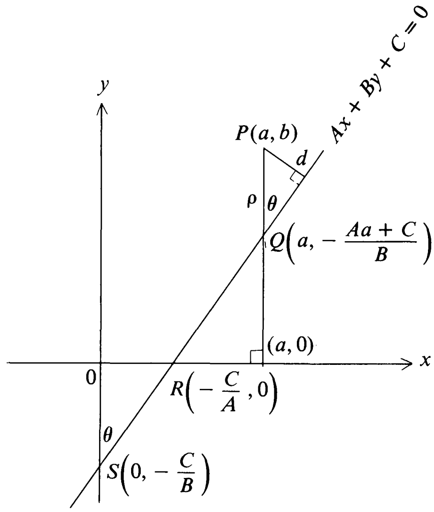

The distance between P and $Ax + By + C = 0$ is

$$d = \rho \sin\theta,$$

where ρ is the distance between P and Q. Computing $\sin\theta$ from triangle ORS, we get

$$\begin{aligned} d &= \left| b + \frac{Aa + C}{B} \right| \left(|B| / \sqrt{A^2 + B^2} \right) \\ &= \frac{|Aa + Bb + C|}{\sqrt{A^2 + B^2}}. \end{aligned}$$

Editor's Note. For ten distinctly different approaches to obtaining this distance formula, see John Staib's article "Problem-solving versus answer-finding," Two-Year College Mathematics Readings, Ed. Warren Page, MAA 1981, pp. 221–227. For Classroom Capsules on this theme, see the derivation given by K. R. Sastry [TYCMJ 12 (March 1981) 146–147] and Warren B. Gordon [TYCMJ 10 (November 1979) 348–349].

The Maximum and Minimum of Two Numbers Using the Quadratic Formula

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The maximum and minimum of two numbers, r and s, may be computed using

$$\max(r,s) = \tfrac{1}{2}(r + s + |r - s|) \tag{1}$$

$$\min(r,s) = \tfrac{1}{2}(r + s - |r - s|). \tag{2}$$

Formulas (1) and (2) are familiar to those who have studied advanced mathematics, but are not typically introduced in the first two years of college mathematics. In this note, it is shown that the formulas arise naturally in connection with the quadratic formula.

The main idea is to apply the quadratic formula to $(x-r)(x-s)=0$, attempting to recover the roots, r and s. Therefore, writing the equation in the standard form

$$x^2-(r+s)x+rs=0,$$

the quadratic formula gives the roots as

$$\tfrac{1}{2}\left(r+s\pm\sqrt{(r+s)^2-4rs}\right)=\tfrac{1}{2}(r+s\pm|r-s|).$$

Clearly, $\frac{1}{2}(r+s+|r-s|)$ is the larger root (i.e., the larger of r and s) and $\frac{1}{2}(r+s-|r-s|)$ is the smaller root (i.e., the smaller of r and s).

There are several features of this derivation that are beneficial in the classroom. First, having derived formulas for obtaining roots from coefficients, and vice versa, it is appropriate to observe the inverse nature of the procedures, and to verify the conclusion by composition. Second, the derivation provides an opportunity to remind students of the identity $\sqrt{x^2}=|x|$ in a setting where the absolute value function has obvious significance. (For example, if $\sqrt{(r-s)^2}$ is simply replaced by $(r-s)$, we appear to find that r is always the larger root!) Finally, the formulas (1) and (2) are of intrinsic interest, and their derivation illustrates how the unexpected can pop up so fortuitously in mathematics.

Editor's Note: Formulas (1) and (2) can be extended to find the minimum and maximum of any n given numbers. For instance, students can now be asked to establish that

$$\max(r,s,t)=\tfrac{1}{2}\left[\tfrac{1}{2}(r+s+|r-s|)+t+\left|\tfrac{1}{2}(r+s+|r-s|)-t\right|\right]$$

$$\min(r,s,t)=\tfrac{1}{2}\left[\tfrac{1}{2}(r+s-|r-s|)+t-\left|\tfrac{1}{2}(r+s-|r-s|)-t\right|\right]$$

for given r,s,t.

Probabilistic Repeatability Among Some Irrationals

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It is well known that most square roots are irrational: their decimal expansions are nonrepeating and nonterminating. Thus, intuition would have us suspect that each of the ten digits would be equally likely to appear in the rth place d_r beyond the decimal point in a given $\sqrt{x}$ $(x>0)$. That such is not the case is the point of this note.

If b is a nonnegative integer, and x is chosen at random in the closed interval from b^n to $(b+1)^n$, what is the probability $P(d_r,k)$ that the rth digit d_r in the decimal representation of $x^{1/n}$ is k?

For b and n given, let

$$B(d_r,k)=\left\{x\in\left[b^n,(b+1)^n\right]:d_r\text{ in }x^{1/n}\text{ is }k\right\}.$$