To the Instructor

Discovering Discrete Dynamical Systems is designed for use in a student-led, discovery-based course for advanced mathematics majors. When used as intended, the structure of this text will enable students to develop persistence and skill in exploration, conjecture, and generalization; to apply principles of real analysis to the study of dynamical systems; to read mathematics independently; and to communicate mathematical ideas with clarity.

The organization of the text into modules is crucial to its success in meeting our objectives. Each module will take three or four class periods to cover, depending on the length of time of a class meeting. The first three modules lay the foundation for all the material that follows. After these modules, the instructor has a great deal of freedom in choosing the topics for the remainder of the course. The table below gives the prerequisites for each module to help with course planning. In addition, Modules 9 and 10 assume familiarity with basic linear algebra and complex numbers respectively. Familiarity with the notion of cardinality is helpful, but not required, for Module 4.

Module 4:	Modules 1–3
Module 5:	Modules 1–3
Module 6:	Modules 1–3
Module 7:	Modules 1–3
Module 8:	Modules 1–3, project requires Module 5
Module 9:	Modules 1–3, project requires Module 7
Module 10:	Modules 1–4
Module 11:	Modules 1–3, 8
Module 12:	Modules 1–4, 7, 8
Module 13:	Modules 1–3
Module 14:	Modules 1–4, 8, 13

Table 1. Module Prerequisites.

Each module starts with an exploration in which the students are asked an open-ended question. The goal of the exploration is to allow the students to make discoveries that lead them to formulate the questions addressed in the exposition of the module. Each exploration can be covered in a class period, with the first half of the period spent with the students exploring in groups. During the remainder of the period, the groups can report their findings and the class can discuss as a whole any questions, ideas, and conjectures inspired by the student discoveries. The text is careful to never follow an exploration with an exposition that overtly answers the questions raised in the exploration.

The module exposition follows the exploration. This exposition is brief, and any student who has the prerequisites for an analysis course can read it independently. When the text is used as intended, students successfully read the exposition, not only because in the

absence of lectures this is the only source of information that they have, but also because the explorations have previously engaged them in the material. The module exposition is followed by exercises designed to reinforce and build upon the reading. The exposition and exercises can be assigned as homework with the students working independently or in groups. The next class period can then be spent with the students asking questions about the exposition and presenting and discussing homework solutions.

Each module concludes with a project that is designed to bring the ideas from the module to bear on a more challenging or in-depth problem. There are many ways instructors can use the projects: for example, students can work in groups on all of the projects, with one group assigned to lead the project discussion; the projects can be divided up between student groups with each group presenting their own project; or some combination of these approaches can be used.

The course can end with students investigating a subject of their own and presenting a brief description to the class. Students can use their own interests to decide on the topics but may also be encouraged to search for relevant papers in *The College Mathematics Journal* or *The American Mathematical Monthly*, for instance.

The explorations and projects will require the use of technology, particularly for the iteration of functions. Past instructors have used various approaches: some have provided the basic coding for their students, using Mathematica, Maple, or Excel. Others have had the students do their own programming, while others have taken advantage of the many web-based applications available on-line.

When using this text for a discovery-based course, the most important thing to do is also the hardest: instructors should step back and let students grapple with their ideas and questions with very little instructor involvement. Students might not learn as many facts on their own as they would with instruction, but they will experience the challenge and thrill of mathematical discovery. They will overcome uncertainty and frustration and grow in confidence and self-sufficiency. To witness that growth is a teacher's best reward!