While not a substitute for the valuable technique of logarithmic differentiation (the rule is a good exercise in it), it allows students to see the harmony in the various rules. Its similarity to the product rule, "take the derivative of f holding g constant plus the derivative of g holding f constant," adds to its appeal for students.

See FFF 47 in *CMJ* 22 (1991) 404.

A Useful Notation for Rules of Differentiation

Robert B. Gardner, Louisiana State University in Shreveport, Shreveport, LA 71115

The freshman calculus instructor is often faced with the task of deciphering the freshman calculus student's versions of the product rule, quotient rule and chain rule. Even when correctly computed, it can be difficult to follow the student's steps, especially when the differentiated function involves several trigonometric functions. I propose a notational convention to deal with this problem.

Several popular calculus books (e.g. Swokowski, Larson et al., etc.) represent the differential operator as $D[\cdot]$ or $d/dx[\cdot]$. With this as motivation, I have used the following notation quite successfully in the classroom. Whenever a function is differentiated using the product rule, quotient rule, or chain rule, put the differentiated parts in square brackets. This means that the product rule would be written as

$$D[(f)(g)] = [f'](g) + (f)[g'].$$

In fact, if the instructor introduces the convention of "always differentiate f (the first function in the product) first," then the product rule can be presented pictorially as

$$D[(\ )(\ )] = [\ ](\ ) + (\ )[\ ].$$

This reduces exercises involving the product rule to nothing more than fill-in-the-blank problems.

Similarly, the quotient rule can be represented as

$$D\left[\frac{(\ )}{(\ )}\right] = \frac{[\ ](\ ) - (\ )[\ ]}{(\ )^2}.$$

The habit of "differentiating f first" developed in the product rule, must be carried over to the quotient rule.

The chain rule is a bit harder to "draw"; however the special case known as the power rule can be illustrated in this manner:

$$D\left[(\ )^n\right] = n(\ )^{n-1}[\ ].$$

My experience has shown that, not only is the students' work easier to follow, but the material is easier to present clearly. Students have reacted quite positively to this notation. In fact, when encouraged to adopt the notation, but not required to, I have found that practically all students choose to use the notation. This has been the case even when the notation was introduced some time after the rules of differentiation, i.e., Calculus 2 and Calculus 3 students quickly "pick up the habit."

An example illustrates the utility of this method. Suppose

$$f(x) = \frac{\sec x \tan x}{(x^2+1)^6}.$$

With the notation,

$$f'(x) = \frac{[[\sec x \tan x](\tan x) + (\sec x)[\sec^2 x]](x^2+1)^6 - (\sec x \tan x)\left[6(x^2+1)^5[2x]\right]}{(x^2+1)^{12}}.$$

Although a formidable derivative, it can be viewed as nothing more than a large fill-in-the-blank problem.

With the square brackets, it is much easier to follow the students' steps and, if applicable, to give partial credit. In fact, give the above problem on your next calculus test. I think, in the absence of the square brackets, you will find it quite a task simply to determine if the students' responses *are* partially correct!

Mathematics the Mother of Writing

A growing body of archeological evidence indicates that the first accountants, beginning perhaps 10,000 years ago in the Fertile Crescent..., kept track of their sheep, cows, grain and wine by counting bean-like pieces of clay.

The use of these tokens led directly to the development of both writing and mathematics, according to archeologist Denise Schmandt-Besserat of the University of Texas at Austin....

Sometime around 8000 BC, a particularly bright individual made what Schmandt-Besserat called a "conceptual leap." That person recognized that accountants could make small clay tokens whose specific shapes identified the commodity they represented....

Archeologists have now found 15 main types of tokens, ranging from cones, spheres, disks, cylinders and tetrahedrons to tool and animal shapes.

...around 3300 BC...someone invented envelopes, called "bullae" to hold the tokens....

The problem with the bullae...was that their contents could not be discerned without breaking [them] open.

To overcome this problem, accountants began pressing the tokens into the clay before closing the envelope and hardening it....

"This simple change, from tokens to their negative impression on the face of a clay tablet, can now be recognized as the invention of writing," she said.

Thomas H. Maugh II, A Page in History for Accountants, *The Los Angeles Times*, March 18, 1991, B7.