# Abstracts of Papers Presented at 

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## Invited Addresses

# Earl Raymond Hedrick Lecture Series 

Bernd Sturmfels University of California Berkeley (bernd@math.berkeley.edu)

Algebraic Geometry: Tropical, Convex, and Applied

## Lecture 1: Tropical Mathematics

Thursday, August 2, 10:30-11:20 AM, Ballroom AB
In tropical arithmetic, the sum of two numbers is their maximum and the product of two numbers is their usual sum. Many results familiar from algebra and geometry, including the Quadratic Formula, the Fundamental Theorem of Algebra, and Bezout's Theorem, continue to hold in the tropical world. In this lecture we learn how to draw tropical curves and why evolutionary biologists might care about this.

## Lecture 2: Convex Algebraic Geometry <br> Friday, August 3, 9:30-10:20 AM, Ballroom AB

This lecture concerns convex bodies with an interesting algebraic structure. A primary focus lies on the geometry of semidenite optimization. Starting with elementary questions about ellipses in the plane, we move on to discuss the geometry of spectrahedra, orbitopes, and convex hulls of real varieties.

## Lecture 3: The Central Curve in Linear Programing

Saturday, August 4, 9:30-10:20 AM, Ballroom AB
The central curve of a linear program is the algebraic curve along which the interior point algorithms travel. We determine the degree, genus and defining ideal of this curve. These invariants, as well as the total curvature of the curve, are expressed in the combinatorial language of matroid theory. This is joint work with Jesus De Loera and Cynthia Vinzant.

## MAA-AMS Joint Invited Address

Thursday, August 2, 8:30-9:20 AM, Ballroom AB

## David Mumford Brown University (dbmumford@gmail.com)

## The Synergy of Pure and Applied Math, of the Abstract and the Concrete

In the mid 20th century, pure and applied math split and, in spite of the financial pressure for collaboration exerted by the NSF, they still largely go their own ways. I believe this is damaging to both. In my own experience, math comes alive through an exciting dialectic between theory on the one hand and examples, applications and experiments on the other. The fantasy of a pecking order topped by the most abstract pure math was canonized by Bourbaki and, as I learned from critical emails last year, is accepted by large segments of the public. I will discuss how I see this affecting both K-12 instruction and the excessive specialization of all branches of math research.

## MAA Invited Addresses

Thursday, August 2, 9:30-10:20 AM, Ballroom AB

Amie Wilkinson Northwestern University (amie.wilkinson@gmail.com)

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## Saturday, August 4, 8:30-9:20 AM, Ballroom AB

Richard Kenyon Brown University (rkenyon@math.brown.edu)

## Random Interfaces and Limit Shapes

We discuss how limit shapes and facets form in simple models of random discrete interfaces. In particular, the "lozenge" tiling model is a model of random stepped surfaces; one can write down and solve a PDE which describes the limiting surface (when the mesh size tends to zero) for given boundary frame. The solutions are parametrized by complex analytic functions, in a similar manner to Weierstrass' parametrization of minimal surfaces (soap films) using conformal mappings.

## Saturday, August 4, 10:30-11:20 AM, Ballroom AB

Robert Ghrist University of Pennsylvania (ghrist@math.upenn.edu)

## Putting Topology to Work

Mathematics implicates motions and machines; computations and colorings; the strings and arrows of life. Perhaps the grandest expression of the beauty and power of Mathematics is revealed in the quantification and qualification that which is not there: holes. Topology the mathematics of holes will be surveyed with a fresh look at the many ways in which topology is used in data management, networks, and optimization.

# AWM-MAA Etta. Z. Falconer Lecture 

## Friday, August 3, 8:30-9:20 Am, Ballroom AB

## Karen King National Council of Teachers of Mathematics (karen.d.king@nyu.edu) <br> Because I Love Mathematics: The Role of Disciplinary Grounding in Mathematics Education

Much like my mentor, Etta Falconer, I enjoy mathematics but have devoted a career to ensuring that students of all walks of life have opportunities to learn important mathematics. The role of the discipline of mathematics, mathematicians' ways of reasoning and participation in the mathematical community, have been a clear part of this work. In this talk, I discuss the recent focus in K-12 mathematics education on the Standards for Mathematical Practice in the Common Core State Standards in Mathematics and the need for teachers and students to be grounded in the reasoning habits of mathematics to ensure opportunities for future students to learn mathematics at the highest levels. Drawing on research on and stories of future high school teachers and current middle and high school students, I discuss the ways in which mathematics as a discipline shapes teachers' views of teaching and students' opportunities to learn. I conclude with considerations for mathematicians and mathematics educators for discussing the important role of mathematics in mathematics education.

## James R. Leitzel Lecture

## Friday, August 3, 10:30-11:20 AM, Ballroom AB

Sylvia Bozeman Spelman College (sbozeman@spelman.edu)

## The Many Faces of Mentoring

Project NExT (New Experiences in Teaching) demonstrated the role of mentoring in a select segment of the mathematics community at a critical point in the career development of those who participated. The Project's success confirmed the added value of mentoring and its effectiveness, even when mentoring expands beyond the one-to-one form. This presentation will explore some of the many forms and benefits of structured mentoring, particularly for students at all levels and for young faculty. It will point out the uses of mentoring, along with other activities, to expand the mathematics community, increase its diversity, and enhance the development of its members.

## Pi Mu Epsilon J. Sutherland Frame Lecture

## Friday, August 3, 8:00-8:50 PM

Melanie Matchett Wood University of Wisconson Madison (mmwood@math.wisc.edu)

## The Chemistry of Primes

We are familiar with the prime numbers as those integers that cannot be factored into smaller integers, but if we consider systems of numbers larger than the integers, the primes may indeed factor in those larger systems. We discuss various questions mathematicians ask about how primes may factor in larger systems, talk about both classical results and current research on the topic, and give a sense of the kind of tools needed to tackle these questions.

## NAM David Blackwell Lecture

## Friday, August 3, 1:00-1:50 PM, Ballroom A

## Carlos Castillo-Chavez Arizona State University (ccchavez@ asu.edu)

## The Marriage Between Disease Dynamics and Mathematics: a History of Success

The concept of threshold or tipping point, a mathematical dimensionless quantity that characterizes the conditions required for the occurrence of a drastic transition between states, is central to the study of the transmission dynamics and control of diseases like dengue, influenza, SARS, malaria, and tuberculosis, to name a few. The quantification of tipping point phenomena goes back to the modeling and mathematical work of Sir Ronald Ross (Ross, 1911; second Nobel Laureate in Medicine) and his "students" (Kermack and McKendrick, 1927, 1932). Ross, in fact, proceeded to confront the challenges associated with understanding and managing malaria patterns at the population level right after the completion of his scientific malaria discoveries. The quantification of the concept of tipping point, in the context of epidemiology, has found countless of applications directly tied in to the design, development, and implementation of public health policy. Ross's writings emphasized the value of mathematical models as integrators of multi-level information and processes and his mathematical framework led to the development of a mathematical theory of infectious diseases (an outstanding review of the field can be found in (Hethcote, SIAM Review, 2000). The overview in this lecture provides a personal perspective on the role of mathematical models in the study of the dynamics, evolution and control of infectious diseases over multiple scales; an area of research that has always attracted mathematicians with a strong desire to address problems at the interphase of the life, mathematical and social sciences.

## MAA Lecture for Students

## Thursday, August 2, 1:00-1:50 PM, Ballroom A

Ivars Peterson Mathematical Association of America (IPeterson@maa.org)

## Geometreks

Few people expect to encounter mathematics on a visit to an art gallery or even a walk down a city street (or across campus). When we explore the world around us with mathematics in mind, however, we see the many ways in which mathematics can manifest itself, in streetscapes, sculptures, paintings, architectural structures, and more. This illustrated presentation offers illuminating glimpses of mathematics, from Euclidean geometry and normal distributions to Riemann sums and Möbius strips, as seen in a variety of structures and artworks in Washington, D.C., Philadelphia, Toronto, Montreal, New Orleans, Madison, Wisconsin, and many other locales.

## Alder Awards

## Alder Awards

## Friday, August 3, 2:00-3:20 PM, Ballroom A

In January 2003 the MAA established the Henry L. Alder Award for Distinguished Teaching by a Beginning College or University Mathematics Faculty Member to honor beginning college or university faculty whose teaching has been extra-ordinarily successful and whose effectiveness in teaching undergraduate mathematics is shown to have influence beyond their own classrooms. Each year, at most three college or university teachers are honored with this national award. The awardees are invited to make a presentation at MathFest on their work.

This year's honorees are:

Kathryn Leonard Californial State University Channel Islands<br>Susan Martonosi Harvey Mudd College<br>Michael A. Posner Villanova University

## Kathryn Leonard Californial State University Channel Islands

## I Failed and No One Died

2:00-2:20 PM
Michael Starbird has described mathematics as the process of becoming progressively less wrong. Indeed, our first attempt at solving a problem almost always fails. The eventual solution depends on our ability to extract meaning from failure, reformulating our mistakes into a new approach. Despite the familiarity with failure implied by that process, we typically drape the word in dour, black tones "this student failed the midterm, that colleague failed to get a grant, the person in question is now A Failure" and follow it with concerned head shaking and averted eyes. Contrarily, this talk will describe some of my attempts to embrace failure, and to help my students do the same.

## Susan Martonosi Harvey Mudd College

## An ORnate ORation on OR

## 2:30-2:50 PM

Operations research (OR) is the use of mathematical thinking to make systems, processes and decisions more efficient. It is naturally appealing to mathematics students who want to understand how the mathematical theory they are learning can be applied to solve important problems. At Harvey Mudd College, student interest in OR has been consistently growing in response to increased course offerings, research opportunities and industry-sponsored capstone projects. In this talk (which, in truth, is unlikely to be ornate) I'll discuss the field of OR and its appeal to students, the OR curriculum we have in place, and best practices (along with pitfalls to avoid) in introducing OR to your students.

## Michael A. Posner Villanova University

## Practicing What We Preach: Evidence-based Evaluation of your Classroom Teaching and Pedagogical Innovations 3:00-3:20 PM

Every teacher is unique. Some of us have impeccable recall, some are experts in applications while other revel in the more theoretical, some are incredibly organized, and others might deserve an off-Broadway production. Therefore, the way we teach should be unique and personal as well. Some lecture very effectively, some use inquiry-based learning, some show videos, some have class projects, some create applets or apps. But the common themes of what makes good teachers are engaging students to master learning objectives and, perhaps, inspire them to learn more. Yet, when we evaluate our teaching or try out something new in the classroom, we rely on personal feelings or voluntary student feedback. I have sought to explore the efficacy of my teaching through classroom-based studies. I will share several of those classroombased studies, describing both the process and the outcomes, and explore strategies that you can use to engage in your own research on your pedagogical innovations.

# Invited Paper Sessions 

## Convex Algebraic Geometry

## Thursday, August 2, 1:00-5:15 PM, Ballroom D

## Bernd Sturmfels University of California, Berkeley (bernd@math.berkeley.edu)

Cynthia Vinzant University of Michigan (vinzant@umich.edu)
Convex algebraic geometry centers around the interplay between algebraic geometry, classical convexity, and optimization. Algebraic geometry provides necessary tools to analyze and develop solutions to optimization problems, and solvers for convex optimization have led to new fast algorithms in real algebraic geometry.

Jordan Ellenberg University of Wisconsin (ellenber@math.wisc.edu)
Why is Sorting Easy but the Traveling Salesman Hard?
1:00-1:30 PM
We will discuss the fact that a wide variety of combinatorial optimization problems (including sorting a list and solving the Traveling Salesman problem) can be phrased in the following form: given a real vector space $V$ with an action of the symmetric group $S_{n}$, a vector $v$ in $V$, and a linear form $V \rightarrow \mathbb{R}$, maximize the quantity $f(s v)$ as $s$ ranges over $S_{n}$. Despite the formal similarity of these problems, some admit solutions by fast algorithms and some are (as far as we know) do not. I will explain what this has to do with the geometry of "orbitopes," convex hulls of orbits of groups in a representation.

Jon Hauenstein North Carolina State University (jhauenst@math.tamu.edu)

## Gradient Descent Homotopies and Real Solving

## 1:45-2:15 PM

The method of gradient descent is a well-known iterative method for solving optimization problems. Since the iterative method can have poor convergence rate due to 'zigzagging,' we consider a modified continuous version that we model using a homotopy. This talk will conclude by using this homotopy to compute real solutions to systems of equations. This approach is joint work with Zachary Griffin (undergraduate student at Texas A\&M).

## Greg Blekherman Georgia Tech (greg@math.gatech.edu)

## Nonnegative Polynomials and Sums of Squares

2:30-3:00 PM
I will review the history and motivation behind the problem of representing nonnegative polynomials as sums of squares. Such representations are of interest for both theoretical and practical computational reasons. However, some basic and explicit questions about nonnegative polynomials and sums of squares remains open. Our journey will begin with Hilbert's theorem, which shows that nonnegative polynomials that are not sums of squares exist, and end with recent results on the nature of the relationship between nonnegative polynomials and sums of squares. On the way we will see a fascinating blend of ideas from convex and algebraic geometry that are used to attack these questions.

## Vicki Powers Emory University (vicki@mathcs.emory.edu)

## Certificates of Positivity: Theory and Practice

## 3:15-3:45 PM

Suppose $S$ is a subset of $\mathbb{R}^{n}$ and a polynomial $f$ with real coefficients is positive, or non-negative, on $S$. By a certificate of positivity for $f$ on $S$ we mean an algebraic expression for $f$, usually involving sums of squares of real polynomials, from which one can deduce the positivity condition immediately. This talk concerns the theory, i.e., results concerning the existence of certificates, and practice, i.e., results concerning finding and counting certificates, of certificates of positivity.

## Amir Ali Ahmadi MIT (a_a_@@mit.edu)

## Joint Spectral Radius and Path-Complete Graph Lyapunov Functions

4:00-4:30 PM
The joint spectral radius (JSR) of a finite set of square matrices-a natural generalization of the notion of the spectral radius of a single matrix-characterizes the maximal growth rate that can be obtained by taking products, of arbitrary length, of all possible permutations of the matrices. Despite several undecidability and NP-hardness results related to computation (or approximation) of the JSR, the topic continues to attract attention because of a wide range of applications, including computation of the capacity of codes, robust stability of uncertain linear systems, localization in sensor networks, and many others. In this talk, we present our novel framework of "path-complete graph Lyapunov functions" which produces several hierarchies of asymptotically exact semidefinite programming relaxations for approximating the JSR with
provable accuracy. Our algorithms are based on new connections between ideas from control theory and the theory of finite automata. They involve optimization over the set of convex and positive piecewise polynomial functions.

Joint work with R. Jungers, P.A. Parrilo, and M. Roozbehani.

## Bruce Reznick University of Illinois at Urbana-Champaign (reznick@math.uiuc.edu)

## Klein's Idea and Identities for Powers of Polynomials

4:45-5:15 PM
In his famous book on the icosahedron, Felix Klein considered the identification of a point on the unit sphere $u$, with the linear form $x-v y$, where $v$ is the image of $u$ under the Riemann map. If you start with the vertices of a nice polytope inscribed in the sphere, and take quadratic forms corresponding to products of linear forms associated with antipodal pairs of vertices, you get interesting sets of quadratic forms. For example, the octahedron corresponds to $\left\{x y, x^{2}-y^{2}, x^{2}+y^{2}\right\}$ (the Pythagorean parameterization), the cube to four quadratic forms whose 5-th powers are dependent and the icosahedron to six quadratic forms whose 14 -th powers are dependent. We'll give more examples and try to explain this phenomenon.

## Applied Computational Topology

Saturday, August 4, 1:00-4:50 Pm, Ballroom D

## Benjamin Mann Ayasdi, Inc (stixmann@gmail.com) <br> Jack Morava Johns Hopkins University (jack@math.jhu.edu)

One of the exciting recent developments in applied mathematics has been the explosion of insights, techniques, and tools from algebraic topology that have been used to great advantage in examining computation problems in data analysis, distributed networks, and dynamical systems. This special session will feature eight presentations on various aspects of a subject that encourages collaborations and synergies between mathematicians, statisticians, biologists, chemists, physicists, materials scientists, and computer scientists. In total, the session should provide a broad introduction to the area with a balance between the theory and applications.

## Henry Adams Stanford University (henrya@math.stanford.edu)

## Evasion Paths in Mobile Sensor Networks

1:00-1:20 PM
Imagine that disk-shaped sensors wander in a planar domain. A sensor can't measure its location but does know when it overlaps a nearby sensor. We say that an evasion path exists in this sensor network if a moving evader can avoid detection. A theorem of Vin de Silva and Robert Ghrist gives a necessary condition, depending only on the time-varying connectivity graph of the sensor network, for an evasion path to exist. Can we sharpen this theorem? We'll consider examples that show the existence of an evasion path depends not only on the network's connectivity data but also on its embedding.

## Andrew Blumberg University of Texas (blumberg@math.utexas.edu)

## Persistent Homology for Metric Measure Spaces and Topological Hypothesis Testing

1:30-1:50 PM
This talk describes work studying the use of distributions of persistent homology barcodes associated to taking subsamples of a fixed size from metric measure spaces. Such distributions can be efficiently computed and provide robust invariants of metric measure spaces. These invariants also supply a basis for applying standard statistical methodology to problems in topological data analysis.

Joint work with I. Gal, M. Mandell, and M. Pancia.

## Justin Curry University of Pennsylvania (curry.justin@gmail.com)

## Cosheaves and Dualities in Generalized Sensor Networks

2:00-2:20 PM
In this talk, I will introduce the computational framework of cellular sheaves and cosheaves, and advocate for a different perspective on Morse Theory, persistent homology, network coding and pursuit-evasion problems in sensor networks. This framework provides local-toglobal results as well as generalizations of Poincare duality. To provide concrete examples, I will introduce a new model for sensing with different modalities (colors, sounds, etc.) and show how a long exact sequence of sheaf cohomology provides forcing results that allow you to infer what you don't know from what you do.

## Rafal Komendarczyk Tulane University (rafko@gmail.com)

## What is Random Homology and Why can it be Useful?

2:30-2:50 PM
We will describe a natural way of thinking about random chains, cycles etc. in simplicial complexes. The outcome of which is the fact that we can express the common topological invariants of random complexes (such as the Betti numbers or the Euler Characteristic) via certain polynomials with symmetries. This approach turns out to be useful when answering questions about random covers of a given space. In particular, we can obtain exact expressions for the probability of a complete coverage by a random cover. The same questions appeared in mathematics earlier in the form of the theory of coverage processes. The novel aspect of our approach is the application of algebraic topology.

## Sanjeevi Krishnan University of Pennsylvania (sanjeevi.krishnan@gmail.com)

## Directed Poincare Duality

3:00-3:20 PM
The max-flow min-cut theorem, traditionally applied to problems of maximizing the flow of commodities along a network (e.g., supply chains) and minimizing the costs of breaking apart networks (e.g., image segmentation), admits an algebro-topological interpretation and generalization as a special case of PoincareDuality between a top dimensional sheaf homology and zeroth sheaf cohomology. In this talk, I will describe the appropriate sheaf homology theory, taking local coefficients and values in semigroups, on directed spaces, and discuss some applications of such a generalized max-flow min-cut duality for constraints more complicated than mere upper bounds.

## Paul Pearson Fort Lewis College (pearson@hope.edu)

## Gene Clusters

3:30-3:50 PM
We will discuss a data clustering and modeling method called Mapper and use it to analyze genetic data sets. Mapper generalizes ideas from Morse Theory to the analysis of discrete data sets. We discuss how to identify clusters of genes that operate together in biological pathways from a Mapper analysis of DNA microarray data.

## Jose Perea Duke University (joperea @ math.duke.edu)

## A Klein Bottle Based Dictionary for Image Represenation, and an Application to Texture Discrimination 4:00-4:20 PM

One of the most celebrated success stories in topological data analysis, is perhaps that of the Klein bottle as a model space for relevant $3 \times 3$ patches from natural images. I will show in this talk how digital photos can be thought of as random samples on this space, whose underlying probability density function can be represented via a variant of Fourier Analysis. We apply this framework to the task of image texture classification, and show its performance on three challenging data sets.

## Michael Robinson University of Pennsylvania (kbldds@gmail.com)

## The Whitney Embedding Theorem in Signal Processing

4:30-4:50 PM
When can you infer the state of a system from measurements of a signal? In a surprisingly diverse set of situations, rather precise bounds can be obtained on the number of measurements needed to constrain a system from the Whitney embedding theorem. This result of differential topology is easy to state, easy to use, and intuitively satisfying. I will discuss its mathematical importance and advocate for its wider application within engineering. Its effectiveness within the context of opportunistic localization, navigation, and other example areas will be discussed.

## Discrete Probability on Surfaces

Saturday, August 4, 1:00-5:15 PM, Ballroom C<br>Richard Kenyon Brown University (rkenyon@math.brown.edu)

## Ivan Corwin Microsoft Research and MIT (ivan.corwin@gmail.com)

## Beyond the Gaussian Universality Class

1:00-1:30 PM
The Gaussian central limit theorem says that for a wide class of stochastic systems, the bell curve (Gaussian distribution) describes the statistics for random fluctuations of important observables. In this talk I will look beyond this class of systems to a collection of probabilistic models which include random growth models, polymers, particle systems, matrices and stochastic PDEs, as well as certain asymptotic problems in combinatorics and representation theory. Focusing on random growth models, I will explain some of the ways in which these different examples all fall into a single new universality class with a much richer (and still not entirely uncovered) mathematical structure than that of the Gaussian.

## Timo Seppalainen University of Wisconsin (seppalai@math.wisc.edu)

## Random Walks in Random Environments and Random Potentials

1:45-2:15 PM
Current research on random walks looks at walks that interact with an inhomogeneous environment in various ways. This talk introduces some of the basic models and describes what is known and what is not known. Sometimes behavior arises that differs radically from the behavior of classic random walk. Such behavior is quantified for example in terms of fluctuation exponents.

## Russell Lyons Indiana University (rdlyons@indiana.edu)

## Random Trees and Surfaces

2:30-3:00 PM
We discuss and show pictures of three connections between random trees and surfaces. Our first pictures reveal surprising properties of random spanning trees in planar graphs via surfaces showing distances in the tree. Proving that these properties hold has not yet been accomplished. Our next picture shows a random spanning tree on a torus; its dual has an interesting property that is conjectured. These probability measures are unified by determinants, which suggest higher-dimensional analogues. A movie shows an example.

## Benedek Valko University of Wisconsin (valko@math.wisc.edu)

## Operator Limits of Random Matrices

3:15-3:45 PM
By the Hilbert-Polya conjecture the critical zeros of the Riemann zeta function correspond to the eigenvalues of a self adjoint operator. By a conjecture of Dyson and Montgomery the critical zeros (after a certain rescaling) look like the bulk eigenvalue limit point process of the Gaussian Unitary Ensemble. It is natural to ask if this point process can we described as the spectrum of a random self adjoint operator. We show that this is indeed the case: for any $\beta>0$ the bulk limit of the Gaussian beta ensemble can be obtained as the spectrum of a self adjoint random differential operator.
(Joint with Balint Virag)

## James Propp University of Massachusetts - Lowell (http://faculty.uml.edu/jpropp/)

Growth, Erosion, and Competition Driven by Random and Non-Random Walk 4:00-4:30 PM
In this talk, I'll discuss different ways to use random walks to drive large-scale changes in a system. The dichotomy between stable and unstable interface dynamics will be explored, as will the resemblances and differences between these random processes and their derandomized analogues.

## Combinatorics and Matrices

## Thursday, August 2, 1:00-3:50 PM, Ballroom C

## Richard A. Brualdi University of Wisconsin - Madison (brualdi@math.wisc.edu)

This session will focus on the substantial interaction between combinatorics, graph theory, and matrix theory.

## T.S. Michael U.S. Naval Academy (tsm@usna.edu)

## Tounaments and their Matrices

1:00-1:20 PM
A round-robin tournament among $n$ players can be modeled by a directed graph known as a tournament. Each vertex represents a player, and a directed edge represents the outcome of the game between two players. The associated tournament matrix $A$ is a $(0,1)$-matrix of order $n$ with the property that $A+A^{T}+I$ has all entries equal to 1 . We examine properties of tournaments, present some applications to other areas of combinatorics, and mention some unsolved problems. This talk is accessible to non-experts.

## Bridget Tenner DePaul University (bridget@math.depaul.edu)

## Providing a Link Between Posets, Topology, and Enumeration

1:30-1:50 PM
In joint work with Ragnarsson, I found that graphs were the best model to use when studying the boolean elements of a Coxeter system. We employed basic graph operations (deletion, contraction, extraction) to prove that the homotopy type of the boolean complex of a Coxeter system is a wedge of spheres. We were then able to describe the combinatorial meaning of these spheres using a bijection between the spheres in the wedge sum describing that homotopy type and derangements of the graph's vertices.

## Eric Egge Carleton College (eegge@carleton.edu)

## Linear Recurrences and the Pfaffian Transform

2:00-2:20 PM
The Pfaffian transform is a function which uses determinants of skew-symmetric matrices to transform a given infinite sequence into another infinite sequence. For example, the Pfaffian transform maps the consecutive powers of 2 to the sequence of all 1's, it maps the Fibonacci numbers to the consecutive powers of 2 , and it maps the tribonacci numbers (which satisfy $T_{n}=T_{n-1}+T_{n-2}+T_{n-3}$ ) to the sequence of natural numbers. I'll define the Pfaffian transform, discuss some of its basic properties, and explain how several of my (undergraduate) students used perfect matchings and paths in a certain directed graph to show that the set of sequences which satisfy a linear recurrence is closed under the Pfaffian transform.

## Adam Berliner St. Olaf College (berliner@stolaf.edu)

## Minimal 2-Matching Covered Loopy Graphs

2:30-2:50 PM
A $\{1,2\}$-factor is a spanning subgraph consisting of pairwise vertex disjoint edges and cycles. Tutte has characterized graphs which have a $\{1,2\}$-factor. We investigate minimal 2 -matching covered graphs, i.e. graphs in which every edge is in some $\{1,2\}$-factor and removal of any edge yields a graph without this property. We will discuss these graphs and their relationship to nearly decomposable matrices. Furthermore, we consider some classes of minimally 2-matching covered loopy graphs (graphs in which vertices may have a loop), where loops are cycles of length one.

## In-Jae Kim University of Minnesota - Mankato (in-jae.kim@mnsu.edu)

## Application of PageRank Centrality to a Survey Instrument

3:00-3:20 PM
In this presentation, we describe a method with which a number of survey questions can be reduced to only a small set of central questions that can be truly representative of the overall purpose of the survey. We do this by constructing a conceptual network of survey questions, and computing PageRank centrality scores of survey questions in the network. The statistical analysis shows that the correlation between the means of the entire survey questions and the small set of central questions is statistically significant.

## Leslie Hogben Iowa State University (lhogben@iastate.edu)

## Parameters Related to Maximum Nullitiy, Zero Forcing Number, and Tree-Width of a Graph

3:30-3:50 PM
Tree-width, and variants that restrict the allowable tree decompositions, play an important role in the study of graph algorithms and have application to computer science. The zero forcing number is used to study the maximum nullity/minimum rank of the family of symmetric matrices described by a graph. Relationships between these parameters and several Colin de Verdière type parameters, and numerous variations (including the minor monotone floors and ceilings of some of these parameters) are discussed.

# Mathematics and Systems Biology 

## Saturday, August 4, 1:00-3:50 PM, Ballroom B

## Timothy Comar Benedictine University (tcomar@ben.edu)

Systems biology is the study of the systems-level understanding and analysis of the biology, behavior and interactions between the biology and behavior at all levels of biological organization from the small scales of molecules and cells up to the large scales of populations and communities. Systems biology is interdisciplinary by nature, and employs significant mathematical and computational techniques to model and analyze complex biological systems. The mathematics involved in modeling complex systems is wide and varied and includes not only differential equations but also Boolean dynamics, contact networks, individual based modeling, and algebraic techniques. Examples of areas of study of gene system analysis and epidemic modeling. This session will focus on the applications of mathematics to research in areas of systems biology.

This Session is Sponsored by BIO SIGMAA.

## Raina S. Robeva Sweet Briar College (robeva@sbc.edu)

Bistability of the Lactose Operon of E. coli: Comparing Differential Equation and Boolean Network Models
1:00-1:20 PM
The lactose (lac) operon is a well-known example of an inducible genetic circuit that encodes the genes regulating the metabolism of lactose in Escherichia coli. Since its discovery in the late 1950s, the lac operon has been serving as a model system for understanding many aspects of gene regulations. One such aspect is bistability, the ability of a system to achieve two different steady states under the same external conditions. We examine and compare several differential equation models of the lac operon and several Boolean network models constructed as their discrete approximations. We analyze the models with regard to their ability to generate bistable behavior and show that Boolean models can elucidate certain qualitative features of the lac operon network related to bistability.

## Brandilyn Stigler Southern Methodist University (bstigler@smu.edu)

## Comparing biological and mathematical models of tissue development in C. elegans

## 1:30-1:50 PM

Complex gene regulatory networks underlie many cellular and developmental processes. While a variety of experimental approaches can be used to discover how genes interact, few biological systems have been systematically evaluated to the extent required for an experimental definition of the underlying network. Therefore, the development of computational methods that can use limited experimental data would provide a useful tool to extract relevant information from existing data, identify unexpected regulatory relationships, and prioritize future experiments.

We have developed a hybrid modeling method that combines two existing methods: an algebraic-geometry and a statistical approach. We reverse engineered a mathematical model from time-course gene expression data collected from wildtype C. elegans embryos and compared it to an existing knowledge-driven biological model based on the same data set. Importantly, the mathematical model predicts more interactions observed in subsequent perturbation experiments than does the biological model. It also provides new insight into the function of a key transcriptional regulator and identifies distinctive activities of two genes previously thought to be redundant. This work provides a strong example of data-driven mathematical models that complement knowledge-driven models to identify non-intuitive network relationships and to guide future experiments.

## Terrell Hodge Western Michigan University (terrell.hodge@wmich.edu)

## Metabolic Pathways and Hypergraphs

2:00-2:20 PM
Hypergraphs have been used to represent and analyze systems of metabolic pathways and to reconstruct their evolution. In this talk, we discuss an algebraic approach using generalized quiver representations and hypergraph algebras. All terminology will be defined.

## Winfried Just Ohio University (mathjust@gmail.com)

## Complex Biological Systems: When are Simple Models Good Enough?

2:30-2:50 PM
Biological systems typically consist of a vast number of interacting components: tens of thousands of genes form a genome, diseases spread through individual contacts of millions of people, and about a trillion neurons think our thoughts. How can we possibly build mathematical models that help us understand how the behavior of the system as a whole emerges from the interactions of these individual components? After all, mathematical models need to be simple enough so that mathematicians can actually study them. And can we ever know that a relatively simple model is good enough to make reliable predictions?

These challenges lead to good mathematical questions. I will formulate some of them, show how they can be solved, and give examples of meaningful undergraduate research projects on this topic.

## Timothy Comar Benedictine University (tcomar@ben.edu)

## Comparison of the Dynamics of Boolean and Continuous Models for Small Gene Regulatory Networks

## 3:00-3:20 PM

We compare Boolean and ordinary differential equations models for gene regulatory networks with three or four genes and up to five connections between the genes. Boolean models can be used as discrete, qualitative models for the behavior of gene regulatory networks and are often simpler than the corresponding quantitative ordinary differential equation models for the same systems. We discuss conditions for which the Boolean models and the ordinary differential equation models exhibit the same dynamical behavior. We also establish conditions for which the differential equation models exhibit stable oscillations. These conditions depend on parameters representing time scales and the cooperativity of the regulating interactions. We also describe the bifurcations which occur between the regions in the parameter space in which oscillatory and stable behavior manifest.

## Walk the Walk, Talk the Talk

## Friday, August 3, 2:00-5:20 PM, Ballroom C

Georgia Benkart University of Wisconsin - Madison (benkart@math.wisc.edu) Tom Halverson Macalester College (halverson@macalester.edu)
This session will feature talks on walks on lattices and graphs and some of their many applications in enumerating tableaux, pattern-avoiding permutations, random walks, Markov chains, and many other constructs.

## Kyle Petersen DePaul University (tkpeters@math.depaul.edu)

## A Funny Thing Happened on the way to Steppenwolf Theatre: from Lattice Paths to Polytopes and Hopf Algebras 2:00-2:20 PM

Steppenwolf Theatre in Chicago is just five blocks east and five blocks south of me, and most of the streets between here and there form a rectangular grid. However, there are some restrictions on how I want to walk there. For one thing, the north branch of the Chicago river runs diagonally from northwest to southeast through the neighborhood, cutting off a number of potential routes. In the first part of the talk, I will show how to count the possible routes to the theatre. (Spoiler alert! There are 42.)

In the second part of the talk, I will show how we can consider both a polytope and an algebra built out of these lattice paths. The polytope is called the associahedron; the algebra is called the Loday-Ronco Hopf algebra.

These examples give a taste for some of the overlap between combinatorics, geometry, and algebra. No background in the subject is assumed.

## Susanna Fishel Arizona State University (sfishell@asu.edu)

## Maximal Length Chains in the Tamari Lattice

2:30-2:50 PM
Fifty or so years ago Tamari defined a lattice, now known by his name. It has Catalan number of vertices-its Hasse diagram, as an undirected graph, is the one-skeleton of the associahedron-and it is a quotient of the weak Bruhat order. Much is known about this remarkable lattice, but, as Knuth says, the number of maximal chains "remains mysterious." This is especially odd, because the numbers of maximal chains in many closely related lattices are known. I will describe recent work, joint with ASU grad student Luke Nelson, on enumerating maximum length chains. We hope it is progress toward finding the number of maximal chains.

## Eric Egge Carleton College (eegge@carleton.edu)

## Pattern-Avoiding Permutations and Lattice Paths: Old Connections and New Links

3:00-3:20 PM
Pattern-avoiding permutations have been known to be connected with lattice paths since before they were called pattern-avoiding permutations, and more such connections continue to come to light. I'll describe some elegant (and now classical) bijections between Catalan paths and certain sets of pattern-avoiding permutations, and explain how they can be used to reveal new information about both the permutations and the paths. Catalan paths use only North and East steps, but they can be generalized to Schroeder paths, which also allow diagonal steps. I'll describe several sets of pattern-avoiding permutations which are in bijection with Schroeder paths, and I'll give several more sets of pattern-avoiding permutations which are conjectured to be in bijection with these paths.

## Kendra Killpatrick Pepperdine University (Kendra.Killpatrick@pepperdine.edu)

## That's The Way The Ball Bounces: The Story of the $\boldsymbol{q}, \boldsymbol{t}$-Catalan Polynomial

3:30-3:50 PM
In the early 1990's, Garsia and Haiman introduced an important sum $C_{n}(q, t)$ of rational functions in $q$ and $t$ which has a nice interpretation in terms of algebraic geometry and representation theory. From the definition of $C_{n}(q, t)$ it was relatively straightforward to prove that $C_{n}(q, t)$ is symmetric in $q$ and $t$. Garsia and Haiman conjectured that $C_{n}(q, t)$ reduces to a polynomial in $q$ and $t$ with nonnegative integral coefficients and called $C_{n}(q, t)$ the $q, t$-Catalan polynomial. Haglund conjectured that it was possible to obtain the $C_{n}(q, t)$ as a generating function for two statistics, called the area statistic and the bounce statistic, on those lattice paths counted by the Catalan numbers. Garsia and Haglund proved this conjecture in 2002 , but it remains an open question to find a combinatorial proof of the symmetry of $C_{n}(q, t)$ by giving a bijection on the lattice paths that takes the area statistic to the bounce statistic and vice versa. In my talk, I will thoroughly describe these two statistics so that the audience can understand this important open question. In addition, I will describe a generalization of the $q, t$-Catalan polynomials to $q, t$-Schröder polynomials with the respective area and bounce statistics defined on Schröder paths.

## Jim Propp University of Massachusetts - Lowell (http://faculty.uml.edu/jpropp/)

## Walks on Walks

4:00-4:20 PM
I'll talk about a couple of ways to take a walk on the set of walks of length $a+b$ from $(0,0)$ and ( $a, b$ ). One of the operations ("promotion") is the cyclic shift on the binary word associated with the path; the other ("rowmotion") is a more complicated dance in which the word gets chopped into pieces and the pieces get reversed in place. In both cases a single walk gives rise to $a+b$ (not necessarily distinct) walks whose average is the straight Euclidean line from $(0,0)$ to $(a, b)$. I will prove this, and will briefly discuss the more general phenomenon of "combinatorial ergodicity".

## Sam Hsaio Bard College (hsiao@bard.edu)

## Random Walks via Quasisymmetric Functions

4:30-4:50 PM
Quasisymmetric functions show up naturally as generating functions in a variety of combinatorial settings. This talk examines the role of quasisymmetric functions in the analysis of random walks arising from several shuffling and sorting schemes. I will discuss applications based on extensions of riffle shuffling, including recent work giving probabilistic results related to tableau combinatorics.

Tom Halverson Macalester College (halverson@macalester.edu)

## Random Walks on Set Partitions

## 5:00-5:20 PM

We begin with a classical walk on permutations given by randomly transposing elements. Then we will add operations that correspond to coagulation and fragmentation. This gives us a random walk on set partitions. The operations of permutation, coagulation, and fragmentation generate an algebraic structure called the partition algebra. The partition algebra is helpful in analyzing the mixing time of this random walk, and the limiting distribution on set partitions has a surprising connection to alternating sign matrices

## MAA-AMS Special Session

# What Mathematics should Every Citizen Know—and How Does K-16 Get us There? 

Friday, August 3, 1:00-4:45 PM, Ballroom D

David Mumford Brown University (dbmumford@gmail.com)
Solomon Garfunkel Consortium for Mathematics and its Applications (sol@comap.com)
William McCallum University of Arizona (wmc@math.arizona.edu)

## Beyond Shoulds and Oughts: Is Consensus Worth It?

1:00-1:30 PM
The constituent states of this nation are currently undertaking to adopt common standards in $\mathrm{K}-12$ mathematics education, in numbers that would have been unbelievable 3 years ago. What are we to make of this and what are we to do with it? Are there advantages to common standards that transcend disagreements about their details? Or do common standards stifle creativity? In this talk I will give some background to the development of the Common Core State Standards in Mathematics, describe concerns about their implementation, and describe efforts under way to mitigate those concerns.

Lynn Steen St. Olaf College (steen@stolaf.edu)
Reflections on Mathematics and Democracy
1:45-2:15 PM
The central message of the 2001 report Mathematics and Democracy is that the mathematics taught in school bears little relationship to the mathematics needed for active citizenship. Other evidence shows that the mathematics learned in school is a sparse sample of the mathematics taught in school. Is there a connection between these two observations?

Hyman Bass University of Michigan (hybass@umich.edu)

## What Mathematics Every American Needs

2:30-3:00 PM
I believe that what adults most need from mathematics are the skills represented by the "mathematical practices" of the Common Core State Standards. These include competencies ranging from persevering with difficult problems, to constructing and critiquing mathematical arguments, to using appropriate tools and models effectively to solve problems. Obviously these practices are used in connection with particular mathematical ideas and contexts, and are meaningless as stand-alone content. Still, being able to reason quantitatively and abstractly, to use language with precision, and to model mathematical relationships matters for mathematical proficiency as an adult, just as the parallel skills with language matter for communicative and interpretive competence.

## 3:15-3:30 PM Break

Joseph Malkevitch York College of the City University of New York (jmalkevitch @york.cuny.edu)

## Mathematical Tools for the Masses

3:30-4:00 PM
What America needs is not a long list of mathematical skills that are considered mandatory for everyone, but citizens who can use a broad array of elementary mathematical tools. These tools are the mathematical equivalents of the hammer, screwdriver, and drill which most Americans have in their closets or basement. Armed with these tools they will be prepared for a wide range of careers, will be informed citizens in a democracy, and will have a richer sense of the nature of mathematics and its applicability, as well as what skills experts in mathematics have.

## Solomon Garfunkel Consortium for Mathematics and its Applications (sol@comap.com)

## Mathematical Modeling as the heart of the Mathematics Curriculum

 4:15-4:45 PMAs the U.S. embarks on a more centralized system of mathematics education we argue against that system being designed to serve the $1 \%$ of mathematically talented and motivated students. Instead we need to focus on the education of the $99 \%$ by demonstrating the important uses of mathematics in their work and their lives. Mathematical modeling is a life skill and it needs to be at the heart of how we present mathematics to students.

## Contributed Paper Sessions

# Incorporating Writing and Editing into Mathematics Classes 

Session 1: Thursday, August 2, 10:40-11:55 AM, Hall of Ideas H

Martin Montgomery Sam Houston State University (mwm014@shsu.edu)
Ryan Stuffelbeam Transylvania University (rstuffelbeam@transy.edu)
Many students earn degrees in mathematics with little practice in writing and editing. Recognizing the lifelong need of graduates to be able to clearly articulate ideas, institutions are placing a greater emphasis on writing throughout the mathematics curriculum. This session invites presentations describing approaches to incorporating writing and editing into mathematics courses. Presenters are asked to discuss any innovative and original projects, papers and problems that involve both writing and editing in their courses. While contributions detailing any form of mathematical writing are welcome, we are particularly seeking examples and approaches where editing is an essential component. The main goal of this session is to highlight various ways writing and editing have been infused into mathematics curricula and inspire instructors to introduce writing and editing into their courses.

## Ted Sundstrom Grand Valley State University (tedsundstrom@gmail.com)

## Developing Writing Skills in an Introduction to Proofs Course

At Grand Valley State University, the "introduction to proofs" course is MTH 210 - Communicating in Mathematics. This talk will focus on our departments approach to the writing requirements for this course.

An important feature of the course is that it is a part of the Universitys Supplemental Writing Skills (SWS) Program. This course has a prerequisite of Calculus I and Writing 150 with a grade of C or better. Students must turn it a total of at least 3000 words of writing during the semester. To be part of this writing program, the instructor must work with the students on revising drafts of their papers, rather than simply grading the finished pieces of writing.

The main assignment that helps satisfy this SWS requirement is the "Portfolio of Proofs." This portfolio consists of up to ten propositions to be proven or disproven. Before submitting these proofs for a grade, students are allowed to hand in drafts of these proofs to the professor to be critiqued. These proofs must be written according to strict writing guidelines developed by the department. Most professors have students submit their proofs electronically. The proofs in the portfolio are chosen to illustrate the various proof techniques studied in the course. Hopefully, the students will be able to use their portfolios to provide examples of various proof techniques when they are needed in later courses.

## Emily Ronshausen UC Santa Cruz (emily063@gmail.com)

## Editing in a Proof-Based Course

After teaching a few proof-based courses, I realized that most math majors cannot write very well. They make major mathematical errors, logic errors and writing errors. The traditional system of collecting homework, grading it and providing a great deal of written feedback, and then returning it to the students did not seem to improve the situation. As a result, I implemented a system in a second semester real analysis class in which every homework assignment was reviewed by myself, returned to the students, edited by the students, and then resubmitted to me for grading.

I plan to discuss the improvements I saw in student writing, ways in which I managed the workload associated with the strategy and the student feedback on the system.

## Kevin A. Roper Cedarville University (roperk@cedarville.edu)

## Writing Mathematics Well

In all of the undergraduate classes I teach I emphasize the importance of writing mathematics well. However as math majors make the transition from the Calculus sequence to upper level math classes the ability to write well becomes paramount. In my "Logic and Methods of Proof" class I emphasize writing. As I read students' work I have two sets of criteria "Is the mathematics correct?" and "Is it written well?" Through a process of resubmitting their work I allow students to edit their work to improve their mathematical writing. In this talk I will discuss what happens in this class and others where mathematical writing is an important element.

## Adam Graham-Squire High Point University (agrahams@highpoint.edu)

## Mat-Rix-Toe: Strategy and writing in Linear Algebra

The game of Mat-Rix-Toe is similar to Tic-Tac-Toe, but instead of placing Xs and Os the players place ones and zeroes into a matrix. If the matrix is invertible the ones win; if not then the zeroes win. Students work on a project where they play the game of Mat-Rix-Toe and have to describe the optimal strategy for each player in a variety of situations: depending on who goes first, the size of the matrix, and other modifications to the game. Students often had trouble explaining their strategy, and we will discuss the editing and revising that the students needed to do in order to better explain their strategies.

## Session 2: Thursday, August 2, 1:00-4:35 PM, Hall of Ideas H

## Brian Lunday United States Military Academy (brian.lunday@usma.edu) <br> Jeremy Riehl United States Military Academy (jeremy.riehl@usma.edu) <br> Victor Trujillo United States Military Academy (victor.trujillo@usma.edu)

## Multimodal Interim Instructor Feedback Mechanisms to Improve Student Writing for a Calculus Project

We present our design and sequencing of student workload for a course-wide differential calculus project that culminates in the submission of a written technical report. For this 20 -instructor, 900 -student course, the critical components implemented to improve the quality of written student work include the phasing of four in-class labs, an oral in-progress review presented by the students, and the formal provision of written instructor feedback on an intermediate submission. Based on a two-semester pilot study, we present empirical instructor and student feedback on the efficacy of this project design on improving written communication of mathematical work.

## Chad Awtrey Elon University (cawtrey@elon.edu)

## Writing and Rewriting Concept Summaries in a First-Semester Calculus Course

Many of Elon University's calculus courses incorporate several types of writing activities. One of these activities, concept summaries, has students crafting an exposition of a given calculus concept for an audience of their peers. In this talk, we describe Elon's implementation of this activity in first-semester calculus courses, and we discuss specific writing prompts, sample student work, possible rubrics, and the editing process students undertake to produce their end-of-the-semester portfolio of summaries.

## Diana White University of Colorado Denver (diana.white@ucdenver.edu)

## A Feedback Cycle on Papers in a History of Math Class

We discuss the feedback cycle used on two projects in a senior-level history of math class. One paper is a biography of a mathematician, generally $5-8$ pages. The other is a 10-15-page paper on the historical development of a mathematical topic. We discuss in detail the five steps of the assignment-initial ideas, small group presentation, submission of complete draft to two classmates for feedback, offering feedback to two classmates, and then the final product. We provide detailed information on the writing assignment and feedback processes, in which students edit each other's papers and critically reflect on the work of their peers We conclude with some sample student comments related to this process from end of course surveys.

## Bonnie Gold Monmouth University (bgold@monmouth.edu)

## A "Writing-Intensive" Real Analysis Course

At Monmouth University, all students must take two writing-intensive courses in their major, which must include at least 15 pages of writing that is revisable for quality of writing as well as content. I have taught one such course, Real Analysis, twice now, using Ken Ross' Elementary Analysis: The Theory of Calculus. I assign five 3-page essays, typically an extended version of a problem from the text in which students prove a theorem, explore what happens when one or more hypotheses are deleted, and perhaps discuss a little historical context. I insist on their revising their papers for deficiencies in use of mathematical terminology, use of English, or serious deficiencies in mathematical content. In my talk I will discuss the logistics of this, provide an example of an assignment, a student response, my request for revision, and the final paper.

## William Cherowitzo University of Colorado Denver (william.cherowitzo@ucdenver.edu)

## Wiki Use for Writing and Editing in Mathematics Classes

Creating pages related to coursework in a private wiki has provided a means to engage students in writing and editing tasks that are collaborative and non-threatening. We describe the use of this technique in two classes; an undergraduate geometry class and a graduate cryptology class.

Brian Katz Augustana College (briankatz@augustana.edu)
Eliazbeth Thoren University of California Santa Barbara (ethoren@math.ucsb.edu)

## Collaborative Writing and Revision: Students Creating a Reference Textbook

I teach an inquiry-based Geometry course. In place of a traditional textbook, the course is directed by a sequence of questions to be answered and theorems to be proved, but the students do not have a reference text like in many other classes. This course design allows me to structure the course around having the students write, edit, and revise a reference textbook collectively. This collaboration is managed through a wiki, so I call their product a WikiTextbook. In this talk, I will describe a course with the construction and revision of a WikiTextbook at its core. I will present some student responses about the development of their writing skills and the coherence in their content knowledge. In addition, the wiki technology allows me to see the students' editing processes because it records every version of each page. I hope to present some preliminary analysis of the students' editing and revision processes.

## Carrie Diaz Eaton Unity College (Ceaton@unity.edu) <br> Stephanie Wade Unity College (Swade@unity.edu)

## Mathematics Writing, Collaboration, and Peer Review using GoogleDocs

Students who take mathematics tend to view mathematics more as a tool than a science. The move in mathematics education towards inquirybased learning has allowed a novel opportunity to address the scientific method and the writing of research findings. In a collaboration with college biology faculty on a common research rubric and with the college writing director on incorporating peer review, I have incorporated various levels of writing activities in a variety of mathematics classes including Statistics and Calculus I and II. In Calculus, current collaborative technology in GoogleDocs is utilized to facilitate group writing, peer review, and instructor feedback. I will share some of the work done in designing writing assignments in the mathematics curriculum at all levels, as well as the practical implementation of using technology to meet these goals.

## Jeff Johannes SUNY Geneseo (johannes@member.ams.org)

## Magazine, Chapter and More: Diverse Writing and Editing Projects

Editing appears in many settings in my courses, from online wiki editing to revising presentations. We will discuss some varied contexts, focusing on two writing projects for a mathematics history course for teachers. The first was a magazine article, and the course culminated in the students writing an additional chapter for their course text. We will include details of the process and the differences in scope for these different situations.

## Jean Marie Linhart Texas A\&M University Department of Mathematics (jminhart@math.tamu.edu)

## Mathematical Modeling is not Matlab: Teaching Students to Communicate

Ask a seemingly easy "why" question on a math exam and you will receive brain dumps from many of your students: they tell you everything that might be relevant to this question, because they can't figure out what to answer! Students in math often master procedures (the "how") long before they learn how to effectively explain reasoning behind procedures (the "why"). Learning how to structure and write effective explanations is crucial for students working on project-based problems, a.k.a. research. This motivated me to design my mathematical modeling course as a project-based writing and communication class. I will discuss how I address my biggest challenges: getting students writing what they know, teaching them how to describe what mathematical modeling is, teaching them how to write an abstract, and persuading them to take the time to edit and revise while using their resources: their peers, the writing center, and me. I will describe how I structure group work, how I get them writing in $\mathrm{IAT}_{\mathrm{E}} \mathrm{Xfrom}$ the start of the semester, how I insure that they get practice, guidance and feedback for each major assignment. I will show how I use a final portfolio instead of a final exam, so that at the end of the semester students revisit, revise, and edit their earlier work and reflect on what they have learned. I will also share students' feedback to me on what activities were the most fun and beneficial in my class.

## Magdalena Luca Massachusetts College of Pharmacy and Health Sciences (magdalena.luca@mcphs.edu)

## Why Do I Need to Write in Biostastics?

In all the degree programs offered at our College, strong verbal and written communication skills are absolutely essential in students' understanding of patients, drug development and public health issues. Students are rarely required to write in math courses, and they do not see the need or purpose of writing in such courses. In this presentation I will address the extensive use of writing in a second Biostatistics course for students enrolled in the Public Health program. In this class, emphasis is placed on scientific reasoning: reading, writing, interpreting and validating statistical analyses found in public health, behavioral and health sciences journal articles. Students use scientific writing during all lectures, tests, and exams. More importantly, assigned projects require students to develop written papers using information acquired from journal article and statistical software. I will present specific examples in which writing is an essential component.

# Emma Smith Zbarsky Wentworth Institute of Technology (emmasz@gmail.com) <br> Writing Mathematics: Investigations in Statistics and Differential Equations <br> I will describe several projects created by my students in two courses: "Probability and Statistics" and "Differential Equations". The statistics projects were entirely open-ended, with several checkpoints along the way, beginning with a one paragraph proposal and finishing with a five page paper. The projects were more varied in style in differential equations, but included two drafts leading to the final version of the paper. 

# Instructional Support Roles for Undergraduates in Mathematics and Statistics 

Thursday, August 2, 1:00-3:35 PM, Hall of Ideas F

Feryal Alayont Grand Valley State University (alayontf@gvsu.edu)
Ksenija Simic-Muller Pacific Lutheran University (cmka@plu.edu)
Undergraduate students in mathematics and statistics departments can assume numerous innovative instructional support roles in addition to the traditional role of grader. They can serve as undergraduate teaching assistants, peer tutors, study session leaders, and statistics lab assistants, to name a few. Assigning undergraduates to these instructional support roles benefits all those involved: undergraduate assistants develop important life and career skills; students receiving the instructional support get additional attention, and have the opportunity to communicate with a peer; and the instructor and the department benefit from the additional help they receive and are able to give students enrolled in their classes more individualized attention.

This session is open to talks aimed to introduce the different ways undergraduates participate in the instructional activities at various institutions. We encourage speakers to include a discussion of the benefits and challenges of their programs and the training/support that students receive while participating in the program. Talks focused on programs based in individual classrooms, as well as those that are department-wide and university-wide are all welcome. We also invite talks focused on improving the efficiency of the more traditional support roles such as grading and common math tutoring.

## David Kung St. Mary’s College of Maryland (dtkung@smcm.edu)

## Undergraduate TAs as the Holy Grail: Work Savings, Educational Benefits, \& Program Development

Nearly every math course offered at St. Mary's includes an undergraduate TA who attends the class, grades papers, promotes the math major ... and loves it! While some students TA for credit and others for pay, all are seen as an integral part of our vibrant mathematics program.

Our introductory level courses typically feature sophomore TAs, only some of whom have committed to a mathematics degree. Junior and senior math majors populate our sophomore-level courses while future graduate students are specifically selected to help with our mandatory Analysis and Algebra courses. At every step, having an undergraduate in the room improves communication between the professor and the students and provides the students with a role model to guide them forward in the department.

Following a brief overview of the history and evolution of the TA program, including cross-campus skirmishes and battles with administrators, we will discuss the benefits (many!) and drawbacks (few!) of our use of undergraduate TAs.

## Jessica M Deshler West Virginia University (deshler@math.wvu.edu)

## The Case of Ann: Our First Undergraduate Mathematics Teaching Assistant (\& Now Math Major!)

The West Virginia University Department of Mathematics has recently started allowing undergraduate students to earn teaching practicum credits for working with instructors in a variety of instructional support roles. In Non-engineering Calculus I, we recruited our first undergraduate teaching assistant (UTA) in the Fall of 2011. This talk will report details of how she got interested in this position (informing us of how we might recruit more UTAs), why she persisted (as both a UTA and now in mathematics) and insights from her about her experience with different instructors and her decision to become a math major based on her experience in the classroom.

## Emily Ronshausen UC Santa Cruz (emily063@gmail.com)

## The ACE Program: Improving Retention in the STEM Majors

ACE is a program designed to increase the diversity of students majoring in the STEM fields at UC Santa Cruz. First-generation college students who elect to participate in ACE are enrolled in special discussion sections of approximately 20 students. In those sections, a staff leader and a student co-leader lead students in group work. Students are also assigned to an hour of mentoring lead by the co-leader. Mentoring sessions are smaller, usually containing five students. The co-leader encourages the students to work on their homework as a group, reviews key concepts and also addresses issues apart from the class material. Some issues that have been addressed are time management, study strategies, managing difficulty with family, internships and planning class schedules for the following quarter.

In this talk, the ACE Program will be described more fully, as will the benefits reaped by students and co-leaders. Some data will be presented which suggests the extent to which ACE improves retention of first-generation college students in STEM majors, as will narratives which describe the successes more personally.

## Joyati Debnath Winona State University (jdebnath@winona.edu)

## Supplement to Instruction Program in Mathematics and Statistics at Winona State University

Supplement to Instruction (S2I) program at Winona State is coordinated by the Tutoring Services Coordinator. The program actively recruits, trains and identifies student leaders. Each student leader then works with a faculty member teaching a particular course. The leader facilitates and plans activities for study sessions twice per week for that particular course. In this talk, I will present how I participated in this program, what I have learned, the challenges and the benefits and what differences it made for my students.

## Monika Kiss Saint Leo University (monika.kiss@saintleo.edu)

## Supplemental Instruction in Undergraduate Mathematics Classes at Saint Leo University

During this presentation, we shall discuss an innovative collaborative program between the Learning Resource Center (The Saint Leo University's Tutoring Center) and the Mathematics Faculty. The name of the program is Supplemental Instruction, and it provides students access to a student tutor or a full time tutor for outside of classroom workshops and one-on-one assistance. We will discuss the program, the instructors involved and their training, the benefits as well as the drawbacks of the program.

## Amanda Hattaway Wentworth Institute of Technology (hattawaya@ wit.edu) <br> Anita Penta Wentworth Institute of Technology (pentaa@wit.edu) <br> Emma Smith Zbarsky Wentworth Institute of Technology (emmasz@gmail.com) <br> Two Forms of Peer-Tutoring in First-Year Engineering Calculus

Our department was recently faced with designing a new calculus sequence for five new engineering majors beginning in the fall of 2011. For the first course, Engineering Calculus I, we brought in approximately one peer tutor for every 10 students each Wednesday-that was usually three tutors in each section-to work with the students on investigations of numerical methods, applications from engineering, and learning to read and understand technical information. In Engineering Calculus II, in the spring, we hired only one peer tutor per section and they were required to attend class every day and hold optional evening review sessions once a week. We surveyed both the students and the tutors, and we shall discuss our results and plans for the future.

## Helmut Knaust UTEP (hknaust@utep.edu)

## Peer-Led Team Learning in Precalculus

For several years the majority of our Precalculus courses have been accompanied by small-group Peer-Led Team Learning (PLTL) workshops, led by an undergraduate peer leader, usually a Mathematics or Engineering major. Attendance of the workshop is mandatory for all enrolled students. Funded by an NSF grant, PLTL is also utilized in our freshmen chemistry and physics courses. We will report about the peer leader program in Mathematics, its implementation, its successes and failures. Additionally we will provide information on the training of the participating peer leaders, and the impact of the program on their academic careers.

## Fariba Nowrouzi Kashan Kentucky State University (fariba.kashan@kysu.edu)

## Is It a Good Idea to Give Undergraduate Teaching Assistants More Responsibilities in Statistics Courses?

Have you ever had any problems or difficulty in your statistics courses? Can you list some of your weaknesses and strengths in those courses? How could you get the best assistance? In my talk I will address some of the problems that are common for students who take probability and statistics courses and about how undergraduate assistants can participate in the instructional activities. I will also discuss some of the benefits and challenges that this program may cause.

# Fostering, Supporting and Propagating Math Circles for Students and Teachers 

Thursday, August 2, 1:00-4:15 PM, Hall of Ideas G

James Tanton St. Mark's Institute of Mathematics (jamestanton@stmarksschool.org)
Tatiana Shubin San Jose State University (tatiana.shubin @sjsu.edu)
Sam Vandervelde St. Lawrence University (svandervelde@stlawu.edu)
A math circle is broadly defined as a sustained enrichment experience that brings mathematics professionals in direct contact with pre-college students and/or their teachers. Circles foster passion and excitement for deep mathematics. The SIGMAA on Math Circles for Students and Teachers (SIGMAA MCST) supports MAA members who share an interest in initiating and coordinating math circles. SIGMAA MCST invites speakers to report on best practices in math circles with which they are or have been associated. Talks could address effective organizational strategies, successful math circle presentations, or innovative activities for students, for instance. Ideally, talks in this session will equip individuals currently involved in a math circle with ideas for improving some aspect of their circle, while also inspiring listeners who have only begun to consider math circles.

This Session Sponsored by SIGMAA MCST.

## James Tanton St. Mark's Institute of Mathematics (jamestanton@stmarksschool.org)

## Dyadic Fractions, Permuations and Paper Folding

In this talk we illustrate some of the spectacular mathematical heights young students can attain in the Math Circle setting. Surprising connections between three seemingly disparate topics will be revealed.
$\begin{array}{lc}\text { Philip B Yasskin } & \text { Texas A\&M University (yasskin@math.tamu.edu) } \\ \text { David J Manuel } & \text { Texas A\&M University (dmanuel@math.tamu.edu) }\end{array}$

## SEE-Math: Six String Probability Activity

Texas A\&M's Summer Educational Enrichment in Math (SEE-Math), now in its eleventh year, is a 2 week summer program for gifted and honors level middle school math students. This talk will discuss a fun activity which allows students to address a simple probability problem intuitively, experimentally and computationally. Specifically, 6 strings are held in one hand with one end of each string sticking out the top and the other end sticking out the bottom, but you don't know which string on the top is connected to which string on the bottom. The 6 strings on the top are then tied in 3 pairs and the same on the bottom. When you let go the strings will be in 3 loops, or 2 loops or 1 big loop. What is the probability they will be in one loop? The generalization to higher numbers of strings leads to a derivation using mathematical induction.

## Amanda Serenevy Riverbend Community Math Center (viajera6@gmail.com)

## Bringing Math Circle Ideas Into Classrooms

The new Common Core State Standards require teachers to provide much richer mathematical experiences compared with traditional pedagogical approaches. Although Math Circles and Teacher Circles should not limit themselves to the content included in these standards, many topics popular in Math Circles lend themselves particularly well to the goals of the Common Core State Standards. The Math Circle movement has an opportunity to assist educators with the challenging transition before them, which has the potential to revolutionize math instruction in this country if sufficient support for teachers is provided. We will review some of the topics which pose the biggest challenges, and talk about Math and Teacher Circle topics that can help to fill in these gaps.

## Edward Keppelmann University of Nevada Reno (keppelma@unr.edu)

## Little Wranglers

A math wrangle is a wonderful opportunity for children to present mathematics, work collaboratively, and listen closely to the presentations of others. The ideal skills for a complete education in mathematics (and for that matter life). As an official time keeper and moderator now at quite a few math wrangles (at the middle and high school level) I have some observations on them-especially with regard to how the format could be adapted to the very young-say K-3. I will also propose some potential problem sets for such children.

## Sam Vandervelde St. Lawrence University (svandervelde@stlawu.edu)

## Path Counting for Math Circles

Practically every well-known sequence of numbers arises as an answer to a path counting question, which for our purposes will ask for the number of ways to move from point A to point B within a certain directed graph. In this talk we will illustrate several entertaining variations on some standard path counting questions and discuss how to incorporate this type of mathematical exploration into a math circle format.

Tatiana Shubin San Jose State University (tatiana.shubin@sjsu.edu)
The Whats and Whys of the AMS MSRI Mathematical Circles Library
What makes a book suitable for a math circle? What kind of supporting materials do leaders and participants of various circles need? We will discuss these and other related questions, and will try to help shaping up the future development of the only book series devoted solely to math circles

## Debra Geddings University of South Carolina (geddings@math.sc.edu) <br> George McNulty University of South Carolina (mcnulty@math.sc.edu) <br> Nieves McNulty Columbia College (nmcnulty@columbiasc.edu) <br> Douglas B. Meade University of South Carolina (meade@math.sc.edu) <br> South Carolina High Energy Mathematics Teachers' Circle (SCHEMaTC): Year 0

A team of mathematicians and educators from the University of South Carolina, Columbia College, and middle school teachers from two public school districts attended the How To Run a Math Teachers' Circle workshop hosted by AIM in Palo Alto, CA in July 2011. The team returned energized and focused on completing the creation of SCHEMaTC. The past year has been spent locating funding and other commitments to start the formal MTC with an immersion workshop. This workshop was held just last week. In addition to reporting on the actual workshop, we will share our experiences preparing for the workshop and our vision for the first full year activities.

Melissa Shepard Loe University of St. Thomas (msloe@stthomas.edu)
Brenda Kay Kroschel University of St. Thomas (bkkroschel@stthomas.edu)

## Collaboration is the Key

The Twin Cities Math Teachers' Circle just completed its second year and is going strong. In this session we share some pointers for fostering high retention and creating a lively, collegial, collaborative atmosphere.

## Brandy Sue Wiegers MSRI, San Francisco State (brandy@msri.org)

## San Francisco Math Circle: Evaluation of Changes in Mathematical Attitude, First Report

This will be a first report of the data from evaluation of San Francisco Math Circle (www.sfmathcircle.org). This after-school program reaches out to 3 different high schools and provides weekly activities to work on problem solving and critical thinking skills with mathematicians and their developing mathematical community. What is the impact of this activity on the students that are involved? Information will be presented about the survey that was developed and results from the last two years.

## Diana White University of Colorado Denver (diana.white@ucdenver.edu)

## Research Results—An Update

We present the latest research results from the national study on the Math Teachers' Circle program, with a focus on how these results can inform other Math Teachers' Circles. We also discuss the research plans for the upcoming year and invite audience members to offer thoughts and feedback.

# Open and Accessible Problems in Number Theory 

Thursday, August 2, 1:00-4:15 PM, Hall of Ideas E

Aliza Steurer Dominican University (asteurer@dom.edu)<br>Tom Hagedorn The College of New Jersey (hagendorn@tcjn.edu)

Undergraduate research in mathematics has become a fundamental part of the mathematics program at many colleges and universities. Number theory is a subject rich with easily stated yet nontrivial problems. This makes it a great source for undergraduate research projects. In this session, we invite presentations about open problems in number theory that are suitable for undergraduate research and/or for joint faculty/student research. We also invite talks that present results concerning these problems. Presentations from elementary, algebraic, analytic, combinatorial, transcendental, and any other branch of number theory are welcome.

## Robert Styer Villanova University (robert.styer@villanova.edu)

## Senior Seminar using Unsolved Problems in Number Theory

In Villanova University's senior seminar, each student chooses a problem from Guy's Unsolved Problems in Number Theory, working on and searching the literature about their chosen problem. Several students have extended bounds beyond those published in the literature; this year one student found the least example of 14 consecutive happy numbers, two others extended the list of P and N positions of Epstein's Put or Take a Square game well beyond the published bounds, a third student calculated the Gaussian primes in 4000 by 4000 squares with centers at radius larger than $10^{100}$, while another greatly extended the range where the persistence of a number cannot exceed 11 . The students conclude the semester with 50 minute presentations and $20+$ page papers. We will look at examples from this spring's student projects.

## Joshua Holden Rose-Hulman Institute of Technology (holden@rose-hulman.edu)

## Variations on a Theme of DLP

The Discrete Logarithm Problem comes up frequently in cryptography as a "hard problem" underlying the security of many cryptographic algorithms. Analysis of the security of these algorithms depends on the assumption that it is statistically impossible to distinguish the use of the map $x$ goes to $g^{x}$ reduced modulo $n$ from the use of a randomly chosen map with similar characteristics. Some proposed algorithms, however, use variations on this map which seem superficially similar but may exhibit very different behavior. Over the past several years, my undergraduate students and I have been studying the structure of the functional graphs produced by these maps and comparing them to functional graphs from randomly chosen maps. The tools involve not only number theory but also combinatorics, analysis, and statistics, in order to predict the structure of a randomly chosen graph and compare it to our experimental data. In some cases the results are as we predicted, while in others they are surprisingly different, and in some cases simply making reasonable predictions has proved surprisingly difficult.

Marc Renault Shippensburg University (msrenault@ship.edu)
Joshua Ide Shippensburg University (JI1574@ship.edu)

## Power Fibonacci Sequences

We examine integer sequences $G$ satisfying the Fibonacci recurrence relation $G_{n}=G_{n-1}+G_{n-2}$ that also have the property that $G \equiv$ $1, a, a^{2}, a^{3}, \ldots(\bmod m)$ for some modulus $m$. We determine those moduli $m$ for which these power Fibonacci sequences exist, the number of such sequences for a given $m$, and also the periods of these sequences. This presentation is the result of joint research with undergraduate student Josh Ide.

## Ryan Stuffelbeam Transylvania University (rstuffelbeam@transy.edu)

## The Best of Both Worlds: Linear Diophantine Equations and Solutions of a Certain Size

Weird fractions are fractions that enjoy a correct invalid digit-based reduction. Attempting to find weird fraction representations of a given fractions results in a consideration of a linear diophantine equation with certain restrictions. In particular, the weird fraction context mandates that solutions to the linear equation be of a certain fixed size-in fact, this fixed size is a building block in the construction of the linear equation. This talk will present approaches to resolving this situation and present other open problems concerning weird fractions.

## Andy Martin Kentucky State University (andrew.martin@kysu.edu)

## Has an Upper Bound for Brun's Constant been Found?

In 1919 Viggo Brun published a proof that the series of reciprocal twin primes converges (say, to B). This is remarkable as it is unknown to this day whether the set of twin primes is infinite. Best current estimates for B (about 1.902) extrapolate from computed partial sums using an unproven conjecture of Hardy-Littlewood. Is even the first digit reliable? What is known for sure?

Michael Brilleslyper U. S. Air Force Academy (mike.brilleslyper@usafa.edu)
Bradley Warner U. S. Air Force Academy (brad.warner@usafa.edu)

## Integer Sequences, Growth Functions, and Growth Limits: Measuring Divergence via Multiplication

As an example, consider the sequence of natural numbers: $1,2,3, \ldots$. We observe that the product of the first 5 terms (120) is greater than the product of the next 2 terms (42), but not the next 3 terms (356). We generalize this and ask: given the product, $P$, of the first $n$ terms, how many terms are in the largest consecutive product, starting with $n+1$ that is less than $P$ ? We call this maximal number of terms $f(n)$, the growth function of the sequence. In this talk we study properties of $f(n)$ for various sequences and prove several theorems. We also consider the growth limit $L=\lim _{n \rightarrow \infty} f(n) / n$ and investigate this limit for several integer sequences. We present a number of open and accessible questions about $f(n)$ and $L$.

## Chad Awtrey Elon University (cawtrey @elon.edu)

## Local Fields with Dihedral Galois Group

An important problem in constructive class field theory is to classify all finite extensions of the $p$-adic numbers by computing important invariants which define each extension. One important invariant is the number of nonisomorphic extensions of a specified degree and Galois group. We consider the case when the degree is a prime $q$ and the Galois group is $D_{q}$, the dihedral group of order $2 q$.

## Jonathan Sondow New York City (jsondow@alumni.princeton.edu)

Open Problems on Lerch Primes and Wieferich-non-Wilson Primes
I will discuss ten open problems posed in my elementary paper "Lerch quotients, Lerch primes, Fermat-Wilson quotients, and the Wieferich-non-Wilson primes $2,3,14771$ ". Here is a summary of it. The Fermat quotient $q_{p}(a):=\left(a^{p-1}-1\right) / p$, for prime $p$ not dividing $a$, and the Wilson quotient $w_{p}:=((p-1)!+1) / p$, are integers. If $p$ divides $w_{p}$, then $p$ is a Wilson prime. For odd $p$, Lerch proved that $\left(\sum_{a=1}^{p-1} q_{p}(a)-w_{p}\right) / p$ is also an integer; we call it the Lerch quotient $\ell_{p}$. If $p$ divides $\ell_{p}$, we say $p$ is a Lerch prime. A simple Bernoullinumber test for Lerch primes is proven. There are four Lerch primes 3, 103, 839, 2237 up to $3 \times 10^{6}$; we relate them to the known Wilson primes $5,13,563$. Generalizations are suggested. Next, if $p$ is a non-Wilson prime, then $q_{p}\left(w_{p}\right)$ is an integer that we call the Fermat-Wilson quotient of $p$. The GCD of all $q_{p}\left(w_{p}\right)$ is shown to be 24. If $p$ divides $q_{p}(a)$, then $p$ is a Wieferich prime base $a$; we give a survey of them. Taking $a=w_{p}$, if $p$ divides $q_{p}\left(w_{p}\right)$ we say $p$ is a Wieferich-non-Wilson prime. There are three up to $10^{7}$, namely, 2, 3, 14771. My paper will appear in the Proceedings of CANT 2011. A preprint at http://arxiv.org/abs/1110.3113 is available.

## Peter Floodstrand Blanchard St. Olaf College and University of Iowa (peter-blanchard@uiowa.edu)

## Pseudo-Arithmetic Super Sets in the Integers, Gaussian Integers, and Eisenstein Integers

A pseudo-arithmetic super set $A$ is one which has the property that a difference in every subset $B \subseteq A$ of size at least $|B| \geq 2$ divides every other difference in that subset. For example, $\{0,6,8,9,12,72\}$ or $\{0,1,-1+i, 3-i,-7+9 i\}$. I will give the classification of all such sets in the Integers, Gaussian Integers, and Eisenstein Integers, and discuss open questions.

## Brian S. Chen learn95.org (bchen@alumni.csupomona.edu)

## Enjoying The Diversity Of Number Theory Research As An Undergrad

First we will warm up with some number theory quickies concerning numerical representations in different bases! Following that, an interesting divisibility algorithm for "most" primes. Furthermore, my personal experience with undergraduate research which consists of ABC-triples and constructible angles on lattices (2nd place and 6th place respectively at regional MAA poster sessions). I will state these final theorems without proof.

# Undergraduate Research Activities in Mathematical and Computational Biology 

Thursday, August 2, 1:00-5:15 PM, Ballroom B

## Timothy D. Comar Benedictine University (tcomar@ben.edu)

This session is dedicated to aspects of undergraduate research in mathematical and computational biology. First and foremost, this session would like to highlight research results of projects that either were conducted by undergraduates or were collaborations between undergraduates and their faculty mentors. Of particular interest are those collaborations that involve students and faculty from both mathematics and biology. Secondly, as many institutions have started undergraduate research programs in this area frequently with the help of initial external funding, the session is interested in the process and logistics of starting a program and maintaining a program even after the initial funding expires. Important issues include faculty development and interdisciplinary collaboration, student preparation and selection, the structure of research programs, the acquisition of resources to support the program, and the subsequent achievements of students who participate in undergraduate research in mathematical and computational biology. The session is also interested in undergraduate research projects in mathematical and computational biology, which are mentored by a single faculty mentor without the support of a larger program. We seek scholarly papers that present results from undergraduate research projects in mathematical or computational biology, discuss the creation, maintenance, or achievements of an undergraduate research program, or describe the establishment or maintenance of collaborations between faculty and students in mathematics and biology.

This Session Sponsored by BIO SIGMAA.

## Meredith L Greer Bates College (mgreer@bates.edu)

## Combining Forces: Math and Bio Students Join to Study H1N1

In 2009, H1N1 spread across the United States and arrived at several college campuses. At Bates College in Lewiston, Maine, several students became ill with H1N1 during the fall semester of 2009. During the winter 2010 semester, a Differential Equations class combined forces with an Epidemiology class to study the outbreak and its patterns. The Epidemiology students described the biology of the disease and used their studies about ways other diseases have spread to make hypotheses what most caused H1N1 to spread. The Differential Equations students contributed what they had learned about modeling change. Small groups, typically of about four students with representatives from both classes, joined together to suggest campus activities that might have contributed to spread, then modeled and simulated these ideas using Mathematica. The Dean of Students and Director of Health Services contributed their perspectives and responded to student questions.

Later, a senior mathematics major with public health interests wrote her thesis seeking further mathematical underpinnings to the patterns that had emerged in fall 2009. This talk discusses the joint class meetings of winter 2010, the thesis of 2011-2012, the contributions to each of a faculty team of both mathematics and biology faculty members, and thoughts for future collaborations.

## Mathew Rissler Loras College (matthew.rissler@loras.edu) <br> Erinn Sanstead University of Minnesota (sans0026@umn.edu) <br> Sam Lampe Loras College (Samuel.Lampe @loras.edu)

## H1N1 and Dictyostelium discoideum: Projects in Math Bio

At Loras College, both Biological Research and Mathematics majors must complete a multi-semester research project before graduating. This presentation will discuss two projects the speaker has worked on with students. The first investigated the spread of H1N1 (a strain of influenza) on Loras' campus using epidemiological models. The second project is collecting and analyzing tracking data of D. discoideum from time-lapse images and comparing to results from simulation. The speaker will discuss insights he has gained from working with these students and obstacles he has faced in these projects.

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James Peirce University of Wisconsin - La Crosse (jpeirce@uwlax.edu)
Greg Sandland University of Wisconsin - La Crosse (gsandand@uwlax.edu)
Barbara Bennie University of Wisconsin - La Crosse (bbennie@uwlax.edu)
Roger Haro University of Wisconsin - La Crosse (rharo@uwlax.edu)
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## Developing Student Understanding of Species Invasions and Disease Transmission at the Interface Between Mathematics and Biology

The UBM-Collaboration on Riverine Ecology (UBM-CORE) program at UW-L is a three-year undergraduate research, learning, and peermentoring experience designed to facilitate the development of twenty-first century biologists and mathematicians with broad, interdisciplinary scientific training. Each year, student teams consisting of one mathematics major and one biology major are integrated into our ongoing collaborative research program focusing on species invasions and disease outbreaks in the upper Mississippi River. As part of this presentation, we will emphasize three key components of the CORE program: education, research and dissemination. Early in the program, students gain a strong understanding of the host-parasite system, experimental design, and mathematical techniques through class-based
lectures (education). Students then engage in a 10 -week research experience which combines empirical methods with differential equation/statistical models. To date these projects have included 1) understanding the role of host competition in parasite transmission and 2) investigating how infected and uninfected hosts allocate resources to different life-history traits. Each year-long experience concludes with student pairs presenting their results in a number of different formats including video learning modules. The benefits of these experiences for students, faculty, and the university as a whole will be discussed.

## Namyong Lee Minnesota State University, Mankato (nlee @mnsu.edu)

## Bottom-up Approach to Mathematical Modeling in Ecology and Epidemiology

One of the main ingredients in undergraduate "mathematical (modeling in) biology" courses is developing class projects. However, often it requires huge effort to develop meaningful and interesting projects. In this talk, we illustrate how some basic mathematical models, such as logistic population model, predator-prey dynamics, and SIR epidemics, can be developed into mathematically rich and biologically interesting models. We include both time and space variables in continuous, discrete, and mixed dynamical systems.

## Robert Allen University of Wisconsin-La Crosse (rallen@uwlax.edu)

## Zombies, Ecology, and Epidemiology ... Oh My!

In this talk, I will discuss ongoing undergraduate research in mathematical ecology and epidemiology concerning the spread of zombiism through a human population. I will discuss several variations of the standard SIR model being considered, and the analytic and computational techniques being used to analyze such models. I will also discuss where these models can be introduced into the math curriculum to attract students to research in mathematical biology. Lastly, I will discuss how the zombie models being considered have real biological interpretations, making the undergraduate research experience worthwhile, and most importantly, fun!

## Talitha M Washington Howard University (talitha.washington@howard.edu)

## Infusing the Flavor of Mathematical Biology Research into the Undergraduate Experience

Many mathematics departments seek feasible ways to incorporate undergraduate research in mathematical biology into the curriculum. However, there is a challenge to ensure a balance between the cost to the faculty and the benefit to the student. The research must be attainable for students to comprehend in a short period of time. It must also encourage the student to be creative with mathematics and, if possible, develop her own mathematics to describe biological processes.

This can be achieved by developing a topics course based on current research articles in the field. These topics courses cater to students ranging from from first-year to advanced graduates in both mathematics and biology. Through these research-based learning experiences, students can begin to understand the research process as well as gain valuable insight on the interplay between mathematics and biology. In this talk, we will present specific examples of how to develop and lead topics courses that infuse mathematical biology research into the undergraduate experience.

## John Zobitz Augsburg College (zobitz@augsburg.edu) <br> Tracy Bibelnieks Augsburg College (bibelnie @augsburg.edu)

## Internationalizing the Mathematics Major with the Calculus of Sustainability

The structured, sequential nature of a mathematics major presents challenges for students to participate in international education experiences. Recently at Augsburg College in Minneapolis, MN we engaged in course design to integrate international education experiences embedded within a Calculus II course. During Spring Break 2012, students traveled to a coffee cooperative in northern Nicaragua to develop and study bioeconomic models of community-based rural tourism.

This presentation will describe elements of the course design of Calculus II, integration with the international experiences, results from student projects, evaluation of the course design process, and describe additional steps to expand international education opportunities for mathematics and natural science majors.

## Thomas Q Sibley St. John's University (tsibley @csbsju.edu)

## Multiple Mathematical Models

Mathematical models help us explore some of the complicated interactions found in biology. My students have, in consultation with biology and biochemistry professors, investigated a variety of systems. I'll discuss their models of the regulation of gene expression, the effects of paired drugs on countering HIV resistance and the kinetics of enzymes. In all of these projects, the students decided more than one model was needed to explore different aspects of the system.

## Timothy Comar Benedictine University (tcomar@ben.edu)

## Student Research Projects with Biological Models Using Impulsive Differential Equations

This talk focuses on undergraduate research projects using impulsive differential equations to model biological phenomena. The examples presented here will be a predator-prey model integrated pest management and an epidemiological model with pulse vaccination. We will discuss the project work, which includes analysis of the dynamics of these systems, and the student preparation needed to engage in this work.

## Marshall Hampton University of Minnesota Duluth (mhampton@d.umn.edu)

## The Other Half of the Genome: Projects in Retroviral and Transposable DNA

The rapidly decreasing cost of DNA sequencing is leading to a flood of publicly available genomic data. The annotations of these genomes usually concentrate on the "genes", protein-coding or otherwise clearly functional transcribed elements. But the majority of vertebrate genomes consists of repetitive DNA of various types, including endogenous retroviral elements and other transposable elements. This somewhat neglected trove of data provides opportunities for undergraduate research in bioinformatics, computational biology and statistics.

## Dan Hrozencik Chicago State University (dhro@att.net) <br> Timothy Comar Benedictine University (tcomar@ben.edu) <br> Undergraduate Research in Gene Regulation Networks

Across several summers, the authors have sustained research projects with multiple students in the area of gene regulation. In particular, the students worked on projects comparing discrete and continuous models of three- and four-gene regulatory networks. The authors will present information regarding the content of the projects, the prerequisite knowledge of the students conducting the research, as well as information about the resources needed to maintain such an ongoing research program.

Istvan Lauko University of Wisconsin Milwaukee (iglauko@uwm.edu)
Gabriella Pinter University of Wisconsin Milwaukee (gapinter@uwm.edu)
Sandra McLellan University of Wisconsin Milwaukee (mclellan@uwm.edu)

## Analysis and Simulation Of Bacterial Pollution on an Urban Beach at Lake Michigan

In this talk we report on an investigation that was conducted by participants of the undergraduate research program in Biology and Mathematics at the University of Wisconsin Milwaukee mentored by mathematics and biology faculty. The project concerns the health of Lake Michigan beaches, in particular, an urban beach in Milwaukee. Our study has three main components: experiments conducted to determine survival and growth rates of E.coli and enterococcus in sand, the assembly and analysis of a long-term dataset, and the building of a mathematical model of the beach that resolves spatial and temporal changes of moisture as well as bacteria and nutrient dynamics in the sand. The experiments gave students the opportunity to experience laboratory work and to understand the inherent difficulties in data collection and interpretation. The data set assembled spans eight years from 2004 to 2011, and includes bacteria counts along different transects of the beach, bacteria counts in water, and environmental factors such as temperature, precipitation and wind. The data has been analyzed to find relationships between precipitation and bacteria levels in the sand and water, as well as for differences that a rain garden installed in 2008 has potentially made. Our mathematical model consists of a system of partial differential equations in two spatial dimensions that represents moisture transport, as well as moisture mediated transport of both dissolved nutrient and mobile biological contaminants in the porous medium under varying environmental conditions. Computational simulations confirm the feasibility of the modeling approach coupling physical and biological dynamics.

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Forrest Mitchell Texas A\&M University (FMitchel@ag.tamu.edu)

## Mathematical Models of Glassy-winged Sharpshooter Population Dynamics in Texas Vineyards

Pierce's Disease (PD) is a threatening bacterial disease of grapevines with the capacity to kill an entire vineyard in one year in Texas. The disease is caused by a bacterium Xylella fastidiosa and is transmitted by a xylem-feeding insect commonly called a sharpshooter. The sharpshooter data with 50 different vineyards has been collected by Dr. Forrest Mitchell, Texas A\&M University since 2003. Among these, Homolodisca vitripennis (Glassy-winged sharpshooter) is known as the most common and abundant insect captured across all vineyards in Texas. Insect abundance varies significantly over seasons and locations.

We have developed the delayed logistic equation with various harvesting and immigration terms to analyze the temporal behaviors of the vector population. We have tried to explain the relation between the annual vector population and the environmental factors such as the insecticide use, information campaigns, weeds cleaning, and temperature changes by developing various models. To obtain the optimal parameter values of the model we applied the optimization algorithm based on the least square method using software, MATLAB. Recently
we use a linear harvesting and a constant immigration. Recently we added another model, Gompertz time-delay model and compared to the logistic model. In the nearest future, we plan to develop an appropriate optimal control problem where the underlying dynamics of the insect population is described by the models mentioned above. We believe that this project can help to understand the insect distribution through time and to set up its valuable management protocols in Texas vineyards.

# Open and Accessible Problems in Combinatorics and Graph Theory 

## Friday, August 3, 8:30-11:05 Am, Hall of Ideas E

## Cayla McBee Providence College (cmcbee @ providence.edu) <br> Lynette Boos Providence College (lboos@providence.edu)

Undergraduate research is more popular than ever, and there remains a high demand for open and accessible problems for students to tackle. Combinatorics and graph theory provide an ideal combination of easily stated, but more difficult to solve, problems. We invite presentations about open problems in combinatorics and graph theory suitable for undergraduate research or joint faculty and undergraduate research. Presentations giving results about these types of problems are also welcome.

## Susan Hollingsworth Edgewood College (shollingsworth@edgewood.edu)

## Tree Packing Conundrums

The tree packing conjecture of Gyárfás and Lehel has been outstanding since 1976. Variations on this conjecture give rise to some interesting and approachable problems. In this presentation we will discuss several such problems.

## Joshua Holden Rose-Hulman Institute of Technology (holden@rose-hulman.edu)

## Stitching Graphs and Painting Mazes: Problems in Generalizations of Eulerian Walks

An Eulerian walk traverses each edge of a graph exactly once. What happens if you want to traverse each edge of a graph exactly twice? If you want to cover the graph with "double-running stitch", then you need to traverse each edge twice but also put conditions on how many edges you traverse in-between. Then you could add conditions on whether you traverse the edges once in each direction or twice in the same direction. Which graphs can you still traverse? Classical algorithms for solving mazes give us some answers to these questions, but others are still open.

## David Jacob Wildstrom University of Louisville (dwildstr@erdos.math.louisville.edu)

## Domination Problems in Families of Graph Products

There are several problems of domination numbers of graph products, most notably Vizing's Conjecture. This talk will present some easy-toverify results and open problems regarding the asymptotic behaviors of the domination number of a product of a fixed graph with a sequence of graphs of increasing size, with particular emphasis on paths and cycles.

## Amanda Watkins The University of Iowa (amanda-watkins@uiowa.edu)

## Consecutive Radio Labelings

Radio labeling is a generalization of vertex coloring, and is a topic that has been a good source of a range of problems, including those appropriate for undergraduate research. Instead of only requiring that adjacent vertices have different labels, we want a labeling $f$ that maps the vertices $V_{G}$ to $\mathbb{Z}^{+}$to satisfy $|f(u)-f(v)| \geq \operatorname{diam}(G)+1-d(u, v)$ for all distinct vertices $u$ and $v$. The analog of the chromatic number is the radio number $\operatorname{mn}(G)$, the minimum span of all possible radio labelings of $G$. Because no two vertices can share the same label in this context, $\mathrm{rn}(G) \geq\left|V_{G}\right|$. We are interested in examples of graphs such that $\mathrm{rn}(G)=\left|V_{G}\right|$, in which case the vertices are labeled with consecutive integers. Complete graphs are trivial examples; examples of high diameter are especially desirable. We construct such examples of arbitrarily high diameter, and investigate further the tool that allowed us to do this-the Cartesian product of graphs-and its influence on radio labeling.

## Zsuzsanna Szaniszlo Valparaiso University (Zsuzsanna.Szaniszlo@valpo.edu)

## Graph Labelings for Undergraduate Research

One of the most famous open problems in graph theory is the graceful tree conjecture made about half a century ago and still very much open. The conjecture generated a new area of graph theory called graph labeling that has been growing ever since and which provides an inexhaustible source of problems suitable for investigation with a variety of backgrounds. We will present the conjecture, a few related problems and provide a few solutions obtained by undergraduates from freshmen to senior.

## Saad El-Zanati Illinois State University (saad @ilstu.edu)

On Some Open Problems in Graph Designs and Graph Labelings
We survey some of the classical questions in combinatorial design theory and discuss how they relate to the area of graph labelings. We report on some recent progress in graph labelings and share some of the resulting conjectures.

Michael Krebs California State University, Los Angeles (mkrebs@calstatela.edu)
Counting Closed Walks in Graphs to Get Nifty Combinatorial Formulas
In any finite graph, the number of closed walks of length $n$ equals the sum of the nth powers of the eigenvalues of any adjacency matrix of the graph. This simple observation opens up a whole supermarket aisle full of pre-packaged, open-and-serve accessible student research projects. For one can start with any particular graph, compute both the number of closed walks and the eigenvalues, and voila! A nifty combinatorial formula pops out. In this talk, I'll discuss some student research projects that have already been conducted in this vein (e.g., recovering the Binet formulae for Lucas and Fibonacci numbers, obtaining an expression for the sum of evenly and symmetrically spaced entries in a row of Pascal's triangle) as well as other avenues to explore, spanning the full spectrum of difficulty levels from the trivial to the unsolved.

Thomas Q Sibley St. John's University (tsibley @csbsju.edu)

## Maximizing Code Words

Hamming developed a family of error correcting codes for dimensions $2^{k}-1$ that had the maximum number of code words, where the Hamming distance between any two code words is at least 3 . What is the best we can do for binary codes in other dimensions? Linear codes are optimal for small dimensions, as well as the special dimensions of Hamming codes, but not all dimensions. Two students and I used combinatorics to find upper bounds on the maximum number of code words. We found, for example, that linear codes in 8 dimensions aren't maximal.

## Inquiry-Based Learning Best Practices

## Session 1: Friday, August 3, 9:30-11:30 AM, Hall of Ideas G

Dana Ernst Plymouth State University (dana@ danaernst.com)
Angie Hodge University of Nebraska - Omaha (amhodge@unomaha.edu)
Stan Yoshinobu California Polytechnic State University (styoshin@calpoly.edu)
In many mathematics classrooms, doing mathematics means following the rules dictated by the teacher and knowing mathematics means remembering and applying these rules. However, an inquiry-based learning (IBL) approach challenges students to create/discover mathematics. Boiled down to its essence, IBL is a method of teaching that engages students in sense-making activities. Students are given tasks requiring them to conjecture, experiment, explore, and solve problems. Rather than showing facts or a clear, smooth path to a solution, the instructor guides students via well-crafted problems through an adventure in mathematical discovery. The talks in this session will focus on IBL best practices. We seek both novel ideas and effective approaches to IBL. Claims made should be supported by data (test scores, survey results, etc.) or anecdotal evidence. This session will be of interest to instructors new to IBL, as well as seasoned practitioners looking for new ideas.

## David Taylor Roanoke College (taylor@roanoke.edu)

## A Modified-Moore Method Approach to Abstract Algebra

The inquiry-based learning (IBL) approach, or Moore method approach, has been popular in mathematics in the recent past; my own undergraduate experience, however, only consisted of one full IBL course, and it was absolutely my favorite! While preparing to teach abstract algebra at Roanoke College for the first time, I realized that some students may not be strong enough for a full IBL class, but I did not want to do a traditional lecture course. In this talk, I will discuss my experiences with a modified-Moore method approach to abstract algebra (two versions, from $66 \%$ student-driven to $33 \%$ student-driven) and my empirical evidence from student feedback over the years.

## Brigitte Lahme Sonoma State University (lahme@sonoma.edu)

## Activities in Upper Division Math Classes: Advantages and Challenges

We know exactly what we are teaching our students; after all, we are preparing very careful lectures and lessons. But what are the students actually learning? Introducing activities in upper division mathematics classes allows us to center the learning in the classroom around the students. Listening to students' group discussions also gives us a glimpse into their thinking and helps us learn about a variety of mathematical issues that would otherwise go undetected.

In this talk, we will look at some of the activities we are using in Modern Algebra, Real Analysis and other math major courses and share what we learned from them.

David Olson Michigan Technological University (daolson@mtu.edu)

## Capstone First: Starting With a Research Seminar

When students arrive at college, they expect that their classes will be harder, challenging, different. Seize the opportunity! We will describe four introductory courses whose central goal is for the class to become an independent research community, where the instructor is (almost) extraneous. In addition to creating a cohesive social support group for future math classes, these courses also develop a wide variety of skills, from public speaking to an appreciation of the central role of definitions. They are very demanding for an instructor, requiring the ability to sit on one's hands and keep one's mouth shut.

## David Clark Michigan Technological University (dcclark@mtu.edu)

## Hands Off! Inquiry Based Learning in Senior Number Theory

How do you get students to work hard to learn Number Theory? Let them hold each other accountable, and stand back! Our department offers a senior level inquiry-based number theory class for majors and minors. Student groups use Mathematica to explore and collect data, with an emphasis on developing skills which working mathematicians need every day: organizing data, creating conjectures, writing clear and convincing proofs, and working closely with colleagues. We will discuss a number of organizational and pedagogical practices which support (and don't interfere with!) these goals. These include student groups which support and encourage each other; pre-labs and reflections which help students come to the surface and see the "big picture" of the course; and communicating course organization via the Canvas Learning Management System.

Patrick Rault SUNY Geneseo (prault@gmail.com)
Olympia Nicodemi SUNY Geneseo (nicodemi@geneseo.edu)

## Co-teaching a Credit-enriched Section of Abstract Algebra

P. Rault and O. Nicodemi team-taught inquiry-based Abstract Algebra this past semester, as a 4-credit-hour version of the standard 3-credithour class. We would like to share our experience team-teaching, the questions it raised, and the answers it provided about inquiry-based learning.

## Susan Bailey Crook North Carolina State University (sbcrook@ncsu.edu)

## Opening the Lines of Communication With Your IBL Students

In any classroom, communicating openly with students is a necessity, but in argue an IBL classroom it is even more vital. From selling the method during the first week of the class to checking in on students' attitudes throughout the semester, talking with students can help prevent a class mutiny. I will discuss some of the many ways I gathered feedback from my students (formal surveys, brief emails, reflections, etc) and their various success rates. I will show actual student feedback and discuss my responses to them. We will end by examining some student reflections and thoughts about the style of the class.

## Session 2: Friday, August 3,1:00-5:00 PM, Hall of Ideas G

## Jessica Audet de la Cruz Assumption College (jdelacruz@assumption.edu)

## A Model for IBL in Mathematics Content Courses for Prospective Elementary School Teachers

A model for inquiry-based learning (IBL) within mathematics courses for elementary school teachers will be discussed. Anecdotal evidence for using this model of IBL to increase conceptual understanding will be shared and specific examples of well crafted tasks designed to lead students to certain conceptual discoveries will be given.

## Diana White University of Colorado Denver (diana.white@ucdenver.edu)

## One Model of IBL for Practicing Teachers

Inquiry based learning is a teaching method that engages students in sense-making activities. Specifically, students are given tasks requiring them to solve problems, pose problems, conjecture, experiment, explore, create, and communicate ... all those wonderful skills and habits of mind that mathematicians engage in regularly. We are familiar with a myriad of approaches to this at the undergraduate and graduate levels. However, to systematically reform mathematics education in this country will require also addressing K-12 mathematics instruction. Many K-12 teachers find teaching in an inquiry-based format to be a daunting task, as most only ever experienced learning mathematics in a primarily procedural, lecture-based format. However, the new Common Core State Standards place a high value on mathematical practices and sense making. Thus, there is a strong need for mathematical professional development for practicing teachers that supports them in becoming more conversant with the mathematical practices.

The Math Teachers' Circle program, developed by the American Institute of Mathematics in 2006, aims to meet this need by providing an innovative form of professional development in which mathematicians directly facilitate mathematical problem solving sessions with middle school teachers. In this talk, we describe a typical Math Teachers' Circle, showing why it can be considered an ideal form of inquiry based
learning for practicing teachers, describing preliminary research results, and detailing how conference attendees can get involved with this program.

Hortensia Soto-Johnson University of Northern Colorado (hortensia.soto@unco.edu)
Gulden Karakok University of Northern Colorado (gulden.karakok @unco.edu)
Stephanie Anderson University of Northern Colorado (stephanie.anderson@unco.edu)

## Teaching and Learning the Arithmetic of Complex Numbers through IBL with Inservice Secondary Teachers

The Common Core Standards emphasizes that students should be able to represent the arithmetic of complex numbers and use this number system in equations and polynomial identities. In an effort to strengthen inservice secondary teachers' own understanding of complex numbers, we developed and modeled IBL-activities as part of professional development. In this presentation, we will describe the activities, the teachers' responses to these activities, and how the teachers independently incorporated their prior knowledge of other mathematical domains to make new discoveries.

## Robert Sachs George Mason University (rsachs@gmu.edu)

## An IBL Enrichment Summer Short Course for BC Calculus Students

This July I will experiment with a short course done in modified IBL mode. Students are high school students who just completed BC calculus. The talk will describe the course and report on some measurements of student attitudes before and after the course.

## Randall E Cone Virginia Military Institute (conere10@vmi.edu)

## Hybridized Methods in Freshman Calculus

In an effort to promote student learning and activity within all ability levels, we create a hybridized learning process for a freshman differential calculus course. The hybridization consists of aspects from traditional mathematics teaching methodologies, procedural reinforcement (through technology channels), and Inquiry-Based Learning techniques. The purpose of this short talk is to describe the successes and pitfalls of such an approach, and to outline a plan of action for successive iterations of hybridized courses in differential calculus.

## Jonas D'Andrea Westminster College (jdandrea@westminstercollege.edu)

## Pwnage!

How do we get students to take ownership of their learning? Especially in lower level classes? I will talk specifically about some recent experiences (good and bad) of trying to incorporate some IBL methods in introductory biostatistics and biocalculus courses from this past year, including a student-generated "SPSS User's Guide" wiki.

## Catherine Beneteau University of South Florida (cbenetea@usf.edu) <br> Zdenka Guadarrama Rockhurst University (guadarrama@rockhurst.edu) <br> Jill Guerra University of Arkansas Fort Smith (jill.guerra@uafs.edu) <br> Laurie Lenz Marymount University (Laurie.Lenz@marymount.edu)

## The POGIL project: Student-Centered Learning in Calculus Preliminary Report

POGIL is a student centered, guided inquiry teaching method that has been shown to increase student engagement and achievement in the classroom by having students work in self-managed teams to analyze data and draw conclusions, modeling the way a team of scientists functions in the research laboratory. This talk is a report of a new NSF-funded project to develop a set of POGIL learning materials for Calculus and Pre-Calculus and to build faculty expertise within the mathematics community to implement these new materials. The talk will include a discussion of the POGIL project as well as examples of learning materials currently being tested in the classroom.

## Bret Jordan Benesh College of Saint Benedict/Saint John’s University (bbenesh@csbsju.edu)

## Creating an IBL/Flipped-Classroom Hybrid

Two of the most promising ideas for improving pedagogy in college mathematics classrooms are "inquiry-based learning" and "flipped classrooms." Inquiry based learning describes a method of teaching where the students are guided through the course material by a solving carefully-crafted series of problems. A "flipped" classroom is one where the transmission of the course content is done outside of class (usually through the textbook and/or videos) and class time is used to assimilate what was learned outside of class.

Wouldn't it be a good idea to try to merge the ideas of IBL with the ideas from a flipped classroom approach? Probably not, but I tried it anyway. I will be reporting on what I learned by teaching such a hybrid approach in a complex analysis class taught in Spring 2012.

## Brian Katz Augustana College (briankatz@augustana.edu) <br> Eliazbeth Thoren University of California Santa Barbara (ethoren@math.ucsb.edu)

## Higher-Order Tasks in an Inquiry-Based Course

Many proponents of inquiry-based learning (IBL) make claims that students in IBL courses spend more time doing higher-order tasks than in other courses. During this term, I have been asking my IBL Modern Geometry students to report on the amount of time they have been spending on the coursework, what percentage of that work they think falls at each of the levels of Bloom's Taxonomy, and how those percentages compare to the percentages in other courses on campus and in the department. I will also ask the students to connect particular course activities with the Bloom's level of tasks they require. In this talk, I will describe a particular IBL course structure and share an analysis of the student responses about the Bloom's level of the work.

## Ryan Gantner Saint John Fisher College (rgantner@sjfc.edu)

## IBL for fun?

There are many reasons why a growing number of mathematics instructors are using Inquiry Based Learning (IBL) practices in the classroom. Some of these include getting the students engaged in the learning process, improving logical reasoning and writing skills, fostering creativity, and understanding big picture ideas. While not void of these reasons, we'll see a course whose IBL format was driven by a different reason: fun. An IBL course in the theory of games will be outlined and we'll see how much more fun can be had in the classroom because of IBL.

## Mindy Capaldi Valparaiso University (mindy.capaldi@valpo.edu)

## Transitioning to Teaching Abstract Algebra as IBL

This presentation will discuss the experience of one professor's transition from teaching abstract algebra as lecture-based to inquiry-based. Descriptions of the delivery of the material, the type of homework assignments, and methods of evaluation will be presented. Also under discussion will be the problems that occurred and plans for fixing them in the future. Some of the, perhaps, more uncommon approaches such as oral exams and individual, or no group-work, homework problems will be emphasized. The success of the transition to inquiry-based practices will be illustrated through grade evaluation, student surveys, and anecdotal evidence.

## Session 3: Saturday, August 4, 9:30-11:30 Am, Hall of Ideas G

## Anders Hendrickson Concordia College (ahendric@cord.edu)

## An IBL Liberal Arts Mathematics Course

The "Exploring Mathematics" course, through which most students at Concordia College (Moorhead, MN) fulfill their mathematics requirement, was recently changed from traditional to inquiry-based pedagogy with great success. We discuss the workbook written for the class as well as strategies employed in the classroom, and we summarize some of the challenges encountered and lessons learned.

## Theron James Hitchman University of Northern Iowa (theron.hitchman@uni.edu)

## Deep Content and Greatest Hits for a Liberal Arts Mathematics Course

We report on an experimental IBL course for non-STEM students. The course focused on widening appreciation for the nature of mathematics by engaging students in challenging topics: Cantor's work on countable and uncountable sets, the classification of surfaces and continued fractions. Results from a SALG survey on perceptions of mathematics will be shared.

## Jacqueline Jensen-Vallin Slippery Rock University (jacqueline.jensen@ sru.edu)

## IBL in a Liberal Arts Mathematics Course

Mathematics educators tend to think that students in liberal arts mathematics courses are not the proper audience for an inquiry-based learning (IBL) approach. At Slippery Rock University I have been teaching a financial mathematics course for liberal arts majors in an IBL style. Due to a grant from the Academy of Inquiry-Based Learning, I was developed a set of course notes for this class, and taught using these notes in Fall 2011 and Spring 2012. In this talk, I will focus on details about this course and the notes. Data on exam scores and examples of student comments will be provided.

## Martha Ellen Waggoner Simpson College (murphy.waggoner@simpson.edu)

## Student-driven symmetry classification

I teach a unit on symmetry in a course that has no mathematics prerequisite and fulfills both a quantitative reasoning and an arts general education requirements. Students typically take the course to avoid "harder" math classes.

In an inquiry-based activity, the students develop their own classifications of symmetry patterns. We first discuss the rigid transformations, and they practice designing their own finite figures, friezes and wallpatterns. They also collect "data" by capturing digital images of as many examples as possible.

They then work on their own classification schemes from their data. Working as a class they divide and conquer, sorting the images by characteristics based on the rigid transformations. The progression from classifying finite figures to frieze patterns and finally wallpatterns gives them the opportunity to build confidence at each step. They demonstrate ownership of the process by creating their own nomenclature.

From the current class I am teaching, I will report on student attitudes towards mathematics before and after this inquiry-based project and excerpts from the student reflections. I will also look for changes in their critical thinking skills due to this activity.

## Wade Ellis West Valley College (Wellis@ti.com)

## Constructing Effective Inquiry Questions for Mathematical Applets

Numerous mathematical applets exist for use in developing student understanding of mathematical concepts and skills. Well-constructed questions for use with such applets enhance the effective use of such applet. Several precalculus and calculus applets will be presented along with techniques for creating inquiry questions that can promote student discourse, reflection and understanding.

## Allen Gregg Harbaugh Murdoch University (allen.harbaugh@murdoch.edu.au)

## Preparing Students for Inquiry-Based Learning: Lessons Learned from the Online Learning Environment

It is not uncommon for instructors to experiment with non-traditional pedagogies and innovative activities. The lessons are well crafted and the activities planned with care. Yet, despite the preparation and the good intention, some student responses can be quite negative. Theories in educational psychology suggest this may be less a consequence of the teaching quality and more an attribute of the academic capital of the students. Research from adult education and online learning proposes reasons for the students' negative reception. Three influential factors emerge: students' level of autonomous learning, epistemic beliefs regarding the nature of authority and justification, and active participation in a community of learning. While many students come to tacitly acquire these skills or beliefs, other students hold expectations about mathematics instruction that hinder them from both establishing this academic capital and engaging fully with the structured learning activities. This presentation discusses research from the online learning environment and describes the parallel nature with inquiry-based learning (IBL). The focus of the talk is to elaborate strategies to support students in math classrooms using IBL. These strategies are designed to engage students in IBL activities while supporting them as they develop skills of autonomous and self-directed learning. In addition, engagement in these activities (1) encourages a more sophisticated belief about the nature of mathematical justification and (2) establishes a community of learners via mathematical discourse.

# The History of Mathematics and its Uses in Teaching and Learning Mathematics 

## Friday, August 3, 1:00-2:55 PM, Hall of Ideas H

## Kelli Slaten University of North Carolina - Wilmington (slatenk@uncw.edu) <br> Scott Guthery Docent Press (sbg@acw.com)

Mathematicians, historians, educators, independent scholars and science writers use the increasingly available corpus of historical mathematical literature to study, understand and elucidate topics mathematical, scientific, historical, intellectual, literary and otherwise. Contributions to this session are case studies in the use of material drawn from the history of mathematics. Speakers describe 1) how they were led to consider this material for their project, 2) how they went about finding, exploring and mining the material, and 3) the impact that the material had on the success or failure of their project.

This Session Sponsored by HOM SIGMAA.

## Bob Palais Utah Valley University (Bob.Palais@uvu.edu)

## How Rodrigues Did It: The Geometric Discovery of Quaternion Multiplication

The math of rotations remains a lively field with applications in computer animation, robotics and many other areas of pure and applied math. The history is exciting and enhances student engagement and understanding in trigonometry, linear algebra and other classes. The
formula for composing spatial rotations in terms of their axes and angles was discovered in a revolutionary 1841 paper by Olinde Rodrigues that anticipated the discovery of quaternions by Hamilton shortly afterward. We will recount the colorful history surrounding Rodrigues and Hamilton and examine the method he used to derive the composition formula. His starting point is a beautiful formula that relates the displacement of $\vec{v}$ under a rotation $R$ to its axis $\vec{u}$, angle $\theta$, and the midpoint of the displacement in the following form:

$$
R \vec{v}-\vec{v}=2 \tan \left(\frac{\theta}{2}\right) \vec{u} \times \frac{\vec{v}+R \vec{v}}{2}
$$

We will show that this formula is equivalent to the Euler-Rodrigues formulas for the rotation matrix, and explain how Rodrigues used it to obtain the formula for unit quaternion multiplication.

## David Richeson Dickinson College (richesod@dickinson.edu)

## Who First Proved that $C / d$ is a Constant?

Open a history of mathematics textbook and you will discover many facts about the history of $\pi$. They begin by asserting that $C / d$ is a constant and that this fact has been "known" for thousands of years. Indeed $C / d$ is generally taken as the definition of $\pi$. But they are all silent on the question of who first proved this basic fact. We discuss this interesting question, which touches on the history of curves and arc length.

## Shenglan Yuan LaGuardia Community College (syuan@lagcc.cuny.edu)

## Using Ancient Egyptian Fractions in Teaching Developmental Math

The ancient Egyptians only used unit fractions. They represented non-unit fractions as the sum of distinct unit fractions. Just why they represented all fractions that way remains a mystery. In this talk, well describe a developmental math course lesson that uses unit fractions to compare the size of fractions. Grasping the size of fractions while using Egyptian fractions seems more intuitive for students and ultimately helps them understand the meaning of fractions. Well describe the process of presenting a fraction as the sum of distinct unit fractions. Well also look into far-reaching problems regarding unit fractions such as Erdss unit fraction conjectures.

## Kathleen Michelle Clark Florida State University (kclark@fsu.edu)

## An MAA/Tensor Project: Young Women Studying the History of Mathematics

Our "Women and Mathematics" (WAM) group has met for two spring semester sessions and during our time together we have both rediscovered and shared an interest in the historical development of mathematical concepts. The group of young women includes staff members of the FSU-Teach Program at Florida State University and students from local middle schools, high schools, FSU, and Tallahassee Community College. The focus of the first spring session was on the mathematical contributions of famous women, including Florence Nightingales statistical analyses, Grace Chisholms geometry text, and Sophie Germains 'grand plan'. Throughout the three "units" we explored each woman's life (some in greater detail than others) and using primary sources, we investigated the mathematics presented by Nightingale, Chisholm, and Germain. This spring we used Berlinghoff and Gouvłas Math Through the Ages to help us focus on fundamental topics that were of interest to the group. We are currently delving into the essentials of algebra from such greats as Descartes, Cardano, and Euler. As a group, we first examine the mathematical ideas discussed in Berlinghoff and Gouvła; then, we turn to primary sources to investigate the mathematics more deeply. Our presentation will include samples of the units from the first session and examples of participants' mathematical work from the second session.

## Charlie Smith Park University (charlie.smith@park.edu)

## Emanuel Lasker: Mathematician and Chess Immortal

For many years I have known that Lasker, World Chess Champion from 1894-1921, also held a doctorate in mathematics and produced new theorems in the realm of commutative ring theory.

As an avid student of both chess and ring theory, it was inevitable that someday I would undertake an investigation of his life and his mathematical achievements. My intention is to incorporate the fruits of my research into my course MA 350 History of Mathematics. My students are well aware of multi-talented mathematicians such as Archimedes and Gauss, but I have never before included such a person as Lasker, who excelled at both mathematics and chess, and furthermore wrote several books in philosophy.

In teaching the history of mathematics, too often I accentuate only mathematical achievements. With an individual like Lasker, I can cover his accomplishments in all three areas and stress that a mathematician can excel in arenas outside of the natural sciences.

## Brian Hopkins Saint Peter’s College (bhopkins@spc.edu)

Finding Euler Cycles: A French Criticism and Solution
Euler's 1736 article on the bridges of K onigsberg is a well-known and accessible introduction to graph theory. There is scant attention, though, to how to find what we now call Euler cycles in graphs when they are known to exist. Textbooks often present two algorithms for this task, sometimes without attribution, from Hierholzer (1873) and Fleury (1883). In this presentation, we will consider Fleury's article, written in response to Euler's work (Hierholzer, in contrast, was addressing figure-tracing puzzles). Students have enjoyed exploring this historical document and have been amused by the tone of Fleury's criticism of Euler's paper.

## Engaging Undergraduates in Geometry Courses

## Session 1: Friday, August 3, 1:00-3:15 PM, Hall of Ideas F

Sarah L. Mabrouk Framingham State University (smabrouk@framingham.edu)
James Hamblin Shippensburg University (jehamb@ship.edu)
M. Brad Henry Siena College (mbhenry @gmail.com)

There are a variety of geometry courses in undergraduate curricula that serve a variety of purposes. They can provide content knowledge in Euclidean geometry for future educators, provide a historical perspective on mathematics, introduce students to the axiomatic method, provide the content for a terminal course for liberal arts majors, introduce students to non-Euclidean geometric systems, provide advanced content for a major course beyond analysis, or form the content of a first course in writing proofs.

In this session, we invite presentations that address the following questions:

- What approaches and pedagogical tools are best?
- What are particularly good topics with which to begin geometry courses?
- What are some of the most enjoyable proofs to share with students?
- What are the best ways in which to explore polyhedra, tessellations, symmetry groups and coordinate geometry?
- How can we help students to develop the visualization skills for two and three dimensions as well as to help them to develop the mathematical reasoning skills that are important for studying/exploring/applying geometry at any level?
- What are the best ways in which to compare and contrast Euclidean and non-Euclidean geometry?
- How can we best convey the beauty of geometry to students?

Presenters are welcome to share interesting applications, favorite proofs, activities, demonstrations, projects, and ways in which to guide students to explore and to learn geometry. Presentations providing resources and suggestions for those teaching geometry courses for the first time or for those wishing to improve/redesign their geometry courses are encouraged.

## Michael Nathanson Saint Mary's College of California (man6@stmarys-ca.edu)

## The Pizza Theorem and the Joy of Discovery

The best mathematics course I took as an undergraduate was Tom Banchoff's student-driven class in geometry. This course began with a list of ten challenging questions and evolved organically based on student efforts at solution. This experience was my first opportunity to explore and research mathematics and had a profound impact on me both as a student and as a teacher. It also introduced me to one of my favorite geometry problems, the Pizza Theorem, which was recently written up in Mathematics Magazine.

I will demonstrate this theorem and its generalizations; and discuss how I have used problems like this to recreate Professor Banchoff's active, exciting classroom culture which I enjoyed as an undergraduate.

## Aaron Hill University of North Texas (aaron.hill@unt.edu)

## Two Geometry Problems

A geometry teacher might ask: "How can I help my students to develop the visualization skills (or the reasoning skills) that are important for studying/exploring/applying geometry?"

We'll discuss two important aspects of an answer to the above question: Rich mathematical problems and substantive student engagement.
Then we'll discuss two geometry problems that are simple to state and naturally interesting (increasing the likelihood that students would be substantively engaged) and that require important visualization and reasoning skills (so in some sense they are mathematically rich).

## J Bradford Burkman Louisiana School for Math, Science, and the Arts (bburkman@1smsa.edu)

## Elementary and Advanced Coordinate Geometry Exercises on a Single Triangle, with Euclidean Connections

In my teaching, I make extensive use of triangles in the plane, and connect the techniques back to Euclidean geometry. Using a single triangle for several exercises shows students the intricate symmetries and depth in a simple figure, and the beauty of the resulting diagrams can encourage students to continue to explore.

In lower-level classes I use the centers along the Euler line, with the associated lines, concurrences, collinearity, circles, and distances, as introductory practice and as a culminating course project. In higher classes I use the segments that cut the area of a triangle in half as an occasion for students to practice parameterization, limits, trigonometry, and conic sections [yes, there are hyperbolas]. Technologically, my students use GeoGebra to explore, Sage to do the heavy symbolic computations, and TikZ to make beautiful diagrams.

We will look at six ways to find the area of a triangle, the Euclidean underpinnings of the formulas for slope, midpoints, distance, and equations of lines, and the rich mathematics we find when we cut a triangle in half. We will look at the technology, and explore the qualities of "good" exercises and how to find them.

## Michael Kerckhove University of Richmond (mkerckho@richmond.edu)

## Geodesic Intuition

According to the Ribbon Test, developed as a teaching tool in David Henderson's book Differential Geometry: A Geometric Introduction, a curve lying on a surface in $\mathbb{R}^{3}$ is a geodesic in that surface only if a stiff ribbon can be laid flat against the surface with its centerline in contact with the curve. Coupling this condition with Clairaut's relation for geodesics on surfaces of revolution and the important idea of local symmetry along a curve offers the opportunity for an engaging and intuitive discussion of the behavior of geodesics on surfaces.

## Ruth I Berger Luther College (bergerr@luther.edu)

## Developing Intuition for Hyperbolic Geometry

I want the students in my Euclidean and non-Euclidean Geometry course to develop a feel for hyperbolic geometry, basically replacing their Euclidean intuition by a Hyperbolic one. I start the course by introducing them to 2 different worlds: "Escher's World" is as a disk populated by inhabitants in which everything shrinks towards the outside. The "Green Jello World", inhabited by fish, consists of Jello that is less dense in one direction, but infinitely dense at the end of the world. Students realize you can get from $A$ to $B$ with much fewer steps/flipper strokes by not necessarily following a Euclidean line. They naturally come up with curved looking paths! Throughout the course, whenever we ask if a certain fact should hold in Hyperbolic Geometry, we first investigate it by drawing pictures in these worlds: "How many lines through $P$ are parallel to $L$ ? Can a line lie entirely in the interior of an angle?". Having this hyperbolic intuition makes it much easier for students to then write formal proofs in hyperbolic geometry.

## Jeffrey Clark Elon University (clarkj@elon.edu)

## Finding a Balance Between Rigor and Exploration in a Non-Euclidean Geometry Course

This presentation will discuss approaches to teaching non-Euclidean geometric content to students whose prior exposure has only been to Euclidean geometry. It will discuss both a rigorous framework for the material as well as software to support student exploration, and will be aimed at first-time instructors.

## John Wasserstrass UW-Rock County (wasserstrass@att.net)

## Imagine This: $\mathbf{6 0 0}$ Cells in 4D

This talk will illustrate how the 4D regular polytopes can be used to challenge the geometry student to think outside the box. I will start by showing how the rectangular coordinates for the tetrahedron, octahedron and cube can be extended into all higher dimensions. Students can be challenged to show that Euler's formula for polyhedra works for these. Then I will show how Descarte's rule for defect can be used to show that besides these 3 , there can be only 2 more 3D and 3 more 4D regular polytopes. Next, by aligning the icosahedron and dodecahedron with the cube, we will obtain the coordinates for them in terms of the golden ratio, using the golden brick with diagonal 2 . Models will be used to show the construction of the 120 and 600 cell 4D polytopes, which are duals of one another. Finally, the coordinates of the layers of vertices in the 600 cell will be derived from the appropriate 3D regular polyhedron

## Session 2, 1:00-3:15 PM Saturday, Hall of Ideas F

## Jason Douma University of Sioux Falls (jason.douma@usiouxfalls.edu)

## Are We There Yet? Distance and Persistence in the Poincaré Model

Mathematics knows no shortage of existence theorems. One benefit of studying geometry is the opportunity it provides for realizing or visualizing the objects whose existence is assured in its theorems. These opportunities come in the form of compass and straightedge constructions, software visualizations, and in some cases through direct analytic calculation. Unfortunately, a one-semester course in geometry offers precious little time to develop the scaffolding necessary for full-fledged analytic calculations in the context of hyperbolic geometry. This talk will examine a challenging exercise from an upper division geometry course which draws on students' knowledge of Euclidean analytic geometry to locate coordinates of points specified by distances in the Poincaré model of hyperbolic geometry. In addition to helping students better grasp key distinctions between Euclidean and non-Euclidean geometry, the exercise has inspired students to persevere in solving novel problems.

## David Eugene Ewing Missouri MAA (ewing@ucmo.edu)

## Hubcap Geometry

What geometry can exist on the surface of a Hubcap, Volcano, Saddle or a Donut? Teach Foundations of Geometry more effectively by having students create their own geometries on these surfaces. Several lessons will be demonstrated, including Creating Definitions, Exploring Shapes \& Their Relationships, Formulating Theories, and Writing Proofs.

## Sam Vandervelde St. Lawrence University (svandervelde@stlawu.edu)

## Propelling Students into the Projective Plane

The concept of the projective plane might come across to students as either arbitrary, unnecessarily complicated, or both. However, there are a variety of ways to naturally motivate both the construction of and the utility of this elegant geometry. In this talk I will share a collection of in-class activities, puzzles, and results that have proven to be effective in helping students to make the transition from the Euclidean plane to the projective plane.

Filiz Dogru Grand Valley State University (dogruf@gvsu.edu)
David Schlueter Vanderbilt University (schlueted@gvsu.edu)
Jiyeon Suh Grand Valley State University (suhj@gvsu.edu)

## Comparison of Quadrilateral Definitions in Euclidean and Non Euclidean Geometries

The students are familiar with the quadrilaterals definitions before starting the Geometry course. While we analyze properties and seek deeper understanding of the definitions of quadrilaterals in Euclidean Geometry, we investigate which one of those definitions works in the non Euclidean geometry (hyperbolic and elliptic). Through class activities students are discovering the answers. In this talk we shall share some of the students' examples and discuss difficulties that they have encountered. Additionally, we shall talk about 9-point circle in Euclidean and Minkowski geometries.

## Brian Katz Augustana College (briankatz@augustana.edu)

## Teaching Mathematical Maturity through Axiomatic Geometry

Mathematical maturity includes the skills to communicate with precision, attend to detail, and interpret results through the epistemologies of the discipline. I will describe an inquiry-based Geometry course structured around an axiomatic development of Euclidean and Hyperbolic Geometry, and I will analyze student products for evidence of changes in the level of mathematical maturity. The evidence will include a comparison of concept maps about mathematical truth from before and after the course as well as student reflection writings about the axiomatic method and their own development in proof construction and communication.

## Marshall A. Whittlesey California State University San Marcos (mwhittle@csusm.edu)

## Topics in Spherical Geometry for Undergraduates

A century ago, spherical geometry was a standard part of the mathematics curriculum in high schools and colleges. Today most mathematicians only learn about it as a short topic in geometry survey courses. In this talk we explore the idea of teaching spherical geometry at greater depth by discussing some key theorems of spherical geometry, short proofs and applications to other areas such astronomy, crystallography, and polyhedra. We discuss how to use different techniques (synthetic versus analytic) to advantage in this subject in the hope that the student will benefit from thinking about when each method is appropriate. We also think that comparison of theorems of spherical geometry to those of plane geometry are a good way for the student to see in a tangible way how changing axioms results in different theorems. We think that exposure to spherical geometry is particularly good for future high school teachers, but also that more mathematicians should be aware of its theorems and applications.

## Recreational Mathematics: New Problems and New Solutions

## Session 1: Friday, August 3, 1:00-3:15 PM, Hall of Ideas E

## Paul Coe Dominican University (coepaul@dom.edu)

Kristen Schemmerhorn Dominican University (kschemmerhorn@dom.edu)
As with all mathematics, recreational mathematics continues to expand through the solution of new problems and the development of novel solutions to old problems. For the purposes of this session, the definition of recreational mathematics will be a broad one. The primary guideline used to determine the suitability of a paper will be the understandability of the mathematics. Papers submitted to this session should be accessible to undergraduate students. Novel applications as well as new approaches to old problems are welcome. Examples of use of the material in the undergraduate classroom are encouraged.

## Colm K. Mulcahy Spelman College (colm@spelman.edu)

## Celebration of Mind: Perpetuating the Legacy of Martin Gardner

Martin Gardner ("the best friend mathematics ever had") died in 2010 after an astonishing publishing career spanning 80 years. His "Mathematical Games" column in Scientific American ran from the 1950s to the 1980s, and introduced thousands of budding mathematicians and
others to elegant problems and magical items which still inspire "Aha!" moments today. We'll discuss how to perpetuate his legacy via the annual Celebration of Mind events held worldwide each October, and the @WWMGT twitter feed.

Michael A. Jones Mathematical Reviews (maj@ams.org)
Brittany Shelton Lehigh University (bcs207@lehigh.edu)
Miriam Weaverdyck Bethel College (mweaverdyck@yahoo.com)

## A Graph-Theoretic Analysis of Rubik's Slide Puzzle

Rubik's Slide is a new puzzle with a $3 \times 3$ planar grid, reminiscent of the top of Rubik's Cube. However, individual squares may light up in one of two colors, or remain unlit. The goal is to use a series of moves, which we view as permutations, to change an initial given pattern to a given end pattern. Different instances of the puzzle have different initial and end patterns. We use a simpler $2 \times 2$ version of the puzzle to introduce a graph-theoretic approach to examine the puzzle. This graph-theoretic approach is used to fully analyze the 36 -vertex graph that represents the easy setting of the puzzle. For this graph, we determine its diameter (which gives a bound for the minimum number of moves to solve the puzzle) and a Hamilton path (which describes a sequence of moves that solves the puzzle regardless of the initial and ending patterns). Further, we consider the hard setting of the puzzle.

## Andrew J. Simoson King College (ajsimoso@king.edu) <br> Mancala as Nim

Mancala may be the oldest board game yet played. A solitaire version, Tchoukaillon, was devised in 1977 to model end-game Mancala strategies. Its board consists of seven pits in linear array, the last of which is the roumba. Play consists in taking all the tokens from a pit, and then seeding the tokens one at a time into the pits nearer the roumba, the last of which is placed in the roumba. Within this coontext-when play alternates between contestants wherein the first player unable to move is declared the loser-the game can be perceived as the classic impartial game Nim, and as such, with a little help from a CAS, is a fun, wonderful example of recursion.

John Kingen McSweeney Rose-Hulman Institute of Technology (john.mcsweeney 80 @gmail.com)

## Graph-based Model of Crossword Puzzle Solution Process

What distinguishes a crossword puzzle from a simple list of trivia questions is the interlocking nature of the answers in the grid - one solution can produce further ones in a cascading fashion. We build a mathematical model of the puzzle where answers are vertices in a graph, and nodes are linked via an edge if the corresponding answers cross, and determine the spread of the solution. Inspired by analogous problems in epidemic modeling, we determine what low-dimensional features of the grid structure and clue difficulty may predict the overall solvability of the puzzle. We also present simulations done on actual puzzles from the Sunday New York Times.

## David Nacin William Paterson University (nacind@wpunj.edu)

## Unique KenKen Over a Set of Complex Numbers

In the October 2010 entry of the blog Bit-player, a four-by-four KenKen over a set of complex numbers was posted with mention that the uniqueness of the solution had not been checked. We show uniqueness of their solution and classify all KenKen with similar conditions having unique solutions. Our criterion is given in terms of both the structure of the clues and the underlying Latin squares that generate such clues.

## Peter Staab Fitchburg State University (pstaab@fitchburgstate.edu)

## How Many Unique 4 by 4 Natural Magic Squares are There?

If we think of a magic squares as matrices, there are 7040 unique natural magic squares that are often classified into one of 12 types, called Dudeney types. Historically, two magic squares are considered equal if you can rotate or reflect one to get the other. In this light each magic square has another 7 related ones, which reduces the total number of unique ones 880 . We seek to reduce this number further by finding families of magic squares that are formed by multiplication by permutation matrices. Depending on the Dudeney type of a magic square, there are between 16 and 384 in its family. The determination of the family of every magic square can determine the total number of unique magic squares. Come join us to find the answer to the title question.

## David Jacob Wildstrom University of Louisville (dwildstr@erdos.math.louisville.edu)

## The Upper Bound of a Triangle-Free Game of Hajnal

Hajnal proposed a game where two players successively add edges to an initially-empty triangle-free graph on $n$ vertices until no more edges can be added without violating the triangle-free condition. One player seeks to end the game quickly, while the other player seeks to extend the game as long as possible. The only proven upper bound on length of a game with rational play is the value $\frac{n^{2}}{4}$, which occurs even with suboptimal play by the player seeking a quick ending. Joint work with Paul Horn and Csaba Biró has constructed a strategy which provides some hope of pushing this boundary down closer to the occasionally cited conjectural bound of $\frac{n^{2}}{5}$.

# Session 2: Saturday, August 4, 9:30-11:45 AM, Hall of Ideas H 

Jay Lawrence Schiffman Rowan University (schiffman@rowan.edu)

## Home Prime Reversals

During a paper presentation on The Home Prime Conjecture at MathFest 2011, one of the editors of The Journal of Recreational Mathematics asked if there was any known research on reversing the order of the factors at each step of the repeated factoring and concatenation process. Since I was unaware of any such research, I have undertaken a journey through each of the odd composite integers less than one thousand. The Home Prime Conjecture asserts that if one takes any composite integer, factors it, concatenates the factors and iterates this process, eventually a prime is obtained called The Home Prime of the original composite integer. For example, consider the composite integer: $35=5 * 7$; $57=3 * 19 ; 319=11 * 29 ; 1129$ is prime, the Home Prime of 35 obtained after three steps. Briefly, $35->57->319->1129$ (Prime). In my investigation, I do the following to arrive at the Home Prime Reversal of $35: 35=5 * 7 ; 75=3 * 5 * 5 ; 553=7 * 79 ; 797$ is prime, the Home Prime Reversal of 35 obtained after three steps. Briefly, $35->75->553->797$ (Prime). Many odd composite integers less than one thousand have their Home Prime Reversals obtained after a few steps but several have become stalled in a similar fashion to the Home Primes of 49 (and hence 77 and 711) where a large composite integer cannot be completely factored stalling the process. Using MATHEMATICA 8.0, I generate the results I have secured.

## William Davidson Saint Mary’s University of Minnesota (wdavidso@smumn.edu)

## Introducing the Peculiar "Davidson Sequence"

A special sequence of integers was constructed by a peculiar method that not only had an interesting pattern but one that was difficult to formulate for subsequent values. These integers grow rapidly and around the 16th value in the sequence they can no longer be handled by pencil and paper (e.g., 521 digits). This paper investigates the foundation of the "Davidson Sequence" by concentrating on the integers $x=2$ and $x=4$ (the other integers in the interval $[2,9]$ are still under investigation). Techniques were developed to calculate the number of digits in each integer of the sequence, as well as, determining the first and last few digits of any large value in this peculiar sequence.

## Vincent J. Matsko IMSA (vince.matsko@gmail.com)

## A Collatz-Like Iteration

Consider the following iterative procedure: begin with an integer written in base 10 . If the integer is divisible by 3 , then divide by 3 ; otherwise, reverse the digits and add 1. It is not hard to show that this process either reaches 1 or gets caught in a cycle. A Mathematica routine shows that all numbers up to $1,000,000$ reach 1 unless they get caught in the cycle ( $14 \rightarrow 42 \rightarrow 14$ ), which happens 90,901 times. Carried out in base 8 , this procedure first results in a cycle, of length 7 , at 269 , 573 . In base 244 , the first cycle encountered is of length 23 and begins with 455,504 . Changing the divisor, 3 , to another number also raises numerous questions. Interesting features of this procedure are graphically depicted with Mathematica.

## Anthony DeLegge Benedictine University (adelegge @ben.edu)

## The Man Who Whammied "Press Your Luck"

Michael Larson, an unemployed ice cream truck driver from Ohio, was a contestant on the popular 1980s game show "Press Your Luck" in June 1984. During the first part of his appearance, he seemed like a normal, excited contestant. But, in the second part of his appearance, he managed to survive over 40 spins in a row at the show's game board and win the game with $\$ 110,237$ in cash and prizes. Both of these were records no one ever came close to matching.

Was he one of the luckiest contestants ever on a game show? Or, was there something more suspicious going on? This talk will help answer this question using some basic probability and statistics and show how this could be used as an activity in an introductory statistics course.

## David Clark Randolph-Macon College (davidclark@rmc.edu)

## Guesses, Metrics, and Basketball

Imagine that you compete in an NCAA basketball tournament bracket pool with some buddies, and that you end up with the most points in your pool-except that you are tied with one other person. One common way to break this tie requires that each player, when entering their bracket picks, must also guess the final score of the championship game. Whichever player guesses closer to the actual score wins the tiebreaker, and thus the pool. But what should "closer" mean here? In other words, how do we determine which is the better guess? In this talk we'll explore some metrics that can help us decide, with possible applications to gambling and voting.

## Todd Mateer Howard Community College (tmateer@howardcc.edu)

## Magic Tricks Involving Error Correcting Codes

Many error correcting code systems can be reformulated into magic tricks. This talk will survey several such magic tricks and introduce several improvements which can simplify the presentation of the tricks. Integrating magic tricks into undergraduate courses involving error correcting codes may help to motivate students to learn these mathematical concepts.

## Christopher T. Sass Young Harris College (ctsass@yhc.edu) <br> Circle Packing: Combinatorics, Geometry, and Computation

The modern topic of circle packing was introduced by William Thurston in 1985. Many areas in this evolving field are highly accessible, and students can acquire intuition through computer experimentation involving visual images. I will discuss the relationship between the combinatorial pattern of external tangencies of a circle packing and the geometry (euclidean, hyperbolic, or spherical) in which the circle packing is realized. I will also introduce a variation of the basic circle packing notion: branched packings. I will conclude with the statement of an important open problem related to the computation of branched circle packings.

# Assessment of Courses for Students in Math-Intensive Majors 

Saturday, August 4, 1:00-2:15 PM, Hall of Ideas E

## Aimee Ellington Virginia Commonwealth University (ajellington@vcu.edu) Catherine Murphy Purdue University (cmmurphy @purduecal.edu)

How does assessment inform the instructor about what students have learned? How can assessment results lead to changes in what content is covered or how it is covered? How can assessment impact what is included in STEM-related degree programs? This session invites presenters to share effective methods for both formative and summative assessment of courses that are part of math-intensive degree programs. Aside from mathematics majors, degree programs of this nature include those in which students take two or more mathematics courses (i.e. economics, business, chemistry,biology, etc.) Talks should include the results of the assessments as well as how those results have been used to make meaningful changes to courses and/or degree programs. The focus of reports should include, but are not necessarily limited to innovative assessment models, ways to analyze assessment results, and course or program improvements based on an implemented assessment program.

This Session Sponsored by Committee on Assessment.

## Amanda Hattaway Wentworth Institute of Technology (hattawaya@wit.edu) <br> Anita Penta Wentworth Institute of Technology (pentaa@wit.edu) <br> Gary Simundza Wentworth Institute of Technology (simundzag@wit.edu) <br> Emma Smith Zbarsky Wentworth Institute of Technology (emmasz@gmail.com)

## Assessing Basic Learning of First-Year Engineers in a Required Calculus Sequence

Our department was recently faced with designing a new calculus sequence for five new engineering majors beginning in the fall of 2011. All engineering students were required to start in Engineering Calculus I. To give students some idea of our expectations and instructors some information on the students' background, we administered an ungraded test on precalculus topics on the first day of class. In addition, there was a midterm Gateway exam on differentiation in Calculus I and a second Gateway exam on methods of integration in Calculus II. Each Gateway was graded on a modified pass/fail basis. Lastly, we administered a common final exam in each course. We shall present our findings, including the correlation between the various exam results and final grades.

Barbara A. Shipman University of Texas at Arlington (bshipman@uta.edu)
Patrick Shipman Colorado State University (shipman@math.colostate.edu)

## Assessment through Targeted True/False Questions

Carefully-constructed true/false questions that target potential misconceptions or errors can be an illuminating mode of assessment, in particular in courses on mathematical foundations and analysis. Here, our focus is on false statements that students affirmed as true, despite simple counterexamples that they easily recognized afterwards. The results draw attention to how students may overlook simple counterexamples, even in an active learning setting where they proposed simple (and more complex) counterexamples daily in class. We provide perspective on the outcomes and suggest ways to help students become more adept at constructing and recalling examples and counterexamples in a variety of mathematics courses. This material is based upon work supported in part by the National Science Foundation through grant \#0837810.

## Kevin Treat US Air Force Academy (kevin.treat@usafa.edu) <br> Jim Rolf US Air Force Academy (Jim.Rolf@usafa.edu)

## Are Oral Panels an Effective Assessment Tool in an Engineering Mathematics Course?

We report on the impact of oral panels on student performance, motivation, anxiety, and long-term retention of mathematical concepts. We compared two groups of students over two testing cycles and the final exam. For each testing cycle, both groups received a brief pre-test quiz. The treatment group took a 15 minute oral panel in lieu of coming to two class periods. The control group took a second quiz and also came to class for the two class periods. Both groups took an extensive exam following intervention. We report on the impact the oral panels during these testing cycles on short-term performance on the exams, long-term retention on the final, and the impact of oral panels on motivation, anxiety, and study habits.

## Aimee Ellington Virginia Commonwealth University (ajellington@vcu.edu)

## An Assessment Plan for Courses Taken By Math-Intensive Majors

Due to administrative and fiscal pressures, a mathematics department at a large university that has offered mathematics classes in small (40 person) sections since its inception will be offering courses for math- intensive majors in large ( $120-150$ person) sections beginning this fall. This presentation will outline the department's plan to assess student understanding in these courses as well as evaluate student success within math-intensive majors. The data that is collected will be used to revise the department's instructional plan for future semesters and will be reported to university administration as plans for future semesters are being made. The results from a previous assessment of lower level mathematics that followed a similar plan will be presented in order to highlight the types of information that can be gleaned from evaluating courses in this manner.

# General Contributed Paper Sessions 

# GCPS Session 1: Research in Graph Theory or Combinatorics 

Thursday, August 2, 8:00-9:10 AM

$\begin{array}{lr}\text { David Housman } & \text { Goshen College (dhousman@goshen.edu) } \\ \text { Nancy Neudauer } & \text { Pacific University (nancy @pacificu.edu) }\end{array}$

Mark Kayll University of Montana (mark.kayll@umontana.edu)

## Counting Matchings using Integrals: a Discrete-Continuous Symbiosis

How many perfect matchings are contained in a given bipartite graph? An exercise in Godsil's Algebraic Combinatorics (1993) solicits proof that this question is answered by an integral involving a rook polynomial. Though not widely known, this result appears implicitly in Riordan's An Introduction to Combinatorial Analysis (1958). It was stated more explicitly and proved independently by S.A. Joni and G.-C. Rota [JCTA 29 (1980), 59-73] and C.D. Godsil [Combinatorica $\mathbf{1}$ (1981), 257-262]. Broadening the formula's reach was one of my goals in writing the article Integrals don't have anything to do with discrete math, do they?, in [Math. Mag. 84 (2011), 108-119]. This talk is based on that article.

Luis David Molina Sam Houston State University (ldm022@shsu.edu)
Luis Garcia-Puente Sam Houston State University (LDG005@shsu.edu)

## On the Sandpile Group of Generalized Book Graphs

In 1987 physicists Bak, Tang, and Wiesenfeld first introduced the sandpile model to study the dynamics of sand avalanches. The abelian sandpile model is represented by a finite connected graph. In particular, a sand configuration is a tuple of nonnegative integers representing the amount of grains of sand at each vertex in the graph. The set of recurrent sand configurations has the structure of a finite abelian group which arises as the cokernel of the combinatorial Laplacian of the graph. An interesting question in the area is to find the structure of the finite abelian groups associated to a given family of graphs. This has been accomplished for some families of graphs including cycle graphs, complete graphs and complete bipartite graphs. In this talk we will present some results and conjectures towards the characterization of the sandpile group of a family of graphs known as generalized book graphs. Traditionally, a book graph consists of triangles sharing a common edge, in other words, a book with $p$ triangles is isomorphic to the complete tripartite graph $K_{1,1, p}$. We generalize this notion by replacing the triangles for any regular graph. Our work centers in the particular cases where the pages of the book are $n$-cycles, complete graphs, and complete bipartite graphs.

Grady Bullington University of Wisconsin Oshkosh (bullingt@uwosh.edu)
Linda Eroh University of Wisconsin Oshkosh (eroh@uwosh.edu)
Steven Winters University of Wisconsin Oshkosh (winters@uwosh.edu)

## Prisoners and Guards on Rectangular Boards

Woolbright introduced the "Prisoners and Guards" game as a two-player game on an $n \times n$ checkerboard. At the beginning of the game, every square of the board has a guard. At each stage in the game, for each prisoner, there must be at least as many guards as prisoners on adjacent squares. The players take turns either replacing a guard with a prisoner in their color or replacing one prisoner (of either color) with a guard, then replacing two guards with prisoners in their color, subject to the rule above. When neither player can adjust the board any further, the player with more prisoners in their color wins. Howard, Ionascu, and Woolbright characterized the maximal and maximum configurations of $n \times n$ boards, that is, the boards for which neither player can make any moves. In this talk, we give formulas for the number of prisoners in the maximum configurations of $n \times m$ boards, where $2 \leq n<m$, for $n=2,3,4$, and 5 , and provide a bound when $n=6$.

## Ezra Brown Virginia Tech (ezbrown@math.vt.edu)

## Sums of Squares, the Octonions, and (7, 3, 1)

A sum of $n$ squares times a sum of $n$ squares is again a sum of $n$ squares if and only if $n=1,2,4$, or 8 . The rules for multiplication in four different systems underlie these four sums-of-squares identities. We'll talk about these systems, both familiar (real and complex numbers), less familiar (the quaternions), and not at all familiar (the octonions). Finally, we'll see what the ( $7,3,1$ ) block design-a.k.a. the Fano plane-has to do with multiplication of octonions.

## Katherine Benson University of Iowa (katherine-f-benson@uiowa.edu)

## Techniques for finding Lower Bounds of Radio Number of Graphs

A radio labeling of a simply connected graph $G$ is a function $f: V(G) \rightarrow \mathbb{Z}^{+}$such that for every two distinct vertices $u$ and $v$ of $G$, $d(u, v)+|f(u)-f(v)| \geq \operatorname{diam}(G)+1$. The radio number of a graph $G$ is the smallest integer $m$ for which there exists a radio labeling $f$ with $f(v) \leq m$ for all $v \in V(G)$. In this talk, we will establish a method of finding a reasonable lower bound for the radio number of certain types of graphs.

# GCPS Session 2: Interdisciplinary Topics in Mathematics 

## Thursday, August 2, 8:00-10:15 Am

## Linda Braddy Tarrant County College (linda.braddy @tccd.edu)

## Regional University vs. Community College: Just how different ARE they?

Are you interested in learning about differences and similarities in a mid-sized regional (rural) university and a large, urban, multi-campus community college district? According to a recent article in the Chronicle of Higher Education, "In any hiring cycle, about 40 percent of the available teaching positions are at two-year campuses. Moreover, a surprisingly large number of Ph.D. students are actually, and actively, interested in community-college careers, perhaps because they've discovered (as I did) that what they really enjoy most is teaching." (http://chronicle.com/article/What-Graduate-Students-Want-to/131600/ April 22, 2012) Topics included in this presentation are: qualifications (both required and "desired"), workload, tenure, salary and benefits, and career advancement.

## Cynthia L McCabe University of Wisconsin - Stevens Point (cmccabe@uwsp.edu)

## Collaboration $\Rightarrow$ Energy

Joe Gallian has told many Project NExT Fellows to "Just Say Yes!" As an example of the benefits that can flow from doing just that, in the context of collaboration with colleagues in the arts, I will describe an energizing project I undertook this year. A colleague from the UWSP Theatre \& Dance Department and I worked together on ideas about how Fibonacci numbers might be used in choreography. Associate Professor of Dance Jeannie Hill incorporated Fibonacci numbers modulo 4, 8, and 12 and the idea of accumulation in her remarkable dance creation "The Sum is Greater." Dance students were challenged to use their left brain and right brain simultaneously to learn and perform the dance, and some of them did Fibonacci projects in their mathematics classes. Mathematics students got to see mathematical topics in a new setting, and the public got to see innovation at the university both during our talk on the collaboration and during the end-of-semester dance performance.

## Mike Pinter Belmont University (mike.pinter@belmont.edu)

## Mathematics and Psychology: A Linked Learning Community

All freshmen at Belmont University take two courses from different disciplines linked together as a learning community experience for them and the course instructors. For several semesters, I have linked my Introduction to Mathematical Reasoning course with an Introduction to Psychological Sciences course. I'll offer a preliminary report on the ways a Psychology instructor and I have linked the courses, including a course project that overlaps both courses. I'll include feedback from the Psychology instructor as well as from students in the learning community.

## Jeff Johannes SUNY Geneseo (johannes@member.ams.org)

## The Flat Earth: The Mathematics of Mapmaking

It is simple to prove that no map of a spherical earth can be perfect. On the other hand, in practical life it is not entirely convenient to have a globe (or part of a globe) of the size we need. In this talk, we will explore the mathematics of choices that are made in making different styles of maps.

## Kevin A. Roper Cedarville University (roperk@cedarville.edu)

## Research Methods—A Transitional Course

In most of our early courses in undergraduate mathematics the instructor with the help of a textbook nicely packages all the information the student needs and hands it to them. In the final year we may have a "Capstone Experience" where the student gives a presentation on a topic that they have researched on their own. What happens in between? Often nothing, we throw them in the deep end with a sink or swim mentality. In a description of a course "Research Methods in Mathematics" this talk shares and solicits ideas about how to start the transition from being a passive receiver of information to one who independently searches for themselves and then, hopefully, continues on to being a creator of new knowledge.

# Kathryn Elizabeth Lenz Department of Mathematics and Statistics, University of Minnesota Duluth (klenz@d.umn.edu) <br> Voting Methods for Political Elections: Propaganda, Experiments and what US Voters Want from an Election Algorithm Within the past two decades various cities across the US have experimented with instant run-off voting (IRV) for political elections. These experiments demonstrate a public desire for replacing plurality voting with a better method and they give insight into what voters want from an election algorithm. This presentation will describe IRV, the Borda count and a score voting algorithm, distortions of the properties of IRV propagating in the public arena and the opportunity this gives mathematicians to engage in civic dialogue and the pursuit of better election algorithms. Examples will be given of city election results, propaganda found on websites, misrepresentations in newspaper opinion pieces and discussions with mathematicians and non-mathematicians. 

## Nicole Breckling Austin College (nbreckling08@austincollege.edu)

Jack Mealy Austin College (jmealy @austincollege.edu)

## Introducing non-Euclidean Geometries at the Public School Levels; the OTP Project

A brief review of various subcategories of Geometry is given, in particular emphasizing a broader definition and scope of the subject. Further remarks are made on the levels of difficulty in the study of these various subcategories, and their grade-level appropriateness. Then we discuss and report on our recent effort, the OTP (Outside The Plane) project, which seeks to introduce various non-Euclidean systems to a much broader audience. Specifically, we discuss the first iteration of this effort, which targeted students in a 10th grade, regular geometry class. The systems that were introduced involved distinct sets of non-Euclidean axioms. One system was modeled on an effort to ease traffic flow in a major downtown area; another was a system based on dynamics resulting from 'transporter gates'. We conclude by mentioning other directions in the project, including the generalization of angle of parallelism formulas to various non-homogeneous systems.

Todd Iverson Saint Mary's University of Minnesota (tiverson@smumn.edu)<br>Roger Kugel Saint Mary’s Univeristy of Minnesota (rkugel@smumn.edu)<br>Paul Weiner Saint Mary's Univeristy of Minnesota (pweiner@smumn.edu)<br>\section*{Energized Integer Partitions}<br>Understanding the way energy distributes among molecules in a system is important because that distribution determines many of the macroscopic attributes of a system, including its thermodynamic, kinetic, and spectroscopic properties. We consider how energy can be distributed over a large collection of particles by adding one unit of energy at a time. The resulting structure illustrates the relationship between the various energy states and Youngs lattice, a partially ordered set of integer partitions. We explore, graphically and numerically, the relationship between Youngs lattice and the most probable energy configuration.

## Andy Martin Kentucky State University (andrew.martin@kysu.edu)

## A Natural Truth about the Natural Numbers Unprovable in Peano Arithmetic

Gödel's Incompleteness Theorems were great mathematics as well as the harbinger of a new kind of insecurity. ("What if the theorem I'm working on is UNPROVABLE?"). Gödel produced a highly artificial "ad hoc" number theoretic sentence which is true but not provable in Peano Arithmetic. It does not seem to be well-known outside of logic circles that there are "real" theorems unprovable as well. This talk will discuss one such: Goodstein's Theorem.

## GCPS Session 3: Teaching or Learning Introductory Mathematics

## Thursday, August 2, 1:00-5:00 PM

## David Easdown School of Mathematics and Statistics, University of Sydney (de@maths.usyd.edu.au) <br> Models of Teaching and Learning in Mathematics

In this session we plan to introduce and discuss some models of teaching and learning, highlighting any general principles along the way, with a view towards developing practical tips towards approaching core undergraduate or service teaching in mathematics, especially where students come from a wide variety of backgrounds and have divergent interests, skills and attitudes. Learn about the ubiquity of the Leunig Model and the resilience of the Passive/Active Interface. Discover The Central Limit Theorem for Rubber Bands as it applies to teaching and learning, and explore the quirky and delicate interplay between communication, syntax, semantics and entropy.

## J. Bradford Burkman Louisiana School for Math, Science, and the Arts (bburkman@1smsa.edu)

## Fraction Addition for Remediation of Arithmetic and Preview of Algebra

Fraction addition exercises can be effective in college algebra courses, reviewing essential operations on integers with applications to algebra topics.

Exercises like $9 / 56-4 / 63$ review the idea of a greatest common divisor and least common multiple while practicing factoring $56=7 \cdot 8$ and $63=7 \cdot 9$, multiplying $9 \cdot 9$ and $4 \cdot 8$, subtracting $81-32$, factoring $49=7 \cdot 7$, and multiplying $8 \cdot 9$.

The Euclidean algorithm reconnects in students' minds division and repeated subtraction, and exercises like $49 / 1711+29 / 1829=$ $49 /(29 \cdot 59)+29 /(31 \cdot 59)=2360 /(29 \cdot 31 \cdot 59)=(40 \cdot 59) /(29 \cdot 31 \cdot 59)=49 / 899$ give students the opportunity (if guided) to practice $31 \cdot 49$ as $(40-9)(40+9), 29^{2}$ as $(30-1)^{2}$, and $29 \cdot 31$ as $(30-1)(30+1)$, previewing differences of squares and squares of binomials.

At the step $2360 /(29 \cdot 31 \cdot 59)$, the student can preview number theory by explaining why $2360=31 \cdot 49+29 \cdot 29$ cannot be a multiple of either 29 or 31 , but could be a multiple of 59 .

All of these topics preview addition of ratios of polynomials. The Euclidean algorithm also works for finding the gcd of polynomials, with "greatest" meaning "of highest degree."

We will look at ways to incorporate fraction addition into introductory mathematics courses, and discuss what makes a "good" exercise and how to find it.

## John Beam University of Wisconsin - Oshkosh (beam@uwosh.edu) Eric Kuennen University of Wisconsin - Oshkosh (kuennene@uwosh.edu)

## Naive Invented Algorithms

Our students know many things that the rest of us don't. For example, $25 \times 15=45$. Or $1 \frac{3}{4}=\frac{13}{4}$. Or the equation $(2 x-1)(3 x+5)=0$ has solutions $x=1$ and $x=-5$. When our work with practicing teachers at the upper elementary and middle school levels brought such student misconceptions to our attention, we began to classify them; and in the process, we realized that children are in fact building naively upon prior knowledge. We have subsequently realized that these misunderstandings persist among students in our introductory college mathematics courses. Our classifications include "Something That Used To Work" and "Like Something That Works." Recognizing that many of our students' mistakes fall into these categories has helped us as instructors, in that rather than discounting such mistakes as random nonsense, we now have a means of addressing them.

## Steven Greenstein University of the Virgin Islands (stevengreenstein @ gmail.com)

## Reconceiving Developmental Mathematics: Embracing the Diversity of Students Mathematical Knowledge

What would Developmental Abstract Painting look like? Or Remedial Jazz Piano? What might traditional developmental or remedial approaches to painting and piano playing yield in terms of expert forms of artistic knowing and participation?

As enrollment in developmental/remedial programs has increased "due in no small part to the narrowing of the K-12 mathematics curricula (as well as the traditionally narrow view of the nature of mathematics itself)" the diversity of students has also increased. Therefore, we must renew our commitment to challenge taken-for-granted assumptions of these students' mathematical understandings, needs, and abilities.

In this session I will present a theoretically-informed, empirically-based developmental/remedial course redesign, one that confronts deficit notions of students' mathematical ability, honors and respects the diversity of knowledge they bring with them to class, and sees that diversity as a resource for the kinds of authentic mathematical experiences each of us enjoys (as opposed to an occasion for remediation).

## Curtis Card Black Hills State University (Curtis.Card@bhsu.edu) <br> Daluss J. Siewert Black Hills State University (Daluss.Siewert@bhsu.edu)

## A Second Chance for Algebra Success (@ No Extra Charge)

Black Hills State University, like so many other universities, has struggled with the problem of students failing Basic Algebra and Intermediate Algebra. Many of these students subsequently failed to graduate. In the Fall of 2009 the mathematics department made structural changes in the way these courses were offered. A BLOCK format was implemented which required students to pass the material taught in a BLOCK prior to moving on to the next BLOCK. In addition to structural changes several instructional changes were implemented. This presentation discusses these implementations and the effect they had on the pass rates of students taking Basic Algebra and Intermediate Algebra. We will also discuss some of the strategic interventions that we have implemented, the surprising results, and provide a brief description of the collaborative research project that has evolved from this work.

## Daluss J. Siewert Black Hills State University (Daluss.Siewert@bhsu.edu) <br> Curtis Card Black Hills State University (Curtis.Card@bhsu.edu)

## Multi-Tiered Systems of Support in Developmental Math

(NOTE: This talk expands on the work discussed earlier in our talk titled "A Second Chance for Algebra Success (@ no extra charge).") This spring we completed the third year of our developmental math project. This talk will focus on the Multi-Tiered Systems of Support
incorporated into our developmental math courses this past year and our success with them. We will discuss the systems of support used and how they were implemented. We will also provide a brief historical overview of this project and how it led to receiving a National Science Foundation TUES (Transforming Undergraduate Education in Science) grant.

## Kristel Ehrhardt Black Hills State University (kristel.ehrhardt@bhsu.edu)

## How Can We Make Mathematics Accessible to Remedial Mathematics Students?

When students enter college with the hopes of obtaining a degree, many are not prepared for the college-level mathematics course(s) required for their degree and are resigned to taking remedial math classes. Is there a way to help these remedial mathematics students succeed and ultimately achieve their goal of earning a degree? My colleagues and I have found ways, using a variety of methods, for these students to be successful in math.

## Holly Gail Stillson Black Hills State University (Holly.Stillson@BHSU.edu)

## Math Bucks Can Mean Money in the Bank for Remedial Math Students

As Black Hills State University addressed the challenge of helping students who struggle in remedial math courses, we discovered some interesting trends. A couple of the strategic interventions that came out of the observations on our BLOCK format to address these trends were attendance and grade scenarios. Paying students 'Math Bucks' for attendance allowed students to purchase extensions on missed or low online assignments on MyMathLab. Discussing different grade scenarios helped students understand that every aspect of the course works together and is important. Math Bucks became money in the bank for students who successfully passed the remedial courses of Basic and Intermediate Algebra the first time.

## Sarah L. Mabrouk Framingham State University (smabrouk@framingham.edu)

## The Final Exam Problem Contest: An Opportunity for Making Connections and Personalizing Mathematics

Helping students to relate course topics to their areas of interest and providing opportunities for students to make connections among course topics, concepts, and methods are important in learning. Accomplishing these enables students to view the mathematics that they study as valuable and applicable in future courses and useful in their anticipated careers. For more than six years, I have conducted an end-of-the-semester Final Exam Problem Contest for which students submit original problems in which they apply course topics to their areas of interest and/or events in their lives. For the contest, students must create exam/quiz quality problems related to real-life scenarios, provide the associated source data when appropriate, and produce the full solution. This contest has been a wonderful way in which to empower students to associate the topics, concepts, and methods that they have examined throughout the course with their interests as well as to entice them to reflect on how the mathematics that they have studied can be applied in real life. In this presentation, I will discuss the implementation of the Final Exam Problem Contest and student reaction to the use of the contest in various courses as well as share some of the submitted and winning problems.

## Matthew Rissler Loras College (matthew.rissler@loras.edu)

## Using a Digital Pen to Enhance Student Learning

Digital pens allow audio and written notes to be recorded simultaneously for review later. This allows students to both look at notes that they took in class and also listen to what was being said at the same time. The multimodal nature helps students of all levels gain more from reviewing class notes. This presentation will provide examples of uses of a digital pen in an introductory mathematics classroom, and some tips to streamline usage of a LiveScribe SmartPen for the instructor.

## Maggie May University of Wisconsin - Fond du Lac (maggie.may@uwc.edu)

## Using Wolfram—Alpha to Accommodate Students with Math-Related Learning Disabilities

The computational knowledge engine Wolfram—Alpha has made powerful mathematical tools accessible to all students. There are currently over 2.5 million students in the $K-12$ system receiving special education services for Learning Disabilities. Approximately $44 \%$ of secondary students with LD are more than 3 years behind in mathematics. When these students enroll in college, they are not sufficiently prepared to meet the expectations of a college-level math course. Many repeatedly fail remedial math courses before dropping out, frustrated and in debt. Wolfram-Alpha provides a powerful new means to provide accommodations to students with math-related learning disabilities, and is the foundation of an alternative mathematics course being developed by the University of Wisconsin Colleges. The course has been successfully piloted for one student. This case study will be presented, along with a consideration of implications for the traditional college classroom.

## Mary Shepherd Northwest Missouri State University (msheprd@nwmissouri.edu)

## A First Look at How Mathematicians Read Mathematics for Understanding

As students progress through the college mathematics curriculum, enter graduate school and eventually become practicing mathematicians, reading mathematics textbooks and journal articles appears to comes easier and these readers appear to gain quite a bit from reading mathematics. Previous research has focused on what early college students do as they read and the difficulties they encounter that interfere with understanding what has been read. This study was designed to help us begin to understand how more advanced readers of mathematics read for understanding. Three faculty members and three graduate students participated by reading from a first year graduate textbook in an area of mathematics unfamiliar to each of them. The reading methods of the faculty level mathematicians were all quite similar and were markedly different from all the students the researcher has encountered so far, including the more advanced students in this study. A Preliminary Mathematics Reading Framework is proposed.

## Alexander G. Atwood Suffolk County Community College (atwooda@sunysuffolk.edu)

## Analyzing Current Economic Growth, Income and Employment in an Introductory Statistics Course

The statistical analysis of economic growth, of changes in income and of changes in employment opportunities provides a powerful way to motivate students to study Statistics in an introductory course. From the years 2000 to 2010, changes in several economic indicators serve to highlight what is happening in the USA. Although US economic output grew significantly in the last ten years, with per-capita Gross Domestic Product increasing by $7 \%$, the average worker did not reap the benefits of this economic growth, with median household income decreasing by $7 \%$. Furthermore, a dramatic increase in the productivity of workers from 2000 to 2010 by $26 \%$, due to automation, computerization and increased efficiency, has led to a steep decline in the number of workers needed to power the growing economy, with $16 \%$ less workers needed to power the economy in 2010 compared to 2000, while the total population of the country has increased by $10 \%$. While we are living in a time of increasing economic prosperity, the average worker is seeing a decline in their standard of living and is chasing after vanishing employment opportunities. What does this mean to students in our Statistics classes, and what enlightened and innovative economic policies might be created to solve these problems?

## Debra Kean DeVry University (dkean@devry.edu)

## Math Ice Breakers

The Challenge: Blended Learning features active learning in the classroom. How can we design in-class activities that will engage all cohorts in MyMathLab (developmental studies to college algebra)?

## Steven B Zides Wofford College (zidessb@wofford.edu)

## Strictly Platonic: Teaching Mathematics through Sculpture

Instructors, tasked with a First Year Experience (FYE) or Liberal Arts Math course, often struggle to adopt a class theme which efficiently balances mathematical rigor and humanistic sensibility. Typically the thematic selection, although interesting, is not easily formatted into a broad spectrum of classroom pedagogies. One motif that shows great promise, in this regard, is Mathematical Aesthetics, the manner in which mathematical ideas get rendered both physically and metaphorically into various works of art, specifically sculpture. Rich with resources, this pairing of math and sculpture allows the instructor to expand their pedagogies beyond the standard lecture-discussion format into areas such as short essay assignments, collaborative group work, experiential trips, and even studio art activities. This talk will start with a brief background in the history of modern sculpture, specifically when did sculptors move away from traditional anthropomorphic subjects and start to render abstract mathematical ideas. Quickly moving into examples of some of the key pieces of mathematical sculpture, I will highlight the work of Morton C. Bradley Jr., a geometric sculpture of the mid twentieth century. With the aforementioned works in mind, we will consider various pedagogical methods designed to get students actively engaged with both the art and the mathematics. If time permits, the talk will end with a brief question and answer session.

## Brandy Sue Wiegers MSRI, San Francisco State (brandy@msri.org) <br> Addie Evans San Francisco State (addieevans@gmail.com) <br> Emiliano Gomez UC Berkeley, San Francisco State (emgomez@berkeley.edu)

## Bridge to Engineering (B2E)- The First Step: Intermediate Algebra to Calculus Ready

The B2E program has the goal of bridging students from calculus and introductory classes at local city colleges to engineering programs at the surrounding university campuses. The pathway from city colleges to universities are quite clear, what is less clear is how to get more students calculus ready without adding two or more years to their coursework. We will discuss the workshops we developed to support the intensive math course work, focused on group problem solving and hands on activities. Specific focus on lessons learned including successes and items to avoid if you try this model.

# GCPS Session 4: Mathematics and Technology 

Thursday, August 2, 2:30-4:25 PM

Anders Hendrickson Concordia College (ahendric@cord.edu)
Beamer, TikZ graphics, and Flash applets in the classroom
Till Tantau's well-known Beamer package has the ability to transform not only the conference talk, but also the classroom. We show examples of Beamer presentations which dramatically increase student comprehension of difficult material, especially by incorporating TikZ graphics and embedded Flash applets. Examples will be taken from courses in linear algebra, abstract algebra, and mathematics for the liberal arts.

## Alexander Basyrov University of Wisconsin - Stout (basyrova@uwstout.edu)

## Making WeBWorK care about Algebra

WeBWorK (webwork.maa.org) is an open source online homework system very well suited for mathematics courses.
The strength of WeBWorK in Mathematics is that it by design accepts as 'correct' any formula that is algebraically equivalent to the correct answer. While it is very convenient in most of Calculus problems, such strength becomes a weakness when one considers the use of WeBWorK in College Algebra or Precalculus courses.

In this not very technical talk we will discuss recent developments that make WeBWorK aware of the algebraic form of the answers submitted by students. This allows one to easily create WeBWorK problems that will recognize if the student answer is in one of the standard algebraic forms (completely factored polynomial, point-slope equation, standard form of equation of circle, etc.). Several examples of such new types of WeBWorK problems will be demonstrated and discussed, as well as an overview of the implemented algorithms.

## Philip B Yasskin Texas A\&M University (yasskin@math.tamu.edu) <br> Douglas B Meade University of South Carolina (meade@math.sc.edu) <br> Matthew Barry Texas A\&M University (mbarry @ neo.tamu.edu) <br> Facilitating Student Mathematical Input on Mobile Devices

The Maplets for Calculus is an award-winning collection of computer applets that help students learn precalculus and calculus. The authors are in the process of adapting the applets for use on mobile devices such as phones and slates. Most of the components are readily available. However, there is no convenient way for students to enter their math formulas, either for computation or as answers to non-multiple choice (free response) questions. The existing graphical input programs are written in java and these will not work on mobile devices. We could ask the students to type in a linear input box using the syntax of MathML, LaTeX or some standard computer algebra system such as Maple, Mathematica or Sage, but students find that syntax confusing. It is important to keep the input as natural as possible. We are developing a linear input system using syntax which is as close to natural math as possible but still preserving accurate content information. A graphical display of the input appears instantaneously below the linear input. This system will be flexible enough to allow other modes of input, specifically MathML, LaTeX, various computer algebra systems or pallet-based input. The input interface will be demonstrated and its syntax will be discussed. We welcome comments on improving the syntax.

## Timothy Lucas Pepperdine University (timothy.lucas@pepperdine.edu) <br> Brian Fisher Pepperdine University (brian.fisher@pepperdine.edu)

## Using iPads to Transform Learning Spaces

Traditionally, research on technology in mathematics education focuses on interactions between the user and the technology, but little is known is about how technology can facilitate interaction among students. We will explore the role that iPads versus traditional laptops play in shaping the learning spaces in which students explore concepts in business calculus. We will present a system for coding student interactions with the technology and fellow classmates during classroom observations. Our preliminary results are that introducing the iPad, a portable device with intuitive applications, enhances collaboration by allowing students to transition back and forth from private to public learning spaces.

## Cecelia Wright Brown, D.Eng. University of Baltimore (cwrightbrown@ubalt.edu) <br> Gargi Bhattacharyya University of Baltimore (gbhattacharyya @ubalt.edu) <br> Kevin A Peters Morgan State University (kevin.peters@ morgan.edu) <br> Professional Development Training for Urban School Teachers in Mathematics, IT and Engineering in Homeland Security

A mathematics, information technology and engineering professional development training program was developed by an urban University to increase the number of teachers in local City Public School with proficient skills, tools, and content knowledge in computer/ information
technology, engineering technology and technical certifications which support students in science, technology, engineering and mathematics (STEM) fields. Through this process, teachers used tools, resources, and training to support homeland security career opportunities in their schools. The program constituted a cohort of STEM teachers from two local urban high schools participating in professional development training in mathematics, information technology, engineering and homeland defense to support students pursuing technical careers in these areas. In this talk, we report on our findings from that work, as well as discuss future directions of the research.

## Michael Karls Ball State University (mkarls@bsu.edu)

## Implementing an Introductory Mathematical Software Course

We will look at the challenges of designing and implementing an introductory mathematical software course; provide an overview of topics covered and resources used for each type of software (Excel, Mathematica, and MATLAB); give examples of homework problems, exam questions, and project topics chosen for the course; and addresses issues that have arisen.

## Dora Ahmadi Morehead State University (d.ahmadi@moreheadstate.edu)

## Getting Ready for College Mathematics

The presenter will discuss results from a project aiming at preparing high school seniors for college level mathematics. The program used the Hawkes Learning System to increase the interest and active participation of high school students during a three year project that has shown its sustainability. Follow-up results of the 2008 cohort of high school students who attended Morehead State University will be shared.

## Jason Molitierno Sacred Heart University (molitiernoj@sacredheart.edu)

## Visualization Projects for a Differential Equations Course

While there are many computations involved in differential equations, there are also many deep ideas that can be visualized and apprecaited by students taking their first course in differential equations. In this talk, I present three projects using Maple that I assign my students. I assign a project involving slope fields, a project investigating families of solutions to initial value problems, and a project analyzing phase portraits for systems of differential equations. I will discuss the details of these projects and how they benefit student learning and enhance student creativity.

## GCPS Session 5: History or Philosophy of Mathematics

Friday, August 3, 8:00-10:30 AM

Andrew Leahy Knox College (aleahy@knox.edu)
Kasandara Sullivan Knox College (ksulliva@knox.edu)

## Evangelista Torricelli's Quadratura Parabolae

Torricelli's 1644 work "de Dimensione Parabolae" presents 21 different geometrical proofs of Archimedes' formula for the area inside a segment of a parabola. In the second part of this work, "Quadratura Parabolae per novam indivisibilium Geometriam pluribus modis absoluta", he focuses on techniques which use Bonaventura Cavalieri's 'new geometry of indivisibles'. While the quadrature of the parabola is outwardly the focus of this work, in this talk we'll describe how Torricelli in fact uses Cavalieri's methods to establish deep connections between Archimedes' quadrature result and much of Archimedes' other work on calculus.

## John F. Bukowski Juniata College (bukowski@juniata.edu)

## The Catenary Problem in Christiaan Huygens's Notebook G

In 1646 the young Christiaan Huygens proved that the shape of the hanging chain, or catenary, was not a parabola. Forty-four years later, no one had yet described the actual shape of the catenary, so Jakob Bernoulli posed the problem publicly in 1690 . Huygens then found a solution that appeared in 1691 along with those of Leibniz and Johann Bernoulli. We will examine some of the work in Huygens's Notebook G that led to this solution.

## Shigeru Masuda Research Institutes of Mathematical Sciences, Kyoto University (hj9s-msd@asahi-net.or.jp)

Poisson's Careful Handling of Transcendental Function, Criticizing Diversion from Real to Imaginary in Definite Integral Why Poisson pays attention of careful handling to the transcendental function, criticizing the diversion from real to imaginary in definite integral? Since 1811, Poisson issued many papers on the definite integral, containing transcendentals, and remarked on the necessity of careful
handling to the diversion from real to imaginary, especially, to Fourier explicitly. To Euler and Laplace, Poisson owes many knowledges, and builds up his principle of integral, consulting Lagrange, Lacroix, Legendre, etc. On the other hand, Poisson feels incompatibility with Laplace's diversion, on which Laplace had issued a paper in 1809, entitled : "Sur le passage réciproque des Résultats réels aux Résultats imaginaires," (On the 'reciprocal' passage of results between real and imaginary), after presenting the sequential papers on the occurring of 'one-way' passage in 1782-3. To these passage, Poisson presents the direct, double integral in 1811, 13, 15, and 20. As a contemporary, Fourier is made a victim by Poisson. To Fourier's main work : "Théorie analytique de la chaleur" in 1822, and to the relating papers, Poisson points the diversion applying the what-Poisson-called-it 'algebraic' theorem of De Goa or the theorem of middle value, to transcendental equation. Moreover, about their disputes, Darboux, the editor of "Euvres de Fourier", evaluates on the correctness of Poisson's reasonings in 1888. In the last pages of the fluid dynamics in 1831, Poisson remembers to put again the restriction, saying that the proving of eternity of time in the exact differential become necessarily defective, if it would include any transcendental.

## Chris Christensen Northern Kentucky University (christensen@nku.edu)

## Copperhead, Mamba, and JN-25

During World War II, US Navy mathematician/codebreakers conceived of several machines to attack the Japanese naval cipher JN-25. We will discuss the mathematical ideas behind the Copperhead and Mamba codebreaking machines.

## Linda Becerra University of Houston-Downtown (becerral@uhd.edu) <br> Ron Barnes University of Houston-Downtown (barnes@dt.uh.edu)

## Alan Turing-A Brief Survey

Alan Turing (1912-1954) is held by many to be one of the key figures of the 20th century. Among his major contributions are his seminal work in the development of computer science, his pioneering work in Artificial Intelligence and his prescient work in Mathematical Biology. In addition, his work in breaking the Enigma Code during World War II earned him the prestigious Order of the British Empire (OBE). The presentation begins with a brief timeline of some of the significant milestones in Turing's life. Each element in the timeline is briefly described. In addition, some sidelights, stories, rumors and apocrypha are included. The presentation concludes with a summary of his impact on the 20th century and beyond.

## Manya Raman-Sundström Umeå University (manya.sundstrom@matnv.umu.se)

## Beauty in Mathematical Proofs

What makes a mathematical proof beautiful? Are such judgements subjective or objective (or neither?)? Is it at all possible to characterize properties that give rise to the sensation of beauty? These are all very difficult questions, and have been considered by some of the greatest thinkers since ancient times. We do not propose to nail down any of these questions, but to use them as a backdrop for a fairly modest set of empirical studies on mathematicians' judgements of beauty of mathematical proofs. Preliminary results indicate there is at least consensus on judgements, and perhaps even some criteria, namely a proofs' ability to explain, which helps predict whether someone will find a proof beautiful or not.

## Brian Hollenbeck Emporia State University (bhollenb@emporia.edu)

## Athens to Istanbul: A Tour of Infinity

For a summer study-abroad course, I led a group of Emporia State University students from Athens to Istanbul on an 18-day journey across two continents and 2,600 years. No other area of the world provides a better backdrop than Greece and Turkey to discuss the interplay between the mathematical, historical, artistic, and theological viewpoints of infinity. It was my goal to give students of any mathematical background a glimpse into these intriguing aspects of infinity, while also experiencing different cultures and visiting sites of significant historical interest. I will discuss these various aspects of the course, as well as the pros and cons of leading such a trip.

## Walter Meyer Adelphi University (meyer1 @adelphi.edu)

## External Influences on Undergraduate Mathematics Curricula: 1950-2000

Between 1950 and 2000, undergraduate mathematics curricula showed significant sensitivity to external influences. Many of the more important proposals and developments of the era illustrate this: the Universal Mathematics Project, the great expansion of service courses, the revival of applications, the institution of discrete mathematics courses, calculus reform and wide-ranging experiments with pedagogy. The external factors behind these curricular movements include: manpower and enrollment issues, the desire to ally ourselves with other disciplines, economic factors, the advance of technology and some hard-to-identify social issues common to many developed countries.

## Brian Beavers Stephen F. Austin State University (beaversbd@ sfasu.edu)

## A Math History Course for Liberal Arts Majors

Motivating students in service courses can be a tough job. In fall of 2011, I taught a special section of math for liberal arts majors, targeted at history majors, where we walked through the history of mathematics. It was my hope that by grounding the mathematics in the context of
culture and history, our liberal arts students would be more motivated. In this talk, I will share the details of the course and how the students responded.

## Susan Foege Kentucky State University (susan.foege@kysu.edu)

## A Brief History of Rigidity

Rigidity theory is accessible to undergraduate math majors. This talk will introduce the idea of rigidity of "structures" and look at a few results, beginning with Euler's conjecture in 1776: "A closed spatial figure allows no changes, as long as it is not ripped apart."

# GCPS Session 6: Modeling, Applications, Probability, or Statistics 

Friday, August 3, 8:00-10:15 am

## Kenneth Driessel Iowa State University (driessel@mac.com)

## A Mathematical Model of Classical Cross Country Skiing

Many people in Wisconsin cross country ski; for example, thousands of them participate in the American Bierkebeiner 50 kilometer ski race which is held annually in northern Wisconsin. Beginning skiers usually first learn the classic style of skiing; almost everyone can learn to walk on skis. But skiing is really about gliding. Advanced skiers use powerful kicks to get long glides. The following question arises: What motions are important for an efficient kick? I shall describe a (fairly simple) mathematical model of classical skiing. In particular, I shall derive the differential equations which govern the dynamics of this model. I believe that the analysis of this model (and its generalizations) will help us understand the complicated motions of cross country skiing.

This is joint work with Irvin Hentzel (Math, Iowa State).

## Lisa Holden Northern Kentucky University (holdenl@nku.edu)

## Assessing the Chaotic Nature of Interstellar Magnetic Fields

The interstellar medium is threaded by a magnetic field that has a strong turbulent component. This magnetic field plays a crucial role in how high energy cosmic rays traverse our galaxy and influence astronomical processes including the formation of stars and planets. Although we don't fully understand the nature of these turbulent fields, several models capturing their important features have been proposed. A proper assessment of these models is therefore warranted.

An important aspect of these fields is the chaotic nature of the particles that move through them. Specifically, it is known that two particles injected into a turbulent field with the same initial velocities but slightly different initial positions will subsequently follow very different paths. Mathematically, we can quantify this effect by calculating what are known as the Lyapunov exponents. We therefore calculate the Lyapunov exponents using a numerical scheme for several different magnetic turbulence models in order to assess and contrast their underlying chaotic structures. Understanding the basic structure of such systems may help to simplify and guide future numerical analyses that are traditionally difficult and computationally expensive to perform.

## Jack Ryder Kean University (jryder@kean.edu)

## Basic Search Pattern Algorithms for Search and Rescue Operations

Search and rescue operations are used by various agencies when something needs to be found and then retrieved. Examples of this include locating missing hikers, searching through debris after an earthquake, looking for lost aircraft or ships at sea and treasure hunting. The search process involves identifying the search area, quickly searching areas with high probability of finding - when time is critical, and then employing a systematic search pattern of the identified area. The United States Search and Rescue Manual recommends various search patterns and more general search and rescue protocols/procedures. This paper develops algorithms for implementing four automated search patterns in a search and rescue operation. The algorithms are intuitive in nature and use concepts from geometry, trigonometry and arithmetic. The search patterns presented have direct application in real life situations, which helps students appreciate an important area of applied mathematics.

The four search patterns to be developed are: Creeping Line, Archimedean Spiral, Rectangular Spiral and Sector Search with their graphs below. Whether using a robot, airplane, boat or human for the search, the algorithms reduce to defining a set of connected waypoints that provide adequate coverage of the search area. Thus, all four algorithms will define a series of coordinates (waypoints) that a search agent must traverse.

All four search pattern algorithms will be developed using both a computer programming language (Java or ALICE) and with spreadsheet software (Excel). The algorithms will be made available to attendees over the web.

## Kodwo Annan Minot State University (kodwo.annan@minotstateu.edu)

## Hollow Fiber Dialyzer: Blood and Bicarbonated Dialysate Flow Characteristics

In an attempt to improve the quality and efficacy of Hemodialysis therapy, dialyzers headers have been redesigned to improve uniform blood and dialysate flows. Any mismatch caused by non-uniform flow distributions in either blood or dialysate compartment would result in inefficiency of a dialyzer. Mathematical modeling technique was used to investigate and provide insights into the flow distributions during dialysis session. A suite of convection-diffusion models for blood and dialysate counter-current flows were deployed. The model coupled blood and dialysate side geometries through a Trans-membrane layer using Kedem-Katchalsky equations. The blood flow distributions were observed to be uniform across each cross-section while a non-uniform dialysate flow distributions were observed near the hollow-fiber packing area at the dialysate-side. The uniform blood-side flow distribution suggested that the new blood header design used in modern hollow-fiber dialyzers was an efficient flow distributor. The non-uniform dialysate flow profile indicated that the collars used in existing hollow-fiber dialyzers to help dialysate uniform flow failed to promote uniform flow. Consequently, the flow distributions mismatch in the dialyzer affected the boundary and bulk bicarbonate concentrations exchange across the membrane. The flow distribution analysis may have practical significances for the design of high efficient hollow-fiber dialyzers.

## Mohammed Yahdi Ursinus College (myahdi@ursinus.edu)

## Modeling the Effect of Diversity in Host Plant-Herbivore-Predator Interactions

Predator and plant diversity can control Potato leafhopper (PLH) pest damage to the host-plant Alfalfa. New models of systems of differential equations were constructed using age structures, Allee eect, the Shannon diversity Index and other modeling approaches. Recent data and results on enemies and diversity hypotheses in field experiments were used to determine parameter ranges and validate the models. Parameters were adjusted to predict outcomes for scenarios not covered by field experiments and to examine their roles. This work provided a frame work for designing cost-effective and environmentally safe strategies to minimize alfalfa damage, and utilize enemies hypothesis and polyculture diversity.

## William Schellhorn Simpson College (william.schellhorn@simpson.edu)

## The Feasibility of Electric Vehicles: Driving Interest in Mathematical Modeling

The study of electric vehicles can be used to promote interest in mathematical modeling in a variety of courses and student projects. In this presentation, I will discuss how the feasibility of electric vehicles can be investigated using fundamental topics in algebra, calculus, and statistics. I will also give examples of how technology can be incorporated into the investigation.

## Audey Shen Henry M. Gunn High School (audeyshen36@gmail.com)

## Valid Range of an Objects Unified Kinetic Quantity

Based on a novel quadratic function which extended the classic Vieta's formulas and found applications in physics, we derive the valid range of a moving objects "Unified Kinetic Quantity" - a recently published quantity that unifies kinetic energy and linear momentum in physics and has a linkage to the energy-mass equation $E=m c^{2}$. We investigate and present both mathematical and physical constraints of this quantity through equations and graphics. In computer simulation, a large number of random samples of the moving object are generated, and the results validate our mathematical analysis and visualize our findings in physics.

## Munir Mahmood Gulf University for Science and Technology (munir_mahmood2000@yahoo.com)

## Explaining Statistical Independence via the Approach of Determinant

The concept of independence and its basic properties of probability are viewed from the perspective of determinant. The aim of this presentation is to explain pairwise independence by utilizing the approach of determinant. This is demonstrated in the context of contingency table and Venn diagram. In an illustration, it is observed that in the case of three attributes in a contingency table, $2 \times 2$ submatrices expose the degree of statistical independence. An alternative expression of Bayess rule is then given as a consequence of this approach. Explaining statistical independence from the approach of determinant has a great pedagogical value as it enhances learning and it can be easily taught in the context of classroom teaching.

## Randall Groth Salisbury University (regroth@salisbury.edu)

## Preparing Prospective Teachers to Implement the Common Core State Standards for Statistics and Probability

More than 40 states have adopted the Common Core State Standards for Grades K-12. Prospective teachers currently in undergraduate preparation programs will be responsible for helping children attain these learning expectations. The statistics and probability content in the Common Core greatly exceeds that of most previous state standards documents. While teachers will be responsible for implementing the Common Core, research in mathematics and statistics education indicates that teachers content knowledge in statistics and probability is in great need of improvement. This presentation will describe a statistics course developed at Salisbury University to prepare prospective teachers to implement the Common Core. The course was designed as the result of collaboration between the mathematics and education departments at the university. The overall scope and sequence of the course will be described. Specific examples of course activities and how they connect to the Common Core will be shared. Sample activities to be shared relate to teaching descriptive statistics, formal inference, combinatorics,
and the distinction between theoretical and experimental probability. Prospective teachers thinking while engaged in the activities will also be discussed in order to identify inroads and obstacles encountered during the course. The results of quantitative assessments of the content knowledge and pedagogical knowledge undergraduates developed during the course will be presented. Attendees will leave with a variety of ideas for enhancing or designing courses that prepare teachers to implement the Common Core Standards related to statistics and probability.

## GCPS Session 7: Teaching or Learning Calculus

Friday, August 3, 1:00-5:30 PM

Denise LeGrand UALR (djlegrand@ualr.edu)
"YouBook": Interactive Editable Mathematics Textbook-the Key to Online Mathematics
Interactivity is one important key to create learning opportunities for online students. What better way to engage students than through their textbook? The most exciting tool I have found for this is the Cengage "YouBook". It is highly customizable and interactive. Teachers may edit/strike-through the actual text, add links and video at a click of the button, insert their own notes, and virtually highlight what they chose to emphasize. Students see the additions immediately and can also individually markup their own ebook without sharing it. Besides all of the tools mentioned, there are WolframALpha demonstrations and "TEC"'s (Tools for Enriching Calculus) already sprinkled in the appropriate locations but teachers can delete or keep these. You Tube videos can be added with a click of the button. Suggested videos are listed or there is an option for searching relevant and related videos. The beginning of each section has a tutorial that includes concepts, examples and mastery. Each of the examples in the text has a video. The Homework Tutorial has homework video examples worked for many of the problems in the homework section. Sections and Chapters can be rearranged or hidden in the "Assigned Readings". There is even a customizable quick link to online chat sessions. I will explain all of these options and show how the YouBook has increased student motivation and participation in my online Calculus courses. Best of all setting up the "YouBook" is easy and fun which translates over to deeper student understanding and learning.

## Douglas B Meade University of South Carolina (meade@math.sc.edu) Philip B Yasskin Texas A\&M University (yasskin@math.tamu.edu)

## On the Role of Assessment in the Design of Online Resources for Learning Mathematics

As more online materials become available for mathematics it is important to understand how these resources impact student learning, and how the these impacts are affected by factors such as the learning environment and the type of hardware.

The authors have assembled a team of evaluators and mathematics educators to assess the NSF-funded Maplets for Calculus (M4C) project. The M4C is a collection of computer applets designed to help students learn more than 100 topics in precalculus and calculus.

In addition to summative evaluations based on attitudinal surveys completed by students and faculty in pre- and post-course surveys, the assessment team is developing protocols for more formative studies. Two questions at the heart of this work are:

- To what degree do the maplets, individually or collectively, impact student understanding of associated math concepts?
- Is there a relationship between cognitive measures (e.g. achievement and conceptual understanding) and the learning environment (e.g. course content system, handheld device)?
This talk includes the discussion of the assessment methodologies and initial results currently being used for the M4C project as well as those that are being developed.


## Barbara Margolius Cleveland State University (b.margolius@csuohio.edu) L Felipe Martins Cleveland State University (luizfelipe.martins@gmail.com) Daniel Gries Hopkins School (DGries @hopkins.edu)

## Flash Applets embedded in WeBWorK homework problems

This is a progress report on our NSF-CCLI grant. For this grant we are developing a library of Flash applets embedded in WeBWorK homework assignments for entry level university mathematics courses including calculus, pre-calculus and differential equations. Applets are self-contained programs that play within the user's web browser. These applets are designed to interact with the WeBWorK online homework system to facilitate the creation of a richer array of assessment and enhancement of student learning than is presently offered by most online homework resources. WeBWorK is a free Perl-based system for delivering individualized homework problems over the web. WeBWorK is partially supported by the DUE Division of NSF and is now sponsored by the Mathematical Association of America. It is used by extensively in the US and Canada and to a lesser extent throughout the world.

Matt Boelkins Grand Valley State University (boelkinm@gvsu.edu)

## Toward a Freer Calculus: a Free, Open-source Calculus Textbook

The purpose of this talk is to give a preview of my current project: writing a free, open-source calculus text. After being pilot-tested during the upcoming 12-13 academic year, I look forward to having the text available for free download in .pdf format by any professor or student, with
the raw .tex and graphics files available for download (upon login) for any professor who wishes to modify them. Licensed under a creative commons "share and share alike, non-commercial" license, my hope is that a community of users will develop that works collectively to improve, grow, and publicize this option for a calculus text. Highlights of the text's approach include high quality full-color .eps graphics generated with PiScript, embedded links to java applets that demonstrate and investigate key ideas, and an active, hands-on approach that includes at least four discovery activities per section; because the text is free, as is the corresponding workbook for activities, it may be also be easily used as a supplement to a traditional text. In addition to sharing some sample sections, I will discuss how the book's electronic format may be used in novel ways in the classroom and offer some thoughts on the benefits of a text that is both free and open-source, for students and faculty alike.

## Allison Henrich Seattle University (henricha@seattleu.edu)

Oral Reviews in Calculus: Improving Student Understanding
Oral reviews are hour-long, ungraded opportunities for small groups of students, working with faculty or learning assistants, to negotiate meaning, make conceptual connections, discuss why procedures work, and draw representations that make concepts clearer. Research done at CU Boulder by Dr. Mary Nelson has shown that incorporating oral reviews into calculus classes significantly improves student performance. In this talk, we'll look in more detail at what oral reviews are, see how they have been implemented both at CU Boulder and at Seattle University, and give evidence of their benefit to students.

## Phillip LaCasse US Military Academy at West Point (phil_lacasse@uwalumni.com)

## Examining the Relationship Between Class Preparation and Student Success on Daily Quizzes

This presentation examines a one-year pilot program conducted in a first year course in integral calculus at the United States Military Academy. The program's goal is to influence students to complete assigned reading and attempt out-of-class problems to prepare for each lesson by providing graded incentives as a reward. We present analysis of twelve class preparation quizzes administered to 78 students over the course of both the Fall 2011 and Spring 2012 academic semesters, specifically focusing on the relationship between class preparation and student success on these class preparation quizzes.

## Dmitry Kondrashov University of Chicago (dkon@uchicago.edu)

## A New Quantitative Modeling Course for First-year Biology Students

In the last few decades biological and medical fields have undergone a profound transformation into data-driven, quantitative sciences. The standard calculus courses which traditionally constitute the quantitative training for biology majors do not adequately prepare students for these needs. A new course developed at the University of Chicago takes the place of the last course in the required calculus sequence. It combines mathematical modeling, computational implementation, and basic statistics, all applied to specific biological and medical problems. I will describe the new course and its role in the biological curriculum, and provide data from surveys and students assessments conducted throughout the course.

## Jane M Mcdougall Colorado College (jmcdougall@coloradocollege.edu)

## Modifications to the Calculus Sequence

A number of schools have experimented with modifications to the traditional calculus sequence. Two years ago we restructured our own calculus sequence, one of our motivations being to allow those students who terminate their mathematical studies with Calculus 2 to apply techniques of differential calculus in a multivariable setting. We give a report on our experiences and discuss challenges and advantages for students with various mathematical aspirations.

## Jeanette Olli Dominican University (jolli@dom.edu)

## Recalling Prerequisite Material to Aid in Student Success in Calculus II

In many disciplines, students have required reading that must be done before class meets. In mathematics, however, students often find that they learn new material better in a classroom rather than out of a book. This does not mean that students do not need to come to a mathematics class with some preparation for the day's lesson. Calculus II requires a student's understanding of algebra, trigonometry, and the basics they learned in Calculus I. This course also contains a lot of new material, leaving little to no time for review of material that some students may not have seen in a long time. This not only can lead to algebraic and trigonometric errors in the process of working through problems, but it can also slow down the pace of the course. To resolve these issues, I assigned questions that reviewed algebra, trigonometry, and basic calculus skills that would be used in the upcoming class. In this talk, I will discuss how I implemented this method and provide examples of assignments. I will also address how this impacted the flow of the course and how students felt about this approach to recalling prior knowledge.

## Nermine El Sissi The American University in Cairo (nelsissi@aucegypt.edu)

## Calculus Across the Atlantic

"Why do I have to take calculus? I will never use it in my life!" Since my first encounter with calculus students in 2002, I have been hearing this statement every semester whether in Dallas, Texas, Washington D.C., or Cairo, Egypt. It was quite intriguing to observe that students’
attitudes toward a college calculus course in the United States and Egypt are very similar. Consequently, such resemblance in students' perceptions of mathematics was the catalyst of my experiment, namely to alter students' attitudes whether in the United States or in Egypt. This talk will discuss pedagogical techniques aimed at increasing students' theoretical awareness as well as developing their analytical skills and mathematical reasoning. More specifically, the former was done by introducing students to "disguised" mathematical proofs via in-class discussions, activities, and numerical exploration of a given concept.

## Michelle Ghrist U.S. Air Force Academy (michelle.ghrist@usafa.edu)

Helping Students Develop Metacognitive Skills in Mathematics
How often do mathematics teachers hear, "I have no idea where to start" from students trying to solve a problem? When students employ metacognitive skills, they demonstrate an awareness of their thinking patterns; this includes being mindful of when and how to use certain problem solving strategies. How can instructors help students develop and hone these essential skills? Certainly this should involve more than just teaching a flowchart approach to mathematics, but what does this look like in the mathematics classroom? How can we design our assessments to help students in this area? In this talk, I discuss metacognitive skills as they relate to helping students learn mathematics, assess their own mathematical development, and ponder deeper mathematical connections.

## Tim Boester Wright State University (timothy.boester@wright.edu)

## Recent Advances in Teaching and Learning the Formal Definition of Limit

How do calculus students conceptualize the formal definition of limit? How can we successfully teach the formal definition of limit to students? A brief review of the history of research on limit conceptions will be followed by an examination of a promising new wave of research in this area, including the researcher's recent work on categorizing and diagnosing student understanding of the logical implications imbedded in the definition.

## Todd King Michigan Technological University (trking@mtu.edu) <br> Allan Struthers Michigan Technological University (struther@mtu.edu) <br> How a Computer Evaluates Trig Functions.

Complete code for the basic math library for NVidia graphics cards is easily accessible. This C language listing completely describes how a specific computer evaluates its basic math library. We evaluate the accuracy and speed of the underlying polynomial/rational approximations for the trigonometric functions (sine, cosine, atan, etc.), outline the range reduction arguments that extend these simple approximations accurately to larger computational ranges, and suggest some exercises incorporating these realistic function approximations into calculus class.

## Rebecca Schmitz Michigan Technological University (rjschmit@mtu.edu)

## Students' Natural Discovery of the Integral Test for Infinite Series

We will share one of the surprising outcomes from a series of interviews of six undergraduate students from a second semester calculus class. This class included an introduction to geometric series and Taylor series, and did not include any of the standard tests for convergence. With the exception of one participant, who had a brief introduction to series in high school calculus, none of the students had studied the convergence of infinite series. We will discuss how several of the participants naturally discovered the Integral Test for Infinite Series and potential implications for instruction.

## Eric Weber Oregon State University (webered2009@gmail.com)

## Students' Ways of Thinking about Directional Derivative

In this presentation, I discuss the results of a teaching experiment focused on students' transitions to thinking about rate of change in space based on their understanding of rate of change in the plane. I focus on one student, Jesse, and discuss how he abstracted the meaning of rate of change to a surface in space and conceptualized a notion of a path. I discuss the implications of this research for studying students' generalizations in mathematics, and how this line of research has important implications for curriculum development and teaching.

## Leon Kaganovskiy Touro College (leonkag@gmail.com)

## Applications of Computer Algebra Systems (CAS) to Multivariable Calculus

In this presentation we would like to explore creating CAS codes which significantly enhance students' learning of more complicated Multivariable Calculus concepts. Among the topics considered are the surface visualization, projectile motion, use of contour and gradient fields in finding maxima and minima, heat seeking particle paths, Lagrange multipliers, multidimensional integration, centers of mass, vector fields, Green's theorem approach to finding areas of complex polygons, divergence and Stokes's theorems.

Michael Kerckhove University of Richmond (mkerckho@richmond.edu)

## PDE in Calc III

We can do a better job of introducing partial differential equations in multivariable calculus. Instead of limiting the discussion to checking that a function satisfies a particular PDE, we should be introducing PDE models for traveling waves and diffusion. In this talk I will show
how I introduce students to PDE in the context of the familiar ODE's for population dynamics from single-variable calculus. The key idea is the time evolution of a profile curve and Mathematica's Manipulate[ ] command allows students to visualize models combining logistic growth, migration, and dispersion in a single lecture and homework.

## Howard Dwyer Monmouth College (hdwyer@monm.edu)

## A Simple Technique for Sketching in Three Dimensions

In many mathematics and science courses, it is necessary to visualize problems in three dimensions. A sketch is often helpful, even if the sketch is not exactly to scale or completely correct. Although modern computer graphics applications provide beautiful three-dimensional figures, such tools are not always readily available. The process of constructing a sketch "by hand" may also provide an improved understanding of the nature of three-dimensional space; doing is often more instructive than watching. Anyone who can draw a straight line using a ruler can create rather impressive three-dimensional images, once they understand a few simple techniques.

## GCPS Session 8: Research in Algebra, Topology, or Analysis

## Friday, August 3, 1:00-4:00 PM

Robert Vallin Slippery Rock University (robert.vallin@sru.edu)

## Preserving Quasi-Metrics

For a space $X$, a quasi-metric on $X$ is a function $\rho: X \times X \rightarrow[0, \infty)$ with the properties

1. $\rho(x, y)=\rho(y, x)=0$ if and only if $x=y$, and
2. for all $x, y, z, \in X$

$$
\rho(x, y) \leq \rho(x, z)+\rho(z, y)
$$

A function $f:[0, \infty) \rightarrow[0, \infty)$ is quasi-metric preserving if for every space $X$ and every quasi-metric $\rho$ on $X$, the composition $f \circ \rho$ is a quasi-metric on $X$. In this talk, we will describe quasi-metric preserving functions in terms of the already established metric preserving functions. In addition, we will completely characterize the functions that preserve weightable quasi-metric spaces.

## Colleen Duffy University of Wisconsin - Eau Claire (duffycm@uwec.edu)

## Algebra Associated to the Hasse Graph of the $\boldsymbol{n}$-cube

The primary goal of this faculty/undergraduate student research collaboration was to determine the structure of the algebra associated to the Hasse graph of the $n$-cube, $\Gamma_{\left\{4,3^{n-2}\right\}}$. The symmetry group of the $n$-dimensional hypercube is the Coxeter group $\left[4,3^{n-2}\right]$, which is isomorphic to $S_{n} \rtimes \mathbb{Z}_{2}$. The symmetries act on the graph and homogeneous subspaces of the algebra. For each element in the symmetry group, we find the Hilbert series of the graded sub-algebra fixed under the action, called the graded trace generating function. We can use these functions to obtain the multiplicities of the irreducible $\left[4,3^{n-2}\right]$-modules in $A\left(\Gamma_{\left\{4,3^{n-2}\right\}}\right)_{[i]}$. Our main result is that we found a nice formula for these graded trace functions dependent only on $n$ and the cycle decomposition of $\sigma$. In the talk we will go over the construction of the algebra, the graded trace functions for this algebra, and mention the methods used to find these functions.

## Lauren Kelly Williams University of Wisconsin - Milwaukee (willia75@uwm.edu)

## Algebraic Combinatorics Related to the Kronecker Coefficients

The Kronecker coefficients are defined as the multiplicities obtained when a tensor product of irreducible representations of the symmetric group is decomposed. In recent years, they have received increased attention in both the mathematics and physics literature since they have applications to quantum information theory. In spite of this fact, a simple combinatorial description remains an open problem. Moreover, their computation is complicated when compared to the Littlewood-Richardson coefficients, which describe the decomposition of tensor products of general linear group representations, rather than symmetric group representations. We present graph theoretic interpretations of certain sums of Kronecker coefficients. We show how these are analogous to the theory surrounding the Littlewood-Richardson coefficients. These results lead to algorithms that are currently being implemented in a program written specifically for the iOS platform, which will also be demonstrated. This program provides a convenient user interface that improves the exposition of these ideas.

## Nick Scoville Ursinus College (nscoville@ursinus.edu)

## Discrete Morse theory and the Homology of Simplicial Complexes

Combinatorial topology is the study of counting techniques in order to gain information about a topological space. One such technique is the use of discrete Morse theory invented by Robin Forman as an analogue of so-called smooth Morse theory. Many topological spaces are
approximated by building them out of discrete "pieces." The resulting object is called a simplicial complex, and the techniques of discrete Morse theory allow one to count the number and kind of holes in the simplicial complex. In this talk, we will discuss the results of an REU project at Ursinus College from Summer 2012 in which attempts were made to generalize several theorems relating a discrete Morse function on a simplicial complex to the homology groups of that complex.

## Christopher Park Mooney University of Iowa (christopher-mooney @uiowa.edu)

## Generalized Factorization in Commutative Rings with Zero-Divisors

Let $R$ be a commutative ring with 1 . We let $\tau$ be a symmetric relation on the non-zero, non-units of $R$. For a non-unit $a \in R$, we look at $\tau$-factorizations of the form $a=\lambda a_{1} \cdots a_{n}$ such that $a_{i} \tau a_{j}$ for all $i \neq j$. We define various forms of $\tau$-atomic as well as demonstrate the relationship between rings possessing various finite $\tau$-factorization properties.

## Adam Graham-Squire High Point University (agrahams@highpoint.edu)

## Explicit Calculations of Local Formal Integral Transforms

Integral transforms play an important role in much of theoretical and applied math. We will focus on the theoretical role the Fourier transform plays in solving differential equations, then look at how this relates to recent calculations of the local Fourier transform on the punctured formal disc. We will also discuss generalizing the techniques to calculate other local integral transforms such as the Mellin transform.

## Matthew Wright Huntington University (mwright@huntington.edu)

## Hadwiger Integration of Functionals

How can we measure the size of a functional? The Lebesgue integral provides a notion of the size of a functional, as does the lesser known Euler integral. Yet there are other integrals that also quantify the size of a functional. I will define integrals with respect to valuations known as the intrinsic volumes. Hadwiger's Theorem states that any Euclidean-invariant convex-continuous valuation on sets is a linear combinations of intrinsic volumes. I will lift this result from sets to functionals over sets, providing a classification theorem of all valuations on functionals, with suitable assumptions about Euclidean-invariance and continuity. This talk will be accessible to undergraduates.

## Samuel Chamberlin Park University (scham003@gmail.com) <br> Irfan Bagci University of California - Riverside (irfan.bagci@ucr.edu)

## Kostant Superalgebras for the Simple Classical Map Superalgebras

We will present Kostant superalgebras for the simple classical map superalgebras, which have the form $\mathfrak{g} \otimes A$ where A is any associative algebra over the complex numbers. We will also present explicit PBW-like integral bases for these integral forms and the straightening lemmas necessary for these bases.

Bruce Atkinson Samford University (bwatkins@samford.edu)

## A generalized Euler constant

Let $D^{*}$ be the domain of analyticity of the principle branch of the $\operatorname{logarithm,~} \log z$. For $d \in D^{*}$ and $a \notin\{0,-d,-2 d, \ldots\}$, define $\Pi_{d}^{a}(a+n d):=a(a+d)(a+2 d) \cdots(a+n d)$, for $n \in \mathbb{N} . \Pi_{d}^{a}(a+n d)$ is the product of successive terms of an arithmetic progression. $\Pi_{d}^{a}$ can be analytically continued to all $z \notin\{-d,-2 d, \ldots\}$. (Note: The classical Gamma function, $\Gamma(z)=\Pi_{1}^{1}(z-1)$.) We can represent $\Pi_{d}^{a}(z)$ in terms of a generalized Euler constant and an infinite product. This product formula can be used for a fairly straightforward alternative proof of the classical Gauss's multiplication formula for the Gamma function.

## Robert Niedzialomski University of Iowa (robert-niedzialomski@uiowa.edu)

## Hilbert Spaces of Measures; and Some of their Applications.

We define inner products of measures. We use the special form of the Wasserstein distance between Borel probability measures defined on the real line and we take advantage of the Hilbert space introduced by Nelson (with the Nelson-Schwarz inner product). Then we provide some applications.

## Chris Ahrendt University of Wisconsin - Eau Claire (ahrendcr@uwec.edu) <br> Kevin Ahrendt University of Nebraska - Lincoln (kahrendt@huskers.unl.edu)

## Introduction to the Time Scale Calculus and the Generalized Exponential Function

The time scale calculus, first developed in Stefan Hilger's $1988 \mathrm{Ph} . \mathrm{D}$. dissertation, is a means to unify continuous and discrete analysis. We will introduce the basics of the time scale calculus, and illustrate how these tools can be used to explain the differences in results one encounters in the study of differential and difference equations. We will then turn our attention to the generalized exponential function on time scales. In particular, we will analyze the asymptotic properties of the exponential function on certain time scales that are isolated. Some of these properties are very similar to classical results of the exponential function when thought of as a solution to certain differential equations. However, we will also see some interesting results that do not have an immediate analog in the classical situations.

## Christopher Frayer University of Wisconsin - Platteville (frayerc@uwplatt.edu)

## The Geometry of Cubic Polynomials

Given a complex cubic polynomial, normalized to have the form $p(z)=(z-1)\left(z-r_{1}\right)\left(z-r_{2}\right)$ with $\left|r_{1}\right|=1=\left|r_{2}\right|$, where are the critical points? Marden's Theorem tells us that the critical points are the foci of the Steiner ellipse of $\Delta 1 r_{1} r_{2}$. This talk will further explore the structure of these critical points. For example, (1) there is a "desert" in the unit disk, the open disk $\left\{z \in \mathbb{C}:\left|z-\frac{2}{3}\right|<\frac{1}{3}\right\}$, in which critical points cannot occur, and (2) a critical point of such a polynomial almost always determines the polynomial uniquely.

## GCPS Session 9: Assessment, Mentoring, or Outreach

## Saturday, August 4, 8:00-10:10 AM

## Beth Schaubroeck U. S. Air Force Academy (beth.schaubroeck@usafa.edu)

## Working with Public Schools: Creating Math Enrichment for Elementary Students

Have you ever considered how difficult it is to teach vastly different children in 4th and 5th grade? In this talk I will discuss how I worked with public school administrators to determine how to be a resource to these teachers. Then I will share mathematics enrichment units that I developed, along with ideas about their use individually or with large groups.

## Elizabeth Yanik Emporia State University (eyanik@emporia.edu) Joe Yanik Emporia State University (hyanik@emporia.edu)

## Si Se Puede—An outreach Program for Middle School Hispanic Students

This presentation will describe the development and evolution over a five year period of the outreach program, Si Se Puede. This program is for students in grades 6 through 8. The objectives and format of the project will be presented along with descriptions of specific hands-on activities which were popular with the students.

## Senan Hayes Western Connecticut State University (hayess@wcsu.edu) <br> Josie Hamer Western Connecticut State University (hamerj@wcsu.edu)

## The Bridge Program Helping Students Transition into College without Remediation

The Bridge Program is a collaboration of Western Connecticut State University and two regional high schools. The main objective of the presentation is to explain what the Bridge Program is and how it has improved student outcomes, reducing the need for remediation upon entering university, in mathematics and other disciplines. It is hoped that attendees will gain insight into some issues of remediation, learn how collaboration improved communication among middle school, high school and university faculty and administration, and understand the basic resources needed to start and continue a program. The presentation will highlight the implementation of mathematics assessments, student and faculty workshops and ongoing dialog among shareholders used to improve student growth and prepare students for post-secondary level work upon entering university. The presentation will showcase how program implementation and improvements, aided by state and local funding, has added a connection within the community and improved student outcomes in mathematics at the high school and university levels.

## Rebecca Metcalf Bridgewater State University (rmetcalf@bridgew.edu) Irina Seceleanu Bridgewater State University (iseceleanu@bridgew.edu)

## A Recipe for Change: A Look at Mathematics Placement for Transfer Students

Higher education institutions are faced with an increasing population of transfer students with varied academic backgrounds and readiness levels. As a result, addressing these students unique needs with standard practices employed for first-time students presents a distinctive challenge. We offer an analysis of the intrinsic difficulties associated with the placement of transfer students in introductory-level mathematics courses. Recommendations to higher education institutions for designing and implementing a placement system that ensures smooth transfer and is in accordance to a students readiness level will be provided.

Maighread McHugh UW-La Crosse (mmchugh@uwlax.edu)
Karoline Auby UW-La Crosse (kauby @uwlax.edu)

## Growing a Learning Center: From a Small, Underfunded Closet to a Popular Learning Space

The Mathematics Department tutor facility has undergone a transition from a few tutors working in a variety of spaces including a classroom, storage area, and student dining area to a large number of tutors in a multidisciplinary Learning Center with permanent space in the University

Library. The center had a limited budget for tutoring that was managed by the Department Academic Assistant for many years. Several years ago, a student funded academic initiative fee began to support additional tutors and Learning Center Director. An emeritus faculty member serving on an interim basis guided the evolution of the Learning Center, which is now led by a permanent director. In this presentation, the evolution of the tutoring center will be discussed by the people involved. This will include a description of the funding initiative, the development of the center with certified tutors, a new process for selecting, interviewing, and hiring student tutors, as well as a continuous improvement plan.

## Jennifer Bergner Salisbury University (jabergner@salisbury.edu)

## A Mathematical Amazing Race: The Radical Dash

Do you want to find something cool for your undergraduate students to do at section meetings or for your math club? The Radical Dash is a 1.5 day mathematical challenge event that runs during the spring meetings of the MD-DC-VA section of the MAA. It involves our undergraduate students in an Amazing Race type game that tests their mathematical creativity in a timed manner. During this talk I will share past activities, the logistics behind running such an event, and some video clips and pictures of the last Dash.

Richard Askey University of Wisconsin (askey @ math.wisc.edu)

## Mathematical t-shirts

Some examples of mathematical t-shirts will be illustrated.
Michael Dorff Brigham Young University (mdorff@math.byu.edu)
Darren Narayan Rochester Institute of Technology (dansma@rit.edu)

## Finding Funding and Support for doing Undergraduate Research

Undergraduate research in mathematics has expanded dramatically during the past decade. But with tightening budgets, funding to support undergraduate research is often difficult to find. We will present some ideas for obtaining funding and support for building an undergraduate research program in mathematics.

Jenna P Carpenter Louisiana Tech University (jenna@latech.edu) Patrick O'Neal Louisiana Tech University (poneal@latech.edu)

## Creating a Culture of Success: A Professional Development Program for Women Faculty

Despite gains in recent years in the number of women obtaining mathematics degrees at all levels, the number of women in tenure-track, tenured, and administrative positions in academia still lag behind their male counterparts. Research points to multiple contributing factors, ranging from climate to family responsibilities. One effective way to address these issues and simultaneously support and encourage the success of women faculty and administrators, is through a regular professional development program. Utilizing campus expertise, as well as external resources, programs can build a sense of community, skills, confidence and expand resources and networks. This paper will review such a program, supported by an NSF ADVANCE grant, and include strategies that can be adopted by other institutions, as well as sources of good material and suggestions for program evaluation.

## GCPS Session 10: Research in Applied Mathematics

Saturday, August 4, 8:15-10:30 AM

## John Anthony Adam Old Dominion University (jadam@odu.edu)

## Zero-order 'Rainbows', Electromagnetic Wave Equations and Poles of the Scattering Matrix: What's All That About?

It is demonstrated that zero-order bows, non-existent in plane wave electromagnetic scattering by uniform dielectric spheres, can nevertheless exist in radially inhomogeneous spheres. An interesting connection between this problem and time-independent quantum scattering theory is exploited to examine the nature of certain poles associated with finite range potentials (corresponding to the refractive index of the sphere).

## Roummel Marcia University of California - Merced (rmarcia@ucmerced.edu)

## Compressive Video Recovery using Bound-Constrained Optimization

This talk focuses on the reconstruction of a video sequence of images where known pixel-intensity bounds exist at each video frame. It has been established that the $\ell_{2}-\ell_{1}$ minimization problem can be solved efficiently using gradient projection, which was recently extended to solve general bound-constrained $\ell_{2}-\ell_{1}$ minimization problems. Furthermore, the video reconstruction can be made more efficient by exploiting similarities between consecutive frames. In this talk, we propose a method for reconstructing a video sequence that takes advantage of the inter-frame correlations while constraining the solution to satisfy known a priori bounds, offering a higher potential for increasingly accurate reconstructions.

## Edward Swim Sam Houston State University (edward.swim@shsu.edu)

## A Coupled Mixed Finite Element Method for Fluid-Structure Interaction

We investigate the interaction between a viscous incompressible fluid and a structure whose deformation defines the interface between the two. Direct numerical solution of the highly nonlinear equations governing even the most simplified models of fluid-structure interaction requires that both the flow field and the domain shape be determined as part of the solution since neither is known a priori. To accomplish this, previous algorithms have decoupled the solid and fluid mechanics, solving for each separately and converging iteratively to a solution which satisfies both. In this paper, we describe a coupled, mixed finite element method which solves the problem simultaneously on each sub-domain.

## Champike Attanayake Miami University (attanac@muohio.edu)

## Convergence of an Immersed Finite Element Method for Parabolic Interface Problems

In this talk we analyze an immersed interface finite element method for second order semi-linear parabolic interface problems. Convergence of the semi discrete solution to the exact solution is shown to be of the optimal order in $L^{2}$ and energy norms. Further, fully discrete scheme based on backward Euler method is studied and optimal error in $L^{2}$ norm is established.

## Allan Struthers Math Sciences, Michigan Technological University (struther@mtu.edu)

## Critical Points of the Non-Seperable Extended Rosenbrock Function

Critical points of the $n$ dimensional, non-separable, extended Rosenbrock function are analyzed and characterized using a two dimensional dynamical system. This method identifies and characterizes previously unknown families of critical points for large $n$ as well as previously unknown individual solutions for small $n$.

## Narayan Thapa Minot State University (narayan.thapa@minotstateu.edu)

## Optimal Parameters for Klein-Gordon Equation with Neumann Boundary Condition

In this paper we study an identification problem for physical parameters associated with damped Klein-Gordon equation with Neumann boundary conditions. The existence, uniqueness, and continuous dependence of weak solution of Klein-Gordon equations are established. The method of transposition is used to prove the Gâteaux differentiability of the solution map. The Gâteaux differential of the solution map is characterized. The optimal parameters are established.

## Darwin Omar Luna Sam Houston State University (DarwinOLuna@gmail.com) Edward Swim Sam Houston State University (edward.swim@shsu.edu)

## Stochastic Models for Heat Flow in a Cylinder

In this talk we will present a model for heat transfer that includes randomness in order to have a more physically accurate representation of what actually happens when a round is fired in a large caliber weapon barrel such as the M256 $120[\mathrm{~mm}$ ] cannon used by the United States Army. When a round is fired from this cannon it causes the steel in the barrel to deteriorate and so the cannon must be retired after it has fired a certain number of rounds or it reaches a state where it is labeled unfit to fire any more rounds. In order to preserve the life of a barrel, coatings are often applied to the interior of the steel that makes up the bulk of the barrel. An open problem in this situation is the estimation of the life span of the new coated gun barrel. Since these cannons are expensive to build, there is no efficient way to fire a cannon multiple times in order to collect data to estimate the life span. Current modeling in this field is built on the ability to accurately simulate experiments conducted with heat pulse lasers and gives results that are in good agreement with these laboratory simulations. However, these experiments are conducted under ideal conditions and do not account for factors such as the temperature of the barrel or the variations in propellant. When a cannon is heated it will fire differently than when it is cold. We will explore different forms of randomness that can be inserted into a model based on classical conservation laws for the heat flow in order to be more effective in modeling the actual performance of the cannon in the field.

## Paul Pearson Fort Lewis College (paultpearson@yahoo.com)

## Visualizing Clusters in Neural Networks

Neural networks are iteratively constructed functions that model functional relationships between a set of input data and a set of target output data. They have been used to solve a wide variety of function approximation and clustering problems. We will use the level sets of a neural network to construct a low-dimensional topological space that models the clusters in the input data formed by the neural network. The method for constructing the topological model uses a generalization of Morse Theory to the discrete setting that is commonly referred to as Mapper.

# GCPS Session 11: Teaching or Learning Advanced Mathematics 

Saturday, August 4, 1:00-3:25 PM

## Donna Beers Simmons College (donna.beers@simmons.edu)

## Preliminary Report: Strengthening Student Understanding of One-to-One and Onto Functions

This presentation describes our efforts to address gaps and misconceptions in undergraduates' understanding of the one-to-one and onto function properties. We look at how these properties are currently developed in grades $\mathrm{K}-16$ and identify what is missing. We describe our ongoing project which includes a pre- and post-questionnaire and use of the equivalence kernel of a function to set the stage for quotient structures which appear in discrete mathematics and in advanced courses in the mathematics major.

## Bharathwaj Muthuswamy Milwaukee School of Engineering (muthuswamy@msoe.edu) <br> Chaos is Fun <br> In this paper, we will discuss chaotic dynamics. Commonly associated with the "Butterfly Effect", we will see chaotic dynamics can be used to gain insights into varied concepts such as steady state solutions of differential equations, dimensions of phase space objects and the Fourier transform. All code in this paper uses the open source SAGE (Software for Algebraic and Geometric Experimentation) tool.

## Debra Czarneski Simpson College (debra.czarneski@simpson.edu)

## Creating Mathematics

In the fall semester of 2009 , I taught a first-year course that focused on skills required to successfully complete undergraduate research including experimentation, pattern recognition, asking interesting questions, creating conjectures, reading and understanding mathematics, and writing and presenting results. This paper will discuss the Simpson College first-year course requirements, my course goals, the graph theory topics covered, student feedback, and instructor reflection. I will be teaching Creating Mathematics again this fall semester and I will share the changes that I plan to make in the course.

Julianna Connelly Stockton Sacred Heart University (stocktonj@sacredheart.edu) Nicholas Wasserman Southern Methodist University (nwasserman@smu.edu)

## Mapping the Common Core State Standards to Advanced Mathematical Knowledge for Teaching

Strong mathematics knowledge is important for good mathematics teaching. Teachers strong in content need less time to understand material, have more time that can be geared towards instructional strategies, tend to be more flexible and confident, and have better substance to their teaching (Brown \& Borko, 1992). Yet while mathematics knowledge is important, the specific advanced content should match the work of teaching and be relevant for explaining concepts, designing activities, and questioning, understanding, and accessing students' thinking. Specification of exactly which areas of advanced mathematics (beyond K-12) are most important for K-12 teachers has not been addressed. Many students in mathematics programs plan to pursue careers as high school teachers and wonder what connection these courses have to their own future teaching. In this study, we attempt to address the question of advanced mathematical knowledge for teaching. We will present preliminary results from a mapping process that utilized a grounded theory approach: working individually and collaboratively from the CCSS-M standards, we identified connected advanced mathematics topics relevant for teaching each standard. We will outline the research process and present the advanced mathematical knowledge relevant for teachers of elementary, middle, and secondary grade levels, including verification of some current common practices as well as some surprising new results. Finally, we will discuss some potential implications for teacher preparation programs and ongoing mathematics education research.

## Christopher Frenzen United States Naval Postgraduate School (cfrenzen@nps.edu)

## On the Cusp in Calculus and Mathematics

The cusp appears in calculus as an example of a continuous function that is not everywhere differentiable. But the cusp occurs in other situations too, some in calculus, some in multivariable calculus and beyond. For example, the evolute of a curve is the locus of its centers of curvature and the 'vertex' of a curve is a local maximum or minimum of its curvature. Evolutes always have cusps. The Four Vertex Theorem says that a non-self-intersecting curve (like an ellipse) has an evolute with at least four cusps. This talk surveys the idea of a cusp and some of the many roles it plays in mathematics.

## Robert Franzosa University of Mainr (robert_franzosa@umit.maine.edu)

## The True-False Binary Cards: A Hands-On Approach to Deductive Reasoning

In the chapter "The Binary System" in New Mathematical Diversions from Scientific American, Martin Gardner presents a deck of punched cards that he uses to demonstrate binary sorting, to demonstrate a number trick, and to solve a logic problem. Building on his approach to the logic problem, we show how to use the deck for a hands-on approach to basic deductive reasoning topics typically seen in an introductory abstract mathematics course. In particular, we show how the deck can be used to compare propositions for equivalence, identify tautologies, identify contradictions, construct truth tables, and construct valid arguments.

## David Presnell Turner Faulkner University (dturner@faulkner.edu)

## An Induction Formula Generator

Students are often introduced to mathematical induction with expressions such as $1+2+\cdots+n=n(n+1) / 2$. Homework problems consist of proving such relationships by induction and sometimes also require the student to conjecture a formula for the sum before providing the induction proof. This talk will consider sums of the type where the general term is a polynomial. The formula for any such sum will be shown to be a linear combination of a particular subcollection of all such formulas.

## Kristen Sellke Saint Mary's University of Minnesota (ksellke@smumn.edu)

## Using Course Capture Technology to Supplement Classroom Learning

There are various methods for classroom course capture which can be implemented using technologies such as tablet computers and interactive whiteboards. One standard use of course capture is to simply record the class "live" to allow for further student viewing. This presentation will examine other uses of course capture that can instead be used to supplement the daily classroom learning such as holding question sessions, presenting exam solutions and flipping the classroom. We will look at student response to these activities, the impact on student learning and some of the best practices in course capture.

## Keith Brandt Rockhurst University (keith.brandt@rockhurst.edu)

## The Induction-Recursion Parallel: An Example for Students

There is a natural parallel between writing mathematical proofs and writing computer programs. This parallel is particularly striking when writing proofs that use mathematical induction and writing programs that use recursion. I will illustrate this parallel with a problem on Fibonacci numbers posed several years ago in Mathematics Magazine.

## Ellen Ziliak Benedictine University (eziliak @ben.edu)

What can you do with Abstract Algebra?
Many mathematics students are required by their major to take Abstract Algebra, a course that is generally difficult. What's worse, there is a common opinion among students that this course has no applications, so why should they study this material? The goal of this talk is to introduce several applications to abstract algebra that undergraduate students studied after their first semester course in Abstract Algebra. These applications include graph coloring, Sudoku, message authentication schemes, cryptography including a new area using non-associative groups, and chemistry.

# GCPS Session 12: Research in Number Theory, Geometry, or Linear Algebra 

Saturday, August 4, 1:00-3:25 PM

## John D Pesek Jr University of Delaware (pesek@udel.edu)

## Bisecting the Simplex: A Tale of Two Symmetries

We generalize the following 19th century result to higher dimensions. Any plane passing through the midpoints of the opposite sides $A B$ and $C D$ of the tetrahedron $A B C D$ divides the tetrahedron into two shapes of equal volume. A five dimensional result is that any 4 -dimensional hyperplane $H$ passing though the midpoints of $A B, C D$ and $E F$ of the simplex $A B C D E F$ divides it into two shapes of equal 5-dimensional volume. In general let $V=\left\{v_{0}, v_{1}, \ldots, v_{n}\right\}$ be an affinely independent set in a real vector space $W$ of dimension $n$ and let $S_{V}$ be the simplex which has vertex set $V$. Let $\left\{B_{j}, j=1, \ldots, m\right\}$ be a set partition of $V$ where each $B_{j}$ has just or one or two elements and at least one of the $B_{j}$ s has two elements. Let $H$ be any hyperplane of $W$ which contains all the points of the one element $B_{j} \mathrm{~s}$ and the midpoints of all the segments determined by the two elements $B_{j}$ s. Then $H$ divides $S_{V}$ into two shapes of equal $n$-dimensional volume. The strategy of the proof is to find an affine involution of the simplex which also preserves the hyperplane. Connections will be given to Varignon's theorem for higher dimensions and to an analogous result for centrally symmetric figures.

## Sean Howe University of Chicago (seanpkh@gmail.com)

## A Geometric Construction of Higher Dimension Counter-Examples to Relative Manin-Mumford

We generalize a construction of Edixhoven on elliptic curves with complex multiplication and work of Bertrand to give a geometric construction of higher dimension counter-examples to relative Manin-Mumford. By describing $\mathbb{G}_{m}$ extensions of the Jacobians of smooth proper curves as quotients of the Jacobians of "pinchings" of the curve, we produce a counter-example on any Jacobian admitting an anti-symmetric polarization. This counter example takes the form of a section of a non-constant extension of the Jacobian over a base of dimension the same as the genus of the curve that is torsion on a dense set which can be described explicitly.

## Adam Coffman Indiana University - Purdue University Fort Wayne (CoffmanA@ipfw.edu)

## Generalizing Eves' Theorem

A 2011 article by Marc Frantz in Mathematics Magazine demonstrated some applications of Eves’ Theorem, which states that certain ratios of Euclidean distances are invariant under projective transformations (a non-Euclidean notion) of the plane. The theorem can be generalized to higher dimensions, and some of the results on invariant ratios of areas pre-date Eves.

## Donald Leigh Hitzl Retired (domarltd@comcast.net)

## Dynamics of the Riemann Zeta Function

Two essential new features of the Riemann Zeta Function will be presented. First, for all values of the parameter $\sigma$ symmetric about $\sigma=1 / 2$, there is a hidden symmetry so that the magnitude of Zeta on the lhs = magnitude of Zeta on the rhs of $\sigma=1 / 2$ to within an easily computable, and slowly changing Scale Factor. Next, every term with $n>1$ in the infinite sum for Zeta will be shown to satisfy a fourth order partial differential equation for an undamped quartic oscillator. This holds for all values of the parameter $\sigma$ and the "time" $t$.

## Chad Awtrey Elon University (cawtrey@elon.edu)

## Galois-Theoretic Invariants of Sextic Extensions of $p$-adic Fields

First introduced over 100 years ago, the $p$-adic numbers have become an important tool in both mathematics and physics. Of particular interest to number theorists is the connection between both the $p$-adic numbers and the rational numbers as well as their respective field extensions. In this talk, we'll discuss a few motivating classical results and then delve into some current computational results related to Galois groups of degree six extensions of $p$-adic fields.

## Nathan Moyer Whitworth University (nmoyer@whitworth.edu)

## Modular Arithmetic on Rings of Algebraic Integers

Modular arithmetic over the rational integers has been utilized for centuries. The theory has been well established, built upon the elementary properties of congruences. In this talk I introduce an analogous definition of congruence for certain rings of algebraic integers. Specifically, I will examine properties of modular multiplication over rings of quadratic integers of the form $a+b \sqrt{r}$ and bi-quadratic integers of the form $a+b \sqrt{r}+c \sqrt{s}+d \sqrt{r s}$. Other issues under consideration will be the matrix representation of these numbers as well as the geometric structure of various complete residue systems. An application to knapsack-based cryptography will also be explored.

## Jay Lawrence Schiffman Rowan University (schiffman@rowan.edu)

## Exploing the Fibonacci Sequence of Order Three

The Fibonacci sequence of order 3 is the sequence of numbers $1,3,10,33,109, \ldots$. Each term in this sequence from the third term on equals three times the term before it plus the term two places before it. This paper will explore ideas such as divisibility and periodicity in the sequence as well as those elements that are prime. In addition, a closed formula similar to the Binet Formula for the Fibonacci sequence will be furnished as well as any palatable number tricks that are discovered.

## Erik R. Tou Carthage College (etou@carthage.edu)

## A Zeta Function for Juggling Patterns

In recent years, a great deal of attention has been paid to the mathematical properties of juggling patterns. Using the so-called siteswap notation, one has an infinite set of juggling patterns with many properties in common with the positive integers. In this talk, we build on the analogy with positive integers by constructing a zeta function for juggling patterns. We will give a meromorphic continuation of the function in addition to identifying its zeroes and singularities. This is joint work with Carsten Elsner and Dominic Klyve.

## Brian Heinold Mount St. Mary’s University (heinold@msmary.edu)

## Divisibility plots

Given a function $f: \mathbb{N} \rightarrow \mathbb{N}$ and an integer $n$, we examine plots of all the points $(x, y)$ for which $n$ divides $f(x, y)$, which we call divisibility plots. The plots are visually interesting and have connections to topics in number theory, such as Fermat's Theorem about sums of squares.

Ron Taylor Berry College (rtaylor@berry.edu)
Jill Cochran Berry College (jcochran@berry.edu)
Eric McDowell Berry College (emcdowell@berry.edu)

## Roots of Polynomials with Fibonacci Coefficients

One of the standard means for deriving the Golden Ratio $\phi$ is by considering the limit of ratios of successive Fibonacci numbers. This leads to a continued fraction representation which can be written as $\phi=1+\frac{1}{\phi}$. Using this relationship, powers of $\phi$ can be written as linear combinations of $\phi$ and 1 with Fibonacci numbers as the weights. By considering $\phi$ to be one of the zeros of the polynomial $f(x)=x^{2}-x-1$ we can construct a sequence of polynomials and a sequence of Laurent polynomials that are derived from these representations of the powers of $\phi$. Both sequences are similar to the generating function for the Fibonacci sequence in that the coefficients are Fibonacci numbers, but the relationship between the Fibonacci numbers and the exponents is reversed. Moreover, the roots of these polynomials have interesting properties related to $\phi$. In the case of the polynomial sequence, the sequence of real roots converges to $-\phi$ and in the Laurent polynomial case, the sequence of real roots converges to $\phi-1$.

# Graduate Student Session 

# Great Talks for a General Audience: Coached Presentations by Graduate Students 

Saturday, August 4, 1:00-5:30 PM, Hall of Ideas K

## 1:00-1:10 PM Joint Introduction / Information Session

Frances Grace Withrow Texas A\&M University (frances.withrow@gmail.com)
Jean Marie Linhart Texas A\&M University Department of Mathematics (jmlinhart@math.tamu.edu)
When Zombies Attack: Round Two
1:15-1:35 PM
A zombie is considered to be the living dead, and of the realm of science fiction. However, in our generation zombies are commonly found in books, movies, and television shows. In this paper we use mathematical modeling to analyze a zombie outbreak. The ultimate goal was to develop a model that had a stable disease free equilibrium. We started with a previous model and developed new models that included concepts such as flesh decay, saturated interaction rates, and human detection rates. We included a sensitivity analysis of one of the new constants. None of the models had a stable disease free equilibrium; however, the most successful models were those that included some combination of saturation and detection rates. This implies that the best way to save humanity would be to decrease the number of humans a zombie can interact with and increase the rate of zombie detection per human being.

## Sami Cheong UW-Milwaukee (cheongs@uwm.edu)

## A Study of the Mathematical Model of Protein Synthesis Initiation

1:40-2:00 PM
The initiation phase of protein synthesis in eukaryotes is known to have the largest rate-limiting effect on the final production of protein. It is also one of the most complex phases of the biological process, involving many reactions with different initiation factors (eIF). In this project, we divide our mathematical investigation into the study of the mathematical model of initiation, and a closer look at the general problem of parameter estimation via a newly developed technique called Proximate Parameter Tuning algorithm (PPT).

## Zhengfu Xu Michigan Technical University (zhengfux @mtu.edu) <br> Chao Liang Michigan Technical University (chaolian@mtu.edu) <br> Franz Tanner Michigan Technological University (tanner@mtu.edu)

## Parameterized Maximum Principle Preserving Flux Limiters for High Order Scheme Solving Scalar Hyperbolic Conser-

 vation Laws2:05-2:25 PM
I will discuss a new strict maximum principle preserving flux limiting technique that was developed by Zhengfu Xu both in finite difference and finite volume framework.

In 1D case, by decoupling a sequence of parameters embedded in a group of explicit inequalities, the numerical fluxes are locally redefined, therefore consistent and conservative. Not only the global maximum principle can be preserved while the high order accuracy of the underlying scheme being maintained, but also the parameterized limiters are less restrictive on the CFL number when applied to high order finite volume scheme. Within the proposed parameterized limiters framework, by relaxing the limits on the intermediate values of the multi-stage Runge-Kutta method, a successive sequence of limiters allow for significantly large CFL number.

We are extending the strict maximum principle preserving flux limiting technique to 2 D case. The parameterized limiters and their determination from decoupling proposed for 1D case is proposed in a more compact way for 2D problem, therefore easy to use. Also, the relaxed successively defined limiters are proposed when the multi-stage TVD Runge-Kutta method is used. With these combined techniques, high order accuracy and strict maximum principle can be achieved without restrict CFL constraint. Numerical tests on two-dimensional problems are presented to demonstrate the capability of this new approach. Potential application and future work will also be discussed.

Crystal Bennett North Carolina Agricultural \& Technical State University (clbennet@ncat.edu)

## Investigation of a Model of Chagas Disease

2:30-2:50 PM
Mathematical biology is an exciting branch of applied mathematics that is vastly becoming more popular. Using math to model what happens in nature or our bodies is not only fascinating but beneficial to the scientific community. This talk will show how dynamical systems can be
used to model the flow of a disease in a human population as it interacts with a vector population. Not only will we use mathematics but also we will show the benefits of using numerical simulations to help compute the equilibrium of our system and interpret its stability.

Mary Therese Padberg University of Iowa (mtpadberg@gmail.com)
Isabel Darcy University of Iowa (idarcymath @ gmail.com)
Stephen Levene University of Texas at Dallas (stephen.levene@utdallas.edu)

## Exploring the Conformation of Protein-Bound DNA: Adding Geometry to Known Topology

2:55-3:15 PM
Understanding the conformation of protein-bound DNA is extremely important for biological and medical research, including improvement of drug creation and administration methods. Many protein-bound DNA conformations have been catalogued in the Protein Data Base; however the process of cataloging larger complexes can prove extremely difficult or unsuccessful. When standard lab techniques fail to determine a conformation, we turn to a branch of mathematical knot theory, tangle analysis, used in conjunction with difference topology experiments to analyze the topology of protein-bound DNA. In this talk, we will discuss these techniques and explain why topology alone is not enough. We will introduce preliminary software which can be used to determine likely DNA geometries consistent with protein-bound DNA topologies. Combining geometric and topological solutions will allow us to more accurately describe conformations for large proteinbound DNA complexes.

## May Mei UC Irvine (mmei@math.uci.edu)

## Modeling Quasicrystals: An Application of Hyperbolic Dynamics

3:20-3:40 PM
Dynamical Systems is, by its nature, useful in describing complex physical phenomena. We will discuss one place it has turned up - in the study of quasicrystals. A quasicrystal is, roughly speaking, ordered but not periodic. Sturmian sequences are aperiodic sequences of minimal complexity, that is, they are the least complicated sequences that are not periodic. This makes them prime candidates for modeling the behavior of quasicrystals. We will discuss a problem involving the spectral properties of Schrodinger operators with Sturmian potential, which has been reduced to studying the dynamical behavior of a polynomial map. This talk is accessible for undergraduate students.

## Scott Kaschner Indiana University Perdue University Indianapolis (srkaschn@iupui.edu)

Population Models and Chaotic Dynamics
3:45-4:05 PM
Dynamical Systems is a fascinating branch of modern mathematics that deals with the patterns, structure, and chaos that arise through repeated processes (for example, the repeated application of a function). The simplest situation where nontrivial dynamics arises is iteration of the logistic map, a quadratic polynomial. While using this map to model changes in biological populations, Robert May discovered that its iteration lead to surprisingly complicated and chaotic behavior. This led to years of deep mathematical research, and this talk will explore some of the results of this research. Specifically, I will explain Robert May's population model, how interesting periodic behavior arises, and how chaotic behavior can also arise.

## 4:45-5:30 PM Panel Discussion / Questions

## Saturday, August 4, 1:00-5:30 PM Hall of Ideas L

## 1:00-1:10 PM Joint Introduction / Information Session (In Hall of Ideas K)

## Donald Charles Sampson Georgia Tech (sampson.dcs@gmail.com)

## Playing with Bubbles: An invitation to Undergraduate Research in Mathematics

1:15-1:35 PM
Students from around the country are doing advanced mathematics research, i.e., playing with bubbles. Come learn about the history of soap bubble problems, thrilling new results, and how you (yes you!) can join undergraduate student researchers in solving new and exiting problems.

## Jennifer Hoxworth Rowan University (hoxwor00@ students.rowan.edu)

## Closed Form Solutions to the Josephus Problem

1:40-2:00 PM
This presentation describes the Josephus problem (a classic number theory problem) and variations of it. A general recursive formula will be developed, followed by the more interesting closed form solutions to the problem and one of its variations, which are derived directly from the
recursive formula. A look at the different patterns visible within the problem will assist in formulating these solutions. Fun visual examples with animations will be used throughout the presentation.

Ryan C. Bunge Illinois State University (rc_bunge@hotmail.com)<br>Saad El-Zanati Illinois State University (saad @ilstu.edu)

On $\lambda$-fold Rosa-type Labelings and Cyclic Graph Decompositions
2:05-2:25 PM
A labeling (or valuation) of a graph $G$ is an assignment of integers to the vertices of $G$ subject to certain conditions. A hierarchy of graph labelings was introduced by Rosa in the late 1960s. Rosa showed that certain basic labelings of a graph $G$ with $n$ edges yielded cyclic $G$-decompositions of $K_{2 n+1}$ while other stricter labelings yielded cyclic $G$-decompositions of $K_{2 n x+1}$ for all natural numbers $x$. Until recently, labelings of the latter type were defined only for bipartite and almost-bipartite graphs. We report on recent progress in this area and show how these concepts extend to $\lambda$-fold labelings. We show that some $\lambda$-fold labelings of a multigraph $G$ with $n$ edges lead to cyclic $G$-decompositions of the $\lambda$-fold complete multigraph ${ }^{\lambda} K_{\frac{2 n}{\lambda}+1}$ while others lead to cyclic $G$-decompositions of ${ }^{\lambda} K_{\frac{2 n x}{\lambda}+1}$ for every positive integer $x$.

## Megan Heenehan Wesleyan University (mheenehan@wesleyan.edu)

## Graphs in Facebook Friendships: How Connected Are We?

2:30-2:50 PM
Suppose I'm looking on Facebook and I want to visualize the friendships. I don't just want to see a list of friends I want to see how the friends are connected. Perhaps by seeing these connections I will find a new friend or other common interests. I can do this by creating a graph in which I have a vertex for each person and I put edges between people that are Facebook friends. I can then color the graph by assigning everyone a color in such a way that if people are friends they are assigned different colors. How many colors will I need? If there is a group of people that are all friends with each other they will each need a different color, this is a clique in my graph. More generally, if there are $n$ people who are all friends with each other then I get a clique of size $n$ in my graph, that is there are $n$ vertices that are pairwise adjacent. So, if I need to use $n$ colors, does this mean that there are $n$ people who all know each other? Or are there $n$ people that are all connected in some other way? In terms of graph theory our question is: Is there a relationship between graphs requiring $n$ colors and a clique of size $n$ ? A clique of size $n$ is the smallest graph that requires $n$ colors, so this is a natural question to ask. In this talk we will look at some attempts to answer this question.

## Sarah Hanusch Texas State University (sarah.hanusch@gmail.com) <br> Daniela Ferrero Texas State University (dferrero@txstate.edu)

## The $r$-Component Connectivity of the Generalized Petersen Graphs

2:55-3:15 PM
In many network applications one wants to know how the network performs after node or link failures: are the remaining nodes still connected if some of the nodes or links break down? The concepts of connectivity and edge connectivity answer this question. However, in practice these measures are sometimes insufficient. In some applications, one may need to know the number of resulting components from such a failure. Thus, the $r$-component connectivity measures the minimum number of nodes or links whose failure results in a network with $r$ components.

A crucial problem in the design of interconnection networks consists of finding with large order with a given maximum degree and diameter. The Generalized Petersen Graphs are a family of 3-regular graphs whose order is close to the upper bound (which is called the Moore bound $M(d, D)$ where $d$ is the maximum degree and $D$ is the diameter). Given two relatively prime integers $n \geq 3$ and $k \geq 1$, the Generalized Petersen Graph $G P(n, k)$ consists of two cycles of $n$ vertices connected by spokes which join the $i^{\text {th }}$ vertex of the first cycle to the $(k i \bmod n)^{\text {th }}$ vertex in the second cycle.

In this work we present upper and lower bounds for the $r$-component connectivity of Generalized Petersen Graphs. While doing so we solved another open problem: finding the girth of Generalized Petersen Graphs. Joint work with Daniela Ferrero.

## Maximiliano Liprandi University of Calgary (mliprand@ucalgary.ca)

## The Game of Blash \& Slash

3:20-3:40 PM
Consider the following problem: given a grid of $n \times n$ squares, how many diagonals can be drawn on these squares such that no two diagonals meet at any point? And what is the least amount of diagonals you can draw without being able to draw any other? This gives rise to the following game, which we will call Blash \& Slash. Left and Right take turns drawing diagonals on a fixed grid, and whoever is unable to draw one loses. This game can be analyzed in both partizan and impartial versions, the latter one bearing resemblance to the game of Node Kayles on Graphs.

Alina Florescu University of Iowa (alina-florescu@uiowa.edu)

## New Factorizations of the Integers

3:45-4:05 PM
Fix a non-negative integer $n$, and let $a$ be an integer that's not 0,1 or -1 . A $\boldsymbol{\tau}_{\boldsymbol{n}}$-factorization of $a$ is a factorization of the form

$$
a=a_{1} a_{2} \ldots a_{k} \quad \text { or } \quad a=(-1) a_{1} a_{2} \ldots a_{k}
$$

where $a_{1} \equiv a_{2} \equiv \ldots \equiv a_{k} \bmod n$ and $a_{i} \neq \pm 1$ for all $i$. These new factorizations generalize the usual factorization of the integers. As new irreducibles emerge, mischief ensues when not all integers can be factored into these new irreducibles.

## Sean Howe University of Chicago (seanpkh@gmail.com)

## Evil Monty Hall

4:10-4:30 PM
You are on a game show with three doors hiding two donkeys and a sports car. You choose one and then the host reveals a donkey behind one of the remaining two doors. You have two options - keep the prize behind the door you already picked, or switch to the other remaining door. Do you stay or do you switch? This is the infamous Monty Hall problem, named for the famous game show host of the 60s and 70s, and the solution to this deceptively simple (and, in this abstract and often elsewhere, incorrectly stated!) riddle has confused and vexed the public and mathematicians alike for over 30 years. In this talk, we introduce an extra variable in order to help understand the old solution-namely, we ask, how's Monty feeling today?

## 4:45-5:30 PM Panel Discussion / Questions (In Hall of Ideas K)

## PosterFest 2012

# A Poster Session of Scholarship by Early Career Mathematicians and Graduate Students 

## Sachith Abeysundara Texas Tech University (sachith.abeysundara@tu.edu)

## A New Approach of Genetic-based EM Algorithm for Mixture Models

Finite mixture models have been receiving important attention over the years from a practical and theoretical point of view, but it is still a challenging task to estimate a reasonable estimator based on the maximum likelihood method. The most widely used technique to solve the problem, to some extent, is the EM algorithm. Researchers have done a lot of work to improve the results of the EM algorithm by modifying its basic idea. This work presents such an attempt to obtain better estimates for a finite normal mixture model. A traditional evolutionary technique, known as the Genetic algorithm, is coupled with the EM algorithm to improve the estimates of the EM algorithm starting with a random initial vector of parameters. The presented method is tested with the availability of a Non-penalized and Penalized likelihood functions. Based on results, we can see that the proposed method is always superior to the classical EM algorithm when concerning the global maximizer in the mixture likelihood function.

Nickolas Jason Hein Texas A\&M University (nhein@math.tamu.edu)
Frank Sottile Texas A\&M University (sottile@math.tamu.edu)
Igor Zelenko Texas A\&M University (zelenko@math.tamu.edu)

## A Mod Four Congruence in the Real Schubert Calculus and the Hermitian Grassmannian

A system of polynomials of degree $d$ with real coefficients has $d$ complex solutions. The number $r$ of real solutions satisfies $r \equiv d$ mod 2 since there is an involution acting freely on non-real solutions (complex conjugation). We present a class of geometric problems we call symmetric Schubert problems with solutions fixed by Lagrangian involution.

When Lagrangian involution and complex conjugation act freely on the solutions to a real symmetric Schubert problem, we have a remarkable congruence: $r \equiv d \bmod 4$. This has connections to linear differential equations and the pole placement problem.

This is joint work with Frank Sottile and Igor Zelenko.

## Erin M Thomas Michigan Technological University (ethomas@mtu.edu) <br> Shari Stockero Michigan Technological University (stockero@mtu.edu)

## Characterizing Novice Mathematics Teacher Noticing

What do prospective teachers (PTs) of mathematics identify as important things that a teacher needs to notice in a classroom setting? How does this match with what they actually do notice in practice? How aware are they of their own noticing in the moment? These are questions that need to be answered in order to design teacher education experiences that effectively prepare new teachers to engage in student-centered mathematics instruction [NCTM, 2000]. Six PTs enrolled in the same secondary school teacher education program were video taped teaching a mathematics lesson three times during their student teaching experience. During these video taped lessons, the PTs documented instances in real time during their instruction that they identified as important to notice. Post interviews were conducted to understand the primary noticing focus of each PT as well as their perception of their own noticing. Each PT's primary noticing was characterized. Changes and trends in noticing were summarized and compared across the six PTs. Possible implications for designing mathematics teacher education experiences are discussed.

Katy Beeler Wake Forest University (beelk11@wfu.edu)
Kenneth Stephen Berenhaut Wake Forest University (berenhks@wfu.edu)

## Deterministic Walks on Graphs

In this poster we consider deterministic movement on graphs, integrating local information, memory and choice at nodes. The research is motivated by recent work on deterministic random walks and applications in multi-agent systems. Several results regarding passing messages through toroidal grids are discussed, as well as some open questions.

## Richard Cho Crescenta Valley High School (Jycho@charter.net)

## Exploring Quadratic Residues and Their Potential Applications

This project explores quadratic residues, a classical number theory topic, using computational techniques. First I conducted computational experiments to investigate the distribution of quadratic residues modulo primes, looking for patterns or evidence against randomness. The experimental data indicates a non-random distribution of quadratic residues. Certain features of such non-random distributions were then further investigated for application in cryptography, especially semi-prime factorization. In this project, I developed a novel approach that uses simply the number of quadratic residues to factor semi-primes. Mathematically, I developed and proved the validity of a function that
returns the factors of a semi-prime given the semi-prime and the number of quadratic residues of the semi-prime. The function reduces the problem of semi-prime factorization to computing the number of quadratic residues of the semi-prime. Computationally, I developed an approach to make the current method of generating quadratic residues more memory efficient while retaining a near linear speed up from parallelization. The next step will be to run my algorithm in the cloud, in a multi-core environment, to validate the efficiency claim of my multi-threaded approach. Future work will focus on computing the number of quadratic residues quickly and efficiently, especially on multi-core platforms, such as servers or clouds. Also, due to some similarities between my theory and the quadratic sieve factoring method, there may be a way to incorporate my findings into the quadratic sieve factoring algorithm to achieve a much faster run-time for factoring semi-primes.

## Zachary Andrew Kudlak Mount Saint Mary College (zachary.kudlak@msmc.edu)

## Exponential and Poisson Random Variables in a Campus Cafeteria

An undergraduate curriculum in probability often includes the study of Exponential and Poisson random variables. We used student entry into the campus cafeteria to collect data which exhibits characteristics of both Exponential and Poisson random variables. The students working on the project used this experience to create posters for an on-campus research symposium.

## Thomas Hoft Tufts University (thomas.hoft@tufts.edu)

## Few-Parameter Camera Response Function Recovery for High Dynamic Range Photography

We describe an inverse problem for determining the response function of a camera system from multiple photographs of a scene captured with dierent exposure times. The irradiance map (amount of light entering the input of the camera) of a natural scene spans up to 810 orders of magnitude but typical digital cameras record only two or three orders of magnitude. Recovering the response function of the camera allows one to combine low dynamic range images into a single image with the original dynamic range. We present preliminary results and analysis.

## Jonathan Weisbrod Rowan University (weisbr90@students.rowan.edu)

## Graphing the Iterations of Selected Polynomials Modulo $n$

We will examine patterns in the directed graphs formed by iterating a polynomial $\varphi(z)$ for all $z \in Z_{n}$. By selecting $n$ values contained within a particular sequence, we can often predict the properties of the graph of iterations of $\varphi(z)$. Each graph is made up of one or more disconnected subgraphs, each containing a cycle of one or more periodic elements. As we vary $n$, we can often predict which elements of $Z_{n}$ have periodic iterations as well as their cycle lengths. With the assistance of Wolfram Mathematica, the graphs of the iterations of $\varphi(z)$ are easily obtained revealing conjectures for further analysis.

## Melanie Pivarski Roosevelt University (mpivarski@roosevelt.edu) <br> Barbara Gonzalez Roosevelt University (bgonzalez@roosevelt.edu)

## Incorporating Students into Curriculum Building

We have recently begun creating a deep learning experience for students in our Calculus II courses by involving them in the development and design of aspects of the course. In Spring 2010 we introduced semester-long projects into our Calculus II courses. In Fall 2010 we began including undergraduates who were "recent graduates" of Calculus II as embedded course assistants. Starting in Summer 2011, we began mentoring students in project development and design. We used a student-developed project modeling population growth in Spring 2012.

Koren Nicole Staat University of Wisconsin - Milwaukee (knstaat@uwm.edu) Albert Milani University of Wisconsin - Milwaukee (ajmilani@uwm.edu)

## Leonardo Da Vinci's Mathematical Investigations

As a famous artist and engineer of the Renaissance period, Leonardo da Vinci has been the subject of many scholarly books and papers. Unfortunately, comparatively few of these resources emphasize Leonardos mathematical pursuits, and even a smaller number of them are in English. The goal of my thesis is to provide an articulate English resource of the various mathematical investigations of Leonardo, a subject in which the majority of resources are in Italian. Leonardo's mathematical skills and explorations in algebra and arithmetic will be assessed, as well as their applications to equivalence of surfaces and volumes such as in the quadrature of the circle and duplication of the cube. Applications to geometry, ballistics, and integration (particularly by method of slices and limits) exemplified in Leonardo's works will also be presented and evaluated.

## Alina Florescu The University of Iowa (alina-florescu@uiowa.edu)

## New Ways to Factor the Integers

Fix a non-negative integer $n$, and let $a$ be an integer that's not 0,1 or -1 . A $\boldsymbol{\tau}_{\boldsymbol{n}}$-factorization of $a$ is a factorization of the form

$$
a=a_{1} a_{2} \ldots a_{k} \quad \text { or } \quad a=(-1) a_{1} a_{2} \ldots a_{k}
$$

where $a_{1} \equiv a_{2} \equiv \ldots \equiv a_{k} \bmod n$ and $a_{i} \neq \pm 1$ for all $i$. These new factorizations generalize the usual factorization of the integers. As new irreducibles emerge, mischief ensues when not all integers can be factored into these new irreducibles.

## Hemalika Abeysundara Texas Tech University (hemalika.abeysundara@ttu.edu)

## Non-Parametric Estimation of Bivariate Survival Function

Estimating bivariate and marginal densities of paired survival data becomes more challenging when only one component is censored. If both components are censored or both are not censored, a bivariate version of Kaplan-Meier remains as a consistent estimator. But if only one variable is censored, Kaplan-Meier fails to take advantage of the information of the remaining variable. The method proposed by Akritas and Keilegom considered the case of single censoring as well as double censoring, a situation that is typical in medical studies. Our objective is to estimate the correlation between two variables in paired survival data at the presence of double and single censoring via nonparametric approaches. We use the estimates of nonparametric bivariate distribution and marginal distribution of each variable proposed by Akritas and Keilegom. These estimates are based on conditional distribution functions considering only those pairs where the value of the conditioning variable is uncensored. We then apply the above method on Diabetic Macular Edema (DME) data to estimate densities and correlation between time to cure for right and left eye.

## Sukanya Basu Grand Valley State University (basus@gvsu.edu) <br> Periodicity and Chaos in Some Planar Discrete Dynamical Systems with Negative Feedback

Oscillatory dynamics occur in many real-life applications such as, for example, circadian rhythms in gene regulatory networks. One way to better understand such oscillatory dynamics is to study mathematical models involving discrete dynamical systems with negative feedback interconnections. In this talk, I will discuss the oscillatory behavior of solutions to a class of discrete dynamical systems in the plane with negative feedback. More specifically, I will give some general yet simple criteria to determine when solutions of such discrete dynamical systems show oscillatory stability in the form of periodicity and when they exhibit oscillatory instability in the form of chaos.

## Ajinkya More University of Michigan, Ann Arbor (ajinkya@umich.edu)

## Symbolic Powers in Noetherian Rings

Given a ring $R$ and a prime ideal $P$ in $R$, the $n$ 'th symbolic power of $P$ is defined to be $P^{n} R_{P}$. I will introduce two open problems based on symbolic powers and discuss my results in this direction. Specifically, I will discuss about the Eisenbud Mazur conjecture which states that in a regular local ring $(R, \mathfrak{m})$ containing a field of characteristic zero, the second symbolic power of any prime ideal $P$ is contained in $\mathfrak{m} P$. Further, I will discuss the question of existence of uniform bounds on the growth of symbolic powers with respect to the ordinary powers of prime ideals.

## David Thomas Heras Radford University (vfhssoccer18@gmail.com)

## The Arithmetic of Elliptic Curves in the Complex Plane.

Guided by L. Washington's book on elliptic curves, we worked to understand three things: the complex analysis, the arithmetic, and the algebraic structure behind elliptic curves over the complex plane. The motivation is that insight into the basic theory is crucial to gaining a deeper and broad understanding of the theory of elliptic curves and, in particular, of forms on the upper half plane. The study was analytical in nature. Comprehension of the subject matter allowed us to reconsider several established theorems and to develop new approaches to the proofs. As a well known result, we showed that every elliptic curve over $\mathcal{C}$ arises from a complex torus.

Dawn Nelson Bates College (dnelson@bates.edu)

## Variation on Leopoldt's Conjecture

Leopoldt's Conjecture is a statement about the relationship between the global and local units of a number field. Informally the conjecture states that the $\mathbb{Z}_{p}$-rank of the diagonal embedding of the global units into the product of all local units equals the $\mathbb{Z}$-rank of the global units. We consider the question: Can we say anything about the $\mathbb{Z}_{p}$-rank of the diagonal embedding of the global units into the product of some local units? We answer the question in the affirmative.

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[^0]:    Chaotic Stability, Stable Chaos
    Viewed from various perspectives, the evolution of a dynamical system over time can appear both orderly and extremely disordered. I will describe some mechanisms behind chaos and stability in dynamics and how in certain contexts this intermixing of behaviors is to be expected.

