A Quick Construction of Tangents to an Ellipse

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Starting from points A and B on circles of radii $a > b$, Figure 1 shows how to construct $C(x_i, 0)$ and $D(0, y_i)$. They become intercepts of the tangent line t to the ellipse $x^2/a^2 + y^2/b^2 = 1$ at $P(x_0, y_0)$. To prove this, observe that

$$\cos \theta = \frac{x_0}{a} = \frac{a}{x_i} \quad \text{and} \quad \sin \theta = \frac{y_0}{b} = \frac{b}{y_i}.$$

Hence

$$\frac{x_0^2}{a^2} + \frac{y_0^2}{b^2} = \cos^2 \theta + \sin^2 \theta = 1,$$

so P is indeed on the ellipse. Then note that $x_i = a^2/x_0$ and $y_i = b^2/y_0$, so the intercept form of CD is

$$1 = \frac{x}{x_i} + \frac{y}{y_i} = \frac{x}{a^2/x_0} + \frac{y}{b^2/y_0} = \frac{xx_0}{a^2} + \frac{yy_0}{b^2},$$

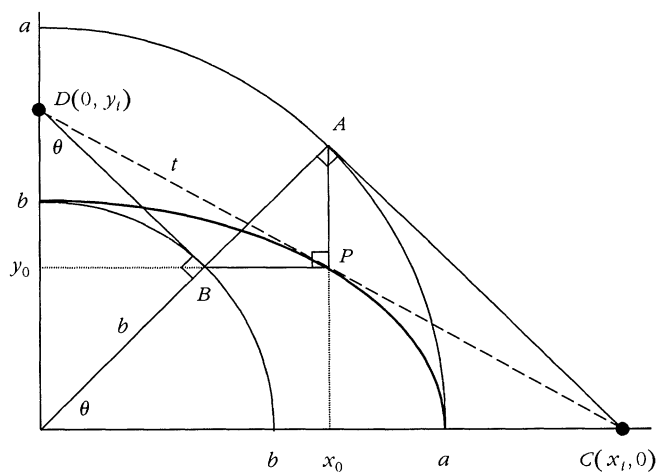a well-known form of the tangent line to an ellipse.

Figure 1