# Preface

Fractals came onto the stage in the 1970's with the emergence of the Mandelbrot set, with its incredibly complicated and interesting boundary. During the 1980's a number of books appeared, including most especially by Mandelbrot, Barnsley and Devaney, that gave a mathematical background for fractals that made fractals accessible to both students and teachers. More recently, as computers and their users have become more sophisticated, the domain of fractals has broadened, from art to scientific application to mathematical analysis. In particular, students in high school as well as college are often introduced to fractals and fractal concepts. The present volume includes six essays related to fractals, with perspectives different enough to give you a taste of the breadth of the subject.

Each essay is self-contained and expository. Moreover, each of the essays is intended to be accessible to a broad audience that includes college teachers, high school teachers, advanced undergraduate students, and others who wish to learn or teach about topics in fractals that are not regularly in textbooks on fractals.

Next is a brief overview of each essay; together these overviews should give you quite different views of the topic of fractals.

The volume begins with "Mathscapes—Fractal Geometry," by Anne M. Burns. Burns, who is an artist as well as a mathematician, discusses several ways of modeling on the computer such fractal objects as plant growth and trees, clouds and mountains. The algorithms that Burns uses to create such fascinating and beautiful fractal scenery include stochastic matrices, simple recursion, and a probabilistic method, all of which are accessible to students in a variety of courses from mathematics to programming to graphics.

The second essay is "Chaos, Fractals, and Tom Stoppard's *Arcadia*," by Robert Devaney. We don't often find mathematical themes in works meant for the theater, but with this essay, Devaney introduces us to a notable exception. Indeed, ideas from fractal geometry and chaos theory are "center stage" in *Arcadia*. As Devaney summarizes the main action of the play, he reminds us of the basic mathematics of iterated function systems (the chaos game) and the dynamics of the logistic function, both of which are very familiar in the study of fractals. In addition, he suggests that by means of the play, teachers can provide a rich interdisciplinary experience for their students.

The third essay is "Excursions Through a Forest of Golden Fractal Trees," by T. D. Taylor. In this article, Taylor explores surprising and beautiful connections between fractal geometry and the geometry associated with the golden ratio. In particular, Taylor discusses four self-contacting symmetric binary fractal trees that scale with the golden ratio. The article includes new variations on such familiar fractals as the Cantor set, the Koch curve and Koch snowflake. And, as Taylor points out, throughout the paper there are "wonderful exercises involving trigonometry, geometric series, the scaling nature of fractal trees, and the many special equations involving the golden ratio."

The fourth essay is "Exploring Fractal Dimension, Area, and Volume," by Mary Ann Connors. In this essay, Connors discusses basic properties of some of the most famous and picturesque fractals. They include the Sierpiński Triangle (or Gasket) and the 3-dimensional analogue of the Sierpiński Triangle, the Harter-Heighway Dragon, Sierpiński's Carpet, the Koch Snowflake and a 3-dimensional analogue of the Sierpiński Carpet, and finally the Mandelbrot Set.

The fifth essay is "Points in Sierpiński-like Fractals," by Sandra Fillebrown, Joseph Pizzica, Vincent Russo, and Scott Fillebrown. The impetus for the essay is the Sierpiński Triangle, one of the most famous of the classical fractals appearing a century ago, which can be defined by three special contractions of the plane. The present essay defines and studies "Sierpiński-like" fractals, which are sets in the plane that also can be defined by three special contractions, however utilizing reflections and rotations of those contractions. The essay shows how to generate rules based on the binary representations in order to determine whether a point is or is not in a given Sierpiński-like fractal.

The final essay is “Fractals in the 3-Body Problem Via Symplectic Integration,” by Daniel Hemberger and James A. Walsh. The 3-body problem, which seeks the positions and velocities of three bodies over time, was studied by Henri Poincaré in the late 19th century, and gave rise to what became the subject of dynamical systems. In this essay, Hemberger and Walsh investigate the 3-body problem through the notion of symplectic maps (defined in the essay). With an elementary approach they discuss why symplectic maps can be very effective for numerical integration of Hamiltonian systems of differential equations. Then they use the information gained to investigate fractals that arise in the study of the 3-body problem.

The editors of this volume would like to thank the MAA for including a contributed paper session on fractals and chaos at the annual meeting in January 2007, from which the six essays included here are derived. The editors would also like to thank the Series editor, Stephen Maurer, and the reviewers, all of whom made many helpful suggestions. Finally, the editors acknowledge with great appreciation the careful work by the MAA production staff, most especially Beverly Ruedi, Elaine Pedreira, and Carol Baxter.

Jon Scott and Denny Gulick
September 2010