze two quantities simultaneously and to understand how those quantities change and *covary*. Proficiency in covariational reasoning enables students to better understand the unique and dynamic problem situations addressed by calculus and related disciplines. Courses in a pathway to calculus should provide students many opportunities to explore dynamic function relationships and should help students more easily conceptualize the notions of an average rate of change—and the transition between an average rate of change and an instantaneous rate of change.

Thinking carefully about the content that is included in courses that lead up to calculus through the lens of function as a process and covariational reasoning skills can help faculty create more focused and coherent curricula, and it can open the door to shortening the pathway to calculus by leveraging corequisite supports.

[26] Oehrtman, Carlson, & Thompson. (2008).

Glossary

Term	Definition	To learn more
Academic mindsets	*Academic mindsets* are the psychosocial attitudes or beliefs one has about oneself in relation to academic work. According to the Mindset Scholars Network, the following learning mindsets play a role in students' educational outcomes: 1. Growth Mindset: The belief that intelligence can be developed 2. Belonging: The belief that one is respected and valued at school 3. Purpose and Relevance: The belief that one's schoolwork is valuable because it is relevant to one's life and/or connected to a larger purpose	See the further readings section, and in particular, Learning mindsets, https://mindsetscholarsnetwork.org/learning-mindsets
Cohort (structure)	Cohorts or a "cohorting" structure in the context of developmental education refers to the practice of designating certain sections of college-level courses exclusively for students deemed to be underprepared (in some situations the cohort of students is then monitored and supported as the cohort progresses through the institution). Compare to *comingling structure*.	
Comingling (structure)	A comingling structure in the context of developmental education refers to the practice of mixing in the same college-level class both students deemed to be college-ready and students deemed to be underprepared. Typically with a comingling approach, the underprepared students are provided additional supports during separate sessions. Compare to *cohort structure*.	

Term	Definition	To learn more
Corequisites, corequisite supports	Corequisite, as in corequisite supports or models, typically refers to the practice of placing students who are identified as being underprepared directly into college-level courses upon enrollment and supporting those students through various "corequisite" structures, such as support courses, labs, or tutoring sessions. Corequisite supports can be separate courses that run before, after, or on opposite days to the college-level courses and that are completed within one semester. The guiding principle of corequisite courses is to meet students where they are academically and to provide them with the content and strategies they need to succeed in their college-level courses. Compare to prerequisite.	See the further readings section, and in particular, *Co-requisite courses: Narrowing the gap between instruction and supports*, https://dcmathpathways.org/sites/default/files/resources/2018-07/Co-req_Supports_2018_07_24.pdf *Launch Years: A new vision for the transition from high school to postsecondary mathematics*, https://utdanacenter.org/launchyears
Gateway momentum	According to the Community College Research Center, *gateway momentum* is taking and passing pathway-appropriate college-level math and college-level English in the first academic year.	See the further readings section, and in particular, *Early momentum metrics: Why they matter for college improvement*, https://ccrc.tc.columbia.edu/publications/early-momentum-metrics-college-improvement.html

Term	Definition	To learn more
Mathematics pathways	A *mathematics pathway* refers to the series of mathematics courses that students take to complete requirements for an academic goal such as high school graduation or completion of a postsecondary program, certificate, or degree. A high-quality mathematics pathway offers students a coherent and consistent learning experience that supports their development as independent mathematical learners and is aligned with their academic and career goals.	See the further readings section, and in particular, the glossary in *Launch Years: A new vision for the transition from high school to postsecondary mathematics*, https://utdanacenter.org/launchyears Note that the Launch Years initiative specifically promotes the concept that educators should align mathematics pathways across secondary and postsecondary education. For additional tools and resources related to the implementation of mathematics pathways, see Dana Center Mathematics Pathways. [Resource website], https://dcmathpathways.org
Multiple measures assessment (MMA) / multiple measures placement (MMP)	Multiple Measures Assessment or Multiple Measures Placement both speak to the growing practice of combining more than one measure, and in particular, measures beyond the typical standardized test score (for example high school grade point average, high school transcript information, non-cognitive assessments, and standardized test scores) to assess a student's readiness for, and likelihood of success in, various higher education programs and pathways. Some MMAs aim to assess college readiness, while others may attempt to measure students' preparation levels for particular academic pathway.	
Prerequisites	In the *prerequisite* coursetaking model, students are required to take developmental or remedial courses before entering into a college-level course. Compare to ***corequisite***.	

Term	Definition	To learn more
Throughput rates, throughputs	In academic contexts, *throughputs* or *throughput rates* refer to the number or percentage of students who complete an entire sequence of courses (starting with first developmental course through gateway-level course) in a specific period of time.	

Further Readings

Corequisite Courses

- Charles A. Dana Center at The University of Texas at Austin. (2018). **Scaling co–requisite supports at the University of Central Arkansas: Perspective from a four–year higher education institution.** Dana Center Mathematics Pathways: Notes From the Field, No. 4. Austin, Texas: Author. Available at https://dcmathpathways.org/resources/scaling-co-requisite-supports-university-central-arkansas-perspective-four-year-higher
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Mathematics Pathways

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Appropriate Content for Statistics

- ASA: American Statistical Association. GAISE College Report ASA Revision Committee (Robert Carver, Michelle Everson, John Gabrosek, Nicholas Horton, Robin Lock, Megan Mocko, Allan Rossman, Ginger Holmes Rowell, Paul Velleman, Jeffrey Witmer, and Beverly Wood). (2016). ***Guidelines for assessment and instruction in statistics education (GAISE) college report 2016.*** Alexandria, Virginia: Author. Available at https://www.amstat.org/asa/files/pdfs/GAISE/GaiseCollege_Full.pdf
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Appropriate Content for Quantitative Reasoning

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Appropriate Content for the Pathway to Calculus

- Boersma, Stuart, and Frank Savina. (2019). **Re-envisioning the pathway to calculus to broaden participation in STEM programs.** Austin, Texas: Charles A. Dana Center at The University of Texas at Austin. Available via https://dcmathpathways.org/resources/re-envisioning-pathway-calculus-broaden-participation-stem-programs or https://dcmathpathways.org/sites/default/files/resources/2019-08/Re-envisioning-the-Pathway-to-Calculus.pdf
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Backmatter

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About CRAFTY

CRAFTY is a subcommittee of the Committee on the Undergraduate Program in Mathematics (CUPM) and is charged with monitoring ongoing developments with the intention of eventually making general recommendations concerning the first two years of collegiate mathematics.

CRAFTY leads national initiatives to renew mathematics course work and instruction offered for students in their first two years of college. In making decisions about timely renewal efforts concerning lower level courses and programs, CRAFTY considers CUPM recommendations including the CUPM Curriculum Guide and information obtained from representatives of employers and of partner disciplines. In undertaking its initiatives CRAFTY collaborates with CUPM and CRAFTY's sister CUPM subcommittees as well as with other MAA committees and special interest groups.

For more information on CRAFTY, see https://www.maa.org/programs-and-communities/member-communities/curriculum-department-guidelines-recommendations/crafty.

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