solution to the cubic. The symmetry of (2) may make its verification somewhat simpler than the verification of (7). Otherwise, either identity provides a simple approach to solving the cubic.

Here is a streamlined version of the Cardano solution, which is particularly simple and memorable. Following the presentation in [**10**], begin with the cubic in the form $x^3 = px + q$ and replace x with $b + c$. That leads directly to the equation $3bcx + b^3 + c^3 = px + q$. Now match the coefficients of x; that is, make $3bc = p$ and $b^3 + c^3 = q$. These conditions are virtually identical to those used earlier, and they allow us to find b and c in terms of p and q. From this point on, the derivation is essentially the same as what was presented before.

The literature on solving the cubic is large; for representative samples, see [**3, 4, 9, 11**]. Besides the presentation in [**10**] previously cited, a recently published version is in [**5**]. A translation of Cardano's solution appears in [**2**], and the history of Cardano's publication and his dispute with Tartaglia [**1, 8**] is quite interesting. Kleiner [**7**] provides a nice discussion of the role of the solution of the cubic in the development of complex numbers.

References

1. Carl Boyer, *A History of Mathematics*, 2nd ed. (rev., Uta C. Merzbach), Wiley, New York, 1991, pp. 282–286.
2. Ronald Calinger, ed., *Classics of Mathematics*, Moore Publishing, Oak Park, IL, 1982, pp. 235–237.
3. H. B. Curtis, A derivation of Cardan's formula, *American Mathematical Monthly* 51:1 (1944) 35.
4. Orrin Frink, Jr., A method for solving the cubic, *American Mathematical Monthly* 32:3 (1925) 134.
5. Jan Gullberg, *Mathematics from the Birth of Numbers,* Norton, New York, 1997, p. 318.
6. Morton J. Hellman, A unifying technique for the solution of the quadratic, cubic, and quartic, *American Mathematical Monthly* 65:4 (1959) 274–276.
7. Israel Kleiner, Thinking the unthinkable: The story of the complex numbers (with a moral), *Mathematics Teacher* 81 (1988) 583–592. Reprinted in Frank J. Swetz, ed., *From Five Fingers to Infinity*, Open Court, Chicago and La Salle, IL, 1994, pp. 711–720.
8. Victor J. Katz, *A History of Mathematics: An Introduction*, Harper Collins, New York, 1993, pp. 330–334.
9. E. J. Oglesby, Note on the algebraic solution of the cubic, *American Mathematical Monthly* 30:6 (1923) 321–323.
10. John Stillwell, *Mathematics and Its History*, Springer-Verlag, New York, 1989, p. 55.
11. Robert Y. Suen, Roots of cubics via determinants, *College Mathematics Journal* 25:2 (1994) 115–117.

MATH and Other Four-Letter Words

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Once parametric equations have been introduced in a calculus course, and after the students have graphed a few examples and seen the usual methods for parametrizing line segments and circular arcs, then the class is ready for this "spelling" lab. Working with the letters of the alphabet is an amusing and engaging way for students to become confident about writing parametric equations and plotting parametric curves.

Since the outlines of the characters in modern computer fonts are actually described by parametric equations, the alphabet is a natural source of exercises for this topic.

First the class is partitioned into an even number of groups with three or four students apiece. Each group receives a copy of the following instructions.

Spelling out a four-letter word:
An exercise in parametric equations

In the first part of today's lab, your group is being asked first to agree on a four-letter word and then to write a set of parametric equations which when graphed will spell out your word. (ANY four-letter word is fine as long as your whole lab group agrees to it.)

In the second part of the lab, you will receive the parametric equations from one of the other groups, and you will be asked to graph them. NOTE: You should graph the equations that you are given and not try to draw the letters that you *think* the group was trying to encode.

Your grade will be based equally on both parts of the lab. Half of your grade will come from your ability to write the parametric equations of the letters in your four-letter word. The other half of your grade will be based on your ability to graph the other group's parametric equations for their word.

As soon as any two groups finish writing parametric equations for their words, their work is exchanged and they start on the second part of the lab. Typically, each part of the lab takes less than one hour.

We can control the level of difficulty by explicit instructions regarding the words and letters used. For instance, we might require that no letter be repeated in the word, or that the first letter be capitalized and the rest be lower case; that the letters appear on the same Cartesian plane, and in correct order; that the domains of the parameter variable for each letter fit together into a closed interval. . . . Clearly, variants abound.

Even without prodding, many students have gone beyond straight line segments and circular arcs in writing their words—choosing parts of conic sections, for example, such as a parabola to draw a C and a hyperbola for an X. Some of the outcome, of course, depends on the students' preparation and creative spirit, and on promised rewards.

This lab is always quite "vocal," as groups decide on their four-letter word, discuss the best way to write certain letters, and begin to figure out exactly what word the other group was trying to spell. The ways in which students benefit from this lab are numerous. Not only do they learn well how to write and to graph parametric equations, but they get to discuss mathematics, to actively engage in it, to exercise creativity, to be mathematically inquisitive, and—depending on their choice of four-letter word—to be downright irreverent. Our students have enjoyed the two hours spent on this lab exercise, and we hope that yours will too.

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A Novel Approach to Geometric Series

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Here is my informal way of explaining the geometric series formula,

$$\sum_{k=0}^{\infty} ar^k = a + ar + ar^2 + \cdots = \frac{a}{1-r}, \qquad \text{where} \quad |r| < 1$$