Carl B. Allendoerfer Awards

The Carl B. Allendoerfer Award, established in 1976, is made to authors of expository articles published in *Mathematics Magazine*. Carl B. Allendoerfer, a distinguished mathematician at the University of Washington, served as president of the Mathematical Association of America, 1959–60.

David J. Hunter and Chisondi Warioba

"Segregation Surfaces," Mathematics Magazine, 94:3, 163-172. doi.org/10.1080/0025570X.2021.1908044

Measuring segregation on a city map is not simple, and such measurements are not defined in just one way. The article "Segregation Surfaces" shares with us several such approaches that have been developed by social scientists. The approaches are illustrated via formulas, full-color maps, and mathematical explanations. A theme across these measurements is that they occur on two-dimensional maps displaying data representing varying concentrations of groups of people. The maps show contours identifying levels of concentration, as well as directions of greatest change, also known as gradients. The mathematical tools used quickly demonstrate not only that this article is showing us mechanisms for indexing segregation, but also that we can study this topic by drawing many of our ideas from a typical course in multivariable calculus.

Early in the article, its authors discuss the conversion of map data into a surface. This process begins with probability density functions describing how two groups, A and B, are distributed, then uses kernel density estimation to determine the surface. The estimation involves parameters that may be chosen in different ways. One system for this process leads to a choropleth map, which shades in the map according to the proportion of white residents and draws contour lines showing the probability that a resident is white. Another visualization on a 2D map shows segregation gradients, with direction indicating greatest increase in proportion of white residents, and length showing how rapidly the proportion changes.

These ideas come together to form two different indexes of segregation. One index computes the average gradient length along the entire 50% contour, that is, along the contour showing 50% white residents. Depending on the data and segregation patterns in a city, such a contour may not be defined, so another index computes the average gradient length across the entire region. The first index makes use of gradients and a contour integral; the second uses gradients and a double integral across a region's area. These formulas bring together calculus concepts, while showing how these ideas can be used in the context of a mean-ingful data set.

The authors end by providing readers with several articles where we can learn more, stating several related problems to try, and linking to their code and data. Throughout, "Segregation Surfaces" makes an excellent case for applying mathematical techniques with care and caution, recognizing that there is no single correct approach and no quick fix. Simultaneously, the article does a marvelous job highlighting how undergraduate data analysis and mathematics techniques can lend insight into how we quantify segregation patterns.

Response from David Hunter

We are honored that our paper on segregation measures and visualization has been selected for a Carl B. Allendoerfer award. We would like to thank the editorial staff of *Mathematics Magazine* and the careful work of anonymous reviewers whose insightful comments improved the paper. We are also grateful for powerful open source tools and open data practices that can support undergraduate research in a range of disciplines. Our hope is that this work will inspire other mathematical investigations into topics that address important questions.

Response from Chisondi Warioba

Math has always fascinated me as a language. It is a universal way to communicate. As someone who speaks English as a second language, the ability to describe the world we live in with such universal descriptors will always take my breath away. It is an honor to contribute to this field and an even greater honor to be recognized as a recipient of this year's Allendoerfer Award.

Biographical Sketches

David J. Hunter received his PhD from the University of Virginia, Charlottesville in 1997 and now teaches mathematics and computer science at Westmont College, Santa Barbara, CA. As a transplanted Chicagoan living in Santa Barbara, he loves to walk around cities and hike in the mountains.

Chisondi Warioba graduated in May 2021 from Westmont College with a bachelor of science in physics, chemistry, and biology. He is currently in pursuit of a PhD in medical physics at the University of Chicago with an interest in MRI physics and cancer biology.