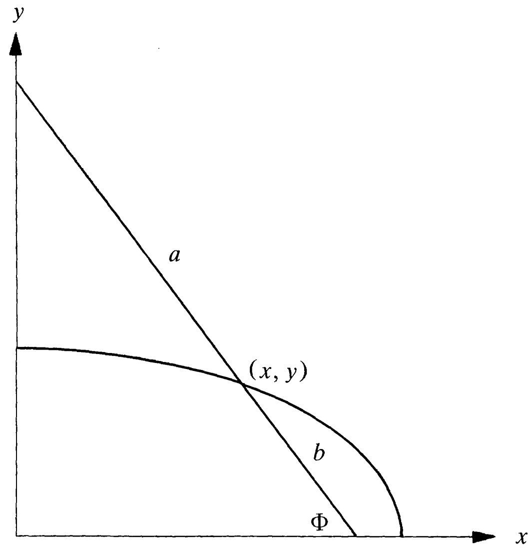

Figure 1

mathematical definition of an ellipse and its consequences. In particular, the notion of foci is completely absent.

Stacking Ellipses

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Let E be the plane region bounded by the ellipse $x^2/a^2+y^2/b^2=1$. Most proofs that the area of E is πab use integral calculus or the fact that E is the affine image of a circular disk. Beginning with the equation of the ellipse, we will prove the area formula by stacking the ellipses and using Cavalieri's principle.

Suppose a/b is rational, say $a/b=m/n$, where m and n are integers. First we stack n copies of the ellipse E side by side as shown in Figure 1. Then a horizontal line at distance y from the center of E cuts the stack in n segments whose combined length is $2na\sqrt{1-y^2/b^2}$. But this is the length in which the horizontal line cuts the plane region E' bounded by $x^2/(na)^2+y^2/b^2=1$. This is illustrated in Figure 1 where, for convenience, we always write the equations of the ellipses as

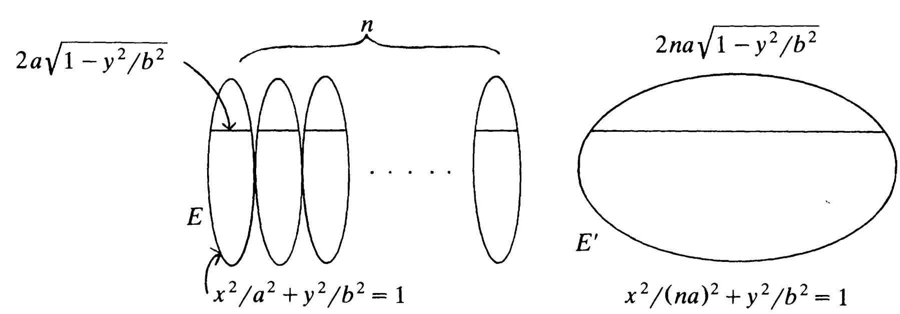

Figure 1

if the origin is at the center. By Cavalieri's principle, the area of the stack of n copies of E is equal to the area of E'.

Now stack m copies of Figure 1, one above the other, as shown in Figure 2. Then a vertical line at distance x from the center of E' cuts the m copies of E' in m segments of combined length $2mb\sqrt{1-x^2/(na)^2}$. But this is the length in which the vertical line cuts the *circular disk* E'' bounded by $x^2/(na)^2+y^2/(mb)^2 = 1$. By Cavalieri's principle the area of the stack of m copies of E' is equal to the area of E'', which is $\pi namb$. This is the combined area of the nm copies of the ellipse E, and so the area of E is πab, as desired.

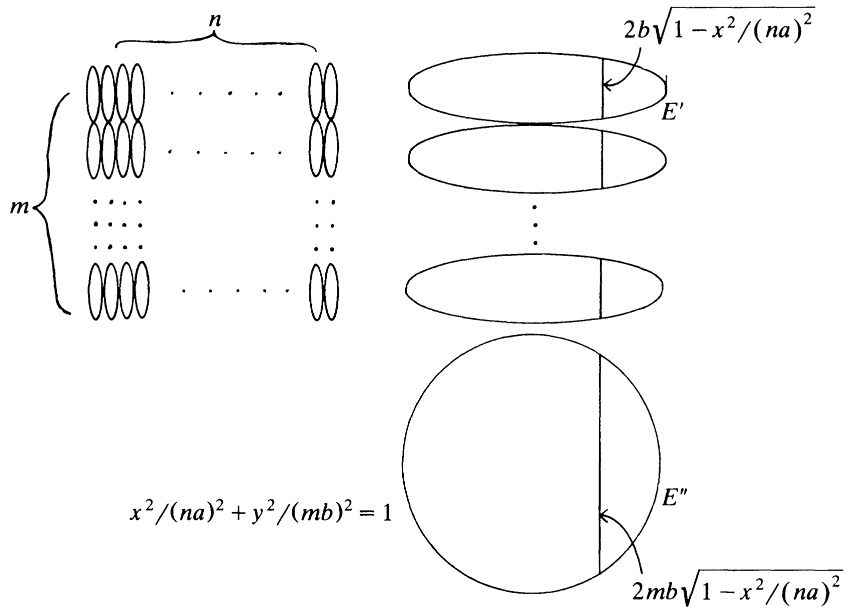

Figure 2

If the ratio a/b is not rational, then ellipse E can be approximated, arbitrarily closely, by ellipses having axes of lengths a and b^* such that a/b^* is rational. Thus, we can show that the area of E is πab for all a and b.

The Isoperimetric Quotient: Another Look at an Old Favorite

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Kouba [1] considered that calculus problem where one cuts a wire into two pieces and forms two figures, each of a prescribed shape, in such a way as to enclose minimum area. It seems worthwhile to pursue this a bit further because of the interesting geometric notions involved. In particular, we shall proceed in the spirit of Niven [2] by dealing with a somewhat more general problem in a manner that avoids calculus and emphasizes the underlying geometry. Perhaps some of these ideas will provide inspiration and motivation for instructors to explore the traditional problems of calculus from more than one viewpoint in an effort to broaden the horizons of their students.