mathematics in "real life," it does give them an opportunity to practice translating something they understand intuitively into a problem they can analyze mathematically (with, of course, ample guidance from their instructor). And, at the very least, this problem works well as an elementary exercise in "mathematical modeling" which can be used to illustrate several of the important theorems of first-year calculus. Indeed, the problem generated favorable results when classroom tested in this manner by the authors.

References

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Reexamining the Catenary

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Some years ago I was called upon to investigate an accident involving the failure of an overhead electrical transmission cable. One of the first tasks was to establish the geometry of the cable's profile, which took the form of a catenary suspended from two supports at different elevations. A search of various textbooks and handbooks of mathematics, mechanics, and engineering practice produced what appeared to be a settled conclusion: when the supports of a catenary are at different elevations, the mathematical complexity precludes a theoretically correct solution, and a parabolic approximation is the recommended approach.

Having since retired, I had the time to revisit this question, convinced that there must be a mathematically correct way to solve the generalized problem. As it turns out, a combination of algebra and a scientific calculator will do the job.

In Figure 1, d, v, l, and g are primary variables; in order to determine one, the values of the other three must be known, since the fixing of any three of them defines the shape and size of the catenary. It is the somewhat elusive parameter, c, however, which governs the relationships between these variables.

If g is the unknown variable, we will find an equation that determines c in terms of d, v, and l.

Although an equation for a catenary may be written in various ways, the simplest form for algebraic purposes is:

$$y = c\cosh\frac{x}{c}, \tag{1}$$

the derivation of which may be found in many texts. An equation for s is readily found by applying the familiar formula for arc length:

$$s = \int_0^x \sqrt{1 + (\sinh x/c)^2}\, dx = \int_0^x \cosh(x/c)\, dx = c\sinh\frac{x}{c}. \tag{2}$$

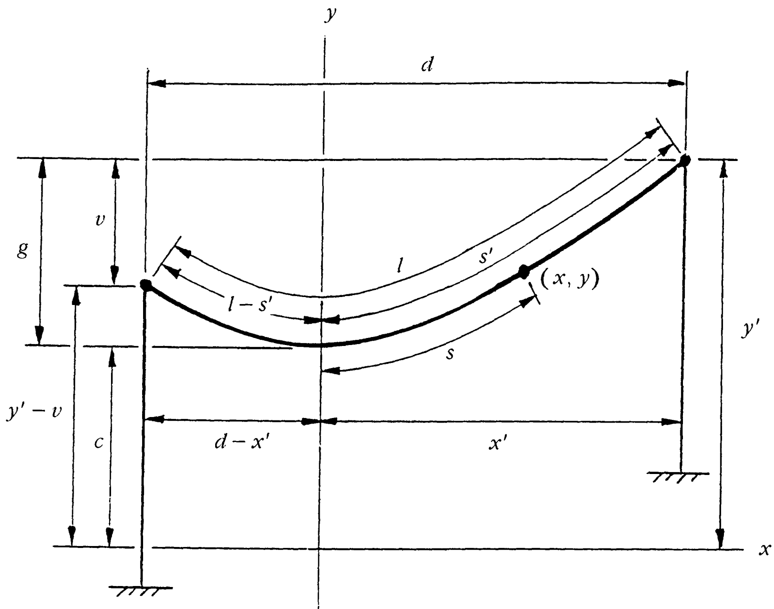

Figure 1

We proceed to derive further equations that will determine c and, ultimately, the sag of the cable.

Applying (1) to the points of support yields

$$y' = c\cosh\frac{x'}{c}, \quad y' - v = c\cosh\frac{d-x'}{c}, \tag{3}$$

and (2) gives

$$s' = c\sinh\frac{x'}{c}, \quad l - s' = c\sinh\frac{d-x'}{c}. \tag{4}$$

From (3),

$$v = c\cosh\frac{x'}{c} - c\cosh\frac{d-x'}{c}.$$

From (4),

$$l = c\sinh\frac{x'}{c} + c\sinh\frac{d-x'}{c}, \tag{5}$$

so

$$\frac{v}{l} = \frac{\cosh(x'/c) - \cosh((d-x')/c)}{\sinh(x'/c) + \sinh((d-x')/c)}.$$

Applying the hyperbolic identities

$$\cosh a - \cosh b = 2\sinh\frac{a+b}{2}\cdot\sinh\frac{a-b}{2}$$

$$\sinh a + \sinh b = 2\sinh\frac{a+b}{2}\cdot\cosh\frac{a-b}{2}$$

we have, after cancellation,

$$\frac{v}{l} = \tanh \frac{2x' - d}{2c}$$

which, when solved for x'/c, yields

$$\frac{x'}{c} = \frac{d}{2c} + \tanh^{-1} \frac{v}{l}. \tag{6}$$

Substituting this into (5) gives

$$l = c \sinh\left(\frac{d}{2c} + \tanh^{-1} \frac{v}{l} \right) + c \sinh\left(\frac{d}{2c} - \tanh^{-1} \frac{v}{l} \right), \tag{7}$$

which is the equation to use for determining c when the unknown variable is g.

From Figure 1, $y' = c + g$. From (3) we get

$$c + g = c \cosh \frac{x'}{c}, \tag{8}$$

so, knowing c, g can be determined.

For example, suppose that we wish to find the sag below the upper support of a cable 110′ long suspended between supports that are separated by 100′ horizontally and 20′ vertically. Then from (7),

$$110 = c \sinh\left(\frac{100}{2c} + \tanh^{-1} \frac{20}{110} \right) + c \sinh\left(\frac{100}{2c} - \tanh^{-1} \frac{20}{110} \right).$$

Here is where the calculator comes into play. Using its ability to solve equations, or to graph them, we get that $c = 72.288$. Substitution into (6) gives $x' = 63.291$ and using this in (8) gives $g = 29.523$ feet.

Similarly, equations may be derived for dealing with the cases where the unknown variable is v, d, or l.

Second Order Iterations

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Introduction

In calculus courses, the concept of iterative computation is introduced to students as they learn to solve problems with the aid of calculators and computers.

In such a computation one starts with an initial approximation x_0 and then iterates an approximative procedure which gradually produces improved approximations of the result. Such iterative processes are similar to parking a car between two parked cars, tuning a guitar, writing a poem, debugging a computer program, fitting a tailored suit to a customer, etc.

The criterion for terminating the iterations is usually met when an appropriate error bound (and therefore the error) is found to be sufficiently small, thereby assuring the desired accuracy.