Mathematics and Literature

Mathematics provides writers with a rich source of themes, images and metaphors.

D. O. KOEHLER
Miami University
Oxford, OH 45056

... with pencil words on your page only
Δt from the things they stand for ...

Gravity's Rainbow

When thinking of disciplines which use mathematics in a meaningful way it is easy to overlook literature. And yet the interface between mathematics and literature is a natural and interesting one, nurtured by the fact that mathematics is a language which evolves from our need to describe the world in which we live. This point is made by John Stark in his discussion of the works of Thomas Pynchon:

> By drawing nonmathematical conclusions from mathematics and by incorporating them into his literary works, he (Pynchon) points out a feature that its technological applications might easily obscure: mathematics is metaphoric because it describes universals. [**48**, p. 69]

That is, mathematics speaks to our human experience and as such provides writers with a means for creating interesting themes and images.

There are diverse types of mathematics that are used in literature as well as diverse ways in which they are used. However, writers most often draw upon areas like geometry, probability and statistics since they are most closely related to the world in which we live. A story may ridicule a mathematically-based solution to a societal problem that fails to take important issues into account. Or it may use mathematics to help characters impose meaning on events which they seem helpless to control. In the eerie world of fantasy and science fiction, mathematical metaphors heighten the intensity of the surreal surroundings. And in the strange worlds of Moebius bands and Klein bottles there exist a multitude of exotic plots and eccentric characters. The possibilities are endless. In this paper we shall explore some of the things writers have done with mathematics as well as illustrate what they might tell us about mathematics.

Satire—some variations on "A Modest Proposal"

Satire is often used as a form of political and social criticism in which the writer ridicules apparent stupidity, folly, or vice. There are numerous examples in which the object of this ridicule is the use of some overly-simplified mathematical law or model to solve a complex problem. Most people in literature would cite Jonathan Swift's *A Modest Proposal* [**49**] as an example of this type. In it, Swift proposed that the problem of poverty could easily be solved if poor people sold their children for food. His matter-of-fact suggestion, supported by carefully evaluated "environmental impact statements," provided a biting and eloquent indictment of the British authorities who administered the Irish people as though they were numbered objects to be manipulated according to appropriate mathematical rules.

There are numerous other examples in which the mathematics involved is more substantial. A good illustration is Ralph Schoenstein's essay "60 Million Projections Can't Be Wrong" [**41**] in which he ridicules the vast wasteland of television programming that results from the use of sophisticated viewer profiles obtained from statistical samples. He suggests the real problem is

By chance, one monkey should average one "d" a minute.

Three monkeys should achieve a "dear" every ten weeks.

10,000 of 'em will average one "dear sir" per 150 years.

The mathematician's obsession in extending a simple thought (having chimpanzees reproduce great literature by chance using typewriters) to absurd limits (calculating how long it would take for "Dear Sir" to appear) is the source of ridicule in Russell Maloney's *Inflexible Logic* [28]. This illustration is reproduced from *How to Take a Chance* by Darrell Huff, illustrated by Irving Geis, with the permission of the publisher, W. W. Norton & Company, Inc., New York, N.Y. Copyright © 1959 by W. W. Norton & Company, Inc.

that the samples are much too large and should be replaced by one consisting of a single typical family he calls the Retinas. Schoenstein describes them as follows:

> The Retinas live in a trailer camp in Dubuque, where their greatest dream is to understand subtle jokes before their neighbors do. They have two television sets, which are side by side and often play together to accommodate diverse tastes. Claude Retina is a 46.3-year-old sometime Presbyterian farmer who sells insurance, votes both Democratic and Republican as a way to fight Communism, loves LBJ but hates what he's doing, and puts his faith in God and a man's deodorant. His wife, Georgia, is a 38.4-year-old junior high school graduate who dreams of someday getting new brakes for the trailer, eats 2.7 pieces of dietetic bread a day, and sneezes 6.5 times whenever she uses the low-sudser that doesn't clog her automatic. She and Claude have 2.6 children: a boy of 9.8 who uses greasy kid stuff, a girl of 8.9 who likes round-the-clock protection, and a basset hound of three. [**41**, p. 280]

Schoenstein goes on to suggest that each day each Retina would be required to complete a questionnaire containing probing questions like "Do you remember any program you saw today?", "Give the plots of three commercials you particularly enjoyed," and "If your children had to live under educational television or Communism, which would you choose and why?" He proposes that the results of these questionnaires "be sent to the heads of networks and advertising agencies, who could interpret them any way they wished, depending on which programs they wanted to justify killing."

A different variation of this theme appears in *The Law* [**14**] by Robert Coates, which describes what happens when the law of averages becomes inoperative. The "meltdown of society" begins as a sequence of isolated variances from the expected: the George Washington Bridge is jammed with drivers out on a moonlight drive; restaurants experience unusual runs on a particular dish; a New York-to-Chicago train leaves with only three passengers aboard; and so forth. In time these perturbations become increasingly widespread, creating catastrophic problems and an inevitable investigation by Congress which is described by Coates as follows:

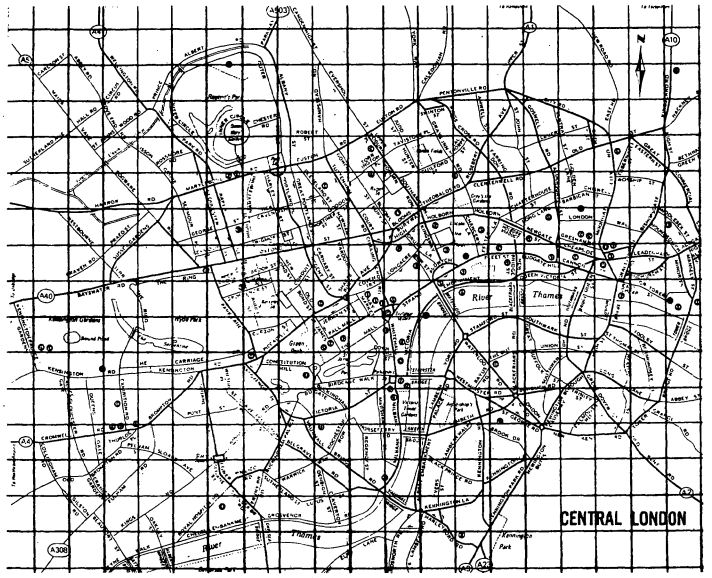
> In the course of the committee's investigations it had been discovered, to everyone's dismay, that the Law of Averages had never been incorporated into the body of federal jurisprudence, and though the upholders of States' Rights rebelled violently, the oversight was at once corrected, both by Constitutional amendment and by a law—the Hills-Slooper Act—implementing it. According to the act, people were *required* to be average, and, as the simplest way of assuring it, they were divided alphabetically and their permissible activities catalogued accordingly. Thus, by the plan, a person whose name began with "G," "N," or "U," for example, could attend the theater only on Tuesdays and he could go to baseball games only on Thursdays, whereas his visits to a haberdashery were confined to the hours between ten o'clock and noon on Mondays. [**14**, p. 19]

Those of us who recall the "odd-even" scheme of rationing gasoline several years ago realize that the "Hills-Slooper" act described by Coates is lurking closer to us than we would like.

Probabilistic imagery in "Gravity's Rainbow"

A different use of mathematics in literature occurs in Thomas Pynchon's *Gravity's Rainbow* [**38**], a complex novel set during the Second World War in which the characters search for meaning in a world dominated by the V-2 rocket. One of the major themes seems to be a debate of whether the universe is deterministic or probabilistic, and Pynchon makes considerable use of probabilistic imagery to personify this debate using a Pavlovian psychologist, Ned Pointsman, and a statistician, Roger Mexico. Mexico has discovered that the distribution of rocket hits about London has a classical Poisson distribution. However this probabilistic model troubles Pointsman.

> "Can't you ... tell from your map here, which places would be safest to go into, safest from attack?"
>
> "No."
>
> "But surely—"
>
> "Every square is just as likely to get hit again. The hits aren't clustering. Mean density is constant."



Nothing on the map to the contrary. Only a classical Poisson distribution, quietly neatly sifting among the squares exactly as it should ... growing to its predicted shape ....

"But squares that have already had several hits, I mean—"

"I'm sorry. That's the Monte Carlo Fallacy. No matter how many have fallen inside a particular square, the odds remain the same as they always were. Each hit is independent of all the others. Bombs are not dogs. No link. No memory. No conditioning."

Nice thing to tell a Pavlovian. [**38**, pp. 55-6]

For Pointsman the world must be described in terms of cause and effect.

If ever the Antipointsman existed, Roger Mexico is the man. Not so much, the doctor admits, for the psychical research. The young statistician is devoted to number and to method, not table-rapping or wishful thinking. But in the domain of zero to one, not something to something, Pointsman can only possess the zero and the one. He cannot, like Mexico, survive anyplace in between. Like his master I. P. Pavlov before him, he imagines the cortex of the brain as a mosaic of tiny on/off elements. Some are always in bright excitation, others darkly inhibited. The contours, bright and dark, keep changing. But each point is allowed only the two states: waking or sleep. One or zero. "Summation," "transition," "irradiation," "concentration," "reciprocal induction"—all Pavlovian brain-mechanics—assume the presence of these bi-stable points. But to Mexico belongs the domain between zero and one—the middle Pointsman has excluded from his persuasion—the probabilities. A chance of 0.37 that, by the time he stops his count, a given square on his map will have suffered only one hit, 0.17 that it will suffer two .... [**38**, p. 55]

Other characters in the book try to find themselves somewhere along the deterministic/statistical scale. One of these is Roger's girlfriend, Jessica.

Roger has tried to explain to her the V-bomb statistics: the difference between distribution, in angel's-eye view, over the map of England, and their own chances, as seen from down here. She's almost got it: nearly understands his Poisson equation, yet can't quite put the two together—put her own enforced calm day-to-day alongside the pure numbers, and keep them both in sight. Pieces keep slipping in and out.

"Why is your equation only for angels, Roger? Why can't we do something, down here? Couldn't there be an equation for us too, something to help us find a safer place?"

"Why am I surrounded," his usual understanding self today, "by statistical illiterates? There's no way, love, not as long as the mean density of strikes is constant. Pointsman doesn't even understand that." [**38**, p. 54]

FLYING-BOMB HITS ON LONDON

k	0	1	2	3	4	5 and over
N_k	229	211	93	35	7	1
$p(k; 0.9323)$	226.74	211.39	98.54	30.62	7.14	1.57

As an example of a spatial distribution of random points consider the statistics of flying-bomb hits in the south of London during World War II. The entire area is divided into $N = 576$ small areas of $t = 1/4$ square kilometers each, and the table above records the number N_k of areas with exactly k hits. (The figures are taken from R. D. Clarke, An application of the Poisson distribution, *Journal of the Institute of Actuaries*, vol. 72 (1946) p. 48.)

The example above is taken from *An Introduction to Probability Theory and Its Applications*, p. 150, by William Feller, and is reproduced with the permission of the publisher, John Wiley and Sons, Inc., New York, N.Y. Copyright © 1957 by John Wiley and Sons, Inc.

Pynchon seems to have created Roger Mexico as a mathematician wanting to be understood and appreciated for the insight his model provides, and yet failing to grasp that a statistical interpretation of death is no less comforting than a deterministic one.

> "The Romans," Roger and the Reverand Dr. Paul de la Nuit were drunk together one night, or the vicar was, "the ancient Roman priests laid a sieve in the road, and then waited to see which stalks of grass would come up through the holes."
>
> Roger saw the connection immediately. "I wonder," reaching for pocket after pocket, why are there never any damned—ah here, "if it would follow a Poisson ... let's see ... ."
>
> "Mexico." Leaning forward, definitely hostile. "They used the stalks that grew through the holes to cure the sick. The sieve was a very sacred item to them. What will you do with the sieve you've laid over London? How will you use the things that grow in your network of death?"
>
> "I don't follow you." It's just an equation ... .
>
> Roger really wants other people to know what he's talking about. Jessica understands that. When they don't his face often grows chalky and clouded, as behind the smudged glass of a railway carriage window as vaguely silvered barriers come down, spaces slide in to separate him that much more, thinning further his loneliness. [**38**, p. 56]

Pynchon's use of metaphors based on calculus

In the quote above, which describes Mexico's growing despair, Pynchon seems to be making use of one of several mathematical metaphors based on calculus. These metaphors mostly revolve about differentiation and integration, and, as Lance Ozier suggests [**33**], seem to represent transformations taking place in the novel. At one point Pynchon is describing what makes it possible to become a political activist.

> Franz was never much in the street. Always some excuse. Worried about security, being caught on a stray frame by one of the leather-coated photographers, who will be always at the fringes of the action. Or it was, "What'll we do with Ilse? What if there's violence?" If there's violence, what'll we do with Franz?
>
> She tried to explain to him about the level you reach, with both feet in, when you lose your fear, you lose it all, you've penetrated the moment, slipping perfectly into its grooves, metal-gray but soft as latex,. ...
>
> She even tried, from what little calculus she'd picked up, to explain it to Franz as Δt approaching zero, eternally approaching, the slices of time growing thinner and thinner, a succession of rooms each with walls more silver, transparent, as the pure light of the zero comes nearer ... . [**38**, pp. 158-9]

Ozier has noted this last sentence is suggestive of the refining of partitions in the formal definition of integration. And, just as letting "Δt approach zero" in this definition transforms continuous functions into differentiable ones, for Franz's wife Leni it is a metaphor for the transformation in

her life that takes place when she finally commits herself to the revolution. For Mexico the "silvered barriers sliding in to separate him that much more, thinning further his loneliness" is a metaphor for his transformation into a state of paranoia and despair.

Similar sequences of metaphors occur when an apprentice architect by the name of Hupla suddenly recognizes the true shape of the tunnels at the rocket assembly plant.

> "It—it's about the shape of the tunnels here, Master."
>
> "Don't flinch like that. I based that design on the double lightning-stroke, Hupla—the SS emblem."
>
> "But it's also a double integral sign! Did you know that?"
>
> "Ah. Yes: Summe, Summe, as Leibnitz said."
>
> All right. But Etzel Olsch's genius was to be fatally receptive to imagery associated with the Rocket. In the static space of the architect, he might've used a double integral now and then, early in his career, to find volumes under surfaces whose equations were known—masses, moments, centers of gravity. But it's been years since he's had to do with anything that basic. Most of his calculating these days is with marks and pfennigs, not functions of idealistic r and θ, naive x and y. ... But in the dynamic space of the living Rocket, the double integral has a different meaning. To integrate here is to operate on a rate of change so that time falls away: change is stilled. ... "Meters per second" will integrate to "meters." The moving vehicle is frozen, in space, to become architecture, and timeless. It was never launched. It will never fall. [**38**, pp. 300-01]

Consequently the process of integration transforms the moving vehicle into a permanent part of history, a transformation from which the world will never fully recover. To strengthen the image, Pynchon goes on to describe how the internal guidance system carries out the double integration electrically in order to locate its position in space and determine the Brennschluss point.

> Brennschluss exactly here would make the Rocket go on to hit 1000 yards east of Waterloo Station. At the instant the charge (B_{iL}) accumulating in flight equaled the preset charge (A_{iL}) on the other side, the capacitor discharged. A switch closed, fuel cut off, burning ended. The Rocket was on its own. [**38**, p. 301]

Thus for Pynchon the Brennschluss point seems to represent the point where the living rocket is transformed from a creation of science into a deterministic instrument of death.

Ozier points out how Pynchon reinforces this idea later when describing the approaching death of Klaus Närrisch as he attempts to cover an escape. Närrisch, like the rocket, is about to reach the point where the charge in the flight accumulator closes the switch and sets him on a path of action from which there is no return.

> ... it meant that this was the Last Day—and, too, with only the grim sixth sense, as much faith as clear reception, that the B of Many Subscripts just over the electric horizon was really growing closer,..., another integrating, not of gyro rate but of the raw current flow itself....
>
> B, B-sub-N-for-Narrisch, is nearly here—nearly about to burn through the last whispering veil to equal "A"—to equal the only fragment of himself left by them to go through the moment. [**38**, pp. 517–18]

Ozier concludes that "the process of double integration that calculates the fatal B_N is like a transformation of life into death as well as a transformation of time into eternity" [**33**, p. 200].

In a recent paper [**47**], Lawerence Stahlberg has suggested that Pynchon's Δt metaphor provides the characters with an opportunity for change. Thus they, like words, "can remain Δt away from what they mean" or, like Franz's wife Leni, they can "penetrate the moment." In fact at the end of the book where Pynchon's readers sit in a darkened movie theatre, he offers a final Δt opportunity:

> And it is just here, just at this dark and silent frame, that the pointed tip of the Rocket, falling nearly a mile per second, absolutely and forever without sound, reaches its last unmeasurable gap above the roof of this old theatre, the last delta-t.
>
> There is time, if you need the comfort, to touch the person next to you, or to reach between your own cold legs.... [**38**, p. 760]

Symmetry as a central image in "Death and the Compass"

Because geometry has been an important part of our human experience for so long, it is only natural that it plays a variety of roles in literature. An interesting illustration of one of these occurs in *Death and the Compass* [**9**] by Jorge Luis Borges. In this unusual detective story, Borges creates a character who weaves a labyrinth from which he cannot escape. The principal imagery hinges on mirror symmetry and the geometric fact that an object and its reflected image are in opposite orientation.

The protagonist is a detective Eric Lönrot, who likes to picture himself as a real-life Auguste Dupin, the famous detective created by Edgar Allen Poe. As the story begins Lönrot is discussing the case with Inspector Treviranus.

> "No need to look for a three-legged cat here," Treviranus was saying as he brandished an imperious cigar. "We all know that the Tetrarch of Galilee owns the finest sapphires in the world. Someone intending to steal them, must have broken in here by mistake. Yarmolinsky got up; the robber had to kill him. How does it sound to you?"
>
> "Possible, but not interesting," Lönrot answered. "You'll reply that reality hasn't the least obligation to be interesting. And I'll answer you that reality may avoid that obligation but that hypotheses may not. In the hypothesis that you propose, chance intervenes copiously. Here we have a dead rabbi; I would prefer a purely rabbinical explanation, not the imaginary mischances of an imaginary robber."
>
> Treviranus replied ill-humoredly:
>
> "I'm not interested in rabbinical explanations. I am interested in capturing the man who stabbed this unknown person." [**9**, p. 77]

Curiously enough, in Poe's *The Purloined Letter* [**36**], there is a similar discussion between the detective Dupin and the chief of police. However, in that story, the roles are reversed with Dupin suggesting a simple solution must exist while the chief of police contends the problem is too complex to have a simple solution.

This curious reversal is amplified throughout the story as Lönrot proceeds to discover a crime worthy of his powers of discernment. Obvious symmetry among clues is rejected in favor of more complex symmetry. For example, three murders occur on the evenings of the third day of three successive months and at locations forming the vertices of an equilateral triangle. An anonymous letter suggests the symmetry is complete and the episode finished. However, Lönrot quickly rejects this suggestion. He points out that at the site of each of the three murders there has been a reference to the Tetragrammaton, the unutterable name of God which contains four letters. He has also learned the Jewish day starts at sundown and hence the murders have all occurred on the fourth day of the month. Consequently Lönrot concludes there will be a fourth murder at a villa named Triste-le-Roy whose location forms a rhombus with the other three.

It is at this point, as Lönrot appears at the villa, that Borges makes excellent use of geometric imagery in portraying eerie and surreal feelings of hopelessness and bewilderment.

> Lönrot advanced among the eucalypti treading on confused generations of rigid, broken leaves. Viewed from anear, the house of the villa of Triste-le-Roy abounded in pointless symmetries and in maniacal repetitions: to one Diana in a murky niche corresponded a second Diana in another niche; one balcony was reflected in another balcony; double stairways led to double balustrades. A two-faced Hermes projected a monstrous shadow. Lönrot circled the house as he had the villa. He examined everything; beneath the level of the terrace he saw a narrow Venetian blind.
>
> He pushed it; a few marble steps descended to a vault. Lönrot, who had now perceived the architect's preferences, guessed that at the opposite wall there would be another stairway. He found it, ascended, raised his hands and opened the trap door.
>
> A brilliant light led him to a window. He opened it: a yellow, rounded moon defined two silent fountains in the melancholy garden. Lönrot explored the house. Through anterooms and galleries he passed to duplicate patios, and time after time to the same patio. He ascended the dusty stairs to circular antechambers; he was multiplied infinitely in opposing mirrors; he grew tired of opening or half-opening windows which revealed outside the same desolate garden

from various heights and various angles; inside, only pieces of furniture wrapped in yellow dust sheets and chandeliers bound up in tarlatan. A bedroom detained him; in that bedroom, one single flower in a porcelain vase; at the first touch the ancient petals fell apart. On the second floor, on the top floor, the house seemed infinite and expanding. The house is not this large, he thought. Other things are making it seem larger: the dim light, the symmetry, the mirrors, so many years, my unfamiliarity, the loneliness. [**9**, pp. 83-4]

Just when Lönrot the discerner has followed the events to their logical conclusion, he begins to become hopelessly lost in a labyrinth he has unwittingly helped to create. In seeing himself "multiplied infinitely in opposing mirrors" we see him, falling as though into a whirlpool, unable to comprehend that the fourth murder will be his own.

Now we learn Lönrot has been lured to the villa by a criminal, Red Scharlach, seeking revenge on Lönrot for his brother's death three years earlier. Scharlach planned the second and third murders after learning Lönrot was dissatisfied with the hypothesis that the first murder was due to chance. Now Lönrot stands before Scharlach.

Lönrot avoided Scharlach's eyes. He looked at the trees and the sky subdivided into diamonds of turbid yellow, green and red. He felt faintly cold, and he felt, too, an impersonal —almost anonymous—sadness. It was already night; from the dusty garden came the futile cry of a bird. For the last time, Lönrot considered the problem of symmetrical and periodic deaths.

"In your labyrinth there are three lines too many," he said at last. "I know of one Greek labyrinth which is a single straight line. Along that line so many philosophers have lost themselves that a mere detective might well do so, too. Scharlach, when in some other incarnation you hunt me, pretend to commit (or do commit) a crime at A, then a second crime at B, eight kilometers from A, then a third crime at C, four kilometers from A and B, half-way between the two. Wait for me afterwards at D, two kilometers from A and C, again halfway between both. Kill me at D, as you are now going to kill me at Triste-le-Roy."

"The next time I kill you," replied Scharlach, "I promise you that labyrinth, consisting of a single line which is invisible and unceasing."

He moved back a few steps. Then, very carefully, he fired. [**9**, pp. 86–7]

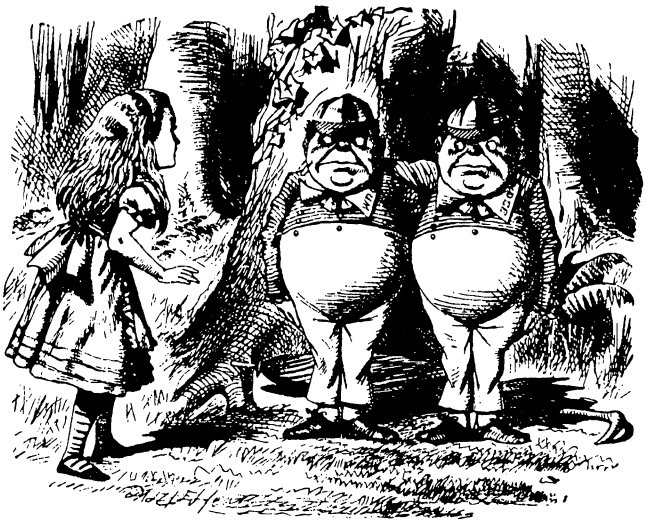
In a postcript Borges suggests that Lönrot and Scharlach may be the same person [**10**, p. 269]. If so, the curious mirror image reversal at the beginning of the story serves to identify the two, and for Borges the story becomes one about the suicide of Lönrot. The suggestion of the straight-line labyrinth at the end of the story (which is Zeno's paradox) becomes Lönrot's acknowledgement that he finally understands what has happened and realizes his reasons for suicide are not as complex as he wants to believe.

Other geometric imagery in literature

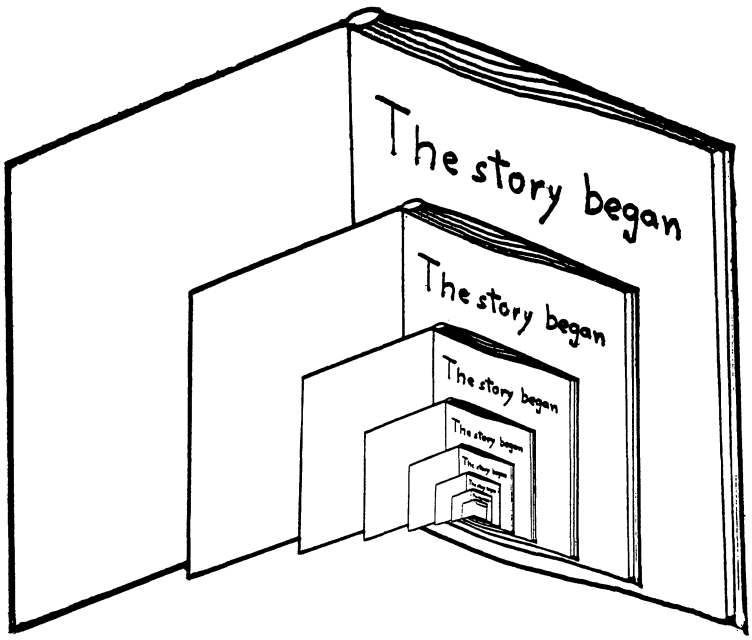
There are many other examples of geometry in literature. For instance, Martin Gardner [**19**] has noted that the geometry of reversal plays an important role in the classic nonsense literature of Lewis Carroll. In *Alice Through the Looking Glass*, Carroll (who in real life was the English mathematician Charles Dodgson) records the adventures of Alice when she enters a world behind the mirror on her mantle. There it is as though she is watching herself in a mirror, for she must continually do the opposite of what seems natural. When she tries to reach the garden by walking toward it she only succeeds in going away from it, and she is only able to approach the Red Queen by walking away from her. The poem "Jabberwocky" must be read in a mirror, and the King's messenger is in jail for a crime he won't commit until next week. Mirror geometry is also present in the characters Tweedledum and Tweedledee who are mirror images of each other. When one extends his right hand to shake hands, the other extends his left, and Tweedledee's favorite word seems to be "contraiwise."

Edwards [**17**] has noted that reversal, which is usually at the heart of nonsense, is deeply imbedded in Carroll's work. In fact, she suggests that in terms of themes, metathemes, structure, and metastructure, *Alice in Wonderland* and *Alice Through the Looking Glass* are reversals of each other. She further suggests that Carroll's stories, where life is forever turning upside down, reflect the effects on Victorian life brought about by industrialization, Darwin's theory of evolution, and other scientific, economic, and political changes. Gardner [**19**, p. 15] has added that the final metaphor of reversal in Carroll's work may be that to accept the Victorian lifestyle rationally and without illusion can only lead to the conclusion that life is irrational. In fact, it may only be when life is viewed irrationally that it can make sense.

The geometric image of a parabola plays a role in *Gravity's Rainbow* [**38**]. In fact, as Ozier notes [**33**, p. 208], underlying the whole novel is the geometric image of the parabolic flight of the rocket, gravity's own rainbow, focusing the attention of the reader and the characters on the deterministic rise and fall of society. This metaphor is suggested by the parabolic cross sections of the rocket assembly tunnels [**38**, p. 298], and repeatedly emphasized by details like the parabolic arches leading to the slums in Berlin [**38**, p. 436], or the abortive German attempt to make a parabolic sonic sound-mirror [**38**, p. 724]. In *Young Archimedes* [**21**], Aldous Huxley uses a young boy's fascination with the Pythagorean theorem to help create a touching story of the exploitation of genius. Finally, in stories like *Flatland* by Edwin Abbot or Norton Juster's delightful *The Dot and the Line*, people are personified by geometric shapes.



Tweedledum and Tweedledee, as drawn by John Tenniel for *Through the Looking Glass* by Lewis Carroll.



Paradoxes of the infinite

Geometry also lends itself to producing images of the infinite, a source of mystery and paradox in literature as well as in mathematics. For example, the parallel mirrors that multiply Lönrot infinitely are an illusion of his life accelerating toward his death. And as Alice shrinks after drinking a magic potion, she describes feeling like a telescope shutting up (much like the terms of a geometric series converging to zero), and fears that it might continue until she goes out like the flame of a candle. This leads her to wonder what the flame of a candle looks like after it goes out. A geometric image of the infinite is also present in Zeno's straight-line labyrinth that is mentioned by Lönrot. Lewis Carroll has invented a clever variation on Zeno in *What the Tortoise said to Achilles* [**64**, pp. 1225–30]. In contrast with Zeno's racecourse where there are an infinite number of distances, each smaller than the previous, the Tortoise describes a three-step racecourse consisting of an infinite number of steps, each larger than the previous. The actual racecourse is a proof:

A. Things that are equal to the same thing are equal to each other.
B. The two sides of this triangle are things equal to the same thing.
Z. The two sides of this triangle are equal to each other.

The Tortoise accepts A and B while suggesting if they don't clearly justify Z one could interpolate

C. If A and B are true, Z must be true.

Then if the argument is still not clear one can continue interpolating

D. If A, B, and C are true, Z must be true.
E. If A, B, C, and D are true, Z must be true, and so forth.

Each of these metaphoric images of the infinite is an example of "infinite regress," something closely related to the "self-referential loops" discussed by Hofstadter in his excellent book *Gödel, Escher, Bach* [**20**]. Examples of these loops occur frequently in literature. For example, at one point in Cervantes' *Don Quixote*, the barber, a creation of Cervantes, questions the merits of Cervantes' writing style. In the middle of *One Thousand and One Nights*, the character Scheherazade begins to tell the story *One Thousand and One Nights*. Lewis Carroll has Alice wonder if her dream is a part of the Red King's dream. And in *Gravity's Rainbow*, Pynchon has one of his

characters begin to wonder about the true nature of the laboratory in which the Pavlovian experiments are conducted.

> From overhead, from a German camera-angle, it occurs to Webley Silvernail, this lab here is also a maze, i'n't it now ... behaviorists run these aisles of tables and consoles just like rats 'n' mice. Reinforcement for them is not a pellet of food, but a successful experiment. But who watches from above, who notes their responses? [**38**, p. 229]

Borges is particularly fascinated with the infinite and continuously probes its interminable mysteries seeking some insight into our existence. In her book *Borges: The Labyrinth Maker* [**8**], Ana Maria Barrenechea discusses the infinite as one of five central themes Borges uses in his writing. She suggests he seems to appreciate the profound and subtle ideas inherent in the infinite as well as to enjoy reflecting on their meaning for us. As evidence she refers to his thoughts in *The Language of Argentinians* where he writes

> I suspect that the word infinite was at one time the insipid equivalent of unfinished; today it is one of God's perfections in Theology, a cause for argument in Metaphysics, as well as a popular point of emphasis in Literature, a revived abstract concept in Mathematics—Russell explains the addition, multiplication, and raising of cardinal numbers to infinite powers and the reason for their almost terrible dynasties—and a true intuition on looking at the sky. [**8**, p. 23]

She then goes on to suggest that his models for the infinite include "the vast spatial and temporal reaches, the interminable multiplications, the endless path (be it lineal or cyclical), and the immobilization through a gesture" [**8**, p. 24].

A representation of the infinite as an endless path occurs when Borges describes the infinite hatred of Scharlach for Lönrot. He accomplishes this through simple circular patterns, capable of unending repetitions, thus forming a labyrinth from which escape is impossible. Scharlach is telling Lönrot of an earlier time when he was critically wounded and his brother was in jail because of Lönrot. Lönrot notes in Scharlach's voice

> ... a fatigued triumph, a hatred the size of the universe, a sadness not less than the hatred. "Nine days and nine nights I lay in agony in this desolate, symmetrical villa; fever was demolishing me, and the odious two-faced Janus who watches the twilights and the dawns lent horror to my dreams and to my waking. ... An Irishman tried to convert me to the faith of Jesus; he repeated to me the phrase of the goyim: All roads lead to Rome. At night my delirium nurtured itself on that metaphor; I felt that the world was a labyrinth, from which it was impossible to flee, for all roads, though they pretend to lead to the north or south, actually lead to Rome, which was also the quadrilateral jail where my brother was dying and the villa of Triste-le-Roy. On those nights I swore by the God who sees with two faces and by all the gods of fever and of the mirrors to weave a labyrinth around the man who had imprisoned my brother." [**9**, pp. 84–5]

In *The Garden of the Forking Paths* [**9**] Borges represents the universe metaphorically as $\mathbf{C}^{\mathbf{C}}$ ($\mathbf{C}$ is the cardinality of the continuum) to illustrate the frail and tenuous nature of the path our lives follow.

> The explanation is obvious: The Garden of Forking Paths is an incomplete, but not false, image of the universe as Ts'ui Pen conceived it. In contrast to Newton and Schopenhauer, your ancestor did not believe in a uniform, absolute time. He believed in an infinite series of times, in a growing, dizzying net of divergent, convergent and parallel times. This network of times which approached one another, forked, broke off, or were unaware of one another for centuries, embraces all possibilities of time.
>
> Once again I felt the swarming sensation. It seemed to me that the humid garden that surrounded the house was infinitely saturated with invisible persons. Those persons were Albert and I, secret, busy and multiform in other dimensions of time. I raised my eyes and the tenuous nightmare dissolved. [**9**, p. 28]

Perhaps Borges' most fascinating metaphoric use of the infinite occurs in *The Library of Babel* [**9**]. Borges, a librarian himself, creates a library containing every possible book that can be obtained by permuting the various letters and punctuation symbols. Such a library, like the infinite, is filled with mystery and paradox for it must contain

> ... the minutely detailed history of the future, the archangels' autobiographies, the faithful catalogue of the library, thousands and thousands of false catalogues, the demonstration of the fallacy of these catalogues, the demonstration of the fallacy of the true catalogue, ... the true story of your death, ... [**9**, p. 54]

A reader familiar with mathematics can easily see this library is literally as well as figuratively filled with mathematics. For example, a Cantor diagonalization argument could be used to show that no discrete catalogue could list all the books. Also the Epimenides paradox must exist in the form of a reference book that only lists reference books that are not self-referential. Such a reference book must be false regardless of whether or not it is self-referential. (Of course the library must contain versions that are self-referential as well as ones that are not self-referential).

In addition there are all the mathematical problems associated with the magnitude of the library—for you could easily spend a lifetime searching for a book containing a meaningful sentence. And consider the problem of a censor wishing to eliminate a particular book, only to discover it is duplicated by an inexhaustible supply of almost perfect copies differing only by a single letter (not to mention those which differ from it by less than a page). Or consider the frustration of those who seek a book A explaining the meanings and secrets of the universe. They know that within the library there must be a book B describing how to find A, as well as a book C describing how to find B, and so forth. Some people, realizing that searching for such a book is fruitless, even attempt to reproduce it probabilistically by the tossing of dice.

It is likely that Borges wrote this story to describe the horrors of working in a library. On the other hand one can read into his account strong mathematical metaphors. Whether or not he intended this is irrelevant, for as he has remarked

> Ana Maria Barrenechea's book has unearthed many secret links and affinities in my own literary output of which I had been quite unaware. I thank her for those revelations of an unconscious process. This means that my best writings are things that were striving to come to life through me, or in spite of me, and not simply allegories where the thought comes before the sign. [**8**, p. viii]

In some sense, this provides ample justification to Stark's claim that mathematics is metaphoric, since the works of authors like Borges, Carroll, and Pynchon allow mathematics to speak to us about nonmathematical things.

Stylistic analysis

Any discussion of mathematics and literature would not be complete without at least mentioning the technical uses of mathematics in analyzing literature. While much of the area of "stylistic analysis" properly belongs to linguistics and automated language processing, some of it is relevant (although not central) to literary criticism.

According to Sally and Walter Sedelow [**42**], stylistic analysis includes indexing, concordancing, author-attribution studies, content analysis, and syntactic analysis. Interestingly enough, Richard Bailey has suggested that the part of stylistic analysis called statistical stylistics originated with a remark by Augustus de Morgan that word-length might be a distinctive trait of an author's style [**7**, p. 217]. Although this hypothesis proved to be insufficient, it paved the way for a variety of quantitative methods for studying the way a writer encodes his or her thoughts. Many of the techniques require a computer in order to detect the subtle patterns of style. For example, Tasman made considerable use of computers to create the indices and concordance necessary to assemble the 40,000 fragments of the Dead Sea Scrolls (representing approximately 400 manuscripts written in three languages) into their original form.

The most frequently cited examples of stylistic analysis seem to be in the area of author-attribution studies. One of the classical examples in this area is the work by Mosteller and Wallace in determining the authorship of the twelve unsigned Federalist Papers [**32**]. This work, which makes use of Bayes' Theorem and a collection of discriminator words, is important because it serves as a model for future work in this area. Another interesting author attribution study by Morton suggests that probably only five of the fourteen epistles attributed to St. Paul were actually written by him [**31**].

Another example of stylistic analysis closer to the content of this paper is the study of thematic structure discussed by Smith [**45**]. He suggests that literary interpretation of a work can be aided by studying its changing patterns of images. To do this, one first decides on the images of interest, and then makes up a list of words and phrases which suggest these images. Then successive 500 word passages of the text are searched (by a computer) to obtain the graphs of the distributions of these images. The changing patterns suggested by these graphs as well as image transition diagrams provide insight into the thematic structure of the work. Smith's own study of Joyce's *Portrait of an Artist as a Young Man* [**44**] found that major developments in Stephen Dedalus's personality correspond to the occurrences of important images, and that the changing personality is mirrored by changing associations between these images. While some may question the importance of such studies, they provide the seeds for future development in this expanding area.

Conclusions

As we have seen, there are many interesting examples of the use of mathematics in literature. It should also be noted that there are a variety of ways these examples and others can be used to enhance the study of mathematics. One way is to incorporate readings in literature as a means of introducing students to ideas in mathematics. Jerry Lenz [**27**] has described the use of science fiction as a means to introduce students to ideas in geometry and topology. This same idea could be applied to other topics such as probability and statistics, logic and foundations, history, codes and ciphers, and so forth.

For advanced students, there are some excellent books that treat a topic in mathematics in an interesting or artistic way, and these could serve as the basis for a seminar or independent study. For example, in the book *Kandelman's Krim* by the British physicist J. L. Synge, the characters develop the real numbers starting from the positive integers. A more demanding book is *Surreal Numbers* by Donald Knuth, in which two students rediscover Conway's development of numbers using a generalization of Dedekind cuts. And a beautiful interdisciplinary seminar could be based on Douglas Hofstadter's Pulitzer prize-winning *Gödel, Escher, Bach: An Eternal Golden Braid.* (A group of students at St. Olaf College took time to listen to recordings of Bach's *Musical Offering* as part of a seminar on this book, and ended their course by producing a puppet play based on the material.)

A more ambitious project would be to create a course on mathematics and literature which could explore different aspects of the two subjects. In seeing how nonmathematical conclusions can be drawn from mathematical ideas, students gain new and interesting insights into mathematics as well as catch a glimpse of how writers view mathematics. By including in the course one or more biographies or autobiographies of mathematicians (such as *Hilbert* by Constance Reid or Norbert Wiener's *I Am a Mathematician*) the students become aware of the lives of famous mathematicians and the milieu in which they worked.

There are undoubtedly many other ways to introduce students to mathematics and literature. Fortunately, no matter which approach is used, most students seem to enjoy it, and are soon discovering examples of their own.

References, with annotations

[1] E. A. Abbott, Flatland, Barnes and Noble, New York, 1963. (A story about life in the plane.)

[2] Isaac Asimov, "Franchise," Earth Is Room Enough, Doubleday and Company, Inc., Garden City, New York, 1957, pp. 58-73. (A sample of size one is used to predict the outcome of the national elections.)

[3] _____, "Gimmicks Three," Earth Is Room Enough, Doubleday and Company, Inc., Garden City, New York, 1957, pp. 74-79. (The fourth dimension is used to break a contract with the Devil.)

[4] _____, "Living Space," Earth is Room Enough, Doubleday and Company, Inc., Garden City, New York, 1957, pp. 98-111. (The problem of overpopulation is solved by having people commute to Earth from different probabilistic versions of the universe.)

[5] _____, "The Feeling of Power," The Mathematical Magpie, Clifton Fadiman, ed., Simon and Schuster, New York, 1962, pp. 3-15. (A computer programmer rediscovers how to do arithmetic in a time when all computations are done by computers.)

[6] Desmond Bagley, The Spoilers, Doubleday and Company, Inc., Garden City, New York, 1970. (The birthday problem, game theory, and the St. Petersburg paradox all play minor roles in this spy thriller.)

[7] Richard W. Bailey and Lubomir Dolezel, editors, Statistics and Style, American Elsevier Publishing Company, Inc., New York, 1969, pp. 217-237.

[8] Ana Maria Barrenechea, Borges: The Labyrinth Maker, New York University Press, New York, 1965.

[9] Jorge Luis Borges, Labyrinths: Selected Short Stories and Other Writings, New Directions, New York, 1964.

[10] _____, The Aleph and Other Stories, 1933-1969, E. P. Dutton and Co., Inc., New York, 1970.

[11] Joan Fisher Box, R. A. Fisher: The Life of a Scientist, John Wiley and Sons, New York, 1978.

[12] Dionys Burger, Sphereland, Thomas Y. Crowell Co., Apollo Editions, 1965.

[13] Arthur C. Clarke, "Wall of Darkness," Other Dimensions, Robert Silverberg, ed., Hawthorn Books, Inc., 1973, pp. 46-67. (The story of an attempt to cross a mysterious wall encircling the entire known world.)

[14] Robert M. Coates, "The Law," The Mathematical Magpie, Clifton Fadiman, ed., Simon and Schuster, New York, 1962, pp. 15-20.

[15] A. J. Deutsch, "A Subway Named Moebius," Fantasia Mathematica, Clifton Fadiman, ed., Simon and Schuster, New York, 1958, pp. 222-236. (Topology helps explain strange events on the Boston MTA.)

[16] Auguste Dick, Emmy Noether, 1882-1935, Birkhäuser Verlag, Boston, 1980.

[17] Shirley M. Edwards, The Scientific Milieu of Lewis Carroll: An Analysis of Theme and Structure in Alice in Wonderland and Alice Through the Looking Glass, M. A. Thesis, Miami University, Oxford, Ohio, 1969.

[18] Bob Elliott and Ray Goulding, "The Day the Computers got Waldon Ashenfelter," 119 Years of the Atlantic, Louise Desaulniers, ed., Atlantic Monthly Company, 1977, pp. 553-57. (The computer discovers a number of unusual correlations and nabs Waldon Ashenfelter.)

[19] Martin Gardner, The Annotated Alice, Bramhall House, New York, 1960.

[20] Douglas R. Hofstadter, Gödel, Escher, Bach: An Eternal Golden Braid, Basic Books, New York, 1979.

[21] Aldous Huxley, "Young Archimedes," Fantasia Mathematica, Clifton Fadiman, ed., Simon and Schuster, New York, 1958, pp. 3-34.

[22] Leopold Infeld, Whom the Gods Love: The Story of Evariste Galois, McGraw-Hill, New York, 1948.

[23] Norton Juster, The Phantom Tollbooth, Random House, New York, 1972. (An interesting children's story that reflects the Newtonian influence on literature during the seventeenth century.)

[24] _____, The Dot and the Line, Random House, New York, 1977. (A delightful story in which a line tries to win the love of a dot who is hopelessly in love with a squiggle.)

[25] Morris Kline, Mathematics in Western Culture, Oxford University Press, New York, 1953, pp. 272-286.

[26] Donald Knuth, Surreal Numbers, Addison-Wesley, Reading, MA, 1974.

[27] Jerry Lenz, Geometry and other science fiction, Math. Teacher 66 (1973) 529.

[28] Russell Maloney, "Inflexible Logic," Fantasia Mathematica, Clifton Fadiman, ed., Simon and Schuster, New York, 1958, pp. 91-98. (A mathematician has a hard time accepting six chimpanzees who start reproducing all the literature in the world by typing on typewriters.)
[29] Harland Manchester, "The Permanent Traffic Solution," The Saturday Review Sampler of Wit and Wisdom, Martin Levin, ed., Simon and Schuster, New York, 1966, pp. 161-163.
[30] George R. McMurray, Jorge Luis Borges, Frederick Ungar Publishing Co., New York, 1980.
[31] A. Q. Morton, A computer challenges the church, The Observer, 3 (1963) 21.
[32] F. Mosteller and D. L. Wallace, Inference in an authorship problem, J. Amer. Statis. Assoc., 58 (1963) 275-309.
[33] Lance W. Ozier, The calculus of transformation: more mathematical imagery in Gravity's Rainbow, Twentieth Century Literature, 21 (1975) 193-210.
[34] _____, Antipointsman/Antimexico: some mathematical imagery in Gravity's Rainbow, Critique, 16 (1974) 73-89.
[35] Alexi Panshin, "The Destiny of Milton Gomrath," Other Dimensions, Robert Silverberg, ed., Hawthorn Books, Inc., 1973, pp. 67-69. ("Probability Central" rectifies an accidental error.)
[36] Edgar Allan Poe, Tales of Mystery and Imagination, The Heritage Press, New York, 1941. (In *The Purloined Letter* Dupin explains why a mathematician could not be a poet. Also *The Goldbug* is a classic story that involves breaking a cipher.)
[37] Arthur Porges, "The Devil and Simon Flagg," Fantasia Mathematica, Clifton Fadiman, ed., Simon and Schuster, New York, 1958, pp. 63-69. (Simon Flagg challenges the Devil to solve Fermat's last theorem.)
[38] Thomas Pynchon, Gravity's Rainbow, The Viking Press, New York, 1973.
[39] Constance Reid, Courant in Göttingen and New York, Springer-Verlag, New York, 1976.
[40] _____, Hilbert, Springer-Verlag, New York, 1970.
[41] Ralph Schoenstein, "60,000,000 Projections Can't Be Wrong," The Saturday Review Sampler of Wit and Wisdom, Martin Levin, ed., Simon and Schuster, New York, 1966, pp. 279-82.
[42] Sally Y. Sedelow and Walter A. Sedelow, Jr., Automated Language Processing, Harold Borko, ed., John Wiley and Sons, Inc., New York, 1968, pp. 181-214.
[43] Walter A. Sedelow and Sally Y. Sedelow, eds., Computers in Language Research: Formal Methods, Mouton Publishers, New York, 1979.
[44] John B. Smith, "Image and Imagery in Joyce's Portrait," Directions in Literary Criticism, Stanley Weintraub and Philip Young, eds., The Penn State Press, 1972, pp. 220-227.
[45] _____, Thematic structure and complexity, Style 9 (1975) 32-54.
[46] _____, Computer criticism, Style, 12 (1978) 326-356.
[47] Lawrence Stahlberg, The calculus of semantics and the possibilities of metaphor: technology and morality in the fiction of Thomas Pynchon, Humanities and Technology, 1 (1980) 18-25.
[48] John O. Stark, Pynchon's Fictions: Thomas Pynchon and the Literature of Information, Ohio University Press, Athens, Ohio, 1980.
[49] Jonathan Swift, "A Modest Proposal," The Prose Works of Jonathan Swift, D. D., vol. 7, Temple Scott, ed., George Bell and Sons, London, 1905, pp. 201-216.
[50] _____, Gulliver's Travels, Louis A. Landa, ed., Houghton Mifflin, Boston, 1960.
[51] John Lighton Synge, Kandleman's Krim, Jonathan Cape, London, 1957.
[52] P. Tasman, Indexing the Dead Sea Scrolls by electronic literary data processing methods, IBM brochure, November, 1958.
[53] S. M. Ulam, Adventures of a Mathematician, Charles Scribner's Sons, New York, 1976.
[54] Bob Vinnicombe, Quiz-mathematics in literature, J. Recreational Math., 10 (1977-1978) 267-269.
[55] Sylvia Townsend Warner, Mr. Fortune's Maggot, The Viking Press, New York, 1927.
[56] Stanley G. Weinbaum, "The Circle of Zero," A Martian Odyssey and Other Science Fiction Tales, Hyperion Press, Inc., Westport, Connecticut, 1974, pp. 271-290.
[57] _____, "The Brink of Infinity," A Martian Odyssey and Other Science Fiction Tales, Hyperion Press, Inc., Westport, Connecticut, 1974, pp. 463-474. (A mathematician solves a mathematical riddle to escape from a madman.)
[58] Norbert Wiener, I Am a Mathematician, Doubleday and Company, Inc., Garden City, New York, 1956.

Collections

[59] Convergent Series, Larry Niven, ed., Ballantine, New York, 1979.
[60] Famous Stories of Code and Cipher, Raymond T. Bond, ed., Rinehart and Company, Inc., New York, 1947.
[61] Fantasia Mathematica, Clifton Fadiman, ed., Simon and Schuster, New York, 1958.
[62] Other Dimensions, Robert Silverberg, ed., Hawthorn Books, Inc., 1973.
[63] The Mathematical Magpie, Clifton Fadiman, ed., Simon and Schuster, New York, 1962.
[64] The Complete Works of Lewis Carroll, Charles Dodgson, Modern Library, Random House, New York, 1976.