Abstracts for the MAA Undergraduate Student Poster Session

MAA MathFest 2022

Philadelphia, PA

August 5, 2022

MAA
MATHEMATICAL ASSOCIATION OF AMERICA

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Organized by

Sara Malec
Hood College

Eric Ruggieri
College of the Holy Cross

and

Jeb Collins
University of Mary Washington

Dear Students, Advisors, Judges and Colleagues,

As you walk around today you will see approximately 75 posters and nearly 125 student presenters, representing a wide array of mathematical topics and ideas. These posters showcase the vibrant research being conducted as part of summer programs and during the academic year at colleges and universities from across the United States and beyond. It is so rewarding to see this session, which offers such a great opportunity for interaction between students and professional mathematicians, continue to grow.

The judges you see here today are professional mathematicians from institutions around the world. They are advisors, colleagues, new PhD's, and administrators. Many of the judges signed up when they registered for the conference, but there are also a number of judges here today who volunteered on site. Their support is vital to the success of the session and we thank them.

We are supported financially by Tudor Investments. We are also helped by the members of the MAA Committee on Undergraduate Student Programming (CUSP) in some way or another. They are: Ximena Catepillan, Emily Cilli-Turner, Paul Fishback (PME Representative), Jason Hardin, Janine Janoski, Emille Davie Lawrence, Nancy Ann Neudauer, Amber Russell, Peri Shereen (Chair), and John Snow (KME Representative). There are many details of the poster session that begin with putting out the advertisement in FOCUS, making sure the online submission system works properly, and organizing poster boards and tables in the room we are in today that are attributed to Twila Cotter (MAA Communities Coordinator), Olesia Romanova (MAA Meetings and Events Program Specialist), and Max O'Hern (MAA Digital Events Strategy Specialist).

Our online submission system and technical support is key to managing the ever-growing number of poster entries we receive. Thanks to MAA staff for their work managing the system this year. Preparation of the abstract book is a time-consuming task. Thanks to Beverly Ruedi (MAA Electronic Production and Publishing Manager) for doing the final production work on the abstract book.

We would also like to thank the members of CUSP who helped to author a new judging form, as well as those who created a video to use in place of a more traditional "judges orientation" session.

Thanks to all the students, judges, volunteers, and sponsors. We hope you have a wonderful experience at this year's poster session!

Jeb Collins
University of Mary Washington

Sara Malec
Hood College

Eric Ruggieri
College of the Holy Cross

Organized by the MAA

Committee on Undergraduate
Student Programming

The MAA gratefully acknowledges the support of

Tudor Investment Corporation.

Their generosity on behalf of the 2022 Undergraduate Student Poster Session enables students to interact with peers and role models in the mathematical sciences during the largest mathematics meeting in the world.

Titles, Authors, Advisors and Abstracts

1. Bifurcus Semigroups With Finitely Many Atoms

Robert Dolan *Grand Valley State University*
Advisor(s): Rene Ardila, *Grand Valley State University*

When algebraic objects such as rings are bifurcus, they exhibit the strong property that every non-atom, non-unit of the object may be factored into two irreducible elements. We study finite semigroups of matrices whose entries belong to congruence classes and expound on their properties. We then use this type of semigroup to construct the first verified finite bifurcus semigroup.

2. Actions and Factorizations

Gradmar Maldonado Marti *University of Puerto Rico - Mayagüez*
Reyes Ortiz Albino *University of Puerto Rico - Mayagüez*
Advisor(s): Reyes Ortiz Albino, *University of Puerto Rico - Mayagüez*

In 2006, Anderson and Frazier defined the notion of generalized factorizations using a restriction of the multiplicative operation with respect to a symmetric relation τ on the non-zero and non-unit elements of an integral domain. This notion can be interpreted as allowing only the elements that are related with respect to τ to multiply. Furthermore, it has been studied since 2003 through 2014 by Anderson's PhD students, but very few results have been published since. Relations are important as they provide information and new ideas to do research, and their study will look for several properties of relations. Anderson and Frazier defined three relations not seen before. However, the nature of the properties of relations need further study. The proposed research project hopes to define a type of action of the domain D on a relation τ, coordinate-wise. We expect to expand our toolset to characterize properties of relations and the potential implications they might have in the theory of generalized factorizations.

3. Zero Divisor Graphs of Certain Matrices

Wiley Debs *Montclair State University*
Advisor(s): Aihua Li, *Montclair State University*

A non-zero matrix A is called a zero divisor if there is a non-zero matrix B such that AB = 0. The zero-divisor graph of certain matrices has the zero divisors as the vertices and there is an edge between two zero divisors A and B if and only if AB = 0. Such a graph is applied to describe coloring of an algebraic structure and it became a hot area of research which involves both theoretical algebra and graph theory. The focus of this research is to classify the zero-divisor graph G arisen from a special type of "tri-Diagonal Matrices". The graph G has 170 vertices and more than 2460 edges. To better describe the graph, the vertices are divided into 7 classes and a class graph is developed. The class graph provides an overall picture about the graph G. Guided by the class graph, the entire structure of G is provided.

4. The Spectrum of Complete Local Rings

Ricardo Garcia *California Institute of Technology*
Mehek Mehra *Harvey Mudd College*
Joelle Ocheltree *Hartwick College*
Rodrigo Smith *Clemson University*
Advisor(s): Cory Colbert, *Washington and Lee University*

If R is a commutative ring with unity, the set $\mathrm{Spec}(R)$ is the set of all prime ideals in R. When the prime ideals are ordered with respect to set-theoretic inclusion, they form a partially ordered set. In particular, given a unital commutative ring R, one obtains a partial order from $\mathrm{Spec}(R)$. So a natural question to ask is, "Given a partial order X, when is it realizable as the prime ideal spectrum of some ring R?" Very surprisingly, the answer to this question is known for general rings, but when one requires that R be a Noetherian ring, there is no known classification despite years of remarkable progress. This project considers a particular class of commutative rings: complete local rings. Intuitively, these are rings where you can take "limits" and do arithmetic with power series, among other important operations. We aim to understand which partial orders can be realized as the spectrum of certain complete local rings coming from families of plane curves. This work was done at Pomona College as part of PRiME (Pomona Research in Mathematics Experience); it was funded by the National Science Foundation (DMS-2113782).

5. Combinatorics of stable surfaces

Adda Fadila Louleid *Lawrence University*
Luke Kovscek *Lawrence University*
Advisor(s): Julie Rana, *Lawrence University*

Algebraic varieties are locally described by polynomials. We study algebraic varieties of dimension two, known as algebraic surfaces. Quotient singularities, which commonly appear on algebraic surfaces, have nice combinatorial constructions. We describe some of these constructions, and give examples of some that we expect will correspond to singular algebraic surfaces.

6. A game of curve contractions

Mohamed Khalil Ben Nasr *Lawrence University*
Kinri Watson *Lawrence University*
Shiyi Yang *Lawrence University*
Advisor(s): Julie Rana, *Lawrence University*

Curve contractions play an important role in the theory of algebraic surfaces. Depending on the types of curves, a contraction might lead to a singularity, or may give a smooth surface which is essentially the same as the original. We describe a game played on curves that corresponds to certain curve contractions. We give examples of solutions that are expected to correspond to singular algebraic surfaces.

7. Nonlocal Optimization Problems

Michael Pieper *University of Nebraska-Lincoln*
Advisor(s): Mikil Foss, *University of Nebraska-Lincoln*

This talk will explore the intersection of the calculus of variations and nonlocal modeling. The calculus of variations is an old and beautiful theory investigating optimization problems in infinite-dimensional function spaces. The field played a crucial role in the historical development of mechanics, and has been applied to optimal control theory, boundary value problems in PDEs, and many other fields. As a natural extension of elementary calculus, the topic makes a wonderful introduction to higher level math for undergraduate students. The other topic, nonlocal modeling, has received a huge amount of attention in recent years. Unlike classical derivatives, nonlocal operators can handle discontinuities and capture long-range interactions, making them ideal tools for fields like fracture dynamics and diffusion problems. I will introduce a class of nonlocal operator and show how it compares to the classical derivative. Then I establish the existence and uniqueness of minimizers for variational problems involving this operator. Finally, I discuss some numerical experiments involving these problems that suggest further results connecting these new problems to the classical theory.

8. An Exploration of Fixed Point Theory

Julianan Sims *Temple University*
Advisor(s): Jeromy Sivek, *Temple University*

Fixed point theory is a research area with many interesting tools, open questions, and applications. We are working on some aspects of metric fixed point theory. We have already finalized details related to our first open question (regarding contractive and fixed point free maps) and have worked through additional, foundational material related to this field of exploration. We are currently exploring open questions about the minimal invariant sets of these maps that could be approached with computational experiments.

9. What Makes a Gerrymander? A Inductive Approach to Identifying Gerrymandered Districts

Win Otamias *University of Alabama at Birmingham*
Ibrahima Diagne *Harvard University*
Alexis Wood *University of California, Berkeley*
Emory Neer *Portland State University*
Christopher Gernon *Middlebury College*
Advisor(s): Courtney Thatcher, *University of Puget Sound*

Electoral redistricting is a complex, politically contentious, and sometimes ambiguous process. When districts are redrawn, they are often challenged along partisan lines as being gerrymandered, or intentionally manipulated to warp

political representation. The criteria for these rulings are often approached from the top-down: courts use a combination of precedent and statutory law to identify a gerrymandered district. Through research conducted during the 2021 SMED REU, we approached this issue of identifying gerrymandered districts inductively, from the bottom-up. Using characteristics that courts have identified as relevant to redistricting (compactness, incumbent protection, local government preservation, partisan bias, racial equality, and equal population), and a combination of standard and novel analyses, we identified a few potential indicators of gerrymandered districts. This unique approach gives way to larger considerations of court adherence to established metrics, as well as the possibility of other correlates associated with gerrymandered districts not explicitly identified by the courts.

10. Determining the Fairness of School Zones in Oklahoma City

Chase Compton *University of Central Oklahoma*

Advisor(s): Bradley Paynter, *University of Central Oklahoma*

The purpose of this project is to research the fairness of OKC public school zones. The aim of my research is to determine if OKCPS zones are drawn fair in relation to race, household income, and population distribution. In 2019, OKCPS consolidated and reconfigured schools as a part of their ”Pathway to Greatness” (PTG) program. After the schools were consolidated, teachers and parents expressed concerns over the negative effects of PTG. Examples include late buses and overcrowded classrooms. An issue of equity regarding fair school zones is present in OKCPS. Modifications to a software known as GerryChain, created by MGGG, are currently underway to account for school capacity. The changes will help analyze if the current OKCPS zones are fair or indicate signs of gerrymandering. Final goal of the research is to develop code for generating accurate proposals of school zone partitions. My research will help ensure K-12 students in OKCPS are not being put at a disadvantage. The techniques developed will help judge the fairness of the current school zones in response to PTG. The research will provide a basis for school districts having to grapple with consolidation and reconfiguration.

11. Existence of Optical Vortex Solitons in Photorefractive Media

Zihan Zhang New York University

Advisor(s): Luciano Medina,*New York University*

Optical propagation and vortices in nonlinear media have been intensively studied in modern optical physics. In this paper, we establish constraints regarding the propagation constant and provide an existence theory and numerical computations for positive exponentially decaying solutions for a class of ring-profiled solitons in a type of nonlinear media known as a photorefractive nonlinearity. Our methods include constrained minimization and finite element formalism, and we study the vortex profile and its propagation by fixing the energy flux.

12. Mathematically Modeling Virus Propagation on Social Media Networks with the SMIRQ Model

Justin Browning *University of North Texas*

Arnav Mazumder *University of North Texas* (TAMS)

Gowri Nanda *University of North Texas* (TAMS)

Noelle West *University of North Texas*

Advisor(s): Jianguo Liu, *University of North Texas*

As social networks grow on a global scale and become more complex, users become increasingly susceptible to digital attacks and risk their accounts being compromised. In the United States, it costs businesses billions of dollars annually to handle the damages of such digital attacks. Furthermore, individuals' personal devices and accounts are at risk, potentially costing them their privacy, personal files, and financial security. Hence, to find better methods of preventing these attacks, we developed the probabilistic SMIRQ (pSMIRQ) model, an extension of the standard SEIR model used in epidemiology. Notably, our model improves upon SEIR models for this class of problem by accounting for the connections between individuals in social media networks by generating a random-scale free node network, via the Barabási-Albert (BA) Algorithm, while maintaining the analytical benefits of SEIR models.

13. The Optimal Age To Start Your Supplemental Security Income (SSI)

Linh Khuat *Washington & Jefferson College*
Genesis Dominguez *Washington & Jefferson College*
Alexander Skowron *Washington & Jefferson College*
Advisor(s): Roman Wong, *Washington & Jefferson College*

The social security system is a keystone principal of American society. In essence, it allows retired, or soon to be retiring Americans to collect a source of steady yearly income for the later part of their life. When workers are deciding the appropriate age to collect this income, they can either start at the full retirement age of 67, wait until age 70, or collect early at age 62. In this project, we use a linear differential equation model that considers the time of collection, tax rate, investment return rate, monthly income, and cost-of-living adjustment to tell us the total account balance after a certain amount of time. This will allow us to calculate the catch-up time of each collection scenario at age 62, 67, or 70. We then conclude that age 62 is the best time to start claiming your Social Security Income (SSI) if you live to age 80. If you expect to live to age 90, then age 67 will be the best age to start claiming your SSI.

14. Bee Colony Optimization for Traveling Salesperson Problem: finding optimal tour route to explore New Orleans

Austin Teter *Xavier University of Louisiana*
Eric Scott *Xavier University of Louisiana*
Timmy Ma *Xavier University of Louisiana*
Charles Burnette *Xavier University of Louisiana*
Advisor(s): Timmy Ma, *Xavier University of Louisiana*

Tourism is an integral part of the economy of New Orleans. Touring companies, in particular, provide tourists of this wonderful city with enjoyment and stimulate the economy by exploring restaurants, historical landmarks, and other points of interest. Touring companies, however, rely on having their routes be optimized to achieve these goals in a timely and efficient manner. Using Google Maps to compute point-to-point distances, various preexisting tour routes are evaluated as a Traveling Salesperson Problem (TSP) to be better optimized using Bee Colony Optimization (BCO) algorithms. This investigation is to utilize a BCO algorithm that solves the TSP to find an optimal path for commercial touring by bus or by walking.

15. An ODE model of yaws elimination in Lihir Island, Papua New Guinea

Presley Kimball *Creighton University*
Amy Moore *Elon University*
Jacob Levenson *Washington and Lee University*
Dewey Taylor *Virginia Commonwealth University*
Advisor(s): Jan Rychtar, *Virginia Commonwealth University*

Yaws is a chronic infection that affects mainly the skin, bone and cartilage and spreads mostly between children. The new approval of a medication as treatment in 2012 has revived eradication efforts and now only few known localized foci of infection remain. The World Health Organization strategy mandates an initial round of total community treatment (TCT) with single-dose azithromycin followed either by further TCT or by total targeted treatment (TTT), an active case-finding and treatment of cases and their contacts. We develop the compartmental ODE model of yaws transmission and treatment for these scenarios. We solve for disease-free and endemic equilibria and also perform the stability analysis. We calibrate the model and validate its predictions on the data from Lihir Island in Papua New Guinea. We demonstrate that TTT strategy is efficient in preventing outbreaks but, due to the presence of asymptomatic latent cases, TTT will not eliminate yaws within a reasonable time frame. To achieve the 2030 eradication target, TCT should be applied instead.

16. Modeling the Digital Health Habits of Families of Individuals with Down Syndrome

Tanith Norris *Simpson College*
Katelyn Smith *Simpson College*
Allison Young *Simpson College*
Advisor(s): Heidi Berger, *Simpson College*

Patients with Down syndrome have significant specialized healthcare needs. Our objective was to understand the digital health habits of caregivers of loved ones with Down syndrome to best understand how to optimize virtual specialty care

for these families. To this effect, we adapted an SIR framework to model the spread of medically relevant information in a network of caregivers. This work was based on survey analyses that we conducted on families in the United States and Mexico through the online platform RedCap.

17. Optimal control applied to wolf and moose population dynamics

Maya Bocanegra *California State University, Northridge*
James Kessinger *Greenville University*
Advisor(s): Norma Ortiz-Robinson, *Grand Valley State University*

In this work we formulate a data driven optimal control model involving differential equations to capture the population dynamics of the wolf and moose populations in Isle Royale, Michigan. A numerical solver implemented in the python language is used to obtain solutions and these are analyzed to identify optimal strategies for augmentation of the declining wolf population. This work was done as part of the REU program at *Grand Valley State University*.

18. 3-Dimensional Agent-based Model of Neural Activity in the Central Nucleus of the Amygdala During Pain

Carley Reith *Duquesne University*
Advisor(s): Rachael Neilan, *Duquesne University*

The central nucleus of the amygdala (CeA) is a region of the brain important in pain processing. Neurons within the CeA expressing protein kinase c-delta (PKC-δ) or somatostatin (SOM) have opposing roles in pain modulation. We created a 3-dimensional agent-based model (ABM) of these neuron populations and their connectivity in the CeA to predict system-level measures of pain. During the model's initialization, a 3-D spatial domain representing the CeA and its sub-nuclei is created and agents representing individual neurons are created with cell-type specific properties and behaviors. Finally, a network of directed links between the neurons is established. During each model time step, neuron firing rates update based on an external stimulus, and a network of directed links sends inhibitory signals between neurons. A measure of pain is calculated as the difference in the cumulative firing rates of PKC-δ and SOM neurons. Results demonstrate the model's ability to produce realistic pain output for constant and non-constant stimuli. The model allows for testing hypotheses about pain-related neurons in the CeA without committing significant time and resources to laboratory experiments.

19. Evolutionary Game Theory Simulations

Kathryn Cantrell *Loyola University Chicago*
Advisor(s): Peter Tingley, *Loyola University Chicago*

While Game Theory often will reward noncooperative and antisocial strategies, the real world exhibits the need for cooperation. In this poster, I examined ways that cooperation can be encouraged through analyzing various setups including distributions of players across a given field. Players are randomly selected to play nearby players and perform a coded version of an altered version of the Prisoner's Dilemma. Players may either form offspring or be removed from play, determined by their scores. As more rounds are played, we note the proportion of cooperative versus noncooperative strategies and how they may change over the course of the simulation.

20. Simulation of Habitat Degradation in Marine Ecosystems

Kate Bove *Loyola University Chicago*
Advisor(s): Peter Tingley, *Loyola University Chicago*

This project aims to utilize elements of evolutionary game theory to predict how certain marine species will interact with each other when their habitats are taken away. Based on a literature review, a simulation will be coded in Python and incorporates a 10 x 10 grid, resembling a coral reef marine habitat, that includes an amount of each species in each square. Species will be allowed to interact between adjacent squares. Evolutionary games between individuals using fitness strategies can be encoded with a payoff matrix, which then reveals payoff equilibriums outcomes. Allowing individuals to reproduce in proportion to their scores will model natural selection. Models for mathematical components involved in this project is the cooperation games, Chicken and Prisoner's Dilemma. Chicken will be used as a basis for the simulation. Based on the payoff matrices for each species in the simulation, a prediction will be made on how species interact in real life scenarios.

21. Music Feature Classification: Using Techniques of Machine Learning to Detect Genre

Ashley Mullan *University of Scranton*
Advisor(s): Murong Xu, *University of Scranton*

Genre is a fluid descriptor used to categorize musical works. Historically, it has been assigned manually due to the challenge of the task. Representatives of a particular genre may share characteristics related to the instrumentation, rhythmic structure, or pitch content of the music. It is of interest to automatically classify music files in a way that aids music information retrieval. Machine learning algorithms, such as neural networks and random forests, have shown remarkable efficiency in classifying music genres. In this project, we created a novel dataset that contains 100 music files spanning 4 musical genres. After extracting sound features such as Mel Frequency Cepstral Coefficient (MFCC), Spectral Centroid, and Zero Crossing Rate from the time series representation of a music file as well as two musical attributes essential to music theory, time signature and key signature, we trained a two-layered convolutional neural network to classify genre. In addition, we used a random forest to evaluate the effectiveness of each feature. The details of this study and the results will be presented in the poster.

22. An Analysis of Social Choice Functions & The Mathematics of Voting

Brandon Calia *Temple University*
Advisor(s): Jeromy Sivek, *Temple University*

Elections are a fundamental component of society, and choosing how those elections will be conducted is of great importance. Ranked choice voting is a common solution to this, where voters are given a set of candidates over which they can rank their preferences. Unfortunately, the different methods used to select a winner under this system are all imperfect, failing to meet certain reasonable criteria. This is asserted by Kenneth Arrow's Impossibility Theorem. It is possible for methods that appear perfectly fair to produce a result that appears unfair, of which we have many documented examples. Using python simulations, mathematical theory, and data from historical ranked choice elections, we look to analyze the potential consequences of implementing various choice functions into our elections.

23. A Statistical Analysis of Stream Litter Accumulation in the Oklahoma Metropolitan Area Considering Levels of Biodegradability

Tianna Samuel *University of Central Oklahoma*
Advisor(s): Emily Hendryx and Matthew Parks, *University of Central Oklahoma*

It is increasingly difficult to go anywhere in the Oklahoma City Metropolitan area and not see litter. This research, supported by an MAA NREUP at the *University of Central Oklahoma*, addresses the growing concern of Oklahoma's litter problem with a focus on small urban-influenced waterways. Using data we collected, we perform a statistical analysis based on the level of long-term biodegradability of litter found in different segments of selected local waterways. We look for relationships between variables including litter type, location within streams, and stream characteristics. The results of this research can lead to a better understanding of litter distribution and long-term trends in accumulation of varying litter types within small metropolitan waterways, informing worldwide efforts to reduce trash in urban streams.

24. Modeling Influence of Urban Development on Litter Accumulation in Metropolitan Waterways

Oday Shwayyat *University of Oklahoma*
Advisor(s): Emily Hendryx and Matthew Parks, *University of Central Oklahoma*

The amount of anthropogenic waste being generated and infiltrating natural environments is a growing concern for current and future generations. Despite mitigation efforts, an increasing amount of litter enters our urban waterways annually. Additionally, the effects of climate change, particularly changing weather patterns, may exacerbate litter accumulation as a direct result of intensifying stormwater runoff. This study uses differential equation modeling to account for surrounding zoning area types, influence of stormwater drainage points, and their potential impact on litter accumulation in waterways. To inform our models, we utilize our collected data from small waterways in the Oklahoma City metropolitan area. In studying these relationships, we quantify the effects of urban development on litter in waterways. This work is completed as part of an MAA NREUP at the *University of Central Oklahoma*.

25. Data Augmentation for Tabular Data Sets

Christina Dietrich *Simpson College*
Jeffrey Roberts *Simpson College*
Jason White *Simpson College*
Advisor(s): Marilyn Vazquez, *Simpson College*

Often times after performing case studies, insufficient data is collected to create accurate predictive models. For this reason, data augmentation has become an increasingly popular research area. The goal of data augmentation is to create new data points without collecting any new data since it can be a costly and time-consuming process. We are developing new methods of augmenting tabular data. To demonstrate the data augmenting capabilities of our methods, we apply them to various data sets. By inflating the data while maintaining the intrinsic patterns, the new synthetic data set helps to get better predictions. This project is part of the 2022 Bryan Summer Research Program in Mathematics.

26. Withdrawn

27. Mathematical Approaches to the Archeological Refit Problem

Nzingha Joseph *Carleton College*
Evelene Zhang *Carleton College*
Advisor(s): Rob Thompson, *Carleton College*

Archeologists often look for evidence in the remnants of the past, like old bones, to better our understanding of human activity. This evidence is growing increasingly quantitative, incorporating more sophisticated tools from applied mathematics, statistics, and computer science to unearth the stories that broken bones can tell: stories of early human and animal activity and interaction.

Working with a consortium of researchers known as AMAAZE (https://amaaze.umn.edu), we aim to create digital reassemblies of broken bones. Using 3D scans of these bones, we isolate the break faces, compare the shapes of these faces, and realign them to form a more complete reconstruction of the original bone. This process blends ideas from linear algebra, computational geometry and optimization. .

28. Applying Optimal Control With SQP to Cycling Performance Represented by Constituent Course Elements

Trevor Loe *Cal Poly San Luis Obispo*
Madison Lytle *Cal Poly San Luis Obispo*
Callan Whitney *Cal Poly San Luis Obispo*
Advisor(s): Charles Camp, *Cal Poly San Luis Obispo*

Improvements in fitness and training for top-performing cyclists have pivoted modern road biking competitions towards contests of strategy, necessitating deeper investigation into optimal approaches over long races. Previous work viewed this problem as a constrained optimal control problem neglecting a set of nonlinear interactions within the bioenergetic model for cyclists' power output. This paper includes these interactions and finds numerical solutions to the optimal control problem via sequential quadratic programming methods. While preserving consistency with previously published simpler models, this model can additionally address environmental conditions such as wind and track curvature. Comparing simulations of various cyclist specializations on real-world courses, the model suggests that climber cyclists exhibit a significant underutilization of energy compared to the utilization by time trial specialists. We explore whether this result represents an inherent advantage for time trial specialists on the simulated courses or a potential shortcoming of the bioenergetic model. This paper was awarded the MAA award for Problem A in the 2022 Mathematical Contest in Modeling.

29. Day Trading in Bitcoin and Gold

Jane Zhang *Columbia University*
Makoto Powers *Columbia University*
Brianna Han *Columbia University*
Advisor(s): George Dragomir, *Columbia University*

Day trading is a delicate predictive problem with a goal to maximize total returns while simultaneously balancing a fee for each purchase and sale. It's easy to look back and recognize signals in a stock's movement that lead to its

eventual rise or decline, but predicting movements before it happens is far more complex. Additionally, frequency trading has at most 40 years of data available - a small data set difficult to glean trends from. Thus, it is important to develop a model that can function using little to no prior data. Two assets of interest are bitcoin and gold, which contrast greatly in stability. Our model is an instantaneous logic model that implements a combination of momentum trading and reversion to mean theories to generate buy and sell commands of these assets. We further improved it by accounting for extreme market conditions that might occur in volatile markets like bitcoin. Various parameters in our model were then optimized through experimental simulations. After implementing our model on daily gold and bitcoin prices from 2016 to 2020, we generated $219,799 from a principal investment of 1000$, beating the market in returns.

30. A Quantum Resource Theory for One-Way Information

Jack Rausch *Creighton University*

Advisor(s): Randall Crist, *Creighton University*

In quantum information theory, the one-way information of the joint evolution of a composite system quantifies the causal relationship between systems. Given a composite two systems, an algorithm is used to create a state $\rho^{A'ABB'}$ which quantifies the one-way information via the measure $R\left(\rho^{A'ABB'}\right) = I\left(\rho^{B} : \rho^{A'AB'}\right) - I\left(\rho^{B} : \rho^{B'}\right)$. A quantum resource theory offers a new perspective to view one-way information. A quantum resource theory examines a problem under a set of physically meaningful limitations which identify certain operations as free (can be used without limitations) and others as resources (operations with limitations or costs). We define a quantum resource theory for one-way information based on the measure $R\left(\rho^{A'ABB'}\right)$, showing that: R is an additive measure, all free states contain 0 one-way information, the free operations contain all unitary operators $U_{AB} = U_A \otimes U_B$, and R is monotonic under free operations, but not under the restricted operations. We also examined if this resource theory is asymptotically continuous.

31. Utilizing Markov Chains to Estimate Allele Progression through Generations

Ronit Gandhi *University of Nebraska-Lincoln*

Clay Cressler *University of Nebraska-Lincoln*

Advisor(s): Bo Deng, *University of Nebraska-Lincoln*

All populations display patterns in allele frequencies over time. Some alleles cease to exist, while some grow to become the norm. These frequencies can shift or stay constant based on the conditions the population lives in. If in Hardy-Weinberg equilibrium, the allele frequencies stay constant. Most populations, however, have bias from environmental factors, sexual preferences, other organisms, etc. We propose a stochastic Markov chain model to study allele progression across generations. In such a model, the allele frequencies in the next generation depend only on the frequencies in the current one. We use this model to track a recessive allele through successive generations. Eventually, the allele will be “cancelled out” by the genotype of an organism becoming homozygous dominant. We estimate the number of generations it will take for this allele to be ”cancelled out” by computing a hitting time in the Markov chain. This will allow us to efficiently communicate the trends of allele frequencies and estimate the speed of growth or decay of alleles.

32. Neuronal Activity and Graph Theory

Ally Larsen *Creighton University*

Advisor(s): Nathan Pennington, *Creighton University*

Neurons, the specialized cells of the brain, communicate with one another via action potentials. This electrical signaling serves as the source of thoughts, movement, respiration, and nearly all bodily functions. Brain networks and neuron signaling are often modeled and studied using graphs. We generated three unique graphical representations of a brain network and simulated neural activity to analyze how network structure impacts neuronal firing.

33. The Silver Bullet of Lycanthropy Interventions: A mathematical modeling study

Anna Morelli *Siena College*
Advisor(s): Scott Greenhalgh, *Siena College*

It's close to midnight and something evil's lurking in the dark! While it could be almost any unnatural creator, we consider the plight of werewolves and develop a mathematical model to describe their inter-species dynamics with humans, in conjunction with the lunar cycle of the moon. Using our model, we inform on how humanity may survive an outbreak of lycanthropy (the disease that causes one to become a werewolf) through the effective deployment of silver ammunition and identify criteria for human-werewolf co-existence. As proof of concept, we apply our model to Hungary (one of the historical homes of werewolves), and determine lives saved by silver weapons under a variety of silver production scenarios.

34. An Agent-Based Model of Environmental Transmission of *C. difficile* in Healthcare Settings

Malen De la Fuente *Lewis University*
Maximus Lewis *Lewis University*
Advisor(s): Brittany Stephenson and Cara Sulyok, *Lewis University*

Clostridioides difficile (*C. difficile*) is one of the most frequently identified healthcare-acquired infections in United States hospitals. Colonized patients, both symptomatic and asymptomatic, shed *C. difficile* endospores that can survive for long periods on surfaces outside the host and are resistant to many commonly-used disinfectants. Transmission pathways can include contact with endospores on fomites, objects likely to carry infection. This project will investigate the relative contribution of environmental pathways to *C. difficile* transmission in healthcare settings using an agent-based model (ABM). Our model simulates the spread of *C. difficile* within a hospital while focusing on frequently-touched surfaces in each hospital room. Simulations allow us to examine the role that surfaces with varying touch frequencies play in patient colonization and determine effective control strategies to mitigate the spread of *C. difficile* in healthcare settings.

35. The effect of non-exponentially distributed infectious periods on cost-effective chlamydia treatments

Jack Farrell *Siena College*
Owen Spolyar *Siena College*
Advisor(s): Scott Greenhalgh, *Siena College*

Pandemics devastate the health and economic well-being of society. Consequently, it is imperative to accurately predict impending infectious disease outbreaks and determine effective disease interventions through the use of epidemiological tools, such as compartmental models. Unfortunately, many ODE compartmental models require the assumption of an exponentially distributed infectious period, despite it being well-known that such an assumption is often biologically invalid. As this in turn could negatively impact the evaluation of disease interventions, we explore the use of an entirely new family of ODE compartmental models that feature non-exponentially distributed infectious periods. Specifically, we investigate their ability to predict the cost-effectiveness of disease interventions. We apply our model to evaluate the effects of different antibiotic treatments on the trajectory of Chlamydia trachomatis in the United States, and illustrate how variation in the infectious period's mean, standard deviation, skewness, and kurtosis alters predictions of incidence averted and disability adjusted life-years saved over a 5-year horizon.

36. An Agent-Based Model of COVID-19 Transmission at *Lewis University*

Austin Kind *Lewis University*
Advisor(s): Brittany Stephenson, *Lewis University*

As new variants emerge, the virus causing COVID-19 continues to spread in the United States and across the world. It is essential to find ways to mitigate its spread, and mathematical models can provide insight into the complex transmission dynamics of COVID-19. In this work, we formulate an agent-based model of the spread of COVID-19 at *Lewis University* in NetLogo. Using this model, we are able to compare the efficacy of a variety of control interventions and combinations of interventions on reducing COVID-19 transmission at Lewis.

37. Dynamics of a Stage-Structured Predator-Prey Model with Cannibalism

Kasey Cooper *Ursinus College*
Ava Dreher *Binghamton University*
Caroline McCrorey *Bellarmine University*
Advisor(s): Eric M. Takyi, *Ursinus College*

Cannibalism, or intraspecific predation, is the act of an organism consuming another organism of the same species and is universally found among different living organisms. Examples observed in nature include cannibalism in polar bears (Ursus maritimus), chimpanzees (Pan troglodytes), brook trout (Salvelinus fontinalis), etc. In real life, living organisms grow in stages. Therefore, in this work, we study the effects of cannibalism in a stage-structured predator-prey model. The system is analytically studied and is found to exhibit various dynamics including limit cycles and a Hopf bifurcation. We perform various numerical experiments to support our theoretical findings and discuss the ecological implications of our results. This is a Research Experience for Undergraduates and is supported by the National Science Foundation via grant number 1851948 at Ursinus College.

38. A Mathematical Model of *C. difficile* Prevention and Control in Healthcare Settings

Justyna Sliwinska *Lewis University*
Advisor(s): Cara Sulyok, *Lewis University*

Clostridioides difficile (*C. difficile*) is the leading cause of infectious diarrhea and one of the most frequently identified healthcare-acquired infections in United States hospitals. Patients receiving antibiotic treatment contribute significantly to the spread of *C. difficile*, as antibiotic use increases the likelihood of developing infection. Patients with a *C. difficile* infections shed endospores which are difficult to eliminate from surroundings. The mathematical model developed in this work uses a system of differential equations representing the spread of *C. difficile* and incorporates several relevant transmission pathways, including patient contact with low- and high-touch frequency fomites as well as through healthcare workers spreading the infection to other patients. Healthcare professionals may apply results from this work into practice to limit the spread of this contagious infection in healthcare settings.

39. Mathematically Modeling the Interactions of Community- and Hospital-Acquired *C. difficile* Infections

Sara Gongora *College of DuPage*
Edmonde Olongo *Dominican University*
Advisor(s): Cara Sulyok, *Lewis University*

Clostridioides difficile (*C. difficile*) is an infection-causing bacterium commonly contracted by patients in medical institutions in the United States. *C. difficile* creates endospores which can survive in harsh conditions for long periods. This bacteria can be spread either through contact from person to person or with surfaces hosting the endospores. Patients who are currently on or have recently taken an antibiotic are susceptible to contracting *C. difficile* as certain bacteria in the stomach become weaker, creating an ideal environment for *C. difficile* to grow as its spores spread faster without competition. This study quantifies the spread of the *C. difficile*, differentiating between community- and hospital-acquired infections. Using a system of ordinary differential equations distinguishing between individuals in the local community and hospital as well as varying environmental surfaces, results can be used by local communities to predict and limit potential outbreaks of *C. difficile*.

40. Entropy along the 1/3 Vein of the Mandelbrot Set

Hayden Lieb *Stony Brook University*
Longzhen Chen *Stony Brook University*
Advisor(s): Yusheng Luo, *Stony Brook University*

Our research, taking place at *Stony Brook University*, aims to study the topological entropy of a quadratic map along the 1/3 vein restricted to its Hubbard tree using Thurston-Milnor kneading theory. We discuss the admissible kneading determinants along this vein and and use this information to identify intervals of constant topological entropy along this vein.

41. A Shortcut to Cutting Triangles

Olivia Sylvester *SUNY Fredonia*
Advisor(s): Robert Rogers, *SUNY Fredonia*

Given a 3, 4, 5 right triangle, what is the shortest cut which will divide this triangle into two pieces of equal area? We will provide the answer to this question and present how this can be generalized to any triangle. The method used to obtain this answer can be generalized to dividing any triangle into two pieces in which the ratio of their areas is any fraction between 0 and 1. For example, the areas could be â. . . " to â. . . " . This question can be extended in a number of ways. For example, suppose one wished to divide a quadrilateral into two pieces whose areas are a given ratio. Another direction is extending this to finding a minimal area plane that divides a tetrahedron into two regions whose volumes have a given ratio.

42. Applying Fractal Geometry to Measure Coastline in Hawaii

Archana-Gayle Delos Reyes *University of Hawaii - West O'ahu*
Advisor(s): Veny Liu, *University of Hawaii - West O'ahu*

A coastline features an endless array of bays and promontories at all scales. Making it almost impossible to decipher an accurate measurement of any coastline. Not only mathematicians, geographers, and philosophers are interested in the coastline paradox, but it is also creating significant challenges for current and future stakeholders in the coastal zone (Stoa, 2020). By applying existing fractal geometry methods, we want to estimate a more plausible measurement of the coastline with consideration of the coastline paradox. The presentation will share our best estimate in measuring the length of the Hawaiian islands coastline and comparing this estimation to the Congressional Research Service (CRS) and the National Oceanic and Atmospheric Administration (NOAA) models.

43. All Tangled Up: Geometry of Tangloid Curves

Kyle Kelley *Kenyon College*
Advisor(s): Carol Schumacher, *Kenyon College*

Children and mathematicians alike are fascinated by the intricate patterns created by Spirographs, a wheel with a pen at a given point rolling around a circle. The mathematical names for Spirograph curves are hypotrochoid and epitrochoid, depending on whether the wheel rolls around the inside or outside of the circle. Our research investigated the properties of curves created by this process, with one major change: what if the outside isn't a circle at all, but a series of connected circular arcs that form a closed loop? The resulting curves we called "tangloids". Some characteristics of tangloids have been studied in the past by former Kenyon students, Seth Colbert-Pollack and Micah Fisher. Our research builds on theirs, exploring how the particular set-up used to create a tangloid curve is reflected in the geometry of the resulting figure.

44. Euclidean Theorems in Spherical and Hyperbolic Geometries

Kai Williams *Swarthmore College*
Michael Perez Palapa *Bucknell University*
Advisor(s): William Dickinson, *Grand Valley State University*

Euclidean, spherical and hyperbolic are three geometries with a common thread. We believe that every Euclidean result has an analogous result in the other two. Many of the basic theorems from Euclidean geometry, like the Pythagorean Theorem and the Law of Cosine and Sines have already be generalized to these other geometries and serve as the tools to help generalize other theorems. In this poster we will explore the generalization of some more advanced results in Euclidean geometry to both spherical and/or hyperbolic geometry. This research was conducted as part of the 2022 REU program at Grand Valley State University.

45. Withdrawn

46. CAT(k) Surfaces

Adam Tsou *Stony Brook University*
Hechen Hu *Stony Brook University*
Saajid Chowdhury *Stony Brook University*
Advisor(s): Matthew Romney, *Stony Brook University*

Alexandrov geometry concerns with geometric properties of special metric spaces where notions like angle are well-defined. One can define a curvature bound on surfaces in two ways: the first is by comparing triangles in with constantly curved surfaces(i.e. spheres, hyperbolic disk, Euclidean space), which gives the notion of CAT(k) surface; the other is by bounds on the sum of angle deficits of finite families of non-overlapping triangles, which gives the notion of bounded integral curvature. In this project we prove that CAT(k) surfaces have bounded integral curvature, which is known colloquially but no satisfactory proof was known.

47. Efficiently Computable, Stable Metrics on the Space of Chiral Merge Trees—Does One Exist?

Zach Schlamowitz *University of Arizona*
Charley Kirk *Juniata College*
Jose Arbelo *Ursinus College*
Antonio Delgado *Ursinus College*
Nicholas Scoville *Ursinus College*
Advisor(s): Christopher Tralie, *Ursinus College*

When analyzing data, it is often of interest to categorize subsets based on similarity. In the case of time series, similarity can be found in key features like the placement and heights of prominent extrema. In this work, we develop a notion of distance between time series to enable such classification. We build this notion by representing time series via merge trees and triangulations of convex polygons, which capture the aforementioned key features. Additionally, such representations abstract away from ”time-warping,” or reparameterization, allowing reparametrized time-series to be classified together. In particular, we seek a distance metric that is robust to noise, as or more discerning than previously proposed distances, and efficiently computable. We show proof of the existence or nonexistence of such a distance metric. While previous work has satisfied the majority of these constraints, no proposed distance metric has simultaneously met them all.

48. The Art of Knot Theory

Isabel Renteria *Loyola University Chicago*
Advisor(s): Carmen Rovi, *Loyola University Chicago*

Knot theory is a field of topology which studies mathematical knots. A mathematical knot is a string that has been knotted with itself and whose two ends have been joined together so as to have one never-ending string. The purpose of this project is to explore knot theory as it applies to the artwork of the Book of Kells, which is held at Trinity College Dublin. The Book of Kells, created around 800 A.D., is an illuminated manuscript of the four gospels of the New Testament. The illustrations in the book contain many Celtic knots, that can be classified mathematically. We discovered recurring patterns of mathematical knots and links that we were able to classify. Further, there were some knots that were not classifiable with basic techniques but are commonly observed in the Book of Kells. Knot classification is still an open problem in mathematics and it has fascinating applications that range all the way from analyzing arts to physics and even understanding the knotted nature of our own DNA strands.

49. Three Colored Bipartite Ramsey Numbers of Small Double Stars

Ella Oren-Dahan *Montclair State University*
Advisor(s): Deepak Bal, *Montclair State University*

Ramsey Theory is a field that studies the mathematical phenomenon that sufficiently large objects unavoidably contain structured sub-objects. In this research, we study the bipartite Ramsey number of double stars. That is, we attempt to answer the following question. How large does an edge-colored complete bipartite graph have to be in order to guarantee the existence of a double star such that all of its edges are the same color? We use mathematical techniques as well as computing to improve the best-known bounds and find exact values in some cases.

50. Spanning Trees of Random Cubic Graphs with Many Leaves

Sarah Acquaviva *Montclair State University*
Advisor(s): Deepak Bal, *Montclair State University*

We study the problem of maximizing the number of full degree vertices in a spanning tree T of a random cubic graph $G(n,3)$; that is, the number of vertices whose degree in T equals its degree in G. In cubic graphs, this problem is equivalent to maximizing the number of leaves in T and minimizing the size of a connected dominating set of G. We provide an algorithm that attempts to maximize the number of full degree vertices, $\varphi(G)$, on random cubic graphs. We analyze our algorithm using the differential equations method to obtain $\varphi(G) \geq 0.437n + o(n)$, an improvement on the previously known lower bound.

51. A New Upper Bound for the Site Percolation Threshold of the Square Lattice

Samuel Oberly *Johns Hopkins University*
Advisor(s): John Wierman, *Johns Hopkins University*

The upper bound for the site percolation threshold of the square lattice is reduced from 0.679492 to 0.666894, providing the first improvement since 1995. The bound is obtained by using the substitution method with new computational reductions which make calculations for site models more efficient. The substitution method is applied, comparing the site percolation model on a self-matching lattice to the square lattice site percolation model in a two-stage process.

52. Using Graph Theory to Analyzing Data

Yash Thakur *Montclair State University*
Advisor(s): Aihua Li, *Montclair State University*

In this research project we will analyze a data table using connectivity theory in graph theory. The data is about the transportation information about certain cities or world happiness report. We will define a mathematical distance between each pair of cities and construct a weighted graph based on the distance. The global and local connectivities of the graph will be calculated to reflect the clustering situation of the data

53. Graph Theoretical Modeling of Web Graphs in Self-Assembling DNA

Austin Schaibley *Lewis University*
Chiara Hurd *Lewis University*
Advisor(s): Amanda Harsy, *Lewis University*

Self-assembly is a term used to describe the process of a collection of components combining to form an organized structure without external direction. The unique properties of double-stranded DNA molecules make DNA a valuable structural material with which to form nanostructures, and the field of DNA nantochnology is largely based on this premise. By modeling nanostructures with discrete graphs, efficient DNA self-assembly becomes a mathematical puzzle. These nanostructures have wide-ranging applications, such as containers for the transport and release of nano-cargos, templates for the controlled growth of nano-objects, and in drug-delivery methods. This research project centers around the exploration of the graph theoretical and combinatorial properties of DNA self-assembly of Web Graphs in three different laboratory conditions.

54. Packing Edge-Colorings of Graphs

Cicely Henderson *Wellesley College*
Camille Kennedy *Northwestern University*
Advisor(s): Michael Santana, *Grand Valley State University*

A proper edge-coloring of a graph is a labeling of the edges so that any two edges receiving the same color cannot be incident to one another. A strong edge-coloring of a graph is a labeling of the edges so that any two edges receiving the same color cannot be incident to one another, nor can they be incident to a common edge. Both of these are well-studied parameters on graphs, and have led to a new notion of graph coloring called packing edge-coloring. More specifically, a $(1^j, 2^{k-j})$-packing edge-coloring of a graph is a labeling of the edges using k colors, in which j of the colors follow the properties of a proper edge-coloring, and the remaining $k - j$ colors follow the properties of a strong edge-coloring. In this poster we will present our results on packing edge-colorings, with an emphasis on graphs

with maximum degree four. This work was completed during the 2022 GVSU Summer Mathematics REU under the mentorship of Dr. Michael Santana.

55. Establishing a tight bound on propagation time of the three-cyclic multicolor forcing process on paths

Nicholas Radley *La Salle University*
Advisor(s): Janet Fierson, *La Salle University*

A 2020 paper introduced the multicolor forcing process, a generalization of the zero forcing process on graphs. Here, we introduce the notion of a cycle contraction for the multicolor forcing process, generalizing the idea of a color contraction. We then consider the three-cyclic multicolor forcing process on paths, and extend the notion of cycle contraction beyond its original definition in order to derive a recursive sequence of vectors which encodes information about the forcing process. We then use this sequence to prove a tight bound on the propagation time of the three-cyclic multicolor forcing process on paths.

56. Fault-Free Tilings Using Polyominoes

Anika Homan *Dordt University*
Samuel Webb *Grand Valley State University*
Advisor(s): Lauren Keough, *Grand Valley State University*

A tiling of a rectangular board is a filling of the board with tiles in such a way that each space is covered by exactly one tile. In 1981, Ron Graham defined a fault-line of a tiling as a straight line (parallel to one of the sides of the board) that cuts through the board but does not cut through any tile. A fault-free tiling is a tiling without any fault-lines. In 2019, Montelius extended the notion of fault-free tilings of $p \times q$ rectangles with 2×1 dominoes to boards that were cylinders, tori, or Mobius strips. We extend this research to tilings with polyominoes, which are shapes formed by connecting unit squares along edges. This research was conducted as part of the 2022 REU program at *Grand Valley State University*.

57. Polytopes Associated to Graphs

Ethan Jakubowski *Lewis University*
Advisor(s): Marie Meyer, *Lewis University*

Polytopes are generalizations of the familiar 2-dimensional polygons and 3-dimensional polyhedra. As geometric objects with combinatorially interesting properties, polytopes have a variety of applications in diverse areas which include coding theory, linear programming, optimization, physics, and topology, just to name a few. Recently, there is interest in studying polytopes associated with graphs. One such construction is the Laplacian polytope, which uses the Laplacian matrix of a simple connected graph to form a polytope by considering the rows of the matrix as vertices of the polytope. We analyze these polytopes using Ehrhart Theory and present new results on the interplay between the discrete structure of a graph and the geometric structure of the Laplacian polytope for different families of graphs.

58. (Drag and) Drop It Like It's Hot: Comparison of Educational Software for Teaching Natural Deduction and Axiomatic Derivation

Mei Rose Connor *Stony Brook University*
Advisor(s): Paul St. Denis, *Stony Brook University*

Natural Deduction and Axiomatic Derivation, two of the forms of proof calculus utilized by logicians, are an essential part of a logic course. Advancements in software have allowed for programs that can help students construct proofs in both Natural Deduction and Axiomatic Derivation. I will be providing a comparison of such software, with a primary focus on the program DragLogic which I developed as a Student Coder at the Teaching and Learning Lab at *Stony Brook University*. I will be comparing DragLogic, a system for Axiomatic Derivation which the user operates via drag-and-drop, with its spiritual predecessor, David Kaplan's Logic 2010, a Natural Deduction software system. A different spiritual successor of Logic 2010, Brian Rabern's eLogic will also be evaluated. Comparison will also be drawn between another piece of drag-and-drop proof software, Terence Tao's QED.

59. Analyzing the Impact of Alternate Assessment on Growth Mindset in Mathematics Courses

Dayanna Sanchez *Lewis University*
Advisor(s): Michael Smith, *Lewis University*

The mindset of a student can help determine how well that student will academically perform both inside and outside of the classroom. Instructors can support student development of a growth mindset and academic growth by encouraging and recognizing students for their growth in learning and by using assessment practices that support the idea that through practice and effort concepts can be understood. Alternate assessment techniques such as mastery grading, specifications grading, and standards-based grading are assessment techniques professors are implementing in order to support a growth mindset of learning. By analyzing pre- and post-surveys with questions adapted from Dweck's Mindset survey, we will explore whether there is a difference in the growth mindset between various cross-sections of student populations between classes and whether the growth mindset of students changed by the end of the semester.

60. The Effectiveness of Three Promising Psychological Interventions on Math Anxiety and Academic Performance

Kim Ngan Huynh *Stevenson University*
Allison Hope *Stevenson University*
Advisor(s): Mark Branson, *Stevenson University*

Math anxiety can be a barrier that prevents some undergraduate students from being successful in college. To help students with their math anxiety and academic performance, we examined the effectiveness of three promising psychological interventions: expressive writing, eye movement desensitization and reprocessing (EMDR), and a short-breathing exercise. Participants were divided into four groups: three intervention groups and a control group. Participants in the intervention groups performed the intervention twice following the lead of the researchers. Math anxiety level of all participants was measured using a standard instrument, the Abbreviated Math Anxiety Survey, before and after each intervention. The short-breathing exercise was shown to produce a significant reduction in math anxiety. We also found a negative correlation between math anxiety level and exam scores.

61. Developing effective supplemental instruction in General Education Mathematics Courses

Dani Ellis *University of Nebraska at Omaha*
Sydney Fuhrman *University of Nebraska at Omaha*
Advisor(s): Michael Matthews, *University of Nebraska at Omaha*

As Learning Assistants in college classrooms, we noticed how some College Algebra students were struggling to keep up in the course and master the material. To confront this issue we planned review sessions to supplement classroom instruction, but without a model to structure the sessions around, they proved to be inefficient and ineffective. We replaced one-on-one instruction with group-based instruction by creating stations for each chapter of the curriculum. With one tutor at each station, this model also helped create more opportunities for peer teaching. This newly created structure allowed the tutors to provide more quality instruction as they were able to prepare for focused content. The transition from individual to group instruction gave students more practice and support not only from tutors but from peers as well.

62. The Matrix Method Approach to the 3x+1 Problem

Rajatava Mukhopadhyay *IISER Thiruvananthapuram*
Advisor(s): Srilakshmi Krishnamoorthy, *IISER Thiruvananthapuram*

Probably one of the most enigmatic unsolved problems, one that can be explained to even a child who knows multiplication is the Collatz Conjecture aka the $3x+1$ problem. While the problem may seem simple to prove, but the solution to the problem has eluded mathematicians for the past few decades. The problem has been already been viewed under the lights of Number Theory; Dynamical Systems; Ergodic Theory; Mathematical Logic and Theory of Computations; Probability Theory and Stochastic Processes; Computer Science: Machine Models, Parallel and Distributed Computation. So far the problem has been verified to hold true for all natural numbers less than $(2^{62} + 2^{60})$. So in this matrix method we are going to use the matrix-vector equations of the form $AX = B$, where A is an invertible (not always) diagonal square matrix and X is the initial vector, where as B is the resultant vector and on top of this we

exploit the previously stated property of natural numbers to come down to a very interesting result which may aid the simplification of the problem's statement and its solution.

63. Sums of k-th powers in ramified 2-adic rings

Irene Deegbe *McDaniel College*
Sarah England *McDaniel College*
Sarah Stewart *McDaniel College*
Advisor(s): Spencer Hamblen, *McDaniel College*

Generalization of Waring's Problem—that for every natural number k there exists an integer $g(k)$ such that every natural number can be written as the sum of at most $g(k)$ k-th powers—have been studied in a variety of contexts from algebraic number fields to non-commutative groups. We will examine values of $g(k)$ for rings of integers of certain wildly ramified extensions of $\mathbf{Q}_2$.

64. Analysis of the continued fraction digits of π

Arpandeep Khatua *University of Illinois at Urbana-Champaign*
Zhuo Zhang *University of Illinois at Urbana-Champaign*
Alex Jin *University of Illinois at Urbana-Champaign*
Xiaotong Li *University of Illinois at Urbana-Champaign*
Shreyas Singh *University of Illinois at Urbana-Champaign*
Efstathios Konstantinos *University of Illinois at Urbana-Champaign*
Advisor(s): AJ Hildebrand, *University of Illinois at Urbana-Champaign*

One of the most important mathematical constants is the number $\pi = 3.1415926\ldots$ A natural question is whether the digits of π behave, in an appropriate sense, like a random sequence of digits. Prior research, based on 60 trillion computed decimal digits of π, showed no significant statistical deviation from the randomness hypothesis. We are interested in the continued fraction representation of $\pi = [3; 7, 15, 1, 292, \ldots]$ It is known that the continued fraction digits of almost all real numbers follow the Gauss-Kuzmin distribution $P(k) = \log_2\left(1 + \frac{1}{k(k+2)}\right)$, but whether this is true for π is an open problem. In our research, we performed a large scale statistical analysis of the first 30 billion continued fraction digits of π to provide compelling evidence that these digits behave like those of a random real number. In addition, we investigated the occurrences of patterns such as a string of k identical digits and the frequencies of special sets of numbers in continued fraction expansions of almost all real numbers. This work is based on research performed at the Illinois Geometry Lab at the University of Illinois at Urbana-Champaign.

65. Developing Solutions For Two Diophantine Equations And Their Behaviors

Arnold Rosas *Montclair State University*
Advisor(s): Aihua Li, *Montclair State University*

This research focuses on the Diophantine equation $x^3 + y^3 + z^3 = 1 \bmod p$, where p is a prime number. Every solution is derived from a 3-part partition of 1 using perfect cubes modulo p. We first categorize the type of solutions into 3 types and then count the number of each type. In case $p \equiv 2 \bmod 3$, we give explicit formulas for each type. When $p \equiv 1 \bmod 3$, we investigate the special solutions involving 1. The number of type I solutions is given. A bound for the number of Type II solutions is estimated. Theorems about quadratic residues and Legendre symbol are the main theoretical tools for this study. The software Python is the computational tool in checking and finding solutions to the equation. Future work includes identifying distribution patterns of the solutions to the equation and developing the exact formula for the number of type II solutions in the case of $p \equiv 1 \bmod 3$.

66. Monodromy Groups of Belyi Lattes Maps

Zoe Batterman *Pomona College*
Ines Chung-Halpern *Yale University*
Ebenezer Semere *Pomona College*
John Clark *University of Texas at Austin*
Advisor(s): Edray Goins, *Pomona College*

A rational map $f : \mathbb{P}^1(\mathbb{C}) \to \mathbb{P}^1(\mathbb{C})$ from the Riemann Sphere to itself is said to be a Lattes Map if there are "well-behaved" maps $\beta : E(\mathbb{C}) \to \mathbb{P}^1(\mathbb{C})$ and $L : E(\mathbb{C}) \to E(\mathbb{C})$ such that $f \circ \beta = \beta \circ L$. We are interested in those Lattes Maps f which are also Belyi Maps, that is, the only critical values are 0, 1, and ∞. Work of Zeytin classifies all such

maps: For example, if $E : y^2 = x^3 + 1$ then $\beta : (x, y) \mapsto (y + 1)/2$ while $L = [n]$ for some positive integer n. In this project, we are interested in the corresponding Belyi Lattes Map f_n. What can we say about such maps? What are their Dessin d'Enfants? Preliminary work shows that this should be a bipartite graph with $3n^2$ vertices. What are their monodromy groups? Preliminary work shows this should be a group of size $3n^2$. This work was done at Pomona College as part of PRiME (Pomona Research in Mathematics Experience); it was funded by the National Science Foundation (DMS-2113782).

67. Valuation Trees and Primitive Prime Divisors

Malike Conteh *Pomona College*
Michaela Fitzgerald *Stonehill College*
Sarah Szafranski *University of Redlands*
Jasmine Camero *Emory University*
Advisor(s): Bianca Thompson, *Westminster College*

A sequence is said to have primitive prime divisors if each term after a certain point always has a new prime divisor. Understanding primitive prime divisors is a step in understanding the backwards orbit of a function. Valuation trees help us visualize the prime structure for a sequence for specific primes. A valuation tree classifies integers via their prime divisibility and organizes the information to better understand how primes might be growing in the sequence. Is there a way to combine information about valuation trees over all primes p in order to understand the primitive prime divisors of a sequence? Understanding backwards orbits of rational maps are of interest in number theory, in particular rational maps where at least two critical points collide in their forward orbit. Thus, can we extend results on valuation trees created for polynomials to rational maps in order to understand their structure? This work was done at Pomona College as part of PRiME (Pomona Research in Mathematics Experience); it was funded by the National Science Foundation (DMS-2113782).

68. Galois Groups of Fields Generated by Points of Curves

Quincy Alston *University of Pennsylvania*
Gesa Nestler *University of Tennessee at Knoxville*
Jack Straus *College of William and Mary*
Kayla Gibson *Rutgers University*
Advisor(s): Renee Bell, *University of Pennsylvania*

There is a deep link between the geometry of a curve defined by a polynomial equation and solutions to the equation over the rational numbers, wonderfully illustrated by Faltings's theorem, which relates the number of rational solutions to the genus of the curve. This result naturally invites one to consider the set of solutions over a finite extension of the rational numbers, and which field extensions give new points on the curve. Along these lines, Mazur and Rubin outlined an approach to the study of a curve by understanding the field extensions of $\mathbb{Q}$ generated by a single point on that curve. We may ask which field extensions arise this way, what their Galois groups can be, how many there are up to bounded complexity, and how all these questions relate to the geometry of the curve. This project builds on the results of Lemke Oliver and Thorne for elliptic curves, exploring these questions for different families of plane curves. This work was done at Pomona College as part of PRiME (Pomona Research in Mathematics Experience); it was funded by the National Science Foundation (DMS-2113782).

69. Good Elliptic Curves

Elise Alvarez-Salazar *University of California at Santa Barbara*
Miguel Aparicio *California Institute of Technology*
Calvin Henaku *Washington University in St. Louis*
Summer Soller *Colorado State University*
Advisor(s): Alexander Barrios, *University of St. Thomas*

By an ABC triple, we mean a triple $P = (a, b, c)$ such that a, b, and c are relatively prime positive integers with $a + b = c$. The ABC Conjecture asserts that for each $\epsilon > 0$, there are finitely many ABC triples $P = (a, b, c)$ such that $\text{rad}(abc)^{1+\epsilon} < c$. Here, $\text{rad}(n)$ denotes the product of the distinct primes dividing n. An ABC triple $P = (a, b, c)$ is said to be good if $\text{rad}(abc) < c$. For instance, we have that for each positive integer k, the ABC triple $\left(1, 9^k - 1, 9^k\right)$ is good. It turns out that the ABC Conjecture is equivalent to a statement about elliptic curves! This equivalence leads to a dictionary between the realms of ABC triples and elliptic curves. In particular, there is a notion of good elliptic

curves. This project will investigate the interplay between good ABC triples and good elliptic curves, with the goal of constructing new infinite sequences of good elliptic curves. This work was done at Pomona College as part of PRiME (Pomona Research in Mathematics Experience); it was funded by the National Science Foundation (DMS-2113782).

70. Does ESG investing affect international capital flows? Some evidence from statistical and machine learning methods

Leon Luo *Wayzata High School*
Advisor(s): Ivy Zhang, *University of California, Riverside*

In response to investors' demand to generate long-term financial profit and make positive societal impact on environmental, social and governance (ESG), ESG investing (i.e., sustainable investing), incorporates environmental, social and governance standards into asset allocation and risk management. Using the sample of U.S. residents' investment in foreign portfolios from 2012 to 2020 and implementing the machine learning LASSO model to address multicollinearity, I find that (1) higher ratings in some country-level ESG criteria, such as less pollution, stronger law enforcement, and more innovations, attract more cross-border capital because ESG investors ultimately seek long-term financial returns. And (2) lower ratings in other ESG criteria, such as poor health and nutrition, also attract more capital, reflecting the ESG impact investing strategy. Among the three pillars of ESG, the governance pillar (especially innovation) is most influential. Overall, the empirical evidence suggests that ESG investing strategies influence international capital flows, and that U.S. investors use ESG principles to align long-term financial performance with societal values.

71. Markov Chain-based Models for Predicting Win Probability of NHL

Miles Mena *Lewis University*
Harvey Campos-Chavez *Lewis University*
Jacob Prince
Will deBolt
Advisor(s): Amanda Harsy, *Lewis University*

Predicting the outcome of a hockey game can be challenging due to the fast paced and physical nature of the sport. Win probability models can be used by teams to increase their likelihood of winning, statistical analysis of teams, as well as the sports betting industry to maximize their likelihood of winning a bet. In this project, we present two continuous time Markov process-based models that will take the certain state the home team is in at any point in the game and give a winning probability for the game. In our first model, the game state is based on the home team's shot and goal differential relative to the opposing team. Our second model uses states based on specific manpower/powerplay situations and goal differential. The outcome was two models that provided the probability that the home team would win depending on the state they are currently in at a given time in the game.

72. Analyzing High-Pressure Moments in Tennis and Competitive Esports

Jakob London *Lewis University*
Leobardo Rodriguez *North Central College*
Advisor(s): Amanda Harsy, *Lewis University*

High-pressure sports moments like shootouts, free throws, and serving can greatly impact the outcome of a game. Past research has looked at psychological, physiological, and behavioral factors in an attempt to understand the phenomena of "choking under pressure." This research seeks to further understand these high-pressure moments specifically in tennis and combat/fighting esports games. We explore questions like how do different professional players perform under pressure? Are top players less likely to choke? Are there any similar trends between tennis players and competitive esport players and how they respond to high-pressure situations within their sport?

73. A Pandemic of Mental Health

Sarah Kircher *Penn State University*
Edward Coleman *West Chester University*
Jhavon Innocent *SUNY Albany*
McKade Trauger *Ursinus College*
Advisor(s): Hugo Montesinos-Yufa, *Ursinus College*

The direct impact of the COVID-19 pandemic and the indirect impact of the ensuing economic and political response have affected the United States on a large scale. During the NSF sponsored REU, the indirect impact of COVID-19 is

analyzed through the stringency of individual states' pandemic policies. Our research utilizes difference-in-difference, linear regression, and correlation matrices to quantify state responses' effect on overall health, particularly mental health. The quantitative results and time-specific analysis enhance our understanding of the direct and indirect effects of the COVID-19 pandemic on public health. Our results indicate a wide variation in mental health-related issues by age group, with a higher prevalence in younger age categories. While we observe a slight decline in the share of the population experiencing anxiety and depression through January 2021–June 2022, the effects of the stringency index on other areas of health are complex and vary by state. We document a potential surge in employment-related anxiety, a reduction in overall quality of life, and a significant increase in unexplained mass shootings (by quarter/year) since the pandemic started.

74. Predicting Customer Retention with Bayesian Networks

Zach Seiss *Francis Marion University*
Advisor(s): Ivan Dungan, *Francis Marion University*

Bayesian networks (BNs) are probabilistic graphical models that can be used to represent complex relationships between many variables. The nodes of the graph represent environment variables and the directed edges between them signify dependency relationships. One strength of BNs is that information inserted on a variable can propagate through the network and change the probability spaces of other variables. This potentially makes them a great choice for modelling messy real-world scenarios. In this summer research project, we used data that a local software company had collected on their clients to construct BN models to predict which clients would be most likely to cancel their subscriptions. We applied clustering methods to discretize environment variables and used 10-fold cross-validation as the primary method to determine the success (or not) of our model.

75. Europe, Natural Gas, and The Risk of Blackouts

Raymond Morehouse *Southern New Hampshire University*
Michael DeGregorio *Southern New Hampshire University*
Steven DiBurro *Southern New Hampshire University*
Advisor(s): Colton Sawyer, *Southern New Hampshire University*

The price of fossil fuels has very far-reaching consequences given how reliant the world has grown on them. With natural gas being a relatively large component in electricity production it can be seen as important to know how the price of natural gas can affect power grids. This research is an attempt at finding the comparative risk of the national power grids of Europe in the event of a large rise in natural gas prices.

76. Modeling Parcheesi Using Markov Chains

Emily Sarro *Juniata College*
Namuka Ishii *Juniata College*
Isabel Dinan *Juniata College*
Advisor(s): Kristin Camenga, *Juniata College*

Markov Chains are a mathematical tool used to model phenomena like the stock market, algorithms, and in our case, board games. Previously, games like Monopoly, Chutes and Ladders, and Hi-Ho Cherry-O have been modeled using Markov Chains; we modeled Parcheesi. Modeling Parcheesi introduces new challenges, including sending players' pawns back to start and keeping track of multiple pawns per player. We kept track of the movements between states of the game, that is, the location of each pawn on the board and which player's turn it is, in a transition matrix. Using this transition matrix, we determined a stationary distribution vector and a fundamental matrix which allowed us to investigate the following questions: What is the probability of landing on a given space? What is the expected number of moves? What is the probability of winning the game?

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