function. In contrast to the differentiable case there may be more minimizing functions; to be precise, note that the one-sided derivatives $f'_+(t/2)$ and $f'_-(t/2)$ exist with $f'_+(t/2) \leq f'_-(t/2)$ [2, p. 199, theorem 4.43] implying that all linear functions

$$g(x) = m\left(x - \frac{t}{2}\right) + f\left(\frac{t}{2}\right)$$

with slope m such that $f'_+(t/2) \leq m \leq f'_-(t/2)$ are all supporting lines of D through $(t/2, f(t/2))$ and hence minimize Φ [2, p. 200, theorem 4.44].

Although the property described looks very elementary we have not been able to locate something similar in the literature.

References

1. James R. Smart, *Modern Geometries*, 3rd ed., Brooks/Cole, Pacific Grove, CA, 1988, p. 91.
2. Karl R. Stromberg, *Introduction to Classical Real Analysis*, Wadsworth, Belmont, CA, 1981.

A Simple Geometric Proof of the Addition Formula for the Sine

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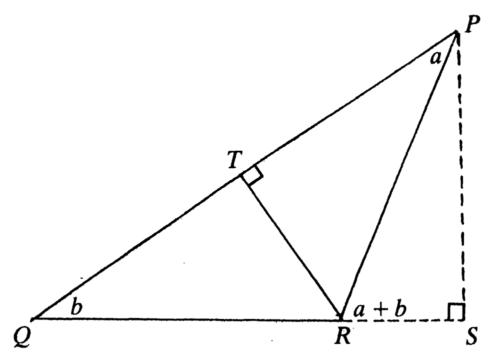
The following is a short, simple, but not well-known proof of the well-known trignometric identity $\sin(a+b) = \sin a \cos b + \cos a \sin b$. The proof can be easily understood by students and uses nothing more complicated than the (triangle-based) definitions of sine and cosine and the area formula for triangles, $A = \frac{1}{2}$(base)(height). In the figure, a, b, and $a+b$ are all acute angles. With appropriate minor modifications, a similar argument applies if b and/or $a+b$ are obtuse.

From the area formula, we see that the product of base and height is independent of which side of the triangle is chosen as the base. So in triangle PQR, we have $\mathrm{QR} \cdot \mathrm{PS} = \mathrm{QP} \cdot \mathrm{RT}$ and thus

$$\mathrm{PS} = \frac{\mathrm{PQ} \cdot \mathrm{RT}}{\mathrm{QR}} = \frac{(\mathrm{QT} + \mathrm{TP})\mathrm{RT}}{\mathrm{QR}}. \qquad (*)$$



Now

$$\begin{aligned}\sin(a+b) &= \frac{\mathrm{PS}}{\mathrm{PR}}\\ &= \frac{(\mathrm{QT}+\mathrm{TP})\mathrm{RT}}{\mathrm{QR}\cdot\mathrm{PR}} \quad \text{by } (*)\\ &= \frac{\mathrm{RT}}{\mathrm{PR}}\frac{\mathrm{QT}}{\mathrm{QR}} + \frac{\mathrm{TP}}{\mathrm{PR}}\frac{\mathrm{RT}}{\mathrm{QR}}\\ &= \sin a \cos b + \cos a \sin b.\end{aligned}$$

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Extending Bernoulli's Inequality

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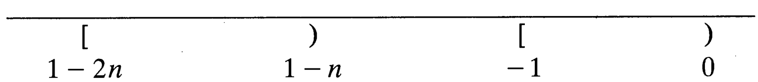
The standard statement of Bernoulli's inequality is: If $x \geq -1$ and n is a positive integer, then $(1+x)^n \geq 1+nx$. Clearly for $x < -2$ this does not hold for large odd values of n. What happens if $-2 \leq x < -1$? Add 1 to get

$$-1 \leq 1+x < 0. \tag{1}$$

Now multiply $-2 \leq x < -1$ by n and add 1, to obtain

$$1-2n \leq 1+nx < 1-n. \tag{2}$$

If $n=1$, Bernoulli's inequality is valid for any x. For $n \geq 2$, the graph



makes it easy to compare the location of $1+x$ of (1) and $1+nx$ of (2). From here it is easy to see why $(1+x)^n \geq 1+nx$. Hence, Bernoulli's inequality is valid for $x \geq -2$.

But Do You Like It?

This paper gives wrong solutions to trivial problems. The basic error, however, is not new.

Clifford Truesdell, *Mathematical Reviews* 12, p. 561.