References

1. F. Demana, B. Waits, and J. Harvey, eds., *Proceedings of the Second Annual Conference on Technology on Collegiate Mathematics*, Addison-Wesley, Reading, MA, 1991.
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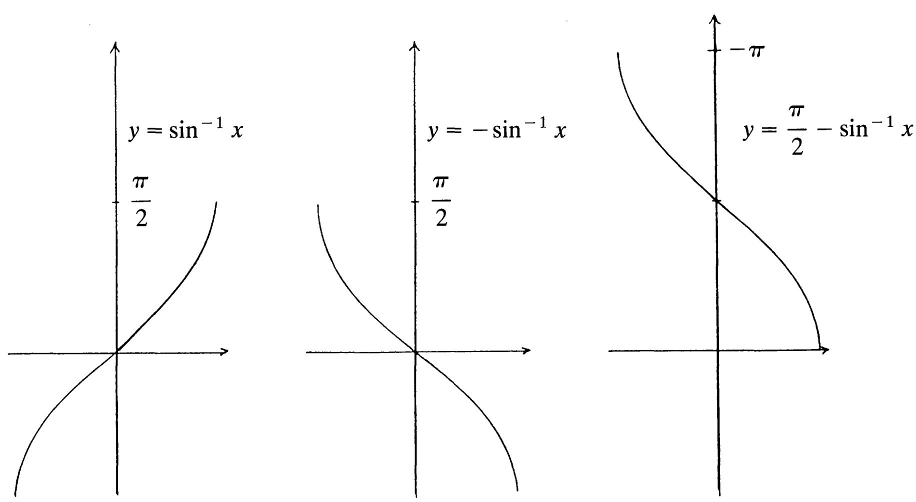
Graphs and Derivatives of the Inverse Trig Functions

Daniel A. Moran, Michigan State University, East Lansing, MI 48824

In a calculus course the differentiation formulas for the inverse trig functions are derived by implicit differentiation (at least for two or three of the functions). To avoid tedious repetition, the formulas for the others are merely stated, and their proofs omitted or left as an exercise.

The approach outlined below gives half of the differentiation formulas as immediate consequences of the others. After the inverse functions are defined, it is established that $f^{-1}(x)$ and $\text{cof}^{-1}(x)$ are always complementary when f is sine, tangent or secant. Along the way, there is an opportunity to use graphics (computer-driven or otherwise) and strengthen the students' grasp of the elementary geometry of reflections and translations. And the whole process takes less classroom time than the conventional method!

The archetypical demonstration:



In the figure, the first graph is reflected in the horizontal axis to produce the second; the latter is then translated $\pi/2$ units upward to yield the third (which is evidently congruent to the graph of $y = \cos^{-1} x$). This establishes that $\sin^{-1} x + \cos^{-1} x = \pi/2$. We can now differentiate to discover that $D_x \cos^{-1} x = -D_x \sin^{-1} x$.

The demonstrations for $\tan^{-1}$ and $\sec^{-1}$ require practically no change from the above.