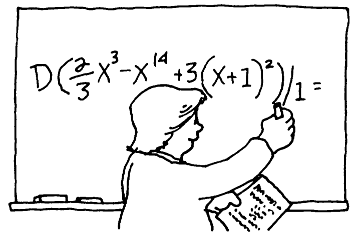
CLASSROOM CAPSULES

EDITOR

Frank Flanigan
Department of Mathematics and Computer Science
San Jose State University
San Jose, CA 95192

ASSISTANT EDITOR

Richard Pfiefer
San Jose State University



A Classroom Capsule is a short article that contains a new insight on a topic taught in the earlier years of undergraduate mathematics. Please submit manuscripts prepared according to the guidelines on the inside front cover to Frank Flanigan.

A Carpenter's Ellipse

Elliot Winston, Englander Millwork Corporation, Bronx, NY 10458

A small window over a door, called a transom, was once a very popular and fashionable style for the front of a private residence. An extra touch of elegance was often added by designing the transom in the shape of the upper half of an ellipse. Many fine examples can be seen in old townhouses located in the hearts of cities and towns in the northeastern United States. There is a small but steady market for replicas of original transoms as people regularly restore their homes. One of many problems encountered in duplicating this type of window is that of creating a pattern for the shape. Occasionally, the old transom can easily be removed, traced, and then replaced while the new one is being built. More frequently, only the lengths of the major and minor semi-axes, a and b, are available. A crude version of an instrument used to draw ellipses, called a trammel, is easily constructed. Two long narrow strips of wood, which serve as coordinate axes, are nailed at right angles to each other on a thin piece of plywood. Another strip of wood is cut to a length of $a+b$, and a pencil is clamped at a distance a from one end. This strip is placed horizontally against the positive x-axis, with one end at the origin, and is brought into a vertical position using the two fixed strips as guides for the movement; that is, the ends of the free strip always touch the axes. The resulting curve is one quarter of an ellipse, which follows immediately from

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=\cos^2\Phi+\sin^2\Phi=1.$$

The quarter ellipse can then be cut out of the plywood and used to create a pattern for the transom, by reflection. This is an especially useful method of drawing an ellipse because in actual practice it is easy to implement and generates rather accurate curves. Also, no theoretical knowledge is needed concerning the

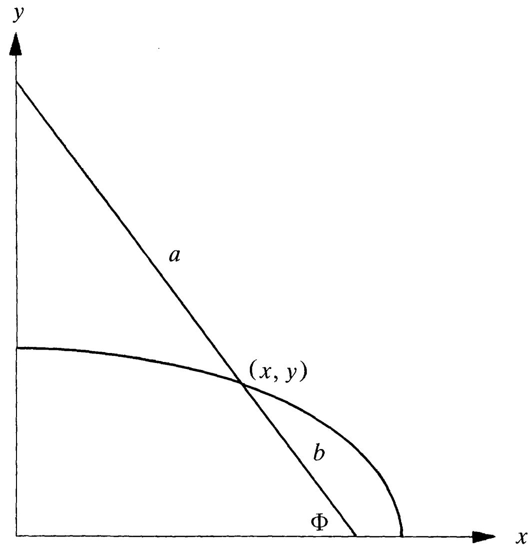


Figure 1

mathematical definition of an ellipse and its consequences. In particular, the notion of foci is completely absent.

Stacking Ellipses

Richard E. Pfiefer, San Jose State University, San Jose, CA 95192

Let E be the plane region bounded by the ellipse $x^2/a^2 + y^2/b^2 = 1$. Most proofs that the area of E is πab use integral calculus or the fact that E is the affine image of a circular disk. Beginning with the equation of the ellipse, we will prove the area formula by stacking the ellipses and using Cavalieri's principle.

Suppose a/b is rational, say $a/b = m/n$, where m and n are integers. First we stack n copies of the ellipse E side by side as shown in Figure 1. Then a horizontal line at distance y from the center of E cuts the stack in n segments whose combined length is $2na\sqrt{1-y^2/b^2}$. But this is the length in which the horizontal line cuts the plane region E' bounded by $x^2/(na)^2 + y^2/b^2 = 1$. This is illustrated in Figure 1 where, for convenience, we always write the equations of the ellipses as

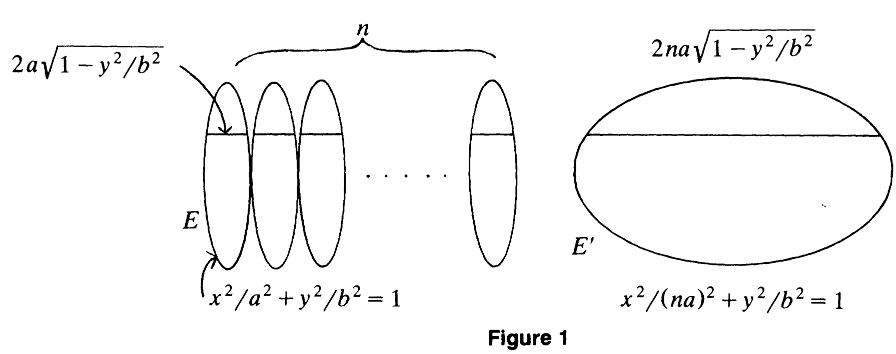


Figure 1