model of Z_{2n-1}, labeling the residue classes $1, 2, 3, \ldots, 2n-1$, rather than the usual $0, 1, 2, 3, \ldots, 2n-2$, and we consider the residue class 1 to correspond to both the top and bottom card positions. Then formula (1) becomes simply

$$R(i) \equiv -2i + 3 (\bmod 2n-1). \tag{2}$$

This, together with $R(1) = 2n$ and $R(2n) = 1$, determines the permutation R on the complete set of $2n$ card positions.

Now, from (2) it is easy to prove by induction that for all $k \geq 1$,

$$R^k(i+1) \equiv (-2)^k i + 1 (\bmod 2n-1).$$

Thus $f(n)$, the smallest integer k such that R^k is the identity permutation, is the smallest *even* integer such that $i + 1 \equiv (-2)^k i + 1 (\bmod 2n-1)$ for all i. It follows that

$$f(n) = \min\{2k | (-2)^{2k} \equiv 1 (\bmod 2n-1)\} = 2\min\{k | 4^k \equiv 1 (\bmod 2n-1)\},$$

which is twice the order of 4 in the multiplicative group in Z_{2n-1}. Using this characterization, a computer program can be written to compute $f(n)$ directly for decks of any even size (up to *longint*).

References

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Finding Volumes with the Definite Integral: A Group Project

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The following group project for a first-year calculus course uses enjoyable hands-on experience and "real world" equipment to help students understand the calculus methods used in finding the volume of a given solid. Before doing the project, in a previous class we set up the definite integral for the volume of a square-based pyramid and the integral for the volume of a solid generated by revolving a line segment about the x-axis. Then for homework the students do some standard problems—volumes of solids of revolution and a straightforward volume of a solid by slices. Two days later I bring the project equipment into class (see Figure 1):

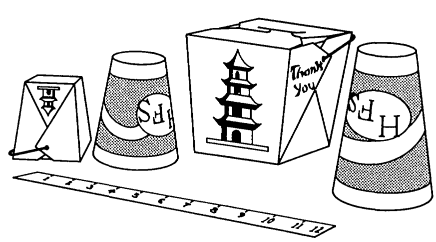


Figure 1

- Chinese restaurant take-out boxes (frustums of pyramids that have regular bases) in three sizes. (They sell for about 50 cents apiece.)
- Flat-bottomed stiff cups in three sizes.
- Rules with centimeters.

To start, I recall how volumes had been introduced:

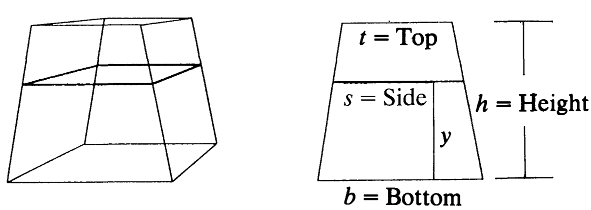
> Remember how a line segment generated a "cup" when it was revolved about an axis? For this project you're going to go the other way. You'll be given a cup; it's up to you to find the line and the integral giving the volume.
>
> And remember the pyramid? These boxes are pieces of pyramids with rectangular bases. The assignment is the same: Using calculus, find the volume by the method of slices.

Next, I divide the class into teams of three students. Each team receives one cup and one box, and each container is assigned to at least two teams. The remainder of the class hour is left for taking whatever measurements the students deem useful.

Team members are allowed to collaborate or work independently; however, each must write up both parts of the experiment individually. The assignment sheet specifies:

> Make a sketch showing a "slice of volume" and include your measurements. Clearly showing the relevant geometry, determine the volume of your slice. Write a definite integral giving the volume. Evaluate the integral analytically and as a decimal.

When each group comes to my office to present their results, I collect the individual papers, which are graded later, and we review their work. I find that all the students manage to set up the integral for the cup. The geometry of the truncated pyramid is more of a challenge, however. Most groups adopt a valid approach: Some determine the theoretical height of the pyramid; others use similar triangles (see Figure 2) to determine the sides of the rectangular slices. But some groups find "similar trapezoids" where they do not exist. All the groups do eventually arrive at a numerical answer in cubic centimeters.



$$s = b - \frac{y}{h}(b - t)$$

Figure 2

Toward the end of their appointment, I point out that the size of each container, in fluid ounces, was printed on the bottom. Since 28.4 cc = 1 fl. oz., we can quickly calculate an approximate answer in cubic centimeters for comparison with the team's own result, an often gratifying corroboration.