# ABSTRACTS of Papers

presented at

# MathFest 2008

## Madison, WI
## July 31–August 2, 2008

# Abstracts of Papers Presented at

# MathFest 2008

## Madison, Wisconsin

## July 31–August 2, 2008

*Published and Distributed by*

The Mathematical Association of America

# Contents

## Invited Paper Sessions



## Contributed Paper Sessions

# Invited Paper Sessions

## Classical Euclidean Geometry

### Thursday, July 31, 1:00 pm–4:00 pm

**Eisso J. Atzema** (atzema@math.umaine.edu) University of Maine

#### On the (History of the) Use of Oriented Angles and Their Extension to n-Sections

In this talk I will discuss some of the history of the use of oriented angles or their equivalents in classical/synthetic geometry (as opposed to analytical/coordinate geometry), both in research and in geometry textbooks. Particularly, I will focus on the period 1890-1927, starting with the first introduction of oriented angles in classical geometry and ending with the introduction of the concept of a cross by Picken and Forder. Finally, I will give an extension of the concept of a cross in the form of n-sections and show how (among other things) n-sections can be used in the study of reciprocal figures.

**Clark Kimberling** (ck6@evansville.edu) University of Evansville

#### High points in the Encyclopedia of Triangle Centers

The first half is planned especially for those who are familiar with the four ancient triangle centers (centroid, incenter, circumcenter, orthocenter) but who may be new to triangle algebra, based on homogeneous coordinates, as represented in the Encyclopedia of Triangle Centers-ETC (http://faculty.evansville.edu/ck6/encyclopedia/ETC.html). The second half concentrates on certain high-numbered points in ETC which involve the Thomson, Darboux, and other cubics.

**James L. Parish** (jparish@siue.edu) Southern Illinois University at Edwardsville

#### Special Quadrilaterals and Special Conics

Call a quadrilateral 'proper' if no three of its vertices are collinear. We associate with each proper quadrilateral a pair of conics, one circumscribed and one inscribed, and examine the connections between properties of these conics and of the quadrilateral. We also consider the polar reciprocals of the quadrilateral with respect to these conics.

**Steve Sigur** (s.sigur@comcast.net) The Paideia School, Atlanta, Georgia

#### Algebraic Ramifications in Triangle Geometry

**Bogdan Suceava** (bsuceava@exchange.fullerton.edu) California State University at Fullerton

#### The Equation of Euler's Line Yields a Tzitzeica Surface

The relative position of Euler's line with respect to the sides of a triangle has raised the geometers' interest since L. Euler's first paper on this topic, published in 1765. We prove that the surface in the three dimensional Euclidean space generated by Euler's line formula in a triangle satisfies Tzitzeica's affine invariance property. This analogy establishes an interesting connection between triangle geometry and the property that lies at the origins of affine differential geometry. Our presentation covers results from works written with A. F. Agnew, A. Bobe, W. G. Boskoff, and L. Homentcovschi.

**Paul Yiu** (yiu@fau.edu) Florida Atlantic University

#### The Circles of Lester, Evans, Parry and Their Generalizations

Beginning with the famous Lester circle containing the circumcenter, nine-point center and the two Fermat points of a triangle, we survey a number of interesting circles in triangle geometry.

# Graph Theory with Connections to Geometry and Topology

## Thursday, July 31, 3:15 pm–5:45 pm

**Alice Dean** (adean@skidmore.edu) Skidmore College

### Recent Rectangle Visibility Results

*Bar-visibility graphs* were introduced in the 1980s as a way of representing graphs with horizontal bars for vertices and vertical visibilities for edges. Since then a number of variations on this theme have been studied: *rectangle-visibility* graphs, in which vertices are represented by rectangles, and visibility can be horizontal or vertical; *unit* bar- and rectangle-visibility graphs, in which all the bars or rectangles must have the same size; and *bar k-visibility* graphs, in which the bars have a certain amount of "translucence," meaning that more than one obstructing bar may be needed to prevent visibility. Only the original class of bar-visibility graphs has been fully characterized. This talk gives a survey of some of the interesting properties that have been proved, as well as some that have yet to be proved, about these generalizations of bar-visibility graphs.

**Stephen G. Hartke** (shartke2@math.unl.edu) Department of Mathematics, University of Nebraska - Lincoln, Lincoln, NE 68588-0130 USA

### Visibility Graphs: Different Models of Sight

Given a collection of regions in the plane, we define a graph $G$ where the vertices of $G$ are the regions and two vertices are adjacent if the corresponding regions can "see" each other. A graph $G$ with such a representation is called a *visibility graph*. When restrictions are placed on the regions or on the type of sight allowed, then different classes of visibility graphs arise. In this talk we will consider different restrictions and their resulting graph classes, including general visibility graphs (when the regions are arbitrary connected regions in the plane and sightlines are arbitrary) and bar visibility graphs (when regions are horizontal line segments and sightlines must be vertical).

**Ellen Gethner** (ellen.gethner@cudenver.edu) University of Colorado Denver

### An Adventurer's Guide to the Colorful Tale of Thickness - Two Graphs

The famous Four Color Theorem addresses the following problem: given any cartographer's map $M$ in which the countries are connected regions, what is the largest number of colors needed to color $M$ so that no two neighboring countries receive the same color? The answer, of course, is four. Is that all there is??

William Tutte is known to have said: *The Four Color Theorem is the tip of the iceberg, the thin end of the wedge and the first cuckoo of spring.*

Here is one possible direction on the journey from spring into summer, as posed by G. Ringel in 1959. What is the largest chromatic number of any thickness-two graph? A thickness-two graph is simply a graph in which the edges can be partitioned in two sets, each of which induces a planar graph. In some sense, a thickness-two graph is one step away from being planar. Not surprisingly, nobody knows a sharp upper bound for the largest chromatic number of the family of thickness-two graphs. We do, however, know that there are "only" four possible answers, namely 9, 10, 11, or 12.

**Arthur Busch** (art.busch@notes.udayton.edu) University of Dayton
**Dave Brown** (david.e.brown@usu.edu) Utah State University
**Garth Isaak** (gi02@Lehigh.edu) Lehigh University

### Recognition Algorithms for Some Generalizations of Interval Graphs

In this talk we review the algorithm for recognizing for recognizing interval graphs and how the algorithm can be extended to recognize the related classes of bipartite tolerance graphs and bipartite probe interval graphs. We discuss the efficiency of this algorithm as well as how this and other algorithms can provide a certificate of correctness.

**Michael J. Pelsmajer** (pelsmajer@iit.edu) Illinois Institute of Technology

### Crossing Numbers

When drawing a graph on the plane or another surface, usually one tries to avoid having any two edges cross. When that is not possible, often people would like to minimize the number of crossings in a drawing, perhaps avoiding "unnecessary crossings". For example, this theorem of Hanani and Tutte:

If a graph can be drawn in the plane such that every two edges cross an even number of times, then it can be redrawn with no crossings at all.

We will discuss this and some more recent results of this nature, mostly due to joint work with Marcus Schaefer (DePaul University), Daniel Štefankovič (University of Rochester), and Despina Stasi (a graduate student at University of Illinois Chicago). Everything will be introduced from scratch; no knowledge of graph theory will be assumed.

# Mathematical Biology

## Friday, August 1, 1:00 pm–3:00 pm

**Gheorghe Craciun** (craciun@math.wisc.edu) Department of Mathematics and Department of Biomolecular Chemistry, University of Wisconsin-Madison, 480 Lincoln Dr., Madison WI 53705

### Multiple equilibria and global attractors in biochemical reaction network dynamics

Modern biological research provides countless examples of biochemical reaction networks. In order to understand the role played by some of the reactions in these networks, one often faces great difficulties in trying to interpret the effect of positive and negative feedbacks, nonlinear interactions, and other complex signaling between the nodes of the network. We will describe connections between reaction network structure and the capacity for multiple equilibria and other interesting dynamic behavior, and will discuss how these connections may impact the interpretation of experimental data.

**Bo Su** (bosu@iastate.edu) Department of Mathematics, Iowa State University, Ames, IA 50010
**Wen Zhou** (riczw@iastate.edu) Department of Mathematics, Iowa State University, Ames, IA 50010
**Karin Dorman** (kdorman@iastate.edu) Department of Statistics; and Department of Genetics, Development and Cell Biology, Iowa State University, Ames, IA 50010
**Douglas Jones** (jonesdou@iastate.edu) Department of Veterinary Pathology, Iowa State University, Ames, IA 50010

### Mathematical Modeling of Immune Response in Tissue

Many infections in both human and veterinary medicine remain difficult to manage clinically or prevent by vaccination. A successful immune response to infections must balance the pro-inflammatory effector actions with the protection of organ function. An anti-inflammatory component to the immune response is proven by the existence of host- derived immunosuppressive cytokines such as IL-10. The complexity of anti-inflammation regulation is revealed by the relatively recent description of CD4+ T regulatory cells. Although the importance of immunoregulation is appreciated in many experimental systems, the characteristics of infection that lead to changes in the balance of pro- vs. anti-inflammatory actions is largely unknown.

Working with immunologists, we have developed a spatial-temporal mathematical model (PDE) to capture fundamental aspects of the immune response to antigen. We have considered terms that broadly describe intercellular communication, cell movement, and effector function (activation or inhibition). The PDE model is robust to variation in antigen load and it can account for 1) antigen recognition, 2) an innate immune response, 3) an adaptive immune response, 4) the elimination of antigen and subsequent resolution of the immune response or 5) equilibrium of the immune response to the presence of persistent antigen (chronic infection) and the formation of a granuloma. Our PDE model not only successfully recapitulates major immune phenomena, but also tests the well-known hypothesis in the immunology community and make interesting predications in immune responses to various of infectious diseases.

**Isabel K. Darcy** (idarcy@math.uiowa.edu) University of Iowa
**John Luecke** University of Texas at Austin
**Mariel Vazquez** San Francisco State University

### Modeling Protein-DNA Complexes Using Tangles

Protein-DNA complexes were first mathematically modeled using tangles in Ernst and Sumners seminal paper, ”A calculus for rational tangles: applications to DNA recombination” (Math Proc Camb Phil Soc, 1990). A tangle consists of arcs properly embedded in a 3-dimensional ball. The protein is modeled by the 3D ball while the segments of DNA bound by the protein can be thought of as arcs embedded within the protein ball. This is a very simple model of protein-DNA binding, but from this simple model, much information can be gained. The main idea is that when modeling protein-DNA reactions, one would like to know how to draw the DNA. For example, are there any crossings trapped by the protein complex? How do the DNA strands exit the complex? Is there significant bending? Tangle analysis cannot determine the exact geometry of the protein-bound DNA, but it can determine the overall entanglement of this DNA, after which other techniques may be used to more precisely determine the geometry. In this talk, we will analyze an experiment by Pathania, Jayaram, Harshey (Cell, Vol. 109, 425-436) in which Mu Transposase binds 3 segments of DNA and a second protein, Crerecombinase, knots the DNA.

**Daniel Beard** (dbeard@mcw.edu) Medical College of Wisconsin

### Physical Chemical Principles for Mathematical Analysis and Simulation of Large-Scale Biochemical Systems

Simulations of cellular systems are optimally realistic and meaningful only when appropriate physical chemical rules are adopted. Since a great deal of information is available regarding the thermodynamic and ion-binding properties of biochemical reactants, it is possible to construct simulations of biochemical systems that properly incorporate these data. Specifically, realistic simulations of biochemical systems require accounting for: (i.) the complex multiple equilibria of biochemical species and dynamic buffering of ions; and (ii.) the pH and ionic dependence of enzyme kinetics and apparent equilibria and thermodynamic driving forces for biochemical reactions. Using a formal method that treats these phenomena, we can develop and validate computational models of systems of unprecedented complexity. In addition, rigorous physical chemical rules facilitate non-ambiguous model integration while reducing uncertainty in parameter estimates and improving the reliability of model predictions. As an example we will show how a model of cardiac energy metabolism that tracks more than 100 species is developed, parameterized, validated, and used to generate hypotheses and understand emergent phenomena.

# Ramanujan's Impact on Number Theory—Then and Now

## Friday, August 1, 1:00 pm–3:00 pm

**George Andrews** (andrews@math.psu.edu) Penn State University

### Gems from Ramanujan's Lost Notebook

In the A.M.S. Notices (January 2008), Bruce Berndt and I published Your Hit Parade: The Top Ten Most Fascinating Formulas in Ramanujan's Lost Notebook. Most of these formulas have great depth or important implications or both. In this talk, we shall back off a bit from these heavy results. Our object will be to consider some of the simpler but still surprising formulas from the Lost Notebook. Also we shall describe some of the frustrations and miscues that attended their elucidation.

**Bruce Berndt** (berndt@math.uiuc.edu) University of Illinois at Urbana-Champaign

### Ramanujan's Series for $1/\pi$

In his famous paper Modular Equations and Approximations to $\pi$, Ramanujan recorded 17 hypergeometric-like series representations for $1/\pi$. These were not completely proved until 1987 when Jonathan and Peter Borwein found proofs. In the past 20 years, several authors have found new hypergeometric-like series for $1/\pi$. In particular, in the past two years, Heng Huat Chan, Nayandeep Deka Baruah, and the speaker have returned to Ramanujan's paper and used his ideas, which are based on Eisenstein series, more so than previous authors to establish proofs of most of Ramanujan's formulas and to discover many new such formulas as well. A historical survey of attempts to prove Ramanujan's formulas will be given, emphasizing the contributions of S. Chowla, R. William Gosper, Jr., Jonathan and Peter Borwein, David and Gregory Chudnovsky, Heng Huat Chan, Nayandeep Baruah, and others. Ramanujan's ideas arising from Eisenstein series will be explained.

**Krishnaswami Alladi** (alladi@math.ufl.edu) University of Florida

### Hardy-Ramanujan and the Creation of Probabilistic Number Theory

In 1917, Hardy and Ramanujan wrote a fundamental paper on the number of prime factors of the integers. Although prime numbers were studied since Greek antiquity, this was the first systematic discussion of the number of prime factors. Two decades later, Erdos, Kac and Turan realized the probabilistic underpinnings of the Hardy- Ramanujan theorems and established important results on the distribution of additive functions. This created the field of Probabilistic Number Theory which is an active area of research today. The study of the distribution of prime factors has had several important consequences in number theory such as in the estimation of the orders of arithmetical functions, and the construction of algorithms for factoring numbers. We will briefly highlight the fundamental ideas of the Hardy-Ramanujan paper and trace some of the major developments emerging from their path breaking work.

**Ken Ono** (ono@math.wisc.edu) University of Wisconsin

### Ramanujan's Incredible $\tau$-function

Ramanujan's $\tau$-function has served as a prototype for some of the deepest works in 20th century number theory. Serre viewed Ramanujan's famous $\tau$-congruences as evidence for his conjectures on Galois representations (ideas which Wiles would later use to prove Fermat's Last Theorem). Ramanujan himself conjectured bounds which turned out to be a famous example of the Weil Conjectures (proved by Deligne). In this talk we will review these two connections, and we shall conclude with a new role for $\tau$ related to deep questions in transcendence theory.

# Implications for Teaching of Research on Learning

## Friday, August 1, 3:15 PM–5:15 PM

**Keith Weber** (keith.weber@gse.rutgers.edu) Rutgers University

### Successful and unsuccessful students' learning strategies in real analysis

In this talk, I discuss the results of a study in which I examined the learning strategies of successful and unsuccessful mathematics majors in a real analysis course. Successful students were more likely to reformulate mathematics definitions in their own words, attempt to connect the new mathematics they were learning to their prior knowledge, and build an understanding of mathematical statementes prior to reading or constructing a proof of these statements. I argue that the roots of these strategies are, in part, based on the epistemological beliefs of the participants and, further, the unproductive epistemological beliefs held by some of the less successful students may be encouraged by traditional instruction.

**Sean Larsen** ( slarsen@pdx.edu ) Portland State University

### From individual interviews to whole-class experiments: Reconstructing a course in abstract algebra

I will describe the reconstruction of an introductory abstract algebra course through multiple cycles of research and development. The core of the course consists of three instructional sequences that engage students in the reinvention of the fundamental course concepts (group, isomorphism, and quotient group). The development of these sequences began with teaching experiments conducted with pairs of students. This approach facilitated the process of identifying students' ways of thinking that anticipated the formal concepts and developing instructional strategies for evoking and capitalizing on these kinds of thinking. The resulting instructional sequences were then tested and refined in a regular abstract algebra course. Currently instructor support materials are being developed for the course curriculum and investigations into the impact of the curriculum on students' learning are ongoing.

**Marilyn Carlson** ( marilyn.carlson@asu.edu ) Arizona State University

### The role of quantitative reasoning in learning word problems in precalculus mathematics

A study of precalculus level students' ability to complete word problems revealed complexities in students' ability to conceive of and reason about the quantities in a contextual situation. Students' inattentiveness to the quantities to be related inhibited their progress toward a solution. In contrast, students who were successful in completing word problems developed dynamic mental models of the problem situation. That is, they formed images of the quantities to be related and were able to imagine how the values of these quantities changed in tandem. Students who developed these dynamic images were also able to construct formula to relate the two quantities. The results provide insight to the complexities students encounter in responding to word problems and the necessity to support students in developing dynamic cognitive models of the quantities in a word problem before algebraically formalizing these relationships.

**Patrick Thompson** ( pat.thompson@asu.edu ) Arizona State University

### Learning without understanding and its implications for the mathematics that teachers teach

Fifteen high school mathematics teachers designed a unit to teach trigonometry to 10th grade geometry and 11th grade algebra students. The study revealed that the understandings of angle, angle measure, and trigonometric function that teachers' had developed in high school and college, incoherent though they be, dominated in subtle and complex ways the mathematical ideas they attempted to convey to students.

# Research with Undergraduates

## Saturday, August 2, 1:00 PM–5:00 PM

**Annalisa Crannell** (annalisa.crannell@fandm.edu) Franklin & Marshall College

### From Chaos to Colleagues

In the spirit of Paul Halmos' advice on writing ("There is no recipe, and what it is"), I will share some of the idiosyncrasies of my own successful research collaborations with students. The advice I will share includes the following dubious gems. Start collaborating with students before you start collaborating with them. Get students to work independently by gathering them into small groups. By doing lots of work, you can get students to do all your work. Carefully choose just the right problem, and then don't stick with it.

**Colin Adams** (Colin.C.Adams@williams.edu) Williams College

### Knot Theory Research with Undergraduates

Knot theory is an area that lends itself to research by students. One can state open problems with almost no need for background and students can play with their shoelaces the first day. We will discuss the kinds of results students have already obtained and directions for further research by students.

**Asuman Aksoy** (aaksoy@cmc.edu) Claremont McKenna College

### Compactness in Metric Trees

A metric tree $(M, d)$, also known as $R$-tree or $T$-Theory, is a metric space such that any two distinct points $P$ and $Q$ of $M$ are connected by a unique arc and that arc is isometric to an interval in . I will present some fundamental properties of metric trees and characterize their compact subsets. These results will be used to show that a map is $k$-set contractive if and only if $T$ is $k$-ball contractive.

**Carlos Castillo-Chavez** (ccchavez@asu.edu) Arizona State University

### Building Communities Through REU Programs

Despite tremendous national efforts to entice American students into graduate programs in the mathematical science the numbers do not seem to add up. As a nation, we continue to have difficulties recruiting and keeping American students in the mathematical sciences despite the efforts instigated by IGERT, VIGRE, AGEP, LSAMP and NIH and NSF Training Grants. Tradition is often the engine behind the growth and survival of scientific and mathematical communities. The birth and growth of American Schools in pure and applied mathematics are closely tied in to the migration of large numbers of extraordinary mathematicians before, during and after WWII. How do we build such communities? How do we establish cultures that promote the growth and vitality of the mathematical sciences? REU programs play a significant role. In this talk, I will describe the model that the Mathematical and Theoretical Biology Institute (MTBI) has developed over the past 14 years and the way it has enhanced through its collaborations with the Institute for Strengthening Understanding of Mathematics and Science (SUMS) that was established in AZ by Joaquin Bustoz Jr, 23 years ago.

**Jacqueline Jensen** (Jensen@shsu.edu) Sam Houston State University

### How to Juggle Seven Undergraduate Student Projects Without Dropping Any

Sam Houston State University (SHSU) provides a Methods of Research course. Undergraduate students participating in this class individually choose research topics of interest to them. Under the guidance of the instructor, students investigate this area, with their results presented as talks at the meeting of the Texas Section of the MAA and as posters displayed at the SHSU Honors Consortium. Individual topics range from expository (The Existence of Infinity Through Transfinite Numbers) to pure research (Fourier Transforms and 4-D Tensor Based Wave Equations). We will discuss some of the projects researched during the Spring 2008 semester, as well as suggestions for other schools interested in implementing such a program. Particular attention will be paid to juggling techniques.

**Michael Orrison** (orrison@hmc.edu) Harvey Mudd College

### The Applied Representation Theory Group at Harvey Mudd College

Each year, I lead a research group of three to five undergraduates. Our group is called the Applied Representation Theory (A.R.T.) group, and is made up of students doing senior thesis, summer research, and independent research projects. In this talk, I'll survey some of the projects we have tackled in recent years (e.g., ”Algebraic Voting Theory” and ”FFTs for the Symmetric Group”), and describe some of the mechanics involved in sustaining what has become one of the most enjoyable ways in which I interact with students.

**Marc Chamberland** (chamberl@math.grinnell.edu) Grinnell College

### Ramanujan's Dream

One can only imagine what Ramanujan would have discovered had he lived a century later with a computer to help him explore his ideas. Today's computing tools allow even undergraduates to make mathematical discovery easier and accessible. Some recent student projects will be detailed, including generalizing a beautiful Ramanujan identity and factoring Hankel matrices.

## Gems in Number Theory

### Saturday, August 2, 3:15 PM–5:15 PM

**Ezra Brown** (ezbrown@calvin.math.vt.edu) Virginia Tech

#### A Tale of Two Curves

This talk will feature two families of elliptic curves over the rational numbers, one from Diophantus and one from the internet. Such curves always have finite ranks as abelian groups. The talk will illustrate how asking questions about ranks, one perfectly natural and the other perfectly outrageous, can get out of hand, and lead to some beautiful and bewildering mathematics.

**Richard K. Guy** (rkg@math.ucalgary.ca) University of Calgary

#### Farey Fractions and Ford Circles

This talk, with input from the audience, focuses on batting and bowling averages, Ford circles, orthogonal circles, and hyperbolic geometry.

**Margaret Robinson** (robinson@mtholyoke.edu) Mt. Holyoke College

#### Two Ways to Count Solutions to Polynomial Equations

The speaker focuses on two ways to count solutions to polynomial equations: solutions in finite fields and solutions modulo powers of a prime. For several polynomials, their cardinalities and how for each counting method they fit together to form a very similar-looking generating function, are investigated. The talk concludes with the tantalizing, sometimes frustrating, questions about how these generating functions are related to one another and to the very different-looking zeta functions of Weil and Igusa.

**Arthur Benjamin** (Benjamin@math.hmc.edu) Harvey Mudd College

#### Biscuits of Number Theory

Based on a collection of articles, collected by Art Benjamin and Bud Brown, the speaker provides an assortment of tasty, easily digested morsels of number theory, immensely satisfying, and leave you wanting more. (It will soon appear in an MAA book.)

## History of Mathematics

### Saturday, August 2, 1:00 PM–3:00 PM

**Janet L. Beery** (janet_beery@redlands.edu) University of Redlands

#### Navigating Between Triangular Numbers and Trigonometric Tables: How Thomas Harriot Developed His Interpolation Formulas

By 1611, Thomas Harriot (1560-1621) was developing finite difference interpolation methods, work that culminated in 1618 or later in his unpublished treatise, "De numeris triangularibus et inde de progressionibus arithmeticis: Magisteria magna", in which he derived symbolic interpolation formulas and showed how to use them to interpolate in tables. This treatise and its influence have been the subject of recent research by the author and Jacqueline Stedall. The interpolation formulas that appear in Harriot's manuscripts vary in notation, structure, and method of application. In the present paper, we use these largely undated manuscripts to show how Harriot may have developed and refined his methods over time.

**Thomas Druckert** (druckert@uww.edu) University of Wisconsin-Whitewater

#### The Best of All Possible Computers

Much of the work that went into early computers can be seen as an attempt to fulfill the vision Leibniz had of a 'calculus ratiocinator'. Even though Leibniz did not pursue the vision in practice, it seemed as though the idea of a computer would be to help in extending human knowledge of the truth. Alan Turing's view of the computer was unsettling and arose from traditions far from Leibniz's philosophical one. This talk will look at the ways in which Turing departed from Leibniz's view and created a more anarchic one, almost too much for those who developed the computing machine subsequently to take.

**Fernando Gouvea** (fqgouvea@colby.edu) Colby College

#### Was Cantor Surprised?

A well-known mathematical anecdote tells that Cantor was so surprised when he proved one of his theorems that he exclaimed "I see it, but I don't believe it!" We will look at the original sources to see what was really going on and what Cantor meant by his phrase.

**Amy Shell-Gellasch** (shellga@plu.edu) Pacific Lutheran University

### Pierre Fermat and the Development of Integration as a Summation

The standard teaching of the Integral Calculus introduces the integral as an area under a curve by the Riemann Integral. Georg Riemann used a sum of rectangles of width $\Delta x$ to approximate the area. Taking the limit as $\Delta x$ tends to 0 gives our modern definition of the integral. Prior to Riemann, several mathematicians were working on the problem of areas, know as quadrature, using similar techniques. In this talk I will outline the developments in quadrature using rectangles that predate Riemann. Special attention will be paid to the work of Pierre de Fermat in the 1650s. In particular I will explore his work on finding the area under the curve $x^{p/q}$ for integers $p$, $q$.

**Deborah Kent** (dkent@hillsdale.edu) Hillsdale College

### The MAA and American Mathematics in WWI

The United States' mobilization to enter World War I impacted the mathematical community in the United States in a wide variety of ways. This talk will investigate the Mathematical Association of America as a possible locus of organization for war-related mathematical activity in colleges and universities, as well as in military and industrial contexts.

# Contributed Paper Sessions

## The History of Mathematics and its Use in Teaching

### Thursday, July 31, 8:30 AM–10:30 AM
### Friday, August 1, 1:00 PM–3:00 PM
### Saturday, August 2, 8:30 AM–10:30 AM

**Doy O. Hollman** (doy.hollman@lipscomb.edu) Lipscomb University

#### Maya Mathematical Presentation

The intriguing history of the Maya will be discussed briefly; the Maya were the Greeks of the New World, the Romans of the Western Hemisphere, the Egyptians of Pre-Columbian America. Attention will be given to astronomical time-focused ideology and especially on their two counting systems, the common, which was purely vigesimal, and the chronological method, which also follows the vigesimal pattern, except in the third position. Also, there will be a PowerPoint slide show of some of the Maya ruins.

**Andrew deLong Martin** ( andrew.martin@kysu.edu ) Kentucky State University

#### Napier's "Logarithms" Weren't

Two popular History of Mathematics texts, *An Introduction to the History of Mathematics* by Howard Eves and *A History of Mathematics* by Victor Katz, disagree on what algebraic form represents the logarithm as defined by Napier. Why don't they agree? Which is correct? Are the representations equivalent? If not, how are they related? Does either exhibit the "logarithmic" properties of turning products into sums and quotients into differences?

After describing Napier's definition, this talk will address these and related questions, which the author has assigned students as a nonstandard way to review properties of logarithms.

**Matt David Lunsford** (mlunsfor@uu.edu ) Union University

#### The Calculation of Galois Groups By Classical Methods with the Aid of Mathematica

Galois' Memoir on the Conditions for Solvability of Equations by Radicals provides a systematic method for calculating the Galois group of a polynomial. Even so, the calculation of Galois groups is a difficult task. As Galois noted, "if you now give me an equation that you have chosen at your pleasure, and if you want to know if it is or is not solvable by radicals, I could do no more than to indicate to you the means of answering your question, without wanting to give myself or anyone else the task of doing it. In a word, the calculations are impractical." The calculation of the Galois group of a quartic polynomial with the aid of Mathematica will be given to demonstrate Galois' original method. The concepts of group, subgroup, conjugate subgroup, and normal subgroup arise naturally from this historical approach.

**Eugene Boman** (ecb5@psu.edu ) Penn State University-Harrisburg Campus

#### Ghosts of Departed Errors: Berkeley's Mathematical Objections to the Calculus

I will present and examine Berkeley's mathematical objections to the foundations of Calculus.

In 1734 Bishop Berkeley criticized the new Calculus of Leibniz and, especially, Newton in a publication titled "The Analyst" where he displayed, very pointedly, the fuzziness of some of Newton's arguments and sarcastically referred to Leibniz's differentials as "the ghosts of departed quantities."

This much can be gleaned by reading the marginal "Historical Notes" of many modern calculus texts.

However the precise mathematical arguments Berkeley used to refute the foundations of Calculus are generally not well known. In particular, one of his claims is that Newton, rather than reasoning clearly, makes two mutually compensating errors in his development of "fluxions" (derivatives). I will examine this argument in particular.

**Jeff Johannes** (johannes@member.ams.org ) SUNY Geneseo

#### "Vedic" Decimal Fraction Patterns

While it is open to debate if the system of "Vedic mathematics" developed by Shri Bharati Krishna Tirthaji in the early twentieth century is an accurate representation of ancient Indian mathematics, there remains interesting components to study from both a historical and mathematical perspective. In this talk we will explore decimal fractions patterns discussed in this systems and connections to classes ranging from elementary and secondary mathematics to number theory and algebra.

**Mohammad Moazzam** (mxmoazzam@salisbury.edu ) Salisbury University

### Mathematician or Poet

"Although I have never made the jewel of your obedience, And I had never swept the dust of the sin from my face; But I have not still lost the benefit of your mercy, Because I have never said that one is two"

Many undergraduate students know about Khayyam's Rubaiat from that often quoted phrase: A loaf of bread, a jug of wine, and thou." Although they know about Khayyam's poetry, few realize that he was quite a mathematician. This Persian mathematician's contributions to mathematics (1048-1131 A.D.) were significant. His work in algebra and geometry contains much mathematical content to enhance mathematics courses for undergraduate students. In this talk some of Khayyam's mathematics will be examined and suggestions for inclusion into college courses will be offered.

**Dave Lawrence Renfro** (dave.renfro@act.org) ACT, Inc.

### Who Discovered Discontinuous Derivatives?

The literature on the history of mathematics contains much about the history of continuous functions that are nowhere differentiable, infinitely differentiable functions whose Taylor series do not converge to the original functions, Cantor sets and other fractal sets, and many other counterexamples presented in undergraduate and beginning graduate analysis courses. However, there seems to be almost no information about discontinuous derivatives in this literature, despite the fact that such examples are sometimes even discussed in high school and college calculus courses. In this talk, I will begin by discussing my search for the earliest mathematicians to consider the idea that a derivative can fail to be continuous and the earliest mathematicians to come up with the example $(x^2)(sin 1/x)$. Following this, I will give a brief survey of the role that discontinuous derivatives have played in the development of analysis.

**Mehdi Radjabalipour** (radjab45@mail.uk.ac.ir) University of Kerman

### Conjectures on Egyptian Fractions

We review almost all the existing conjectures on the use of ancient Egyptian fractions and conclude that none of them justifies the taboo forbidding a simple representation such as $1/29 + 1/29$ for the fraction $2/29$ in the favor of a complicated one like the Egyptian fraction $1/24 + 1/58 + 1/174 + 1/232$. By a re-examination of the Ahmos Papyrus, we conjecture that Ahmos, just like Archimedes treating the infinitesimals, would treat the fractions in the same way we do today, but for the respect of the rituals established in the minds of Egyptians for centuries, had to translate them in the formalities of Egyptian fractions. We will give details of our reasons for such a conjecture.

**Joyati Debnath** (jdebnath@winona.edu ) Winona State University

### Treasures in Three

We count in ten's (decimal system), computer counts in two's (binary system) but who counts in three's (ternary system)? This presentation will focus on the history of counting in three's. It has been found that the notion of counting in threes was already implicit in Hindu Vedas (1500 BC) making the idea to be very old. Thomas Fowler in 1840 constructed a wooden calculating ternary machine to ease his calculations as a treasurer. In the numbering system, it appears that when the base 10 is too big and base 2 is too small, base 3 is just right. There are multiple uses of base-three systems. It is used in counting Islam prayers, to denote fractional parts of an inning in baseball, and to understand the self-similar mathematical structures like Sierpinski Triangle or a Cantor set. History of ternary system is both fascinating and intriguing. It makes students to appreciate the number system and enhances mathematical thinking.

**Amy Shell-Gellasch** (shellgae@plu.edu ) Pacific Lutheran University

### Fermat's Shrinking Rectangles

Integration is traditionally taught via the Reimann integral. Prior to Reimann, Pierre de Fermat, as well as several other mathematicians, found the area under a curve summing the areas of rectangles. However, Fermat's method was not restricted to rectangles of constant width, but used widths that decreased in geometric proportion. In this talk I will give a brief overview of Fermat's work on finding the area under a curve and how it can be used in the calculus classroom.

**Ciarán Mac an Bhaird** (canbard@maths.nuim.ie ) National University of Ireland-Maynooth

### Positive Impacts from Using the History of Mathematics When Teaching Students with Weak Mathematical Backgrounds

The newly opened Mathematics Support Centre (MSC) in the National University of Ireland, Maynooth (NUIM) is the busiest in the UK or Ireland.

The MSC has been overwhelmed by the number of students who struggle to master many concepts that they were first introduced to at school, these include, for example, zero, infinity, limits and quadratic equations. We have discovered that using the history and development of these topics to explain the context and background to the students has really had a big impact on their understanding. The students are encouraged by the fact that they can see where these topics originated, and that they developed into their current form over time. The students have responded very well to this initiative, however, it can take a considerable amount of time to go through the details involved in these topics.

Although there is a wealth of material available on the history of mathematics, the sheer volume of information can be overwhelming for weaker students. In conjunction with SIGMA (The Centre for Excellence in Mathematics and Statistics Support, based in Coventry and Loughborough Universities), we are developing a series of paper based and online resources.

These resources are a series of double sided A4 pages which contain the core information on a particular topic, and also a concise history of the material, with some common problems and worked examples. Similar leaflets in other areas of mathematics are very popular with students. I will go through the resources we have developed to date.

**Sarah Mabrouk** (smabrouk@frc.mass.edu ) Framingham State College

### My Mathematician is Better than Your Mathematician

Since fall 2002, I have included a history of mathematics assignment in each of my courses. Initially, students researched the development of various concepts in mathematics and how these concepts are used in various areas of mathematics as well as in other disciplines. However, learning about the development of concepts and methods as well as getting a glimpse into the lives of those who contributed to the development of these concepts and methods can enhance one's understanding of and appreciation for course material. Thus, I changed the assignment to focus on mathematicians who contributed to the development of the concepts and methods that students study in various courses, for example, Sir Ronald Fisher, Karl Pearson, and, Florence Nightingale for *Introduction to Statistics*, Friedrich Wilhelm Bessel, Jacques Charles Francois Sturm, and Olga Alexandrovna Ladyzhenskaya for *Differential Equations*, and Gottfried Wilhelm Leibniz, Benoit Mandelbrot, and Maria Gaetana Agnesi for *Calculus III*. The assignment, which initially included an in-class presentation as well as the creation of a web page or writing a paper, has evolved to include an online presentation and an extended online discuss in which students compare, contrast, and brag about their mathematicians in comparison to mathematicians selected by other students in the class. In this presentation, I will discuss how I have used this assignment in various courses as well as share some of the students' insights into the lives, contributions, and accomplishments of the mathematicians that they studied.

**Pragati Jain** (pragatijain2@gmail.com ) SIMS, Indore

### The Role of History of Mathematics

History of Mathematics enables us to know how the Mathematical ideas were conceived and how Mathematics helped the development of human society. The best way of learning Mathematics is by getting the knowledge; the process of development of mathematical fundamentals. A person can never be frustrated if he knows the steps of determining the mathematical results. Students should know about the struggles, failures and success in arriving at the final results. There is no wonder that students begin to feel that Mathematics is made by some superman or God and the role of man is only to explain. In fact, with this type of experience, a feeling grows that there is no manufacturing or production in Mathematics, but only useful products which continue to be sold, generation after generation, by teachers to students.

**Robert Myers** (robert.myers@bethelcollege.edu ) Bethel College

### Motivating Infinite Series Through Modified Leap Years

Among the most common examples used to introduce calculus students to infinite series are the paradoxes posed by the Greek philosopher Zeno of Elea. While these paradoxes help students grapple with the relationship between the finite and the infinite, they often lack motivation or relevance.

In this talk, we will give a different historical motivation for infinite series that is familiar to all calculus students: the leap year. (The year 2008 is a great example of one!) After some background about the development of the Julian calendar, we will look at the modifications introduced by mathematicians in 1582 under Pope Gregory XIII that led to better approximations of the year. We will see how atomic clocks further confused the issue, and how alternating series came to the rescue.

**Janet L. Beery** (janet_beery@redlands.edu ) University of Redlands

### Thomas Harriot's Pythagorean Triples: Could He List Them All?

English mathematician and scientist Thomas Harriot (1560-1621) gave the usual formula for Pythagorean triples using his new algebraic notation but he also started to list them in a systematic way. If he could have continued his list indefinitely, would he have listed all of the Pythagorean triples? In exploring this question, students can recognize and describe patterns, write and use algebraic formulas, and construct proofs, including proofs by mathematical induction. This material could be used in an undergraduate number theory course, in a "proofs" or "transition" course, or as enrichment for bright algebra or general education students. At least part of it could be used in college algebra or other general education courses.

**Tom McMillan, Jim Fulmer** ( tcmcmillan@ualr.edu, jrfulmer@ualr.edu) University of Arkansas at Little Rock

### Connecting Pascal's Triangle to Geometry and Trigonometry

Pascal's Triangle was known by the Chinese some 400 years before Pascal lived. The triangle appears in many different contexts at nearly all levels of mathematical endeavor. This session will present a replica of the Chinese Pascal's triangle and will explore connections to results in combinatorics, geometry and trigonometry. Emphasis will be on counting geometric objects under different constraints and linking the triangle to trigonometric identities. We will also explore the link between Pascal's Triangle and the Catalan numbers and see a representation of the Sierpinski triangle.

**Shawnee McMurran** (smcmurra@csusb.edu ) California State University San Bernardino

### It's Just Thin Air: Resistance and Projectile Motion

When introducing the dynamics of projectile motion in a first year calculus course it is common to follow Galileo's example of making the simplifying assumption of negligible air resistance. This makes sense as it creates a problem that novice modelers are able to solve analytically. At the same time, we do want to impress on students the dramatic effect that air resistance has on projectiles in the real world. One way to do this is via a discussion of how the invention of the ballistic pendulum in the early 18th century influenced understanding of the true effect of air resistance on projectiles and how Leonhard Euler applied this knowledge to improve Galileo's model.

**Anne E Edlin** (edlin@lasalle.edu ) La Salle University

### Edlin's Enigma

Assign your students more work, enhance their understanding of mathematical proof and improve your student evaluations all at the same time. This talk will show you an easy way to use the History of Mathematics in a Math Major transition course to do all of this and more, with a little help from Simon Singh.

**Alexander G. Atwood** (atwooda@sunysuffolk.edu ) Suffolk County Community College

### Brain Injury and Mathematical Discovery: the Enigma of Stanislaw Ulam

In 1946, the mathematician Stanislaw Ulam was struck down by viral encephalitis. After emergency brain surgery, he underwent a recovery in which he regained many of his non-mathematical skills. However, his mathematical powers were substantially transformed. Although he was unable to concentrate on one subject for more than a few minutes and had significant difficulty in performing simple mathematical operations such as solving quadratic equations, his creative mathematical powers were substantially enlarged. In the next ten years, Ulam, in conjunction with von Neumann, Teller, Fermi and others, would create some of the outstanding achievements of applied mathematics in the twentieth century. These include the creation of the Monte Carlo Method, the formulation of the breakthrough principle which led to Thermonuclear Fusion Weapons, and seminal discoveries of non-linear dynamics in the Fermi-Pasta-Ulam problem. How was Ulam able to create these monumental mathematical achievements in the face of crippling mathematical limitations? How might have Ulam's brain compensated for his technical weaknesses? How did his ability to work with other mathematicians enable him to overcome his inability to concentrate? What can we learn from Ulam about the way in which the brain discovers and creates mathematics?

**Peter Schumer** (schumer@middlebury.edu ) Middlebury College

### What's Interesting About the Number 1729?

In late 1918, S. Ramanujan impressed G.H. Hardy by mentioning offhand that 1729 is the smallest integer expressible as the sum of two cubes in two different ways. What is the situation for squares? How about fourth powers? Is 1729 really the smallest such sum? Is 1729 interesting in other ways as well? Such questions can lead to fruitful generalizations and much interesting mathematics.

**Charlie Smith** (charlie.smith@park.edu) Park University

### Obsession from the Greeks: Doubling Cubes, Squaring Circles, Trisecting Angles, and Constructing Regular Polygons

These ancient Greek problems contain a richness of content which is both vast and beautiful. They require the combined forces of several branches of our discipline to research and resolve. The study of these problems involves an extensive array of topics and subtopics. In the classroom, the material can be used to supplement and enrich various courses, including geometry, abstract algebra, number theory, calculus, analysis, and of course history of mathematics.

# Teaching Mathematics and Statistics Through Current Civic Issues

## Thursday, July 31, 9:30 AM–10:20 AM
## Thursday, July 31, 1:00 PM–3:00 PM

**Adam Molnar** (amolnar@bellarmine.edu) Bellarmine University

### The Homeless Average Age Nine? Examining a Bad Statistic

In December 2007, a commencement speaker cited a figure that drew gasps from the audience: “The average age of a homeless person is nine.” Surprised, I looked into the claim, which turned out to be untrue. That said, this seemed like a great class example for an Introductory Statistics course; ask the students to research the number, discuss their discovered answer, then examine how the error was made and propagated. In the first attempt, some parts of this lesson succeeded, while others, well, did not. This talk will discuss the problem and lessons learned, as an example of how to better integrate real life claims into first courses in statistics.

**Michael Catalano** (micatala@dwu.edu) Dakota Wesleyan University
**Thomas Pfaff** (tpfaff@ithaca.edu) Ithaca College
**Tanya Leise** (tleise@amherst.edu) Amherst College

### The Gini Coefficient: Measuring Inequality in Resource Allocation

Inequality in resource allocation, whether the resource is income, wealth, land, energy, etc., is a common topic in social discourse. In this paper, we discuss the Gini Coefficient, introduced in 1921 by Italian statistician Corrado Gini as a measure of inequality, and how it can be used as an example in calculus courses. We focus on applying the Gini Coefficient to data related to energy resources. The mathematics behind the Gini Coefficient is accessible to students who have studied differentiation and integration as the representation of area between two curves. This paper stems from work done by authors Michael Catalano, Tanya Leise, and Thomas Pfaff, at the *Mathematics for Social Justice Workshop* held in June of 2007 at Middlebury College.

**Dianna Spence, Robb Sinn** (djspence@ngcsu.edu, rsinn@ngcsu.edu) North Georgia College & State University

### Authentic Discovery Learning Projects in Statistics with Constructs from Environmental and Social Science Disciplines

This presentation will describe discovery learning projects for students in introductory undergraduate statistics courses for non-majors. The projects and related teaching methods are the cornerstone of work supported by a National Science Foundation grant, “Authentic, Career-Specific Discovery Learning Projects in Introductory Statistics”. Authentic research constructs were developed with input from an interdisciplinary team of faculty, including professors representing biology, ecology, psychology, sociology, and criminal justice. These constructs form the basis for projects in which students explore statistical relationships between constructs of their choosing. These projects focus on linear regression analyses and on various types of simple t-tests. Preliminary data suggest that students involved in these projects end the course with higher content knowledge and better attitudes about statistics than do students in sections where these projects are not used. Research constructs and sample projects will be shared.

**Michael Catalano** (micatala@dwu.edu) Dakota Wesleyan University

### College Algebra in Context: A Report and Examples from an NSF Project

A progress report on a National Science Foundation’s CCLI-EMD project (DUE #0442979) supporting the development and implementation of a learner-centered, inquiry-intensive, data-driven, activity-oriented college algebra course, incorporating realistic problem situations emphasizing social and economic issues, including hunger and poverty, energy, and the environment. The project, now in its final year, seeks to address two national needs, namely a need for U.S. citizens with a greater level of quantitative literacy, and a need for improved mathematics education for K-12 teachers. Pilot sections of the course have been taught at Dakota Wesleyan University using tablet laptop computers and Fathom Dynamic Data software, supplemented with the use of graphing calculators. Materials will be available for those interested in offering pilot sections in the fall of 2008.

**Milo Schield** (milo@pro-ns.net) W. M. Keck Statistical Literacy Project

### Confounder Influence on Cases Attributed

Civic issues increasingly involve cases that are attributed to an associated factor. These issues include second-hand smoke, radon cancers and deaths due to obesity. The percentage and number of cases attributed to an associated factor are readily calculated. But since this data is typically acquired in observational studies, the underlying rates or percentages are easily influenced by confounders. This paper uses a new graphical technique to analyze the influence of a binary confounder on an association between a binary predictor and a binary outcome to analyze the influence on the percentage and number of cases attributed to the predictor. To be statistically literate, students need to see that these cases attributed to an associated factor need not be caused by that factor and may be attributed to other related factors.

**Lisa Marano** (lmarano@wcupa.edu) West Chester University of PA

### Developing "Mathematics and Social Justice": A Course Which Incorporates Social Justice Issues and Service-Learning Projects

For the past two summers, I participated in a workshop where participants developed methods to incorporate social justice and service-learning projects into the mathematics curriculum. At these workshops, mathematicians from across the country met to brainstorm on what a course dealing with mathematics and social justice may look like. After two years of hard work, I will see my version of this course come alive when I teach it for the first time this fall. In this talk, I will discuss the topics which will be covered in my "Mathematics and Social Justice" course and the service-learning projects which I have planned. In addition, I will discuss methods for dealing with student and potential colleague resistance to the course.

**John Zobitz** (zobitz@augsburg.edu) Augsburg College

### Integrating Service Learning Projects in a Differential Equations Course

Service learning can be a powerful curricular tool to accomplish learning outcomes by actively engaging students with course content. A service learning project was integrated with a semester-long modeling and differential equations course. Students partnered with a community organization to investigate the environmental feasibility of an urban garbage incinerator in Minneapolis and Saint Paul, Minnesota. Students utilized input-output mixing models as well as systems of coupled differential equations. Students were responsible for formulating an appropriate model, determining reasonable values for model parameters, using numerical techniques to generate a solution, and interpreting model outputs. Project results indicated that the garbage incinerator could be environmentally feasible provided incinerator emissions are offset with a 600 square kilometer forest (twice the size of Minneapolis and Saint Paul). This presentation describes a) design of the course, b) methods to structure course content effectively around a service learning project, and c) results of students' projects.

**Bradford Bynum** (bbynum@westminstercollege.edu) Westminster College

### Global Consciousness, Social Responsibility, Ethical Awareness (and MATH!) at Westminster

To meet program goals for students to learn to "effectively communicate mathematical ideas" and to develop "curricular and co-curricular opportunities for student leadership", the mathematics department of Westminster College developed a tutoring program at East High School, a neighboring school with a diverse student body and many challenges. The tutoring program, nicknamed POD-Math (Portable-On-Demand Mathematics) offers an organized, pre-planned service-learning activity that professors can easily incorporate into their courses. As an effective way to connect course and program learning goals with needs in the community survey results suggest our calculus students almost universally consider tutoring a positive and useful experience while data linking tutoring participation with success in learning calculus is inconclusive. On the other side of the equation, the community partner identifies the POD-Math tutoring program as the most successful of a number of initiatives they have participated in. This paper shares suggestions for gaining program and institutional support, ideas for unleashing positive student energy in organizing such a program, techniques for assessing the impact of service learning, and strategies for communicating results. The evidence after one year of implementation indicates this model tutoring program is valuable to both the college and high school participants, energizing to both mathematics departments, and is not only successful, and responsible, and sustainable, but also portable.

**Lawrence M. Lesser** (lesser@utep.edu) The University of Texas at El Paso

### Equity: Not Just a Goal, But a Vehicle in Introductory Statistics

Social justice, equity, and service learning are powerful vehicles for motivating or supporting statistics education for non-majors. Research suggests that all instructors (regardless of one's personal or political beliefs) need to be aware of how students' equity beliefs can affect how they process certain ideas in statistics (e.g., bias, random selection, random assignment). With support from Project ACE (funded by the US Department of Education; PI: J. Tinajero), I applied the groundwork of my 2004 and 2007 Journal of Statistics Education papers by redesigning an introductory statistics course I taught in fall 2007 to 52 pre-service elementary and middle schoolteachers (mostly Latinas) at a mid-sized research university on the US-Mexico border. We'll discuss quantitative and qualitative evidence of impact on students' knowledge and engagement, such as their significant (p ¡ 0.001) gain in how knowledgeable they rated themselves in being able to use concepts and tools from statistics to explore equity concepts. We'll also share examples (and associated implementation strategies) of connecting equity themes with standard course topics. The course used a statistical literacy approach (Utts 2005). Beyond including more theme-specific examples in class discussion, a major addition to the course was a required project paper with 3 options: standards-based lesson plan involving data, an original data collection and analysis, or a service learning experience. While it was more time-consuming than studying for a final exam, all 21 students who mentioned the project in end-of-course evaluations viewed it favorably and for some it was deeply affirming or transformative.

**Matthew Tom** (tomma@emmanuel.edu) Emmanuel College

### Statistics Through Current Events in an Online Environment

In response to the expanding demand for introductory statistics, Emmanuel College is now offering an online version of its statistics course for non-mathematics majors. The online, asynchronous environment provides unique opportunities to present examples and applications found in current events and civic issues. Lecture notes and other learning materials include links to recent news articles and research papers. In this election year, we are using the latest commentary on demographics, campaigning and polling results. Instead of being constrained by 50- or 75-minute lecture periods, students continue discussions of issues on message boards and threads as long as they desire. In this talk, we will show different ways of introducing and presenting current events and headlines. We will look at different methods of assessing student participation in discussions. This talk will include access to Emmanuel College's online course in progress.

# Advances in Recreational Mathematics

## Thursday, July 31, 1:00 PM–3:00 PM
## Friday, August 1, 4:15 PM–5:15 PM

**Michael A. Jones, John Stevens** (jonesm@mail.montclair.edu, stevensj@mail.montclair.edu) Montclair State University

**Redistributing and Reconstructing Probabilities in Horse Races, Voting Theory, and Poker**

Can knowing the likelihood that each horse will win a race be used to determine the likelihood of the order of finish? In general, this is not true, despite an implicit assumption in a molecular dynamics textbook. We review the origins of the implicit assumptions about conditional probability and how these assumptions explain the relationship between first place finishes and ordered finishes. We also investigate this relationship from a voting theory perspective by examining what happens when a candidate drops out of an election. Finally, we develop a test to determine when the assumptions hold and demonstrate its use with data from the 2006 World Series of Poker final table.

**Charles Ashbacher** (cashbacher@yahoo.com) Mount Mercy College

**Problems From the Pages of the Journal of Recreational Mathematics that are Still Unsolved**

The Journal of Recreational Mathematics (JRM) has been published for over three decades and a problem column has always been an integral part of the journal. While the vast majority of these problems have been solved and the solutions published in JRM, some remain unsolved. As co-editor of JRM in charge of content, I have begun a program to catalog these problems, publicize them and solicit solutions. In the past, solvers of problems have been noted in a subsequent problem column. The new program will be to consider each solution of a problem that has remained unsolved for 5 or more years as a submitted paper. This program as well as some of the more intractable problems that remain unsolved will also be presented.

**Andrew James Simoson** (ajsimoso@king.edu) King College

**Falling Through the Earth in the Mirrour of the World**

In England's first illustrated, printed book, Caxton's 1481 *Mirrour of the World*, a short encyclopedia of what every man should know, is an account of the classic recreational mathematics problem of falling through the earth. Within the context of Caxton's discussion is a woodcut of a curve through the earth that closely resembles a hypocycloid, which is in fact the resistance-free, falling path of an object when dropped at the equator of a rotating earth, as we show *without* using the calculus of variations.

**Helen Schroeder**
(schroederh@uwstout.edu) University of Wisconsin - Stout

**Stuck in a Sudoku**

Nikoli Sudoku puzzles are hand-crafted works of art. They are symmetric in design and in my experience almost always have a single, unique solution. Computer-generated puzzles, on the other hand, can present their own beauty. The art to solving these may not be to find **the** solution, but to narrow down to **one** of many possible solutions.

Multiple solutions to these Sudoku puzzles arise when, given any one particular solution, subsets of numbers can be found which may be permuted to yield new solutions.

A particularly interesting example will be presented whose permutable subsets range from the most basic and straight-forward to the rather complicated. This puzzle has more than 24 distinct solutions and was found in the very first book of Sudoku puzzles I ever attempted.

**Doug Chatham** (d.chatham@moreheadstate.edu) Morehead State University

**$N + k$ Queens Reflections**

The classic $N$ Queens Problem asks for an arrangement of N queens on an $N$-by-$N$ chessboard such that no two queens attack each other. It is easy to see that reflecting an $N$ Queens solution vertically, horizontally, or diagonally always produces a distinct solution (if $N > 1$).

The $N + k$ Queens Problem asks for an arrangement of $N + k$ queens and $k$ pawns on an $N$-by-$N$ chessboard where no two queens attack each other. We show that reflecting an $N + k$ Queens solution vertically, horizontally or diagonally always produces a distinct solution (if $N > 1$). We finish the talk with some data on the rarity of rotational symmetry of an $N + k$ Queens solution.

**Dylan William Helliwell** (helliwed@seattleu.edu) University of Wisconsin - Stout

#### Voting Off: How Bad Can it Be?

After briefly discussing common voting methods and fairness criteria, a particular method, namely "voting off" (where people vote to eliminate candidates until one candidate remains) will be introduced and a single voter profile will illustrate just how bad this method can be.

This example will be discussed in the context of the "Mathematical Reasoning and its Applications" course satisfying the core math requirement for liberal arts majors at Seattle University.

**Matthew Menzel** (mmm002@marietta.edu) Marietta College

#### Combinatorics, Probability, and the NCAA Basketball Tournaments

Single-elimination tournaments, such as the NCAA Men's and Women's Basketball Tournaments, provide many interesting questions in combinatorics and probability. Problems range in difficulty from simply counting the number of games that will be played in a single-elimination tournament to determining the expected value for the number of games picked correctly by a person filling out an entire bracket. We will address some basic questions, and we will advise you how to respond when somebody tells you that their co-worker's nephew's sister correctly picked every game in last year's tournament. We will look at some counting techniques, some ways to address relevant probability questions, and a couple of nice combinatorial proofs.

**Hossein Behforooz** (hbehforooz@utica.edu) Utica College

#### Mathematical Thinking with Magic Squares

No doubt that recreational mathematics always appreciates the study of magic squares. Different types of my own magic squares will be presented together with some magic squares with U.S. election years. Can we predict the results of the coming U.S. election years? Come and join us and enjoy with my interesting magic squares. Remember, math is fun and that is why we have recreational mathematics.

**Jay Lawrence Schiffman** (schiffman@rowan.edu) Rowan University

#### First Occurrence Counterexamples in Odd Abundant Number Sequences

A positive integer $n$ is designated abundant if $\sigma(n) > 2 * n$ where $\sigma(n)$ denotes the sum of the divisors of n. To cite an example, 12 is the initial abundant number; for $\sigma(12) = 28 > 2 * 12 = 24$. In this paper, a discussion of the more interesting set of odd abundant numbers will ensue. Numerous conjectures that seemingly can be formed related to this class turn out to be false although the first counterexample often lies far out in the sequence. Even the initial odd abundant number does not appear until one reaches 945. Some misconceptions associated with odd abundant numbers include the following: 1. All odd abundant numbers terminate in the digit 5. 2. All such integers are divisible by 3. 3. There are no members of the Fibonacci sequence in this class. We will prove that there indeed exist infinitely many odd abundant Fibonacci numbers, show how to construct odd abundant numbers that are not divisible by any one of the first $n$ primes and determine a sharp lower bound for the number of divisors possessed by odd abundant numbers. Please join us to witness some dynamic excursions in recreational mathematics.

## Integrating Biology and Mathematics

### Thursday, July 31, 1:00 PM–3:00 PM

**Ben Fusaro** (fusaro@math.fsu.edu) Florida State University

#### Biofuels, Useful Arithmetic and Embodied Energy

The move to replace fossil fuels by ethanol produced from grain-based biofuels promises to have a short half-life, and its backers (and many environmentalists) suggest that replacing grain by cellulosic will do the job. A combination of a few examples, arithmetic, some qualitative reasoning and the concept of *embodied energy* is sufficient to allow a judgment on whether this notion is sound.

**Atabong Timothy Agendia, Awunglefack Dominic Fobellah** (agendia@yahoo.com, tibasrinfojou@yahoo.co.au)

#### Biology, Discrete Mathematics, and Theoretical Computer Science

Students most often neglect future specialization in areas like mathematics because they do not see an immediate application of this course of study in real world. As they advance into their fields of interest, they start seeing the critical role that mathematics plays. Biological science is one area where mathematics is playing an active part in the explanation of real-life issues. To try and increase the interest of computer science students in Discrete Mathematics, we initiated a special course, “MTH326: Biology, Discrete Mathematics, and Theoretical Computer Science” in the computer science curriculum of Madonna University, during the second semester of 2006/2007 academic year. In this course, the epidemiology of infectious diseases, the migration of insects, the division of cells and others were explained using discrete equations. The equations were solved by simple straight forward methods and using simple numerical algorithms which the students were aware of. From the structure, methods, dissemination and results of the course, we found among other things that, instituting such a course into the computer science, mathematics and biological sciences programs, not only increases the interest in mathematics of biology students and vice versa, but also help them to understand biomedical concepts better. This way, the mode of transmission and consequently eradication of existing biomedical phenomena such as Bioterrorism, HIV/AIDS, Malaria, and others were easily understood. This paper presents: 1) the course objective and contents; 2) the methodology in ensuring topic flow; 3) some examples used; 4) the level of coverage desired; and 5) the lessons learned concerning the course.

**William Schellhorn** (william.schellhorn@simpson.edu) Simpson College

#### Using Knot Theory to Model DNA: An Undergraduate Research Project

Deoxyribonucleic acid (DNA) is a molecule that contains the genetic information necessary for living organisms to develop and function. The double helix structure of DNA allows it to be studied topologically using knot theory. In this talk I will discuss my experience of advising a two-semester undergraduate research project in which the student investigated the use of a tangle calculus to model the actions of enzymes on DNA. I will introduce some of the topics she studied and offer perspectives about advising her project.

**James Fulton, Sabatino Linda** (fultonj@sunysuffolk.edu, sabatil@sunysuffolk.edu) Suffolk County Community College

#### Teaching Mathematics to Biology Majors Using the Scientific Method

A major challenge exists for many college biology departments. How can their majors take all the necessary biology courses, as well as complete the mathematical requirements for their degree in a timely manner? This is especially challenging at community colleges, when students entering the biology program often lack a precalculus level of mathematics training. As part of an NSF grant, we looked at ways to make mathematics more relevant and accessible to biology majors at the precalculus level. However, to be even more effective at reaching incoming biology students, we realized that we needed to begin at the Algebra II level. As we did in our NSF Grant, we are developing a Scientific Method Approach to teaching mathematics, and are closely tying it to concepts studied in their introductory biology courses, The intent is to take students at the beginning algebra level and enable them to learn the necessary mathematics, without having to delay their progress in their major. The approach is to tie the mathematics to their biology lab course, via the scientific method, and give them all the prerequisite mathematical skills when they need them, but at the same time give them a coherent introduction/presentation of the mathematics as well. The general philosophy on how to do this, as well as some specific examples, will be presented and discussed.

**Lee Stemkoski** (stemkoski@adelphi.edu) Adelphi University

#### Analyzing the Interaction of Species: Game Theory in a Calculus Course

Some animals hunt together in packs, while others resort to stealing. This conflict of behaving in a self-centered versus an altruistic manner is captured by the “Prisoners’ Dilemma” model in game theory. The dilemma may be partially resolved by extending the model to allow repeated interactions (multiple rounds of the prisoners’ dilemma). In such a scenario, complicated strategies may arise. We will discuss how techniques from both differential and integral calculus reveal optimal strategies, and how these ideas have been successfully implemented in a course module in calculus.

## Actual Problems, Actual Mathematics—Applied Mathematics in Science and the Classroom

### Thursday, July 31, 3:15 PM–6:15 PM
### Saturday, August 2, 8:30 AM–10:30 AM

**Mostafa Ghandehari, Siamak Ardekani** (ghandeha@uta.edu, ardekani@uta.edu) University of Texas at Arlington

#### Euler’s Constant in Interstate Highway Driving

Interstate highway design standards in the U.S. require a milepost marker each mile. By convention, milepost numbers in each state increase from south to north and from west to east. Suppose a driver traveling south at milepost 80 decides to travel at the same speed as the milepost number, namely 80 mph at milepost 80, 79 mph at milepost 79, etc. How long does it take for the driver to reach the southern border of the state? In general, if a driver adjusts his speed between mileposts $X$ and $Y$ according to the above rule, how long would it take to travel the distance $X - Y$? We will show that the use of harmonic series and Euler’s constant occur in calculation of the travel time.

**Scott Searcy** (searcys@waldorf.edu ) Waldorf College
**Paul Bartelt** (kimberly.swetz@usafa.edu) Waldorf College

### Using Simulation to Verify Biological Model Fit with Field Data

Limited filed data in Biology makes it difficult to verify model fit to actual field data. This paper investigates how simulation is used to generate evidence that a Geographical Information System (GIS) environmental model of amphibian habitats is a reasonable predictor of actual observed field data.

**Angela Vierling-Claassen** (avierlin@lesley.edu) Lesley University

### Gendered Division of Labor in Parenting: A Game-Theoretic Perspective

In this talk, we will analyze conflict situations faced by parents of young children through the lens of game theory. We will explore several different two-player games and analyze goals, strategies, and outcomes using tools easily accessible to undergraduate students. This analysis suggests some strategies that might help parents negotiate conflict for mutually beneficial outcomes. We will also consider the case of parenting in the animal kingdom and look at evolutionary stable strategies utilized by different animal species in caring for offspring.

**Alfred Paul Lehnen** (alehnen@matcmadison.edu) Madison Area Technical College

### The Brachistochrone Revisited: A Timely Consideration

One of the origins of the Calculus of Variations is the brachistochrone problem posed in 1696 by Johann Bernoulli. The problem is to determine for a frictionless bead accelerated from rest the path that would minimize the time of descent between two specified points. The well-known solution is an inverted cycloid. The fundamental independent variable of the problem is r, the ratio of the horizontal to vertical displacement between the two points. Properties of the cycloid solution as r varies, including its uniqueness, are elaborated. In particular, if r exceeds pi/2, then the least time path attains an absolute minimum below the terminal point of the trajectory. Explicit asymptotic expansions for the time of descent for both large and small r are developed. Relatively simple and computationally efficient formulas that allow for an accurate graph of the minimizing cycloid for any value of r are derived. In addition, variational calculations that minimize the time of descent for trial function trajectories are presented. The first trajectories considered were piecewise linear segments. Their ability to approximate the solution of the brachistochrone is summarized. Finally, a rather thorough analysis of the least time parabola is given. Despite the fact that a parabolic trajectory between two points has only one free parameter, it is able to come very close in matching the time of descent of the minimizing cycloid.

More details are available at faculty.matcmadison.edu/alehnen/brach/brach.pdf

**Chulin Likasiri** (julin.likasiri@gmail.com) Chiang Mai University

### Problem Formulation for Sustainability

Food insecurity is a state of food scarcity happening in certain regions of the world caused by poverty and the imbalance between food production and demand. Insufficient arable land and poor distribution of water resource, poor policy making as well as weak resource managements are the main reasons for inadequate food production. A region's rapid population growth and urbanization raises the consumption of food, water and energy, causing environmental hazards and, as a result, health risks in that region. Global environmental problems from these are still increasing and intractable.

Science as well as mathematics, technology and good policy constructions should become keys to improvement for those seeking help. With this in mind, a course entitled "Problem Formulation" has been constructed for the newly developed Integrated Science and Mathematics program at Chiang Mai University. Resources such as food, water and energy as well as wastes are the main focus of this course. The course objectives are to enable students to construct models and solve problems mathematically.

**Doug Shaw** (shaw@math.uni.edu) University of Northern Iowa

### What Else Can You Do with an Open-topped Box?

Starting with the classic Open Box optimization problem, we present extensions of this problem that can be used in high school mathematics classes. Some extensions require calculus, and some don't. Surprises and beauty abound! We also challenge high school teachers to use this process of problem analysis in their own practice as a way to enrich the content of their lessons and as a means of individualized professional development.

**Benjamin Galluzzo** (bgalluzz@math.uiowa.edu) University of Iowa

### A Modular Approach to Teaching a Course in Application-Based Problem Solving

"Introduction to Applied Mathematics Research," first offered at The University of Iowa during the 2007 fall semester, explores how mathematics is currently being used to interpret and solve real-world problems. The one-semester course is composed of five independent modules, each focused on the development, implementation, critique, and analysis of a model relating to a particular area of current research interest. For example, previous topics covered in one semester were: collisions, subsurface fluid flow, traffic flow, epidemiology, and financial option pricing. Despite substantial differences in subject material, each module is approached using the same methodology: several introductory lectures, followed by computer simulation and experimentation, classroom discussion, and ultimately group projects and presentations. In this talk we will discuss the structure of the modular framework and its pedagogical effectiveness as determined by anonymous surveys, discussions with students, and our own personal observations.

**Michael Quentin Rieck** (mrieck@drake.edu) Drake University

### Quartic Equations Applied to 6-DOF Wiimote Tracking

Recent efforts to track the location and orientation of a Wii remote controller lead to a system of quadratic equations. Current implementations rely on numeric methods to approximate the solutions to this system. This results in difficulties with performance and stability. However, it is possible to reduce the problem to solving single-variable quartic equations, which can be handled using classical formulas. This approach is fast and stable. The geometric interpretations of the quartic polynomials, and their cubic resolvents and discriminants, are also explored.

**James S Rolf, Kimberly Swetz** (jim.rolf@usafa.edu, kimberly.swetz@usafa.edu) United States Air Force Academy

### Modeling the Buoyancy Properties of Virginia Class Submarines

We describe a group project used in an Integral Calculus course that asked the students to explore the buoyancy properties of a Virginia Class Submarine. Specific topics explored in the project include the concept of neutral buoyancy, the development of a differential equation model of the diving capabilities of the submarine, and the computation of terminal velocity encountered when diving. We will demonstrate technology that provided a quick method for numerically solving the differential equation model and enabled students to easily change parameters that influenced the submarine dive. Finally, we will discuss student and instructor reaction to the project.

**Michael Lundin** (lundin@cwu.edu) Central Washington University

### Energy Efficient Flight: Decision-Making Mathematics Glider Pilots Use To Soar Great Distances

Glider pilots fly hundreds of miles in airplanes without engines by efficiently using the sun as an energy source. In doing so, they frequently adjust airspeed to balance altitude with average ground speed, while planning to “land out” in case the energy dissipates. Highlighted here is the mathematics behind two types of decisions glider pilots commonly make: choosing optimal flying speeds and choosing optimal emergency landing sites. All activities suggested here have been tested at several levels of the college mathematics curriculum, and several activities were published in the April 2008 volume of Math Horizons.

**George Rublein** (gtrubl@math.wm.edu) College of William and Mary

### Elementary Physical Chemistry Problems in Multi-variable Calculus

We describe a collection of exercises that draw on standard models for chemical thermodynamics. These exercises, requiring a minimum acquaintance with the vocabulary of chemistry, are accessible by multi-variable calculus students. A (correct) account of the use of differential forms helps outsiders to decipher the somewhat arcane approach used by chemists to the mathematical structure of thermodynamics. The role of theideal gas as a basis for the subject will be emphasized.

**William Joseph Satzer** (wjsatzer@mmm.com) 3M Company

### Four Stories in Applied Mathematics

Real examples of applied mathematics in industrial applications are remarkably various, in application area, in mathematical content and in depth. I’ll present four stories of applications, two of them I believe to be absolutely unique.

**Chen-Han Sung** (drsung@tamiu.edu) Texas A&M International University

### The Math behind a Roll-Call Vote

What influential power a voter would gain or lose in a roll-call vote? Who is the most powerful voter in a roll-call vote? Which voter will determine the outcome in a close vote of a roll-call vote? Those and more will be addressed. Experiences with some past important voting records and our simulation results will be shared as actual problems and actual applications.

**Leon Kaganovskiy** (lkaganovskiy@ncf.edu) New College of Florida

### Predator-Prey Models—a Generalization of Lotka-Volterra Model

We consider Holling’s extension of classical Lottka-Volterra model for predator-prey systems. We discuss ecologically unrealistic assumptions of classical models and more realistic Holling’s model. The main feature is the applications of Matlab and Maple software packages to create programs which are easy for students to study and modify to investigate different cases and model extensions. Equilibrium conditions and limit cycles are discussed. The resulting models are applied to ecological pest control.

**Jane Beauchamp** (beauchampj@stu.easternct.edu) Eastern Connecticut State University

#### Wavelet Transforms: The Linear Algebra Approach

Wavelet technology encompasses the idea of taking large amounts of data at one end of a process, and being able to store that massive amount of information in a much smaller, more space efficient form at the other end, while at the same time retaining what is necessary to keep the transformed data accurate. This presentation provides an overview of how this process is realized in real world applications, and demonstrates how matrix algebra is used in the calculation of the transforms themselves. To demonstrate the accuracy of this process, an elementary example of a wavelet transformation using the matrix algebra approach will be presented. This will further emphasize how and why wavelet technology has proved to be one of the most influential mathematical developments of the twenty-first century.

## Incorporating Humanities and the Arts into the Mathematics Classroom (and Vice Versa)

### Thursday, July 31, 3:15 PM–5:15 PM
### Friday, August 1, 8:30 AM–10:30 AM
### Saturday, August 2, 8:30 AM–10:30 AM

**James Richard Hughes** (hughesjr@etown.edu) Elizabethtown College

#### Mathematics in Music: An Interdisciplinary Honors Course

The use of mathematical models and methods to enhance the creation and analysis of music is a well-established and growing interdisciplinary area, increasingly recognized as part of the academic mainstream by music theory researchers and composers via professional organizations, conferences, and scholarly journals. It is therefore in the interest of all undergraduates with serious involvement in music to become familiar with the contemporary applications of mathematics to music. We report on a new, roughly sophomore-level honors course that addresses the need for mathematical awareness among musicians. At the heart of the course are several “Creative Experiences” that allow students to experience first-hand how mathematics plays a vital role in music. Examples of such experiences include sight-singing, tuning and temperament, instrumental technique, and post-tonal analysis of musical compositions.

**Tracey McGrail** (tracey.mcgrail@marist.edu) Marist College

#### Honor-ing Mathematics History

The Marist College Honors Program in Liberal Arts offers a seminar entitled Science, Technology, and Society. I will describe one version of the seminar, satisfying the core requirement in mathematics, in which students will explore recent significant results in mathematics. Students will study the impact of these results in terms of their historical context and future ramifications, as well as the public perception of their significance through various forms of media, such as film, news reports, and magazine articles.

**David Edward Boliver** (dboliver2@cox.net) University of Central Oklahoma

#### Mathematicians Can Participate in Faculty Development for the Arts & Humanities

Mathematics faculty have in common with those in the Arts & Humanities the task of teaching service and general education courses to a population which includes few if any students from their own major. Beginning with this premise and 40 years of faculty experience at two universities, a presentation was developed for faculty from other disciplines and shared with a very positive response last August. A basic outline will be shared and the full power point developed will be available via e-mail to any attendee who desires it.

**Michael Wodzak** (mawodzak@viterbo.edu) Viterbo University

#### Running Circles ’round the Truth: A Mathematical Perspective on Deconstruction

One of the more influential forms of modern literary criticism is Deconstruction, and one of the most important works in the history of that branch of the discipline is J. Hillis Miller’s seminal Stephens’ “Rock and Criticism as Cure”, in which he analyses language using Wallace Stephens’ poem “The Rock” as a point of departure. In this essay, Miller makes arguments based on a less than complete understanding of certain mathematical structures, most notably different types of spirals and their relationship to the mythological Ariadne and Arachne. I discuss these mathematical structures and how they relate to mythology, history and literature, and find that both Miller’s arguments and his conclusions are fundamentally unsound. While students of the Humanities are quite likely to be familiar with Miller’s thesis, and acquainted with Ariadne and Arachne, it is less likely that they will be able to spot the lack of mathematical rigor that Miller exploits.

**Anna Davis** (davisa@ohiodominican.edu) Ohio Dominican University

### Perspective Drawing Experiment in a Freshman Interdisciplinary Seminar

Leonardo's treatise on painting includes an extensive discussion of perspective. Leonardo suggests that to study perspective we use "a pane of glass through which are seen various objects which you draw [directly] on [the glass]". Based on Leonardo's description I devised an experiment that I assigned in my interdisciplinary freshman seminar. The experiment involved students drawing model railroad tracks viewed through a pane of glass. Students were asked to collect measurements from their drawings and to plot their data. This talk will address the data collection process and the function that arises from the theory behind this experiment.

**Michelle Ghrist** (michelle.ghrist@usafa.edu) U.S. Air Force Academy

### MathTV: A Curiously Funny Mathematical Production

In the spring of 2002, I led an honors seminar at Belmont University entitled, "Mathematics in Prose and Poetry." Our culminating activity was a mathematical play produced by myself and the five students from a wide variety of non-mathematical backgrounds. In this talk, I discuss the course, including the many sources I used and some of the novel ideas I implemented along the way. I focus on our play, which we deemed "Musical Aesthetic Taboo Humorous Entertaining Mathematics All Together In Chaotic Skits". It included many readings and original skits as well as original musical pieces based on the Fibonacci numbers, pi, and Euler's constant. This course showed me how non-technical students could grow to understand, appreciate, and be excited by mathematics at a deep level.

**Samuel A Lopes** (slopes@fc.up.pt) University of Porto

### A Musical Journey Through Abstract Algebra

In the 1930s, G. Polya obtained a beautiful result combining group actions, generating functions and equivalence class counting. Through the example of musical chords, his ideas can be illustrated, thus providing a familiar context for the mathematical notions of equivalence class, group action, modular arithmetic and dihedral groups. This type of combinatorics yields not only a plausible generalization of the binomial coefficients as well as numerical evidence of certain subjective qualities of musical chords.

This talk has been given to audiences of diverse backgrounds including upper level high school students and working musicians. The mathematical baggage can be easily adjusted to suit the target audience.

**Zdenka Guadarrama** (guadarrama@rockhurst.edu) Rockhurst University

### An Artistic Exploration Within an Introduction to Measure Theory and Dimension

I will describe an interdisciplinary exploration in mathematics, computer science and art. This project brings together students and faculty with different backgrounds to use math concepts and computer science to produce works of art that express the mathematics learned in the process.

**Rick L. Spellerberg** (rick.spellerberg@simpson.edu) Simpson College

### Math Modeling Can Help Bridges to the Humanities

During my recent sabbatical I gave a talk at the Iowa Philosophical Association section meeting that highlighted the work of researchers interested in explaining the evolutionary dominance of mankind. These researchers incorporated game theory models to support their arguments. This topic led to much discussion during the conference and provided me and a colleague of mine in the department of Philosophy at Simpson College the motivation to start thinking about ways we can provide our majors opportunities for interdisciplinary study. This talk will illustrate how math modeling can start building a bridge between Mathematics and Philosophy.

**Atabong Timothy Agendia** (agendia@yahoo.com) not affiliated

### Finite, Infinite and Countable Infinite

Finite and infinite are words used commonly in Art, Humanities, and Science. These words undoubtedly originated from mathematics in the days of the early people. Many people used these words alternatively in alternate situations. To convey these notions in a classroom, most people will do this using the notion of set theory. The traditional African *bitter kola* eaten by both the young and old can convey these notions in a classroom to the understanding of even those with learning disabilities. As a whole, in its parts, and within color, this presentation links it to finite, countable infinite and infinite.

**Russell Goodman** (goodmanr@central.edu) Central College

### Leading a Book Discussion in a Liberal Arts Mathematics Class

One of the purposes of Central College's liberal arts mathematics class, Contemporary Mathematics, is to explore the use of mathematics to better understand the world. The presenter is currently teaching the course and is leading a class book discussion of "The Curious Incident of the Dog in the Night-Time" about a fictional young boy, Chris, with autism, who has a love for mathematics.

Through Chris' narration of the book, the presenter hopes his students will experience a unique perspective on mathematics that will enhance their appreciation of the discipline, but also open their eyes a bit more to the world around them.

In his talk, the presenter will provide a brief overview of the book and then describe how he led this class book discussion. He will also present the results of his efforts.

**Gizem Karaali** (gizem.karaali@pomona.edu) Pomona College

### An "Unreasonable" Reading Component to a Reasonable Course: Readings for a Transitional Class

A standard student question mathematics instructors often need to respond to is "Where is this concept used in real life?" This talk will focus on a reading component that was incorporated into a traditional transitional linear algebra course, which seeks to address this issue. The readings were chosen around the main theme of Eugene Wigner's famous article on the unreasonable effectiveness of mathematics in the natural sciences and included a personal memoir and a short story. The concept of such a reading component fits within the general liberal arts college classroom, providing a natural way to engage students' interest in the place of mathematics in our modern world.

**Cynthia Elizabeth Chin** (cchin@madison.k12.wi.us) Madison East High School
**Jeffrey Sanders** (jeffrey.sanders@oberlin.edu) Oberlin College
**Jonathon Giam** (jonathon.giam@vance.af.mil) Air Force
**Mary Paulson** (mpaulson@madison.k12.wi.us) Madison East High School

### This World and Others: Flatland in the Classroom, Rehearsal Studio, Theater, and Beyond

Students at Madison East High School engaged with Edwin Abbott's novel Flatland from a variety of perspectives during 2005–2006 and 2006–2007. Geometry students reviewed the mathematics, examined and debated the relevance of socio-historic context and themes, and created their own original episodes in many different media. Concurrently, several older students began work on a theatrical adaptation, including student-written music and lyrics, which was ultimately produced (and filmed) for a combined school and community audience. As the text, score, and staging were developed, students identified key mathematical and dramatic features of the story, and decided how to convey these orally, musically, and visually. Advanced mathematics students who began the project put their varied artistic talents to work. Several students who did not consider themselves talented in mathematics were attracted to the project's theatrical aspects. For them, declaiming or singing about number, form, and the problem of proof was a (nearly) painless route of entry to these topics. Students and faculty from a variety of departments, including Mathematics, Music, Drama, Science, Art, and even the prom committee, collaborated on the logistics of this production. Audience members responded appreciatively to the dramatic arc of the tale. They also found themselves invited (as were Abbott's original readers) to consider parallels between the experiences of the frustrated "Apostle of the Third Dimension", attempting to prove a newly-discovered mathematical truth, and the difficulties encountered by anyone seeking to establish new scientific or social norms.

**Rachel Hall** (rhall@sju.edu) Saint Joseph's University

### Teaching the Mathematics of Music

Music has many connections to mathematics. The ancient Greeks discovered that chords with a pleasing sound are related to simple ratios of integers. Ancient Indian writers discovered the Fibonacci sequence in the rhythms of Sanskrit poetry. Other connections between math and music include the equations describing the sounds of musical instruments, the mathematics behind digital recording, the use of symmetry and group theory in composition, the exploration of patterns by African and Indian drummers, the application of chaos theory to modeling the behavior of melodies, and the representation of families of chords by exotic geometric objects called orbifolds.

I will discuss a course I created for sophomore-level math majors on the mathematics of music. This course introduces a number of mathematical topics and investigates their applications in the analysis and creation of music. I use the medium of music to (1) explore mathematical concepts such as Fourier series and tilings that are not covered in other math courses, and (2) introduce topics such as group theory and combinatorics covered in more detail in upper-level math courses. Along the way, we discuss the role of creativity in mathematics and the ways in which mathematics has inspired musicians. Students complete a semester-long project that explores one aspect of the course in depth.

I will provide a sample syllabus and worksheets from the course.

**Mike Pinter** (pinterm@mail.belmont.edu) Belmont University

### Viewing Mathematics as Human Endeavor

In several courses I teach at Belmont University, including an Honors Program Analytics course and a first-year seminar non-mathematics course, I incorporate several readings, discussions and assignments that encourage students to consider the study and enjoyment of mathematics as part of what it means to be human. The courses are structured to make connections between mathematics and humanities (as well as other academic areas). For example, *The Man Who Loved Only Numbers*, by Paul Hoffman, is a required Analytics course text that provides a rich look at Paul Erdős. Also in Analytics, we view and discuss "The Proof" video (from the PBS Nova series), which examines from a human interest perspective the story of Fermat's Last Theorem and Andrew

Wiles' proof of the theorem. In several courses, students read about Robert Moses and discuss his work with the Algebra Project. Voting Methods and Social Choice Theory lead to discussions not only of current issues associated with how we determine winners but also of the post-WWII mindset about planning to survive a nuclear war, along with direct references to John Nash and some excerpts from the book *A Beautiful Mind*. Work with modular arithmetic finds connections ranging from music theory to the historical significance of encryption during WWII. Some of my courses include *The Curious Incident of the Dog in the Night-Time* as a required reading. In those courses, we explore several mathematical components of *The Curious Incident*.

**Patricia Oakley** (poakley@goshen.edu) Goshen College

**Mathematical World: Liberal Arts Mathematics with an Art Emphasis**

A liberal arts mathematics course can be taught with a variety of emphases. This talk will present an outline for a liberal arts mathematics course with an emphasis on connections between mathematics and art. Selected student work will be shown and the final portfolio project will be discussed.

**Julie Barnes** (jbarnes@email.wcu.edu) Western Carolina University

**Teaching Art, Literature, Science, Theater, Speech, and Computer Science All in One Mathematics Course**

At the 2004 MAA Southeast Section meeting at Austin Peay, Dr. Sue Goodman presented information in her invited address about a liberal studies course she had developed on fractals for an honors class at Chapel Hill. Because I also have an interest in fractals, I talked with her about the course and the possibilities of doing something like that at my school. Since then, I have used her materials, adapted the framework to our liberal studies program, and added several activities. In this talk, I will share how I implemented a fractal course into our curriculum, how I dealt with teaching writing, literature, poetry, music, art, theater, science, speech, and computer science all in a fractal setting, and the benefits of teaching such a course for primarily non-majors. Although the framework of the course comes from Sue Goodman's presentation, this talk will emphasize what I've added to the course and how I made it work at a smaller school.

**Susan Jane Colley** (sjcolley@math.oberlin.edu) Oberlin College

**What Is Mathematics and Why Won't It Go Away?**

We describe a low-prerequisite seminar for first-year students that intersperses various topics in mathematics (e.g., elementary number theory, proof by induction, some geometry, and topology) among discussions about more humanistic, philosophical, and cultural aspects of the discipline. The student audience to date has consisted both of those without a significant affinity for mathematics, as well as those having a background in calculus and an interest in the sciences. Thus, class time has been very lively. Reading and video lists and sample assignments will be presented and the challenges and rewards of teaching such a seminar will be discussed.

**Pamela Warton** (warton@findlay.edu) University of Findlay

**Group Theory is Child's Play!**

From childhood, people are fascinated with kaleidoscopes. Along with the geometry of the placement of the mirrors in a closed two-mirror system (the traditional toy kaleidoscope), the mathematics of kaleidoscopes involves linear algebra, group theory and a bit of graph theory. These 'kaleidoscope groups' can be completely classified! Attendees will be provided with project sheets for students that can be used in a traditional undergraduate abstract algebra course.

**Paul F. Stang** (paulstang@gmail.com) not affiliated

**Mandala as the Key to Teaching Trigonometry to Multiple Intelligences**

It's been said that we are teaching today to only $1/2$ of one chakra, emphasizing math and science in education. When we recognize the multiple intelligences in the classroom, and find ways to tap into them, we develop a more humanistic method, see much better results with students and open ourselves as teachers.

To begin teaching trigonometry, beyond proportional understanding of sin, cos, and tan, let us start artistically, with Mandalas. We will engage our "non"-mathematical students with techniques that will broaden the skills of all, as we use particularly 6-, 12-, and 8-fold divisions of the circle to make art, and along the way become familiar with key mathematical relationships and angles. We will applaud the best efforts, usually meaning success for the normally unsuccessful, further interest from people of intelligence (not necessarily mathematical), provide balance for the math-wizzes, and create more inspiring math classrooms.

This work will continue toward 3D, particularly Platonic forms, using the Pythagorean theorem en route to real application of this formula, with the non-use of the calculator (but with a keen eye on the lookout for patterns, which is what mathematics is about, after all).

**William L Bloch** (bbloch@wheatoncollege.edu) Wheaton College

**Combinatorics, Topology, and Graph Theory in Jorge Luis Borges' short story,** *The Library of Babel*

According to Nobel-prize winning author Carlos Fuentes, the modern Spanish novel would not exist without the prose of Argentine writer Jorge Luis Borges. Borges, an autodidact, was enamored with mathematics and read, for example, works of Bertrand Russell over and over to help him as he grappled with ideas and paradoxes surrounding the notion of infinity. Borges' affinity for math works itself out in many of his remarkable short stories, but most especially in "The Aleph" and "The Library of Babel." In a course (*The Edge of Reason*) for humanists that I teach in conjunction with an English professor, we unpack a number of literary and mathematical ideas. In particular, "The Library of Babel" provides a natural springboard to discuss combinatorics (How many distinct books are there in the Library?), topology (What manifolds best model the Universe that is the Library?), and graph theory (How might we best understand the interior of the Library?). In this talk, I'll describe the unique(?) structure of the course, provide a brief synopsis of the story, and discuss some of the mathematics we cover.

## Creative Uses of Emerging Technologies for Mathematics Teaching

### Friday, August 1, 8:30 AM–10:30 AM and 1:00 PM–4:00 PM

**Erick Hofacker** (erick.b.hofacker@uwrf.edu) University of Wisconsin - River Falls
**Kathryn Teresa Ernie** (kathryn.t.ernie@uwrf.edu) University of Wisconsin - River Falls

#### Using Clickers to Encourage Discussion and Communication in Mathematics

Over the past couple of years, personal response systems have been used to encourage and promote more active learning in classes. Personal response systems can be used to gain an understanding of what all of the students believe about a concept. Results of student polling on the possible answers to a question can be revealed to the class after everyone has had an opportunity to vote on their choice. This leads to small group discussion over differing answers.

Recently our university purchased personal response systems in the form of i-clickers. This paper describes our experiences over the last year using i-clickers in our respective collegiate mathematics courses: a liberal arts math class, a math for elementary teachers class, and a pre calculus class.

The paper presents our methodology for including i-clickers into our classes. Conceptually based questions were written for each of the three courses and asked of the students at the beginning of class. One of the goals of our use of the i-clickers was to encourage discussion and communication amongst students in our courses. Another goal was to give students an opportunity to self-reflect and gain an understanding of concepts they may have thought they understood, but did not. Examples of the questions used will be shared from each of the three courses. Details of everyday classroom experiences and the long-term effects of using the i-clickers will also be shared.

**Christopher K Cartwright** (cartwright@ltu.edu) Lawrence Technological University

#### Using Tablet PCs in Calculus 1

This spring semester I used a Tablet PC with Windows Journal to deliver lectures to an on-ground Calculus 1 course. Some but not all students in the course also had Tablet PCs available for taking notes. For students who did not have Tablet PCs or Windows Journal, I converted the lecture notes to PDF format and posted the notes on-line using our Blackboard course management system. I will provide some preliminary assessment of using the Tablet PC compared to the traditional chalkboard from both the instructor's and students' points of view.

**Douglas A. Lapp** (lapp1da@cmich.edu) Central Michigan University

#### Using technology to promote reflective discourse: Combining dynamically connected representations with video reflection.

This session describes the use of technology to promote mathematical exploration. Although the use described occurred in a new program designed for pre-service secondary teachers, the use of technology and approaches to teaching are also applicable to typical undergraduate courses such as abstract algebra.

Experiences will be shared from the first course in a four-course sequence that is taken as a precursor to several upper-level undergraduate courses providing early exposure to concepts from abstract algebra and analysis. Approaches for infusing technology into this course to promote mathematical discourse will be shared. Specifically, this course combined the TI-Nspire CAS™ (hand-held and software) with video sharing through iTunes University.

The TI-Nspire CAS™ combines a computer algebra system, dynamic geometry system, spreadsheet, and dynamic statistics system into one device where all representations are dynamically connected. Changes to any created object in any problem page of a document results in real-time changes in other connected objects on any page within the same problem. This has been a powerful tool in student exploration and promotes discourse among students.

The second prong of this approach is the use of iTunes University for posting a video podcast immediately following class. The students are able to download video from class using iTunes. This provided students with an opportunity to "relive" the classroom discussions and results shared in class. The use of the video promoted discourse among students and helped create the atmosphere of a mathematical community within the course.

**Denise LeGrand, Sally Robinson** (djlegrand@ualr.edu, sarobison@ualr.edu) UALR

### "Tabulating" Mathematics

Teaching mathematics has evolved over the past decade due to the capabilities of enhanced instruction via technology, particularly with the tablet PC. Many new approaches in content delivery offer today's student the opportunity to further their education through non-traditional ways. Online courses, web enhanced courses, video lectures, live chats, distance learning options, testing alternatives, virtual office hours, online resources and pod-casting are making it easier for today's student to juggle their busy lives with their university studies. The tablet PC is being utilized to provide audio and video lectures beyond the capabilities of any traditional classroom lecture. It is also being used for grading homework, quizzes, and labs as well as posting previous lectures and homework solutions. In this presentation, we will demonstrate how we use the tablet PC to make video lectures and other readily available resources along with course management software to deliver full course content virtually to students who can not always attend the lectures. With mathematics, the presentation of material has been difficult in the past but the tablet PC has made this extremely easy. The pros and cons of going completely "online" will also be discussed along with adaptations that have been made over the years. In addition, our research about online courses and student comments will be shared.

**Jeff Gregg** (gregg@calumet.purdue.edu) Purdue University Calumet

### Interactive CaluMath Web Pages for College Algebra and Pre-Calculus

CaluMath is mathematics web page construction software that will run on any computer with access to the Internet and is freely available. It was developed by Professor Peter Turbek at Purdue University Calumet. This session will focus on CaluMath web pages that have been constructed for use with college algebra and pre-calculus courses. The pages pose problems, allow students to create and manipulate graphs, tables, and symbolic representations, and respond to student actions with feedback or additional questions. These pages fall into three categories: a) those intended to facilitate classroom discussions, b) those that can serve as a tutorial for students to use at home, and c) those designed to support students' work on applied problems. Examples of each type of page will be provided with the primary emphasis on how the web pages can help students develop their understanding of functions. Links to CaluMath pages used at Purdue Calumet are at ems.calumet.purdue.edu/mcss/psturbek/CaluMath/college_algebra/MA153

**Allan Struthers** (struther@mtu.edu) Michigan Technological University

### Popcorn Statistics: Audio Data Collection

When students collect large datasets they own the data,connect statistics to the real world, and may appreciate the ubiquity of large data sets.Audio software provides a straightforward way for students to collect substantial data sets from numerous interesting phenomena. the free software *Audacity* is used to record popping corn and automatically extract thousands of pop times. We will compare microwave to hot air popping for various popcorns. The normality of the resulting poptime distributions will be assessed to judge their accuracy as exemplars of the normal distribution.

**Chen-Han Sung** (drsung@tamiu.edu) Texas A&M International University

### ICLICK the MATH in Class

How to engage students of all kinds in learning and encourage their classroom participation without embarrassment concerns? What kind of instantaneous feedback can an instructor receive from and provide to students in class during discussions or after quizzes in mathematics? Those and more will be addressed. Experiences with some emerging technologies will be shared.

**Lila F. Roberts** (lila.roberts@gmail.com) Lawrence Technological University

### Mathematics on the iPhone—Closer to Reality

The February release of the iPhone/iPod Touch SDK has made it possible for the public to participate in developing applications for these multi-touch interfaces. This presentation will give an update of available applications for mathematics and some new applications that are under development. In addition, the presentation will provide a short demonstration of the robust development environment.

**Teri Jo Murphy** (tjmurphy@ou.edu) University of Oklahoma

### Using Clickers Toward Conceptual Understanding: Experiences in Statistics Courses

Classroom response systems (CRSs, aka clickers) allow for immediate feedback to both students and instructors. Students use a clicker to transmit their answers to a question (often multiple choice) posed by the instructor. The software captures student responses and offers a bar graph to display the distribution of answers. Instructors can then make decisions about how to proceed: move on to new content or discuss the question topic some more. This presentation will offer some experiences using this technology in statistics courses. As with other users of this technology, the presenter believes that the greatest learning happens during student-student discussion after a question has been asked. As this talk is part an NSF-funded project (DUE-0535894) to write clicker questions, the presenter can also discuss the iterations involved in producing a meaningful question.

**Peter Garst** (pgarst@enventra.com) Enventra Inc.

### Mathematical Handwriting Recognition for Education

We introduce and demonstrate MoboMath, a system which recognizes handwritten mathematics and allows handwriting based editing operations on mathematics. The system works on tablet PCs, ultra-mobile PCs, and desktop computers with attached tablets. MoboMath interoperates with other applications, such as Word, and works in a number of formats, such as TeX and MathML.

We describe computational linguistic techniques which allow us to adapt it to different courses, or different fields which use mathematical notation.

The system has not been operational long enough to be able to report on the results in education, but we are working on a number of educational applications and would like to explore these and others the session attendees may be interested in.

The current targets include:

1. Online learning systems—students could submit work in symbolic form, and could use, edit and reuse equations from course materials.
2. Note taking—class notes could be used directly in a computational engine, in homework, or in other contexts.
3. Highly interactive texts.
4. Lab notebooks—there are many ways to use and manipulate quantitative data using this technology.

—as well as much improved word processing for technical notation.

More information is available at www.enventra.com/mobomath.htm.

**Kevin William, Chad Kjorlien** (kdennis@smumn.edu, ckjorlie@smumn.edu) Saint Mary's University of Minnesota

### Capturing a Class: Adventures with Tegrity

Tegrity is a classroom capturing software that allows students to replay parts of classes efficiently. Saint Mary's University of Minnesota has been using this software for two years and is in the process of evaluating its effect on student learning and attitudes. In this presentation, we will discuss our experience with implementing this technology. We will also discuss results from surveys administered by the Director of Instructional Technology which are being used to assess the software.

**Andrew S Leahy** (aleahy@knox.edu) Knox College

### YouTube in the Classroom: video screencasts of new software packages

Over the past several decades, new computer software packages have transformed the way undergraduate mathematics courses are taught. However, for students, each new software package presents a new set of computer skills to acquire and, for teachers, each new class presents the challenge of teaching students those skills while using class time as efficiently as possible. Screencasting (making a video recording of a computer desktop session) is one way to address this challenge. This talk will discuss how screencasting and YouTube-like flash videos can be used in both introductory and advanced mathematics courses to make classroom technology demonstrations readily available to students wherever they can find a computer or a portable video device.

**Peter Turbek** (turbek@calumet.purdue.edu) Purdue University Calumet

### Using CaluMath Software to Create Interactive Web Pages for College Algebra, Pre-Calculus, and Calculus

Developed at Purdue University Calumet, CaluMath is open-source mathematics web page construction software. It is designed to be compatible with all web browsers and is freely available. Its goal is to allow instructors to create interactive mathematics web pages for use with their students or to tailor existing pages for their particular classrooms. This session will demonstrate how the software can be used to develop web pages that enable students to create and manipulate mathematical objects and receive feedback concerning their interactions with the page. We will illustrate how to add graphs that students can manipulate, boxes that will accept student input, and buttons that enable students to perform various actions. We will also illustrate how "replies" to students' actions may be incorporated into the web pages and how the software can randomly generate new versions of a given problem. No programming is necessary since the software provides the web page developer with an array of menu-driven options and boxes used to construct the interactive web pages.

**Elisha Peterson** (elisha.peterson@usma.edu) United States Military Academy (West Point)

### Online Collaboration with a Wiki in Real Analysis

A wiki is a collection of webpages with streamlined authoring, linking, and collaboration capabilities. This emerging technology has forever changed the nature of encyclopedias and "collective knowledge", and its utility in the classroom is just beginning to be explored. This talk will focus on two ways in which wiki capabilities were used to enhance a real analysis course: students (i) collaborated on a glossary of terms that they were permitted to use on examinations, and (ii) collaboratively wrote and submitted papers directly to the wiki website. In the end, the wiki encouraged student collaboration, as was expected, but there were also several unexpected positives.

**Jennifer Kosiak, Rebecca LeDocq, Robert Hoar, Jon Hasenbank** (kosiak.jenn@uwlax.edu, ledocq.rebe@uwlax.edu, hoar.robe@uwlax.edu, hasenban.jon@uwlax.edu) University of Wisconsin - La Crosse

#### Using Video Podcasts to Model Critical Thinking in Undergraduate Mathematics

With advancements in screen and voice capture technology and the explosion of portable media players, podcasts are impacting the teaching and learning in higher education. This session will focus on two projects that utilize Tablet PCs and software packages to generate faculty and student-created video podcasts for several mathematics classes (e.g., Math for Elementary Teachers, Applied Calculus, and Calculus II). The presenters will share how these podcasts served as supplementary course material to introduce concepts, remediate misconceptions, present alternate coverage of the material, or allow students to review course presentations. Enhancement in the critical thinking of the student designers and users for these projects will be discussed. Session participants will be introduced to the process and technology involved in these projects.

## Projects and Demonstrations that Enhance a Differential Equations Course

### Friday, August 1, 1:00 PM–3:20 PM

**Sarah Mabrouk** (smabrouk@frc.mass.edu) Framingham State College

#### Exploring Differential Equations Using MS Excel

Graphical exploration of solutions for second order linear homogeneous equations enhances one's understanding of mechanical systems such as the mass-spring and the simple pendulum. Textbooks usually include some graphs for functions corresponding to damped and undamped motion. However, exploring the effect of even subtle changes in the mass, the length to which the spring is stretched, and the initial velocity for a mass-spring system or to the mass and length of the string for the simple pendulum, for example, can help one to better understand motion as well as help one to make connections between the differential equation and its solution. In this presentation, we will examine and discuss an interactive tool, created using MS Excel, that allows the user to explore the graphs of solutions for second-order linear homogeneous differential equations and their applications by changing the coefficients of the equations using scroll bars and buttons.

**Terry Jo Leiterman** (terryjo.leiterman@snc.edu) St. Norbert College

#### Getting Your Hands Wet in a Differential Equations Course

A laboratory component in a mathematics course provides an opportunity to pique interest and engage students with a wide range of majors in a non-traditional way. A first course in differential equations has been planned to include a 20- to 30-minute (out of 220) weekly lab session in fluid mechanics. The first part of these labs is devoted to introducing core topics in differential operators, integral forms of differential equations, transforms, numerical methods, and modeling through the lens of fluid mechanics. In addition, key concepts and theorems in multi-variable calculus are revisited. This review allows the traditional lecture material to be presented outside the lab without interruption. In the second part, students investigate concepts first-hand, in the convention of a lab section in a science course. In this talk, several of the laboratory sessions will be discussed and cost-effective ideas for lab equipment will be offered. Undergraduate students often report that "getting their hands wet" imparts the most in-depth understanding of an idea. In this course, students got their hands wet each week! Student reflections and plans for future semesters are shared.

**Joe Latulippe** (jjlatulippe@csupomona.edu) California State Polytechnic University at Pomona

#### Integrating Synaptic Models Into a Differential Equations Course

Incorporating exploratory projects or applications into a differential equations course can enhance student understanding of the course content. Drawing project examples from disciplines such as biology, neuroscience and engineering can reinforce the importance of differential equations to students in science and engineering fields. The main goal of this presentation is to discuss the application of synaptic current for the transmission of neurological information and how it can be incorporated into the curriculum as a project or an interactive example for undergraduates. We will formulate simple mathematical models for single synapses and demonstrate how students can solve these models. The models vary in difficulty but can be accessed with only a basic knowledge of differential equations. The solutions presented will include content related to exponential functions, first-order linear differential equations, unit step functions, and Laplace transforms. Understanding of the models and solutions is further improved through computer simulations that allow students to vary model parameters, leading them to distinctly different solution patterns.

**Howard I. Dwyer** (hdwyer@monm.edu) Monmouth College

### The Laplace Transform: Motivating the Definition

Most introductory textbooks on ordinary differential equations contain a chapter devoted to the Laplace transform. Typically, the definition for the transform (as an improper integral) appears in the first section of this chapter, often on the first page. In later sections, the important properties of the transform are derived from the definition, but without any insight as to what might have motivated the "rabbit from a magic hat" definition in the first place. In this presentation, we will outline a sequence of topics or concepts which "discover" the Laplace transform as the answer to a well-motivated design problem, based on an analogy with the familiar properties of the natural logarithm. Experience shows that presentation of the Laplace transform in this manner takes only a little more class time than the conventional, "magical" presentation, since the extra time spent laying the groundwork is recovered from the time usually spent deriving the basic properties, because the transform has these properties by design.

**Wen Liu, Gro Hovhannisyan** (wliu3@kent.edu, ghovhann@kent.edu) Kent State University

### Fundamental Solution of Dirac System

We construct the fundamental system of solutions of time-dependent Dirac systems by using Euler's method. Using the fundamental system of solutions, we describe the conservation law for Dirac system. We also establish the error estimates for representation of solutions via an approximate fundamental system. Using this representation of solutions, we find out approximate conservation laws as well.

**Richard Marchand** (richard.marchand@sru.edu) Slippery Rock University

### Stabilizing Elastic Beams

The uniform stabilization of vibrating elastic beams is an important engineering application involving both ODEs and PDEs. The goal is to damp unwanted vibrations uniformly to zero as quickly as possible. A collection of beam models and an analysis of their behavior will be presented. In all cases, it will be necessary to compute or approximate the eigenvalues of the equation or system of equations. The primary model to be discussed involves a thermoelastic beam described by a system of partial differential equations in which the eigenvalues can be computed analytically in the undamped case but must be approximated numerically when damping is involved. In this particular model, the damping can be done thermally or mechanically. Both types of damping will be discussed along with their relative merits.

**Allan Struthers** (struther@mtu.edu) Michigan Technological University

### Desktop Circuits: Audio Data Collection

Standard computer audio hardware and software provides an inexpensive signal generator and recorder to collect data from real circuits. This presentation is intended to demonstrate that this is can be extremely inexpensive (materials cost less than $10) and straightforward. The behavior of inexpensive physical circuit elements (resistors, capacitors, inductors, diodes, and op amps) will be compared to their mathematical idealizations. A standard Inductor, Resistor, and Capacitor LRC resonant circuit will be demonstrated and compared to the theoretical response presented in differential equations courses. Finally, a nonlinear chaotic circuit will be demonstrated.

# How to Get Students to Read the Text and Does This Matter?

## Friday, August 1, 3:30 PM–5:30 PM
## Saturday, August 2, 8:30 AM–10:30 AM

**Amy Wehe, Rala Diakite** (awehe@fsc.edu, rdiakite@fsc.edu) Fitchburg State College

### Journal Forms to Test Students' Understanding

When asked to teach History of Mathematics this spring, we chose to require our students to read *Fermat's Enigma*, a book beyond the textbook. Since we would not be discussing this book very much in class, we wondered how we would make sure our students had read the material. We recalled a conversation with a fellow faculty member and asked her for her "journal forms". This talk will discuss the reception and results of using the journal forms in this course with examples of student work and survey responses.

**Rachel Esselstein** (racheLesselstein@csumb.edu) California State University at Monterey Bay

### Learning Styles and Measurable Reading Assignments

People preferentially take in and process information in different ways: visually, kinesthetically, socially, and aurally. Thus it is only appropriate that they might read and process textual information according to their learning style. Unfortunately, in their K–12 education, our students are rarely given the opportunity to explore how their individual learning styles might influence how they read. In this talk, we will discuss a list of reading exercises I have been assigning to my students in a variety of math courses. These exercises are divided by learning styles and the students are encouraged to find one that works best for them. The best part is that each exercise can be collected and assessed to keep the students accountable.

**Bonnie Gold** (bgold@monmouth.edu) Monmouth University

### Assessing Student Growth in Reading Mathematics

One of Monmouth University's goals for its mathematics majors is that they develop "the ability to read, discuss, and write about and orally present mathematics," with the specific reading objective being, "Students read mathematics and can demonstrate their understanding of the material in some way." In this talk, I will discuss the range of activities (preliminary homework assignments, applications projects, writing assignments based on readings) and courses (from freshman year through senior) in which we develop this ability. I will then discuss how we are assessing whether students have developed this skill during their time with us.

**Clark Wells** (wellsc@gvsu.edu) Grand Valley State University

### An Analysis Sketchbook: Rethinking Texts to Generate Student Interest

I believe the short explanation of why students don't read the text is that they don't believe the benefits of doing so outweigh the opportunity cost. If we denote the utility of reading, time spent reading, utility of not reading, and time spent not reading by $(u_r, t_r, u_n,)$ and $(t_n)$, respectively, a student's concept of their total utility function might be $(u_T = u_r t_r + u_n t_n)$ subject to a constraint on $(t_r + t_n)$.

Approaches to getting students to read the text that I have seen focus on increasing $(u_r)$. The approach I suggest here is to help students see that the relationship is not this simple and that increasing $(t_r)$ actually leads to an increase in $(u_n)$ that more than compensates for the decrease in $(t_n)$. I suggest we can do this by generating student buy-in through rethinking the structure and role of the text.

In the summer of 2006, Dr. Jonathan Hodge and I created a collection of discovery assignments and activities that we have compiled into what we refer to as a "sketch book" in analysis for use in our undergraduate introductory analysis course. Using some of our activities as examples, I will explain how using a sketch book for a text can generate buy-in for reading the text. I will also discuss results of using the sketch book in our analysis courses.

**Allen Hibbard** (hibbarda@central.edu) Central College

### Read It: Techniques to Get it to Happen

I will discuss several different techniques to prod students to the read the text. Two of these are intended to encourage the reading of the text before we discuss a section and is typically used for lower-level courses. Another technique is meant to help upper-level mathematics students learn how to read their text, particularly focusing on reading the proof of a theorem.

**Matt Boelkins** (boelkinm@gvsu.edu) Grand Valley State University

### Learning to Read and Reading to Learn: The Value of Reading the Text Before Class

In www.maa.org/features/readbook.html, Tommy Ratliff and I argued for the value and effectiveness of before-class reading assignments in a variety of classes. We also documented our approach to these assessments in which we have students submit their responses via email prior to class. In the seven years since that article was published, I have regularly employed a variety of similar assignments in a wide range of classes. My goals in this talk are to share an overview of options that my GVSU colleagues and I have used for different types of reading-based assignments in courses other than those discussed in the aforementioned article (for example, in a bridge course, a differential equations course, a voting theory course, and a senior capstone), to present new ideas for the streamlined receipt and grading of such assignments via email, and to offer further evidence of the effects of this approach in its support of student learning.

**Mary D Shepherd** (msheprd@nwmissouri.edu) Northwest Missouri State University

### What Students Read is Not Necessarily What They "Get"

This paper reports on five years of observations as students read their college mathematics textbooks. I have instructed and observed students reading primarily first-year college mathematics textbooks in a variety of situations. I have used reading homework, quickie quizzes at the start of class, reading guides and propaganda to convince students that they really can read a mathematics textbook. I have observed students in reading situations including without reading guidance, with reading guidance, reading to each other, and with interruptions to check for understanding. These readers often do not come away from their reading with the understanding we as teachers would like them to have. These observations have lead me to propose a framework for reading mathematics textbooks with understanding. The framework combines reading and learning theories and will be the focus of this presentation.

**Satish C. Bhatnagar** (bhatnaga@unlv.nevada.edu) University of Nevada at Las Vegas

### Students and College Algebra Textbooks

Students' attitude of not thoroughly studying textbooks partly comes from high schools where the books are often left in school lockers before returning home. High school homework and tests are geared towards minimum dependence on textbooks. Based upon the author's multi-cultural experience over five decades, the paper details five remedies used this spring semester for motivating the students towards their textbooks. Include at least one example in every test and final exam. Take all test problems straight from the textbooks. Give one or two open book quizzes. Discuss more word problems in the class. Include at least one write-up quiz involving essay-type writing, but no problem solving. In addition, the author will share ideas collected from the students in a recent survey.

**Andrea Frazier** (afrazier@noctrl.edu) North Central College

### Do Real Mathematicians Read the Book?: Encouraging Mathematical Maturity via Reading Checks

Can mathematical maturity be developed through "Reading Checks?" To practice and reinforce the skills necessary for each new level of mathematical reasoning, my students (from integrated calculus through abstract algebra) regularly submitted electronic quizzes. These "Reading Checks" encouraged mastery of theorems in integrated calculus, introduced critical reading skills in courses for math minors and sophomore majors, and attempted to facilitate discussion and inquiry in an abstract algebra course. In this presentation, I will discuss advantages and pitfalls of various forms of electronic quizzes, and two questions that are useful at every level. Results from student surveys will be included.

**Richard Maher** (rmaher@luc.edu) Loyola University at Chicago

### Homework can Motivate Students to Read the Book

This paper discusses how changing the way in which homework is assigned can increase students' motivation to read the book in courses such as "An Introduction to Statistics" or "Quantitative Literacy". It also discusses how to implement the process, which is described below, offers comments on its overall effect on student performance, points out how in-class group assignments can extend the benefits even further, and how a restricted form of this approach can be used in courses like Calculus I and II.

Using this approach, the course ground-rules/syllabus indicates that the standing homework assignment for each student is to do as many problems as they wish that have answers in the back of the book. Students also are assigned a number of problems in each class meeting that are not in this category, and hint/solution sheets are provided periodically. Finally, the course syllabus states that questions on examinations during the semester will be "chosen either directly or indirectly" from these problems. This last sentence usually provides the motivation that most students need to work a lot of problems and as a consequence to read the book.

Textbooks that provide solutions that include explanations and not just numbers are widely available for both courses. The examinations that result from this approach are quite similar to those prepared in a standard setting. Finally, the large number of potential questions involved (150-200 for each examination) eliminate any concerns about students "guessing" what problems will appear on the test.

**Bill Rybolt** (rybolt@babson.edu) Babson College

### What Do Students Actually Read? And Its Educational Implications

Several years ago it became clear that students, in both my freshmen calculus and freshmen probability and statistics courses, were only reading a fraction of the course material. This paper describes efforts to determine how they spend their time, and attempts to focus their attention. For several years, I have experimented with quizzes before, during, or after class to encourage students to read assignments. Electronic surveys have been administered both at the beginning and the end of the semester to measure their time budgets. Although I present the highlights of this material as background information, the focus of this paper is more limited. To what extent do students actually read and follow simple instructions, and is there any relationship between this and their academic performance? This work has taken two forms. In one type of longitudinal experiment, I embed simple commands, such as, if you read this, draw a box around that, in exams. In a complementary set of experiments, I placed the same instructions in the assignments for each class. During the semester, I monitored the extent to which each student followed the instructions. In this paper I present an exploratory analysis of the results and draw a few simple conclusions.

# Projects, Applications and Demonstrations to Enhance a Numerical Analysis or Computational Mathematics Course

## Friday, August 1, 3:15 PM–6:15 PM

**Matthew Glomski** (Matthew.Glomski@marist.edu) Marist College

### Truth, Justice, and the Search for Rigorous Error Bounds: A Case Study in Interval Analysis

A standard computer algebra system will evaluate $\sin(\pi)$ and tell you the truth. Ask it to evaluate $\sin(\pi + 1)$, and it fudges. Errors in computation arise, and worse yet, without warning. When will an error occur? How large will it grow with subsequent computations? One method we can use to answer these questions lies in interval analysis, an arithmetic system which provides rigorous error bounds and even suggests methods to tighten these bounds. In this talk, we will discuss the basics of interval analysis, and apply it to a problem in classical hydrodynamics.

**Anthony Tongen** (tongenal@jmu.edu) James Madison University

### If I May Make a Generalization, Generalizations are Generally Good (In Numerics)

Patterns are not only ubiquitous in nature and quilts, but also in mathematics. The ability to generalize patterns and convert them into algorithms is one aspect of numerical courses that is a foundational tool for mathematicians. This talk will present examples across the numerical curriculum through which students can gain experience in generalizing patterns to develop algorithms.

**Muhammad Usman** (muhammad.usman@notes.udayton.edu) University of Dayton

### Some Interesting Applications of Numerical Analysis in Science and Engineering

Numerical analysis is considered as a new approach to scientific discovery. Experiments give insight into possible theories, theories inspire experiments and experiments reinforce or invalidate theories. Data obtained from the experiment has created the importance of numerical analysis and collaboration between the mathematicians and scientists. I will talk about some interesting applications of numerical analysis in different areas of science and engineering.

**David Coulliette** (david.coulliette@asbury.edu) Asbury College

### Muscle Contraction Modeling: This Will Pump You Up!

Modeling muscle contraction requires a range of computational skills that complement several topics in an undergraduate numerical analysis course. This presentation will introduce several of these topics as potential applications for these courses. The basic problem is to model the interaction of muscle fibers (the myosin and actin fibers) as they create stress and force. A general approach has been to model these fibers as containing an array of interaction points called heads and binding sites. Within the fibers, these points are connected via a system of linear springs. With the proper assumptions, these points generate a linear system of equations that may be solved with direct or iterative methods from a basic numerical analysis course. The interaction of the myosin and actin is further complicated by the unknown nature of when the heads and binding sites activate. Current approaches in the literature combine stochastic models with systems of differential equations, and the computational mathematics of these approaches are accessible to the undergraduate numerical analysis student. Recent approaches have modified the fiber model to account for deformation of the fibers themselves, in addition to the spring-modeled interaction. These modifications give rise to an interesting differential equation system that may have application for a more advanced computational course or perhaps as an individual project for an advanced student. Validating these models with experimental data is another fruitful area involving optimization and Monte Carlo simulation.

**Nicoleta Eugenia Tarfulea** (ntarfule@calumet.purdue.edu) Purdue University Calumet

### Computational Applications in Mathematical Biology

Any mathematics course has an immediate goal of conveying concrete concepts and techniques, such as addition of fractions, solving differential equations, proving theorems, and so on. However, teaching mathematics must amount to more than simply substituting numbers into equations and memorization. In this talk we will address combining traditional and nontraditional ways of teaching mathematics to students majoring in biology by presenting some examples to illustrate the use of the computer in presenting different topics and applications; the learning is different, the material is appropriate to students' needs and level of sophistication, the use of the numerical tools opens up new ways of understanding and representing mathematics.

**Olga Brezhneva** (brezhnoa@muohio.edu) Miami University

### Projects and Illustrations that Can be Used in a Numerical Analysis Course

In teaching a Numerical Analysis course, one big question is how to motivate students' learning and make the course interesting for a variety of majors. Since I started teaching a Numerical Analysis course in August 2006, I have been looking for interesting examples, projects, and illustrations that relate to different topics of the course. In this talk, I will present some of my findings and describe some illustrations and projects that I use while teaching the Numerical Analysis course. For the open-ended projects, I will discuss topics chosen by students and analyze students' experiences with working on the projects.

**Andrew John Miller** (millera@mail.belmont.edu) Belmont University

#### Providing Options: Making Projects Work for a Diverse Audience

At Belmont University, 'Numerical Methods' is cross-listed as both a mathematics and a computer science course. Consequently, the students taking this course can range from math majors with an aversion to coding to computer science majors with an aversion to theorems. "One size fits all" projects will not work with such an audience. We will describe the successful alternative approach we developed in fall of 2007, including a look at the projects themselves, featuring applications to planetary motion, engineering, image and sound compression, and font design.

**Brittany Terese Fasy** (brittany@cs.duke.edu) Duke University
**David Millman** (dave@cs.unc.edu) University of North Carolina at Chapel Hill

#### Exploring Computational Mathematics: Unfolding Polyhedra

The polyhedral unfolding problem can be described as follows: Can a polyhedron in $\mathbb{R}^3$ be unfolded into a simple polygon in $\mathbb{R}^2$? A vertex unfolding finds a solution such that the faces are connected at vertices, but the interiors are potentially disjoint. In the paper *Vertex Unfoldings of Simplicial Manifold*, Demaine *et al.* present such a method for any triangulated two-manifold. We have implemented this algorithm, and we have also explored the possibility of reconnecting disjoint interiors. Our modular implementation of the algorithm allows it to be presented as a sequence of mathematical units supplemented with bi-weekly assignments. Since the data structures involved are simple (graphs and trees), the programming requirements are accessible to students with only a basic knowledge of computer science. In this paper, we propose a timeline for a semester-long project in an advanced undergraduate mathematics course. The proposed project incorporates the areas of geometry, topology, numerical analysis, graph theory, visualization, and computer science. The student would see the implications of a concrete application of computational mathematics, and could continue developing related ideas as a summer research project. Furthermore, the student would acquire a deep understanding of an open problem and would be introduced to research at the graduate level.

**Jason Howard Martin** (jmartin@ou.edu) University of Oklahoma

#### Expert vs. Novice Understanding of Estimation Using Taylor Series

Taylor series is a topic briefly covered in most college calculus sequences. In many cases it constitutes only one or two sections of a calculus textbook. Yet, the estimation properties of Taylor series can be viewed as a very important precursor into future numerical methods. With this limited exposure, what do calculus students really understand about the estimation properties of Taylor series? If our goal is to turn novices into experts, then we should also consider how experts comprehend these properties. Therefore, this presentation will offer some insight into the understanding of estimation of Taylor series by experts and shed light on how the understanding of novices may differ. It will also show some visual tools that may aid in instruction. This presentation is a portion of an ongoing qualitative study into student understanding of the convergence of Taylor series.

## Fascinating Examples from Combinatorics, Discrete Mathematics and Graph Theory

### Thursday, July 31, 8:30 AM–10:30 AM
### Friday, August 1, 8:30 AM–10:30 AM
### Saturday, August 2, 8:30 AM–10:30 AM and 1:00 PM–4:15 PM

**Doug Shaw** (shaw@math.uni.edu) University of Northern Iowa

#### Activities Exploring the Collatz Conjecture: An Unsolved Problem in Fifth Grade Arithmetic

Do your students think mathematics is "dead"? If so, consider sharing the Collatz Conjecture (also known as the $3n + 1$ conjecture) with them as an example of an unsolved problem that demonstrates how mathematics is still growing and evolving. We present camera-ready group activities that should be appropriate for high-school students, although you might argue they would also work well for grades 6 to 8 as well.

**Isaiah Lankham** (ilankham@simpsonuniversity.edu) Simpson University

### Permutation Patterns and Patience Sorting: Sophisticated Combinatorics From a Simple Card Game

If you've ever wanted to know how someone could simultaneously play games and perform real mathematical research at the same time, then this talk is for you.

The study of Permutation Patterns (and in particular permutations avoiding certain patterns) has recently become a hot research topic because of its many applications to fields ranging from Algebraic Combinatorics to Statistical Learning Theory. In this talk we begin with a gentle introduction to such patterns and then discuss how they naturally arise when studying a simple yet mathematically sophisticated card game called 'Patience Sorting'.

**T. S. Michael** (medley@erols.com) United States Naval Academy

### Hard to Guard Art Galleries

Which polygons (art galleries) with $n$ sides require the most stationary guards to protect? What if each guard must also be visible to at least one other guard? What if each angle of the polygon is 90 or 270 degrees? What if each guard has only a 180-degree field of vision?

These questions arise in computational geometry and are answered through a mixture of geometric and combinatorial techniques.

**Matthew G Hudelson** (mhudelson@wsu.edu) Washington State University

### Adapting Hosoya's Topological Index as a Local Vertex Environment Descriptor

Introduced in 1971 by Haruo Hosoya, the "Hosoya index", also called the topological index or Z-index, of a structure enumerates the number of ways of partitioning the set of atoms into singletons and bonded pairs. We demonstrate adaptations of the Hosoya index that characterize the local environment of vertices within graphs. One such adaptation will be shown to link leaves of caterpillar graphs with continued fraction representations of rational numbers. We show how to link together two local Hosoya indices to facilitate quick sorting and comparison of local environments, specifically applied to analysis of organic chemical compounds.

**Mike Krebs** (mkrebs@calstatela.edu) California State University at Los Angeles

### Building Fast Communications Networks: A Brief Introduction to Expanders and Ramanujan Graphs

Think of a graph as a communications network. Vertices represent entities (e.g., people, computers) that wish to communicate with one another. Two vertices are connected by an edge if they can communicate directly with one another. Putting in edges (e.g., fiber optic cables, telephone lines) is expensive, so we wish to limit the number of edges in the graph. At the same time, we would like messages in the graph to spread as rapidly as possible. Assume that information travels instantaneously across edges, but that there may be delays at the vertices. We will see that the speed of communication is closely related to the eigenvalues of the graph's adjacency matrix. Essentially, the smaller the eigenvalues are, the faster messages spread. It turns out that there is a bound, due to Serre and others, on how small the eigenvalues can be. This gives us a rough sense of what it means for graphs to represent optimal communications networks; we call these "Ramanujan graphs." Techniques from group theory allow us to constuct regular graphs with many vertices but relatively few edges, and to estimate their eigenvalues. Families of k-regular Ramanujan graphs have been constructed in this manner by Sarnak and others whenever k minus one equals a power of a prime number. No one knows whether families of k-regular Ramanujan graphs exist for all k.

**Keith Brandt** (keith.brandt@rockhurst.edu) Rockhurst University

### Using Recursion to Study Mathematical Induction, Schur Numbers, and the Pill Problem

I will present three topics suitable for classroom presentations and student projects in discrete mathematics. The use of recursion is central to all three. One studies the parallel between writing recursive algorithms and certain types of mathematical induction proofs. The other two, a coloring problem and a question regarding probabilities, require significant computation.

**Rachel Marie Robertson** (rmrobert@mtu.edu) Michigan Technological University

### Solving Instant Insanity Without Going Insane

Instant Insanity is a puzzle consisting of four six-sided dice with each side colored red, blue, green or yellow. The goal is to stack the dice so that each side of the column displays each of the four colors. Solutions (found by a backtracking search and graph decomposition technique) have been known since the late 1960s. This puzzle illustrates multiple algorithmic solution techniques on an interesting combinatorial problem making it ideal for an introductory combinatorial algorithm course.

**Michael Nathanson** (man6@stmarys-ca.edu) St. Mary's College of California

### The Guessing Secrets Problem: Classical and Quantum Algorithms

Motivated by applications in internet routing, Chung, Graham, and Leighton introduced the Guessing Secrets problem, a variation on the game of 20 Questions. This game involves 2 players and a fixed parameter $k$. Player A selects $k$ objects from a known set $\Omega$, with $|\Omega| = N >> k$. Player B then attempts to learn about these objects by asking Yes-No questions, which we write as functions $q : \Omega \to \{Y, N\}$. For each question, Player A must select one of the $k$ objects and answer truthfully on its behalf.

In the case $k = 1$, this is simply a standard game of 20 questions, which may be efficiently solved in $\log N$ steps. When $k = 2$, this problem has a natural restatement in terms of graph theory that is simple enough to discuss with undergraduates but that still demonstrates the challenges that persist for all higher values of $k$. Successful applications of coding theory have recently been shown to solve this problem optimally in time that is quadratic in $\log N$ (though exponential in $k^2$).

The purpose of this talk is to provide an introduction to this fun, rich problem and to describe briefly the speaker's work in showing how it could be solved efficiently on a quantum computer.

**Pallavi Jayawant** (pjayawan@bates.edu) Bates College

### Minimum Spanning Trees: An In-Class Project

In a connected weighted graph, a minimum spanning tree is a connected acyclic subgraph that uses all the vertices of the graph and has minimum total weight. Prim's algorithm and Kruskal's algorithm are well known algorithms to find minimum spanning trees. I will discuss these algorithms and then talk about an in-class assignment that led to the development of an algorithm different from these algorithms. I will describe the context for the assignment and then give the details of the algorithm and the proof developed by my class.

**Mark Joseph Logan** (loganm@morris.umn.edu) University of Minnesota at Morris

### Egalitarian Matching and Normalized Matching

The college admissions problem studies many-to-one matching in bipartite graphs (each college is matched to the many students who will attend it). Traditional formulations of this problem assign a quota (the maximum enrollment allowed) to each college, and a great deal of research has focused on matchings which are stable in a game-theoretic sense with respect to the preferences of the colleges and the students. In this talk we will look at cases where the number of students enrolling at each college is desired to be equal (within 1 student) across colleges, without regard to preferences. This problem is connected to current undergraduate research in partially ordered sets with the normalized matching property.

**Bruce Walker Atkinson** (bwatkins@samford.edu) Samford University

### Numerals Based on the Golden Ratio

We use the *golden ratio* $\tau = \frac{1+\sqrt{5}}{2}$ as a base for expanding real numbers. Since $1 < \tau < 2$ and $1 + \tau = \tau^2$ then any $x > 0$ is a sum of distinct powers of $\tau$ which contains no two consecutive powers. $\tau-$*integers* have sums involving only non-negative powers of $\tau$; 0 is also a $\tau-$*integer*. (Note: $\tau-$integers appear in wavelet constructions used to analyze diffraction patterns in crystals.) Each $\tau-$integer determines its binary $\tau-$*numeral*; e.g. $\tau^4 + \tau^2 + 1$ determines 10101. The $\tau-$numerals $0, 1, 10, 100, 101, 1000, \ldots$ generate all binary words that do not contain the word 11. We present interesting patterns relating the $\tau-$integer sequence and the Fibonacci sequence. E.g, using $\tau-$numerals, it follows that the difference between consecutive $\tau-$integers is either 1 or $\frac{1}{\tau}$. Assigning $a$ to 1 and $b$ to $\frac{1}{\tau}$, the sequence of successive differences forms an infinite word with alphabet $\{a, b\}$, $f = abaababaabaababaababa\ldots$which is invariant under the substitution : $a \mapsto ab$, $b \mapsto a$. $f$ is the "limit" of iterates of this substitution starting with the word $a$. Counting the number of $a$'s and $b$'s in the iterates we get the Fibonacci sequence; the above substitution is called the *Fibonacci substitution* and $f$ is the *Fibonacci word*. (Note: Many of the results of this talk were part of an undergraduate research project directed by the speaker during the past academic year.)

**Joshua Holden** (holden@rose-hulman.edu) Rose-Hulman Institute of Technology

### The Pohlig-Hellman Exponentiation Cipher as a Bridge Between Classical and Modern Cryptography

The Pohlig-Hellman exponentiation cipher is a symmetric-key cipher that uses some of the same mathematical operations as the better-known RSA and Diffie-Hellman public-key cryptosystems. First published in 1978, the Pohlig-Hellman cipher was never of practical importance due to its slow speed compared to ciphers such as DES and AES. The theoretical importance of the Pohlig-Hellman cipher comes from the fact that it relies on the Discrete Logarithm Problem for its resistance against known plain text attacks, as does RSA and several other modern cryptosystems. For this reason, the Pohlig-Hellman system can play a very important role pedagogically, since it also shares many features in common with classical ciphers such as shift ciphers and Hill ciphers. Thus, it allows the instructor to introduce the important concepts of the discrete logarithm and known plain text attacks separately from the more conceptually difficult idea of public-key cryptography.

**Brad Bailey** (bbailey@ngcsu.edu)
**Dianna Spence** (djspence@ngcsu.edu)
**John Holliday** (jeholliday@ngcsu.edu) North Georgia College & State University
**Peter D. Johnson** (johnspd@auburn.edu) Auburn University

### Edge Cut Cycles and Cutting Numbers of Cycles and Graphs

Let $C$ be a cycle in a connected graph $G$. We define the cutting number of $C$ to be the number of components in the graph obtained from $G$ by deleting the edges of $C$. Whenever a cycle has cutting number at least 2, we call such a cycle an edge-cut cycle. In this talk, we discuss the motivation for these definitions, characterization of graphs with edge-cut cycles, and the necessary and sufficient conditions for a cycle to be an edge-cut cycle with respect to its graph $G$. We examine the cutting numbers of some bipartite graphs, and we also explore the relationship between $|V(G)|$ and the cutting number of $G$. In particular, we explore bounds on $|E(G)|$ and cutting number of $G$ for different orders and classes of graphs.

**Amy Mihnea** (amalyamy@yahoo.com) Florida Atlantic University

### Changing Dimensionality in Data Processing

Bringing data to lower dimensions is an important tool in reducing the complexity of a problem. I analyze some ways of doing that and establish a connection to clustering and image compression.

**Kay Smith** (smithk@stolaf.edu) Saint Olaf College

### Applications of Graph Theory to Conservation Biology

Ecological landscapes can be modeled using graphs. For example, nodes may represent habitat patches or roosts for birds, while edges represent movement between the patches. By using these models to study the structural organization of a landscape, biologists are able to analyze properties such as the relative importance of certain nodes in the network and the degree of connectivity between nodes. These properties affect the spread of disease, the vulnerability to disturbance of the landscape, and other issues related to conservation. In this talk we describe how these methods were applied in some studies of bird populations.

**Robert Dobrow, Rebecca Ferrell, Miranda Fix, Michael Duyzend** (rdobrow@carleton.edu, ferrellr@carleton.edu, fixm@carleton.edu, duyzendm@carleton.edu) Carleton College

### Cover Times for Stars, Sparklers, and the Petersen Graph

Given a random walk on a graph, the cover time is the time to visit all the vertices. In principle, the expected cover time can be computed by solving a system of linear equations. But even for small graphs, the number of such equations quickly becomes intractable. For the Petersen graph, by exploiting the symmetries of the graph and the isomorphisms of its subgraphs, we give an exact expression for the mean and standard deviation of the cover time. We also present exact results for the $n$-cycle, the star graph (a tree with a central vertex and rays of equal length), and the "sparkler" (a star with one long ray). Analyzing cover times of random walks gives an interesting application of combinatorial and linear algebraic methods accessible to undergraduates.

**Benjamin V. C. Collins** (collinbe@uwplatt.edu) University of Wisconsin at Platteville

### Fibonacci Trees: A Dream Come True

In the mathematical literature, there are two different structures that are called "Fibonacci Trees." Such ambiguity must not be allowed to stand! I will present the two different structures, and attendees at the talk will help me to decide which is appropriately called a Fibonacci Tree, and what we shall call the other one.

**Paul Weiner** (pweiner@smumn.edu) Saint Mary's University of Minnesota

### Viewing the Hamming Code in its Natural Habitat

Because the Hamming code lives in a high dimensional realm, it is difficult for us low-D folks to picture. However, the presenter has managed to get some candid shots of the Hamming code in its home environment, revealing an interesting geometric pattern.

Necessary background on the Hamming code and hypercubes will be provided.

**Kristina Garrett** (garrettk@stolaf.edu) St. Olaf College

### MIT vs Harvard: The Mathematics of Good Will Hunting

In the 1997 movie "Good Will Hunting" one math problem catapulted the main character into the mathematical limelight, thus setting the stage for the entire film. The problem, posed by a fictional MIT professor, combines elementary graph theory, linear algebra and some basic calculus in a fascinating way. In this talk we will examine the problem scrawled on a hallway blackboard at MIT and see if the movie consultants earned their fee by providing Matt Damon with a correct solution!

**Todd CadwalladerOlsker** (tcadwall@fullerton.edu) California State University at Fullerton

### Paint By Numbers: Constructing a Map That Needs Nineteen Colors

Most undergraduate mathematics students have heard of the Four Color Theorem concerning maps on a sphere. However, students may not be familiar with the generalization of the Four Color Theorem on other surfaces. In 1968, Ringel and Youngs proved a formula for the chromatic number of any surface (other than the sphere) using current graphs. In this presentation, we will see how these current graphs can be used to construct two nice, symmetric maps requiring exactly the chromatic number of colors: on the torus, we will construct a map requiring seven colors, and on the twenty-torus, a map requiring nineteen colors. Interested undergraduates may be able to extend these methods to create similar maps on other surfaces.

**David Molnar, Adam McDougall** (dmolnar@ups.edu, acmcdoug@math.uiowa.edu) University of Iowa

### An Unexpected Appearance of Continued Fractions

The Catalan numbers have a well-known interpretation as the number of paths of a given length constrained to a particular region of a lattice. The Fibonacci numbers have better-known interpretations, but can be described in this way as well. Some of my Discrete Mathematics students were looking at different interpretations of a generalization of the Catalan numbers, including a lattice-path interpretation. For one of those students, that project grew into a summer research project, during which we found another counting problem in a lattice, whose answer was surprisingly the numerators and denominators of the convergents of the continued fraction expansion of $\sqrt{3}$. I will give some details about how we stumbled upon this discovery, how we know it to be correct, and some possible generalizations.

**William Dickinson** (dickinsw@gvsu.edu) Grand Valley State University

### Equal Circle Packing On A Square Torus

The study of maximally dense packings of $n$ disjoint equal circles into various containers has developed over the past forty years. The optimal densities and arrangements are known for many packings of small numbers of equal circles into containers including squares, equilateral triangles and circles. In this presentation, we will explore packings of small numbers of equal circles into a special torus. We will illustrate the globally maximally dense arrangements in each case and describe the proof technique involved in demonstrating their optimality. The technique rests on identifying properties of possible packing graphs and an exhaustive enumeration of unlabeled embeddings of these graphs on the torus.

**Vince Matsko** (vmatsko@imsa.edu) Illinois Mathematics and Science Academy

### Edge Nets of Cubes

When making a paper cube, it is convenient to design a net; that is, a connected set of squares which may be cut out and folded together to form a cube. When the squares are folded together, pairs of edges from the squares come together to form edges of the cube. If one edge from each of these pairs is removed from the original net so that the remaining edges form a connected graph, these twelve edges form an edge net of the cube. In other words, an edge net is a connected graph with twelve edges of the same length which may be folded to make a cube.

A complete enumeration of the edge nets of the cube will be given. The enumeration involves an interesting interplay of algebra, geometry, and graph theory.

**Alan Alewine** (jaalewine@mckendree.edu) McKendree University

### A Graph Theoretic Proof That There Are Only Five Platonic Solids

The Platonic solids are convex polyhedra whose faces are made up of congruent convex regular polygons. We give a graph theoretic proof that that there are only five such polyhedra. This proof is accessible to undergraduates and has been used in both geometry and graph theory courses.

**William Griffiths IV, Albert Bush** (solo2987@yahoo.com, abush2@spsu.edu) Southern Polytechnic State University

### Generalized Permutation Descents and Inversions Controlled by a Drop Sequence

Descents and inversions have long been studied as basic properties of permutations. A descent is any entry of a permutation which is bigger than its successor. An inversion is counted for each pair of elements in the permutation, regardless of distance, which have a bigger ancestor element. We generalize these concepts to be controlled by a separate drop sequence, which determines the maximum distance between two elements of the permutation on which an inversion can be counted. Enumeration of these properties will be discussed for various drop sequences, as well as unimodality, log-concavity, and symmetry of the number of permutations with k-generalized inversions. This talk is the result of work done with an advanced undergraduate, and the focus will be on the counting and generating function arguments used to get some nice results.

# Innovations in Mathematics Teacher Education

## Saturday, August 2, 1:00 PM–3:00 PM and 3:15 PM–5:15 PM

**Alan P. Knoerr** (knoerr@oxy.edu) Occidental College

### Math 201: Mathematics, Education, and Access to Power

“Math 201: Mathematics, Education, and Access to Power” is a seminar course with a significant community-based learning component taught every semester in the Mathematics Department of Occidental College. Students include, but are not limited to, prospective teachers. The course was created jointly by mathematics faculty at Occidental and at nearby Franklin High School, in Highland Park, which is an economically-disadvantaged, predominantly Latino community in Northeast Los Angeles with a high percentage of English-language learners and a low rate of mathematical achievement. In addition to describing this course, we highlight the application of community-organizing principles in creating, teaching, and further developing this course. It also places the course and its effects within the context of a broader effort to change the culture of education in Northeast Los Angeles through a community-organizing approach that emphasizes long-term relationships and reciprocity.

**Jane Ries Cushman** (jcushman@math.buffalostate.edu) Buffalo State College

### Development of a Survey to Assess Pre-Service Teachers' Views and Uses of Problem-Solving in Mathematics

Mathematical problem solving is now a process strand in many state and National standards. Experiencing and valuing problem solving by our pre-service teachers is important in order for teachers to emphasize it in their future classrooms. A survey used to determine if pre-service teachers value problem-solving strategies and if pre-service teachers will use problem-solving strategies in their classrooms was developed. The following is the method used to develop the survey. On the last day of two sections ($N = 47$) of a non-lecture upper division mathematical problem-solving class, the students listed at least three ways their class experience was different from a traditional lecture-based mathematics class. The responses were compiled into 10 questions about problem-solving strategies and 10 questions about the use of problem solving in teaching of a mathematics class. Verification and validation used the following procedure: pre-service teachers (n=10) took the survey and then wrote what they thought each question meant. Their responses were coded positively if the statements matched what the researcher meant the questions to ask or negatively if not. One question was coded negatively for nine of the 10 responses (that question was rewritten).

**Michael B. Scott** (michael_b_scott@csumb.edu) California State University at Monterey Bay

### Supplementing Pre-Service Mathematical Content Courses with Online Homework

The lack of mathematical content knowledge is a significant issue in the training of pre-service elementary school teachers. Many pre-service teachers arrive in such programs with poor conceptual knowledge of K-8 mathematics and weak computational skills that often prevent them from acquiring the depth of mathematical proficiency needed to teach mathematics. At CSUMB we attempt to address this problem by supplementing our mathematics content courses with an online homework system. The online homework is designed to coincide with the material covered in each course and can be modified if the content changes. A natural question to ask is how do such online systems affect student learning of mathematics? The online homework system captures large amounts of data, such as the number of attempts, time accessed, all saved entries by the students and their grades earned. It also provides the students with access to help tutorials. In this talk, we will demonstrate the key features of the online homework, how our pre-service mathematics students have been interacting with the system and a preliminary analysis of the data generated by the system.

**Jon Hasenbank, Jennifer Kosiak** (hasenban.jon@uwlax.edu, kosiak.jenn@uwlax.edu) University of Wisconsin at La Crosse

### The Impact of a Teaching for Understanding Experiment in 8-12th Grade Mathematics

We report on the impact of a year-long professional development program designed to help middle- and high-school mathematics teachers improve the depth of students' knowledge of mathematics procedures. Students from nine treatment groups and five comparison groups were tested three times during the year to assess the impact of the program. The data collection scheme allowed for a detailed chronology of the changes in student skill and understanding both over time and between groups. We also triangulate the student impact by presenting the results of our classroom observations of teacher practice, which were obtained using a digital classroom observation instrument developed for the study.

**Theresa A. Jorgensen** (jorgensen@uta.edu) University of Texas at Arlington

### Vertically Connecting College and Middle Grades Mathematics: The Two-Problem Comparison Paper

We report on an innovative course element designed as a summative evaluation for use in mathematics courses for both pre-service and in-service teachers: the two-problem comparison paper. The goal of the two-problem paper is to facilitate the understanding of the vertical links between middle grades mathematics and college-level mathematics. Participants select a rich, college-level problem from the course, and a related problem from grade 4–8 mathematics. They must then place the problems within mathematical and pedagogical context at both levels, solve the problems, and highlight the mathematical connections between the two problems. The connections could be algorithmic, structural, and/or strategic in nature. The process aims to develop a better understanding of the unifying mathematical ideas across the grade levels, and to stimulate teacher reflection and learning. This course element has been piloted in both precalculus and calculus undergraduate courses for pre-service middle grades teachers, and in a Master of Arts program for K–8 mathematics. We will give initial results on the realization of the goals for this implementation.

**Melissa A Stoner** (mas606@lehigh.edu) Lehigh University

### Constructing a World of Mathematics in the Middle School

Through the National Science Foundation's GK–12 STEM program at Lehigh University, graduate students in the STEM (Science, Technology, Engineering, and Mathematics) disciplines are paired with K–12 teachers in the Lehigh Valley to enhance student learning and increase awareness and enthusiasm of the STEM disciplines. Students at the middle school level begin to question the motivation for learning mathematical concepts and are apathetic in their pursuit of mathematical knowledge. To combat this lack of enthusiasm and effort, the mathematics team, consisting of graduate students, professors, and K–12 teachers) at Broughal Middle

School have begun to construct lessons that fit within the existing curriculum that engage and attract students to mathematics. To achieve this we have grounded the mathematical concepts in real world problems, in particular the construction of their new middle school being built on the premises. Through assignments and written assessments we have seen students use drawings and diagrams to solve abstract problems that they understand at the concrete level. By pairing with teachers, graduate students help teachers to expand their concept of a traditional book-based teaching style to a complex learning environment that is stimulating and exciting.

**Tim Hendrix** (hendrixt@meredith.edu) Meredith College

### Putting a STAMP on Mathematics Teacher Education

The Science, Technology & Assessment integrated with Mathematics Partnership is a local Mathematics-Science Partnership grant designed to bring in-service middle and secondary mathematics and science teachers together for collaborative learning over three years. The report addresses both the challenges and the impact of designing and delivering mathematics and science content to participants with such a wide breadth and depth of background knowledge. The interactions between grade levels, the integration of content areas, the infusion of emergent classroom technologies, and the incorporation of formative assessment have created a collaborative learning community that can impact both classroom practice at the K-12 level and methods course instruction at the collegiate level. One particular focus has been to examine the ways in which mathematics can support science learning and in turn, areas of science that can provide a context for learning mathematics. The focus on technology and multiple representations of concepts has promoted this interdisciplinary approach. One theme that has emerged is the role of formative assessment in changing curricular decision-making and instructional practice. Implications for teaching middle and secondary methods and examples of significant curricula from the project will be shared.

**Donna Beers** (donna.beers@simmons.edu) Simmons College

### Preparing Elementary School Teacher Candidates to Meet Changing Licensure Requirements in Mathematics

The purpose of this presentation is to report how a small, private, comprehensive university in New England is responding to changing licensure requirements in mathematics. We will outline the differences between the old and new licensure requirements and describe recent revisions and additions to our undergraduate and graduate mathematics offerings to meet the changes, describing their content, format, and course materials.

We will specifically address: (1) how the courses have been designed to align with national and state standards, and to cover the content strands of elementary school mathematics; (2) how the courses meet the Massachusetts Commissioner of Education's report, Guidelines for the Mathematical Preparation of Elementary Teachers, whose goal is to assist college and university mathematics departments in providing pre-service teachers "...the scope and depth of mathematical knowledge - both skills and understanding - that are expected of elementary teachers and that will be assessed on the (MTEL) test;" and (3) how we will measure students' increase in knowledge of mathematics for teaching, using such instruments as the Mathematical Knowledge for Teaching measures developed by the Learning Mathematics for Teaching project of the University of Michigan-Ann Arbor.

**Jeff Gregg, Gayle Millsaps** (gregg@calumet.purdue.edu, millsaps@calumet.purdue.edu) Purdue University Calumet

### Focusing Pre-Service Elementary Teachers' Thinking on Children's Thinking in Order to Deepen Their Mathematical Understanding

The Mathematics Education faculty at Purdue University Calumet is developing materials for pre-service elementary education majors which address children's mathematical thinking. This session will highlight some of the activities we have developed and discuss their rationale. This initiative forms the primary component of our efforts to revise our three-course Mathematics for Elementary Teachers sequence so that it is consistent with professional standards in the field.

Having found most textbooks wanting in their attention to children's mathematical thinking, we are drawing upon our experiences in elementary and middle school math classrooms to modify inquiry-supporting math activities so that they provide an appropriate challenge for our college students. Using our first course on number and number operations as a source of examples, we will discuss how these activities a) enable us to focus on children's mathematics, b) promote a conceptual understanding of the mathematics our students will be teaching, c) allow our students to experience an inquiry approach to teaching and learning mathematics, d) provide the opportunity to discuss mathematics teaching and activity design strategies, and e) help students realize the importance of possessing a deep subject matter understanding that includes consideration of how children think about mathematics. We expect that these activities will enable teachers to construct pedagogical content knowledge and to develop what Liping Ma calls a "profound understanding of fundamental mathematics". Finally, we will discuss an assessment instrument we are using to evaluate our efforts.

**Abd Rahim Abd Salim** (rahim_salim@petronas.com.my) Universiti Teknologi Petronas

### Geometer's Sketchpad Software (GSP) as a Teaching Aid for Secondary School

In this era of educational development, the world of education is not lagging behind but at par with the progress of the country. There are a variety of softwares and methodologies used in teaching and learning mathematics with the current state-of-the-art technology. During this development and modernization of urban areas, a secondary school in Sarawak, Sekolah Menengah Kebangsaan Kapit No. 2, of Kapit district, that is remotely located in the rural areas of Sarawak, has attained some educational progress. The author uses the Geometer's Sketchpad Software (GSP) to teach mathematics on the topic of functions and graphs. This topic is the main focus because students are facing difficulties to comprehend the basic concept.Overall, the use of this software contributes towards the improvement of students' comprehension on the concept of functions and graphs. The purpose of this research is to study the effectiveness of using GSP in improving the students' understanding on the basic concepts of functions and graphs. The author selected 80 science students of that school as a study sample. Two groups were formed with 40 students each. The first group used the GSP while the other group used traditional method of learning mathematics. The author interviewed three of the best students from each group. The finding from this research shows that the application of GSP displays a significant improvement in the students' achievement in the topic.

**Bonnie Saunders, Janet Simpson Beissinger** (saunders@math.uic.edu, beissing@uic.edu) University of Illinois at Chicago

### Using Cryptography to Teach Number Theory to Future Middle School Teachers

Cryptography is a motivating setting for learning and applying mathematics. In an innovation for teaching our standard 'Number Theory for Future Middle-School Teachers' course, we incorporated the book *The Cryptoclub: Using Mathematics to Make and Break Secret Codes* (Beissinger, Pless, 2006). This book, written for use in middle schools, provided inspiration, motivation, and teaching experiences for our undergraduate students as they grappled with ideas from classical number theory. The college students prepared and presented lessons from the book. In the process, they learned how this engaging topic applies middle-grade mathematics such as negative numbers, percents and decimals, prime numbers, factorization, common factors, division with remainder, and exponents. Meanwhile, the instructor conducted a discovery and problem-solving approach to college level topics, including divisibility, modular arithmetic, Euclidean Algorithm, factoring large numbers, and Fermat's theorem. The two efforts combined for a culminating experience with RSA coding techniques. Results from exams and student evaluations indicate this was an effective and enjoyable way for pre-service teachers to learn.

**Kathryn Shafer** (kshaferlax@aol.com) Bethel College

### Technology, Conjecture and Proof: Exposing the Thinking of Teachers

This session will outline the methodology of a research study planned to "push" practicing (or in-service) geometry teachers to consider how technology facilitates student learning of conjecture, and ultimately, proof. The researcher plans to facilitate an action research project with teachers, paying specific attention to the teacher's reflective thinking while assessing student work. The methodology is grounded in theories on teacher change, professional development and technological, pedagogical and content knowledge (TPACK). Feedback from participants will be appreciated.

**Angela Marie Hodge** (Angela.Hodge@ndsu.edu) North Dakota State University

### University Geometry: Pre-Service Teachers' Views on its Role in the Classroom

As part of the requirements to complete their undergraduate degrees pre-service secondary mathematics teachers (PSMTs) complete many mathematics courses, including a geometry course. The role of this coursework in their classroom teaching, however, is not always apparent to PSMTs. This presentation describes a study in which the instructor was able to get the PSMTs thinking about the connections between classroom teaching and axiomatic geometry throughout the course. As a final project, all members of the class reported on how the course would help them in their future careers. This study reports on the responses from PSMTs and consequent implications for other mathematics courses populated in part by PSMTs.

**Kathryn Teresa Ernie; Erick Hofacker** (kathryn.t.ernie@uwrf.edu, erick.b.hofacker@uwrf.edu) University of Wisconsin at River Falls

### PRAXIS Project Wisconsin: Interactive Learning Objects

This paper describes our team's involvement on a statewide project that focused on the creation of web-based materials that will be disseminated to future teachers that need additional learning experiences related to their mathematical understanding before taking the PRAXIS exam. Completion of the PRAXIS exam is required of pre-service teachers in the state of Wisconsin before being admitted into their teacher education program. The web-based materials will also be available to practicing teachers at the K-8 level that would like to make use of them in their current classrooms.

The paper presents our team's methodology, which included undergraduate student involvement, to create the materials. Created materials were based upon released mathematics items from previous PRAXIS exams. Our undergraduate student experts were responsible for creating static storyboards in power point, which were turned into dynamic learning objects. Each completed problem includes visual tutorials, podcasts for solving problems, additional problems with feedback, and an interactive applet that allows the user to do some exploring and conjecturing about the problem presented to them.

Novice mathematics students assisted in the process in order to prepare our undergraduate student experts to understand why students may have misconceptions with the problems presented to them. The novice students participated in team meetings, and were interviewed while solving the math problems, in order to provide feedback and knowledge to the undergraduate student experts during multiple phases of the creation and editing process.

**Kien Hwa Lim** (kienlim@utep.edu) University of Texas at El Paso

### Using Prediction Items and Clickers to Address Misconceptions

Students tend to overgeneralize, such as with "multiplication makes bigger" and "division makes smaller." In a pre-test, 14 out of 23 middle-school preservice teachers inserted "<" between $98765432/12345678$ and $1357/2468$ while 11 students chose ">" where only one is a proper fraction. To help these students address such misconceptions, a sequence of five prediction items were designed and used in conjunction with clickers and PowerPoint slides. For each item, students were asked to make a prediction, write their reasoning for their prediction on a piece of paper, and input their prediction using clickers into a personal response system. They were then asked a rhetorical question such as, "Many children will choose 'Never True' for the statement: '$N$ is a natural number, $67/89 \times N < N$.' Why?" The rationale underlying such a response and the explanation for the correct response were presented using animated PowerPoint screens. The items were designed to make explicit the difference between multiplying/dividing a value by a proper fraction and that by an improper fraction, as well as to review mathematical ideas such as the difference between multiplication as repeated addition and multiplication as enlargement. There is an improvement from 49% in the pre-assessment to an average of 80% in post-assessments.

**Jeff Johannes** (johannes@member.ams.org) State University of New York at Geneseo

### Place Value Arithmetic via Polynomials

In our secondary capstone course, we discuss a month-long analogy between polynomials and place value arithmetic. We use this analogy to review arithmetic of integers, fractions, and decimals, all in arbitrary base. Furthermore, we pursue connections unique factorization, and conversions between fractions and decimals. This discussion concludes with two open mathematical questions arising from the analogy between numbers and polynomials.

# General Contributed Paper Sessions

## 8:30 AM–10:30 AM, 1:00 PM–3:00 PM and 3:15 PM–5:15 PM each day

**Scott Dillery** (dillerys@lindsey.edu) Lindsey Wilson College

### Report on Progress in Developmental Algebra at Lindsey Wilson College

Lindsey Wilson College has seen significant improvement in the success rates in its developmental algebra courses over the last four years. The improvement has come with the addition of an online homework component and a restructuring of the courses to allow for restarts mid-semester. In this talk I will share what we have done and the lessons learned in implementing our strategy.

**Erin M. Hodgess, Carol Vobach** (hodgesse@uhd.edu, vobachc@uhd.edu) University of Houston - Downtown

### RcmdrPlugin.epack: A Menu Driven Package for Time Series in R

R is an excellent open source statistical software package. Many undergraduate students prefer using menu driven software to command line software, so they often balk at using R. We have put together a plug in package for use in time series and forecasting classes, called RcmdrPlugin.epack. Nearly all of the typical material in an undergraduate time series class is available via menus in this package. Students have used the new package and found it very useful.

**Michael Edward Zeidler** (c_zeidler@ameritech.net) Retired

### An Idealized Mathematical Model of a Runner Built-up from the Angle-of-Lean

Four measures of a runner (weight in kilograms, angle-of-lean in radians, stride in meters, and velocity in meters per second) are used in this model. The model arose while answering the question as to why ice skaters and runners lean. The runner's weight is used to obtain the vertical force. The runner's angle-of-lean is used to obtain the horizontal force. The runner's axis-of-lean serves as the center-of-rotation of damped harmonic motion. From these forces it is possible to calculate the components of energy expenditure by a runner. Under this model, the stride consists of three mutually exclusive stages: acceleration, inertial flight, and deceleration. These stages are under supervision of a control-and-communication system that appears to manipulate timing. This model provides a mathematical framework for analyzing a specific runner or for comparing runners.

**Jiten C. Kalita** (jiten@iitg.ernet.in) Indian Institute of Technology Guwahati

### A Streamfunction-Velocity Based Simulation of the Laminar Flow Past a Square Cylinder

We investigate the confined flow around a cylinder with square cross-section mounted inside a plane channel with blockage ratio $B = \frac{1}{8}$. A recently developed transient $\psi - v$ formulation on compact uniform grid has been used. Numerical results are presented for Reynolds number $Re$ up to 200. For the $Re$s considered in the study, the computed results on relatively coarser grids are extremely close to the experimental and established numerical results. We also able establish the periodic nature of the flow in the range of $60 \leq Re \leq 200$.

**Maocheng Liu** (drule@163.com) China Petroleum University

### Probe into Goldbach Problem

A new concept named root prime number is raised, to formulate a concise formulae calculating accurately number of primes not larger than $N$ and help to reveal the real quality of distribution of the primes. It is revealed out that the Goldbach's problem of a even number N corresponds to some kind of binary distribution of the primes from 3 to $N - 3$, having the same topological structure and corresponding to one same probabilistic model, under which we obtain the solution $D(N)$ and its infinitely asymptotic expression. It is pointed out that essence of the Goldbach's problem is binary generalization of the prime number theorem.

**Jim Sullivan** ( jjjjteacher@juno.com ) Dallas ISD

### The Box Method for Teaching Ratio and Proportion Problems

While engaged in teaching students how to solve ratio/proportion problems in high school algebra, the author recognized that his students could perform the arithmetic calculations flawlessly but had great difficulty setting up the problems. As a result, the students computed the right answers to the wrong problems!

In order to rectify this, the author developed a methodology to allow students to properly set up such ratio/proportion problems. Once students understand the proper method to set up this class of problems, obtaining the correct answers is easy for almost all students.

This paper discusses the methodology of the "box method" and demonstrates how it is used with several problems. Such problems arise in engineering and various other disciplines on a frequent basis.

**Roland Shen** (rolandshen@yahoo.com) Olympia Institute

**Beyond Classic: New Solution to Generalized Quadratic Function and Equation**

Based on the quadratic function used in deriving the prominent and widely-applied Steepest Descent and Conjugate Gradient optimization methods, we first propose a new generalized complex quadratic function, and then take a novel approach in complex analysis to derive the properties and solutions for the new function. Computer simulations are performed to visualize the analytical results and provide further analysis. In addition, we prove that the classic solutions of the quadratic equation are special cases of the new theoretical findings and show the condition under which the generalized solutions exist.

**Pamela B. Pierce, Jeffrey Willert** (ppierce@wooster.edu, jwillert09@wooster.edu) The College of Wooster

**Dissecting $2n$-Gons to Approximate the Circle-Squaring Process**

A famous problem of Alfred Tarski asks whether a circle can be decomposed into finitely many pieces which can then be rearranged to form a square. Laczkovich proved in 1990 that this is indeed possible, but his proof is non-constructive and requires non-measurable sets. Using the idea that a circle can be approximated with as much precision as we desire by an appropriate $2n$-gon, we provide a concrete method for "approximate circle-squaring". We begin by outlining a method for dissecting any $2n$-gon. In the spirit of Laczkovich's proof, our rearrangement process will use translations only. No rotations or reflections will be necessary to form the final square.

**Scott Hochwald** (shochwal@unf.edu) University of North Florida

**The Harmonic Series: Used, Abused, and Confused**

We offer several proofs that the harmonic series diverges. Each of these proofs and others in the talk have some sort of WOW factor. We then decimate the series in various ways. Some decimations leave convergent series (remove all terms with the digit 9 in the denominator) and others still diverge (the reciprocals of the primes). We conclude by moving from decimations to truncations, and view numerators through a number theory lens.

**John E. dePillis** (jdp@math.ucr.edu ) University of California

**The Lorentz Transformation as a Visual Consequence of the Swiveled Line Theorem**

Special Relativity is based on the assumption that $c$, the speed of light in a vacuum, is the same for all observers, regardless of the relative speeds of the light source or of the observers. This physical assumption is visually modeled as a theorem in plane geometry (the Swiveled Line Theorem) which, using basic linear algebra, finally produces the classic Lorentz Transformation.

**Nancy Rodgers, Doug Anewalt** (rodgers@hanover.edu, douganewalt@uky.edu) MAA

**Illuminating Group Lattices**

We will demonstrate how to use the software program, Groups Unlimited, (www.groupsunlimited.net) so that students can have hands-on experience with finite groups in a colorful and exciting format. By working visually with subgroup lattices, generating sets, mod $n$ groups, dihedral groups, permutation groups, quotient groups, and direct products from an extensive group library, students can deepen their understanding and appreciation of basic concepts in group theory.

**Mike Krebs** (mkrebs@calstatela.edu) Cal State LA

**The "No WAY!" Moment in Mathematics**

Much has been written, and many tales told, of the so-called "Aha!" moment in mathematics. In this talk, we discuss a subtly different but no less pleasurable instant, namely the "No WAY!" moment. By way of illustration, we consider an unexpected turn of events the speaker once encountered while exploring a small variation on Cantor's original proof of the uncountability of the real numbers.

**Jennifer McLoud-Mann** (jmcloud@uttyler.edu) University of Texas at Tyler

**Lessons Learned from a Calculus Redesign Project**

In this talk we will discuss the lessons learned from a Calculus Redesign project involving on-line instruction. Unlike traditionally redesign projects, both the redesigned course and the regular course were taught at the same time. We will focus the talk on comparative data for students in both kinds of sections. Questions to be addressed include: How do the student outcomes compare (especially on the common final exam)? How do students feel about taking these courses? What adjustments can be made in the future to improve the courses we offer?

**William Calbeck** (billc@lsua.edu ) LSU-Alexandria

### Dual Enrollment Using mathxl

I will discuss how the dual enrollment program at LSU-Alexandria is implemented using high school teachers and mathxl software. I will primarily focus on college algebra and trigonometry, explaining how assignments are handled and credit awarded.

**Vince Matsko** (vmatsko@imsa.edu) IMSA

### Challenging Gifted High School Students

At Quincy Senior High School in Quincy, Illinois, an increasing number of juniors complete calculus. The Creative Problem Solving in Mathematics course was designed for these students as seniors.

The course primarily focuses on the study of polyhedra and geodesic structures using spherical trigonometry. Additional topics include envelopes, inversion, taxicab geometry, four-dimensional geometry, matrices, and graph theory. Hands-on work, include the building of geometrical models as well as individual and group projects, is emphasized. Come to hear about the course and see if it might be appropriate for a high school near you.

**Sayel Ali** (alis@mnstate.edu u) Minnesota State Univ. Moorhead & The Petroleum Institute/Abu Dhabi
**Radwan Al-jarrah** (radwan.aljarrah@swosu.edu ) Southwestern Oklahoma State University

### Modified Taylor Polynomials

For an infinitely differentiable function on $[-a, a]$, we will find a unique polynomial of degree $n$ that equals $f$ and its first $m$ derivatives at 0 and equals $f$ at a finite number of non-zero points in $[-a, a]$. We will show that this polynomial converges to $f$. We will give examples that will show that the graphs of these polynomials converge to the graph of $f$ fast.

**Sayel Ali** (alis@mnstate.edu u) Minnesota State Univ. Moorhead & The Petroleum Institute/ Abu Dhabi
**Radwan Al-jarrah** (radwan.aljarrah@swosu.edu ) Southwestern Oklahoma State University

### A Generalization of Taylor's Theorem: A Determinant Approach Using Mathematical

In this presentation, we utilize Cramer's Rule in constructing the interpolating polynomials of Taylor, Lagrange, Hermite, and beyond. We use the power of Mathematica in evaluating some determinants and arriving at conjectures that would then be proved mathematically. The Vandermonde determinant and a generalization will be highlighted in the construction process.

**Glenn Hurlbert** (hurlbert@asu.edu) Arizona State University

### Facilitating IBL Classroom Issues with Technology

We will describe the integration of a number of technologies, hard and soft, new and old, that we used recently in a Moore method course on problem-solving and proof-writing. These include a laptop tablet, windows journal, pdf annotator, and data projector, among other things. The ways in which these technologies changed my delivery of the method were significant and robust, and the benefits to students, I believe, were many.

**Linda Becerra, Ron Barnes** (becerral@uhd.edu, barnes@dt.uh.edu) University of Houston-Downtown

### Mathematical Myths: Some Interesting Facts and Fictions

This paper considers a number of key individuals in the history of mathematics. Some interesting facts and fictions that have accrued to their reputations will be briefly discussed. The cast of characters ranges from Thales, Pythagoras, Aristotle, Euclid and Hypatia to Sophie Germain, Gauss, Bolyai, Hilbert and Russell. Special mention of some mathematical Martians, i.e., mathematicians with ideas that radically altered mathematical history, will also be discussed, including Kepler, Newton, Leibniz, Galois, Ada Byron Lovelace, Cantor, Ramanujan, Gdel, and Turing. On the lighter side, the talk will conclude with the Cardano versus Tartaglia quarrel and the prolific publication exploits of Nicolas Bourbaki.

**Sanjeev Sharma, Manoj Sahni** (sanjiit12@rediffmail.com, manoj_sahani117@rediffmail.com) Jaypee Institute of Information Technology University, Noida

### Creep Transition of Transversely Isotropic Thin Rotating Disc

Creep stresses and strain rates have been obtained for a thin rotating disc using Seth's transition theory. A disc made of transversely isotropic material rotating with higher angular speed increases the possibility of fracture at the bore as compared to a disc made of isotropic material and possibility of fracture further decreases with the increase in measure $N$. The deformation is significant for transversely isotropic disc for the measure $N = 7$.

**Sanjeev Sharma, Manoj Sahni** (sanjeev.sharma@jiit.ac.in, manoj_sahani117@rediffmail.com) Jaypee Institute of Information Technology University, Noida

### Elastic-plastic Transition of Transversely Isotropic Thin Rotating Disc

Elastic-plastic stresses have been derived for transversely isotropic thin rotating disc by using Seth's transition theory. Results obtained have been discussed numerically and depicted graphically. Thin rotating disc made of transversely isotropic material yields at a higher angular speed as compared to disc made of isotropic material. Rotating disc made of isotropic material required high percentage increase in angular speed to become fully plastic from its initial yielding as compared to disc made of transversely isotropic material. Rotating disc made of transversely isotropic material is on the safer side of design as compared to rotating disc made of isotropic material.

**Sanjeev Sharma, Manoj Sahni** (sanjeev.sharma@jiit.ac.in, manoj_sahani117@rediffmail.com) Jaypee Institute of Information Technology University, Noida

### Elastic-plastic Analysis of Thin Rotating Disc Having Variable Thickness and Variable Density With Edge Loading

Elastic Plastic stresses have been derived for a disc having variable thickness and variable density with edge loading by using transition theory. The effect of edge loading has been discussed numerically and depicted graphically. It is concluded that a rotating disc having variable thickness and variable density with edge loading requires high percentage increase in angular speed to become fully plastic than to its initial fielding as compared to a rotating disc having variable thickness and variable density with edge loading.

**James Hamblin** (jehamb@ship.edu) Assessing General Education Mathematics Courses

### Assessing General Education Mathematics Courses

This talk will follow the development and implementation of an assessment program for General Education Mathematics Courses. I will give suggestions for how to develop a program that meets typical assessment guidelines and is also meaningful and useful to the faculty in the department.

**Gary Chartrand** (gary.chartrand@wmich.edu ) Western Michigan University
**Futaba Okamoto** (okamoto.futa@uwlax.edu) University of Wisconsin - La Crosse
**Zsolt Tuza** (tuza@sztaki.hu) Hungarian Academy of Sciences / University of Pannonia
**Ping Zhang** (ping.zhang@wmich.edu ) Western Michigan University

### Three Colorings in Graphs

The best known vertex coloring of a graph is a proper coloring. Two related colorings are complete and Grundy colorings. Relationships among these three colorings are described.

**Duokui Yan** (duokuiyan@hotmail.com) Brigham Young University

### Simultaneous Binary Collision for the collinear four body problem

In this paper, we use canonical transformations to collectively analytically continue the singularities of the simultaneous binary collision solutions for the collinear four-body problem in both the decoupled case and the coupled case. An important first integral constant C is discovered and it helps us see a clear picture about the behavior of the decoupled case. All the solutions are found and more importantly, we describe the relationship between the decoupled solutions and the coupled solutions.

**Mulatu Lemma** (malatulemma@bellsouth.net) Savannah State University

### Milking The Fascinating Applications Of Geometric Power Series

The Geometric Power Series is a series of the form $\sum_{k=0}^{\infty} ax^k$. The purpose of this study is to investigate some important properties and applications of the geometric power series $\sum_{k=0}^{\infty} x^k$ in $|x| < 1$. We will show that the Geometric power series has surprisingly many applications. Impressive problem solving techniques will be discussed. By considering different examples, we will show that the fascinating applications of the geometric power series are everywhere.

**Dan Kalman** (kalman@american.edu) American University

### Return to Polynomia: Uncommon Excursions for the Seasoned Visitor

Math students and teachers have visited the Province of Polynomia many times. They are intimately familiar with all the main routes and attractions, including factorization of quadratics, graphs, synthetic division, properties of rational roots, and the divisor and remainder theorems. But the Province has many other interesting diversions for the return visitor, just a short way off the beaten track. Return with me to Polynomia to take several short uncommon excursions. On our itinerary are Horner's form, Palindromials, and Lill's method, among other topics.

**Jason Joseph Molitierno** (molitiernoj@sacredheart.edu ) Sacred Heart University

### Teaching Statistics from A to Z

In any course, the more often you teach it, the more of a rhythm you get into. After having taught Business Statistics several times, I have settled into a routine that really works. In this talk, I will describe the syllabus I use and the topics that I cover in a course that meets 26 class periods (75 minutes) per semester—hence the title "A to Z." I will describe lecture topics, group assignments, and assessment tools that I give and how they fit into my overall pattern for teaching a Business Statistics course.

**Patricia Kiihne** (pkiihne@ic.edu) Illinois College

### Using a Homework Notebook in College Algebra

"We learn mathematics by doing mathematics." Math professors tell their students this, but students don't always follow it, particularly in lower level courses. Using homework notebooks can not only encourage students to work the problems but can also help them organize their work. This talk will describe how homework notebooks were used in a college algebra course, including advantages and disadvantages.

**Kate McGivney** (kgmcgi@ship.edu) Shippensburg University

### Frontloading Statistical Inference Topics in an Introductory Statistics Course

In many introductory statistics courses, inferential statistics is a topic which is introduced during the later part of the course. In this session we will discuss one strategy for moving this important (and often challenging) concept to the front end of the course. We will also share data-collection projects and writing assignments that have been used to strengthen students' knowledge of statistical inference.

**Tony Weathers** (tonyweathers@adams.edu) Adams State College

### Open Source Software: What can it do for you?

The term "Open Source" refers to a software development method that incorporates community peer review and participation via freely redistributable source code. As such, it provides free alternatives to proprietary software and operating systems. Among mathematicians, the most well-known example of open source software is the TeX typesetting system. Perhaps the most widespread example of open source is the Linux operating system, developed in the early 1990s to mimic the Unix operating system. Once the haven of hardcore computer junkies, Linux has become a popular, user-friendly, and powerful alternative to proprietary operating systems such as Windows or Mac OS.

Linux is just the tip of the iceberg, however. There are hundreds of open source programs available, many of which provide functionality equal or superior to proprietary programs. We will take a look at several of these that are of particular interest to teachers and students of mathematics.

**Rebecca S. Wills** (wills@roanoke.edu) Roanoke College

### Google's PageRank for Beginners: A Directed Graph Example for Liberal Arts Math Courses

Google's reputation for superior search engine results is well-known. A component of Google's overall ranking of query results is PageRank, a probability score computed for each indexed webpage. The algorithm developed by Google founders Larry Page and Sergey Brin to determine PageRank scores models the link structure of the Web as a directed graph. Although the mathematics behind the PageRank algorithm is an excellent topic for any linear algebra course, the details of computing PageRank are readily accessible to students in liberal arts math courses. By presenting the PageRank algorithm as a model of Web surfer behavior, we provide an introduction suitable for inclusion with any discussion on directed graphs.

**Terry Jo Leiterman** (terryjo.leiterman@snc.edu) St. Norbert College

### The Ultimate Class Project...and No Grade!

To the students, the task sounded impossible. Build a square wheel bicycle that moves smoothly; one you could pedal the whole day through. With an ah-ha moment, a short loss of interest, many hours, several mistakes, and no grade, a square wheel bicycle was built (and rode!) by a class of twelve Mathematical Modeling students. In this talk, a teacher with plans to strengthen her students' problem-solving skills via the spirit of mathematics shares a fortunate success with a pedagogical experiment. A story with objectives, uncertainties, surprises, and missteps is told. Student reflections confirm that the goal, and more, was met!

**Michael Wodzak** (mawodzak@viterbo.edu ) Viterbo University

### Who was that Bearded Polack? Shakespeare's Response to the Copernican Revolution

The evidence that Shakespeare was familiar with the mid Sixteenth Century English mathematics textbooks of Robert Recorde can be easily seen in such plays as "As You Like It", "Henry V" and, most notably, "The Merchant of Venice", which play demonstrates a remarkable familiarity of much of what was the Modern Mathematics at the late Sixteenth Century. We analyze this evidence and use this viewpoint as a lens through which to view the themes in "Hamlet" and suggest that, at one level, the Prince of Denmark should be viewed as an allegorical Everyman set adrift in the new heliocentric universe.

**Trae Holcomb** (trae.holcomb@usafa.edu) United States Air Force Academy

### Developing Mathematical Thinkers—Laying the Foundation for Successful Math Majors

We discuss useful tips on developing and running a first course in mathematical thinking and communication (to include proof writing). We will cover a number of novel techniques successfully used to spur student development. These include a point-free (but not pointless) grading system based on specified criteria and rubrics, reading assignments that are clandestine writing assignments, midterm and final oral exams, and student self-assessments against the course objectives.

**Charlotte Schulze-Hewett** (hewettj@aol.com ) Harper College

### Compass and Straightedge Constructions, without a Compass

Can you bisect an angle, using just a paper clip, a straightedge and a couple of pencils? In this talk, we will explore which compass and straightedge constructions can be done using a paper clip in place of a compass.

**Christopher K. Cartwright** (cartwright@ltu.edu) Lawrence Technological University

### Applying Active-Cooperative Learning in Calculus 1

This project applies Active-Cooperative Learning (ACL) theory to Calculus 1. Research shows that these techniques increase student progress and content retention. Particular ACL techniques implemented were team projects, code of cooperation, team process check, and peer-rating. Teams of at most four students were formed based on research recommendations. Academic progress and effectiveness was assessed and quantitative and qualitative outcomes are presented.

**Ranjith A. Munasinghe** (ranjith.munasinghe@mail.wvu.edu) WVU Institute of Technology

### An Integral Representation for $\zeta(n)$

In recent literature we find several proofs for Euler's formula that evaluates Zeta function, $\zeta(n) = \sum_{k=1}^{\infty} \frac{1}{k^n}$, for positive even integer values of n. Other publications have shown recursive methods for evaluating $\zeta(2n)$ . In this paper we present an alternative method to find $\zeta(2n)$, $n = 1, 2, \ldots$. Techniques used in this method are simple and not beyond the comprehension of an undergraduate student who has completed a first course in differential equations. Exact value of $\zeta(n)$ is not known for positive odd integer values of $n \geq 3$ . We find an improper integral that can be used to approximate $\zeta(2n + 1)$ .

**Mandi Shea Maxwell** (mandi.maxwell@trnty.edu) Trinity Christian College

### Visual Mathematics for the Visually Impaired: Reflections and Strategies

Liberal Arts Math classes are exciting to teach because of their visual nature, but how do you instill this excitement in a student if s/he cannot see? In this paper, I will reflect on strategies and techniques that I utilized in my Liberal Arts Math class to help a blind student "visualize" mathematics. I'll discuss the challenge of balancing the needs of those who need to "see" in order to comprehend with the needs of students who cannot see. I'll discuss specific strategies that we employed along with critiques of our successes and failures, as well as suggestions to help those who may face similar challenges.

**Steven Morse, Elisha Peterson** (steven.morse@usma.edu, elisha.peterson@usma.edu) United States Military Academy

### Signed Graph Coloring, a Theorem of Jacobi, and the Art of Linear Algebra

Signed graph colorings can be used to define a functor between specific classes of labeled graphs and multilinear functions. Under this correspondence, the "simplest" graphs correspond precisely to standard constructions of linear algebra such as the determinant, the trace, and the adjugate of a matrix. The result is a beautiful yet powerful calculus in which multilinear functions can be manipulated with ease. Using this graphical notation, we provide a one-line proof of a well-known theorem of Jacobi regarding matrix minors and determinants whose classical proof spans several pages.

**Karen Rhea** (krhea@umich.edu) University of Michigan

### Calculus: Where we've been; Where are we going?

What changes have the past two decades brought to Calculus instruction and undergraduate education? Curriculum and pedagogical adjustments have affected mathematics classes from elementary school to university. In this talk we will look at the impact of some of those changes on Calculus and begin a discussion of how we can categorize and capitalize on any gains we have made as we all move forward.

**Mahmoud F Almanassra** (mahmoud.almanassra@uwc.edu) UW-Marinette

### The Explicit Solution to an Infinite Linear Differential Equation System

Considering the mathematical model that estimates the optimal damage of cancer cells by adding genetic drugs, a system of infinite linear differential equations has been posed. In particular, we are interested in finding the explicit solution to the system. I applied mathematical techniques and the Mathematical Induction method and have found the explicit solution to the infinite linear differential equation system.

**Kenneth Cary Millett** (millett@math.ucsb.edu) University of California

### The Effect of Knotting on the Shape of Polymers

Momentary configurations of long polymers at thermal equilibrium usually deviate from spherical symmetry and can be better described by, on average, by an ellipsoid. The radius of gyration, asphericity, and prolateness, which describe their shape, have been studied both theoretically and experimentally for linear, circular, and branched polymers. We describe the effects of knotting on the shape of circular polymers as their length increases. We observe random polygons forming different knot types reach asymptotic shapes that are distinct from the ensemble average. For the same chain length, more complex knots are, on average, more spherical than less complex knots.

**Robin Lydiann Anderson, Southwestern Illinois College** (robin.anderson@swic.edu) Southwestern Illinois College

### Math Chats: Engaging Students in Mathematics Outside the Classroom

As educators, many of us face the challenge of generating student interest in mathematics. To address this issue, Southwestern Illinois College created "Math Chats," a series of informal talks about various math topics and careers that use mathematics. Southwestern, a two-year community college, currently has six chats a year with attendance averages of 90 students per talk. The students' ability levels range from developmental algebra to differential equations. Come hear their secrets!

**Jonathan Paul Lambright** (lambrij@savstate.edu) Savannah State University

### The Next Phase In Developing Effective Teaching Techniques For Undergraduate College Algebra Courses

Since the summer of 2005, a core of mathematics faculty at Savannah State University have been involved in a pil0t project that focuses on changing the was college algebra is taught. The project was initiated at The United States Military Academy in West Point, NY. under the direction of Dr. Don small. Several other institutions are involved in the effort and have been seeing positive results. Students continue to arrive in the college classroom under prepared in mathematics and they struggle one enrolled in college algebra and follow-on courses. This pilot project emphasized a refocus of course content, presenting problems from a real world perspective, and in class group exercises and board work. The pilot project at Savannah State University has been effect for four semesters and we have five sections being taught. This paper presents the structure of the project, results to date, challenges, and the future direction.

**Salim Wehbé Salem** (salim.salem@fi.usj.edu.lb ) Ecole Suprieure d'Ingnieurs de Beyrouth

### Teaching Mathematics to Lebanese Engineers : The Experience of the Ecole Supérieure d'Ingénieurs de Beyrouth

Students enrolled at the Ecole Supérieure d'Ingénieurs de Beyrouth, a Lebanese school of Engineering at the Saint-Joseph University, follow a two years preparatory curriculum, during which their Mathematics study program is a French (Boubakist) one. In this paper, we discuss the main topics of this program, its benefits, its shortcomings and the problems encountered by students.

**Yossi Elran, Michal Elran** (yossi.elran@weizmann.ac.il, michal.elran@weizmann.ac.il) Weizmann Instutute of Science

### Math-by-Mail - A Correspondence Program in Recreational Math

Math-by-mail is an extracurricular math enrichment program that gives students in school the opportunity to correspond with mathematicians and discuss topics in recreational math. The program is run by the Davidson Institute of Science Education at the Weizmann Institute of Science. The correspondence is done through a series of math booklets dedicated to topics in recreational math. The booklets are available either in hardcopy or online via a special purpose website dedicated to the program. The correspondence with the mathematicians is conducted via an online forum available at the website. The completed booklets are sent to the mathematicians by mail or submitted electronically for review. Math-by-Mail provides the talented student with enrichment that stimulates creative thinking and expands the student's knowledge way beyond the limits imposed by the curriculum and their peer environment. Teachers who run the program in their schools have found Math-by-Mail useful for talented students, providing them with mathematical challenges and broadening their math horizons beyond the scope of the regular class curriculum. The program is currently run in five countries and four different languages (Hebrew, English, Spanish, Arabic) with over 2000 participants. Feedback from the program, both from teachers and students is very positive.

**Laszlo Erdodi** (lerdodi@emich.edu ) Eastern Michigan University
**Zsolt Lavicza** (zl221@cam.ac.uk) University of Cambridge

### Statistics'r'us—Reshaping Students' Attitude from Aversion and Anxiety to Curiosity and Confidence

This paper proposes possible ways to resolve two important issues in relation to students' preparedness and motivation that instructors face in introductory university-level statistics courses. A large number of students enter introductory courses with both the lack of theoretical foundation in formal quantitative reasoning and the lack of motivation stemming from an inability to see the relevance of the material to one's chosen field of study. Creating lively analogies grounded in everyday experiences has the potential to ease these problems. After years of qualitative data analysis (student feedback), the utility of examples, metaphors and stories illustrative of core statistical concepts has become apparent. Two specific psychological mechanisms of action are hypothesized: 1) Demystifying (well-chosen analogies demonstrate that seemingly complex concepts can be understood using simple logic if their similarity to some already familiar phenomena is pointed out); and 2) Self-referencing (emphasizing the commonality between quantitative thinking and student's life experiences). Examples range from basic visual aids to computer modeling of sampling distributions, word problems that capitalize on current events, making concepts relevant to students. Strategic use of humor is an important catalyst in creating a pleasant atmosphere in which lasting memories can form. The implicit message in the described methodological adjustment is that statistics is not an abstract, counterintuitive science detached from the practicality of everyday life, but merely an attempt to formalize decision making rules people spontaneously apply. The heuristic value of the method lies in revealing the connection between typical human thinking and statistical reasoning has both cognitive and psychological advantages. Demonstrating that students know more about statistics than they are aware facilitates the learning process by uncovering new connections with the material, and provides a sense of mastery at the same time.

**Janet E. Mertz** (mertz@oncology.wisc.edu) University of Wisconsin - Madison
**Titu Andreescu** (txa051000@utdallas.edu ) University of Texas - Dallas
**Joseph A. Gallian** (jgallian@d.umn.edu) University of Minnesota - Duluth
**Jonathan Kane** (kanej@uww.edu ) University of Wisconsin - Whitewater

#### Cross-Cultural Analysis of Females Identified with Exceptional Mathematical Talent

Women are scarce among the faculty in the very top-ranked graduate mathematics departments in the USA. Lawrence Summers speculated that a major contributing factor for this might be the scarcity of females who possess extremely high intrinsic aptitude for mathematics. To test this hypothesis, we determined the ethnicities and birth countries of females identified over time as having truly exceptional aptitude in mathematical problem solving as measured by the Putnam Mathematical Competition, International Mathematical Olympiad (IMO), and USA Mathematical Olympiad. These data were compared with the ethnicities and birth countries of the women currently on the faculty of the top 5-ranked graduate mathematics departments in the USA. The findings from these analyses will be presented.

**Siamack Bondari; Monika Vo** (siamack.bondari@saintleo.edu, monika.vo@saintleo.edu) Saint Leo University

#### Academic Program Review

Many institutions must undergo an academic program review as part of preparation for accreditation. The review process will help the faculty to recognize the strengths of the program and to expose its weaknesses. The process of review is very complicated and time consuming, and requires careful planning and organization. In the last year our department has been working together with the office of assessment and research on all aspects of the program review. We will share the outline and the resources that we used to complete the procedure.

**Joy Becker, Christopher P Bendel, Helen Schroeder** (beckerjoy@uwstout.edu, bendelc@uwstout.edu, schroederh@uwstout.edu) University of Wisconsin-Stout

#### Calculus: One Lesson Study at a Time

The Lesson Study process provides an opportunity to collaboratively investigate student learning. The presenters have participated in multiple Lesson Study projects in a first semester calculus course and are currently involved in one on the Fundamental Theorem of Calculus. We will briefly describe the Lesson Study process, the benefits of engaging in the process, and some results from our current study.

**Murphy Waggoner** (murphy.waggoner@simpson.edu ) Simpson College

#### CAGD, Geometer's Sketchpad and Secondary Mathematics

Introductory topics in Computer Aided Geometry Design (CAGD) are accessible to high school students, but few pre-service secondary mathematics teachers take courses in CAGD. As part of their senior projects in spring 2008, the pre-service teachers at Simpson College studied introductory topics in CAGD and developed Geometer 's Sketchpad sketches designed to expose high school students to topics such as Bezier curves and barycentric coordinates. I will present the work of some of those students, and show how these topics in CAGD can be used in a high school classroom.

**Jerome Dancis** (jnd@math.umd.edu) University of Maryland

#### Common misconceptions in middle school math textbooks

We read several middle school mathematics textbook series last year. Now, we understand better how students arrive in college with misconceptions. Many textbook writers appear to have had too little training in how to write mathematics correctly. For example: Wrong use of negative numbers in real world problems: "Death Valley is $-282$ feet below sea level." This type of error was made consistently; not a typo. Such conflicts between common English usage and textbook mathematics must be confusing to students.

**Jay Stine** (jstine@misericordia.edu ) Misericordia University

### Induced Topologies and Separation Axioms

There are many well-known ways to form a topological space from a given family of topological spaces. For instance, one can form products, quotients, subspaces, etc. These, and many other spaces, are instances of two general constructions; namely, the induced topology and (its dual) the coinduced topology. In this talk we explicitly describe these topologies, and discuss their universal nature. We show how the familiar structures mentioned above are instances of these constructions. By way of application, we give a novel description of several classical separation axioms in terms of the induced topology.

**Jay Lawrence Schiffman** (schiffman@rowan.edu) Rowan University

### Exploring Prime Gaps with CAS (Computer Algebra System) Technology

The paradoxical nature of the primes is manifested in their infinitude on the one hand and the ability to construct gaps as wide as one pleases on the other. Our goal is to discuss the idea of a prime gap of length $n$ consisting of $n-1$ consecutive composite integers between two successive primes. We define $d_k = p_k - p_{k-1}$ where $p_k$ is the $k$-th prime number. For example, $p_5 = 11$ and $p_4 = 7$. Thus $p_5 - p_4 = 4$ and one has three consecutive composite integers ( 8, 9, 10 ). Technology will be displayed in both the form of a scatter plot as well as via the command $|NextPrime(x) - x|$ to determine the distance between an integer and the next prime. If $x$ is indeed prime, then a prime gap is displayed. We will also discuss the first occurrence of prime gaps of a certain length as well as maximal prime gaps. Thomas Nicely defines a maximal gap as one strictly exceeding in measure all the prime gaps preceding it (those between consecutive primes). For example, the prime 89 starts a gap of length 8 and this gap is larger than all gaps between smaller primes. On the other hand, while a gap of 10 occurs initially with the prime 139, the gap is not maximal; for a gap of 14 occurs between the smaller primes 113 and 127. Finally we discuss the merit of a prime gap which compares the size of the gap to the local average gap. The merit is given by $\frac{p_n - p_{n-1}}{\log p_n}$. The prime gap with largest known merit will be given.

**Jack Mealy, Cicily Smith** (jmealy@austincollege.edu, clsmith@austincollege.edu) Austin College

### Paracycles in Snell Geometries

A category of geometries, dubbed 'Snell Geometries', is defined. Basically, a system is declared to be a Snell Geometry if it consists entirely of regions of locally constant curvature, wherein Snell's Law (of optics) is in play across the boundaries between these regions of constant curvature. An important and interesting subcategory consists of those with the following further restrictions: i) the parameter space is a subset of Euclidean n-space; ii) the regions all have curvature zero, but may have different indices of refraction; iii) the boundaries between the regions of constant curvature are appropriately smooth $(n-1)$-manifolds. After noting some very basic characteristics of such systems, the freedom to choose the shapes, sizes, and indices of refraction of infinitely many sub-regions is exploited for the purpose of the construction some interesting objects. In the 2-dimensional category, a "paracycle" (a limit of circles) is constructed which has infinite circumference, but (unlike horocycles in Hyperbolic geometry) has everywhere constant radius. Further, by tweaking various indices of refraction, the set of such objects is seen to be large.

**Gowribalan Ananda Vamadeva** (gowribalan.vamadeva@uc.edu ) University of Cincinnati

### Resourceful Session Before a Developmental Math Test

Developmental Math students are most attentive and engaged when they are Active participants in the learning process. This session introduces innovative measures taken to make a Test Review class most productive for students. Faculty will be given numerous strategies and tips to have an informative and compelling classroom environment.

Upon attending this session, participants will be able to conduct a Developmental Math test preparation class with:

1. A complete student centered environment.
2. Numerous student friendly strategies to make class learn important aspects of the material and yet, make it fun to be in class.
3. Motivated students regardless of their Mathematical ability.
4. Nifty techniques to facilitate different learning styles.
5. Adequate expertise in making a positive influence on each student before the test.

**Aaron Wangberg** (awangberg@winona.edu ) Winona State University

### Let's Do This Once: Folding Vector Calculus into Multivariable Calculus using Geometry and Language

How can students learn Lagrange multipliers in under 20 minutes, understand divergence and Stokes' theorem, and use the gradient of a function correctly even as they are just beginning to learn to calculate mixed partial derivatives, double integrals, and the mathematics of three-dimensional space? Although vector calculus and multivariable calculus share many concepts, many procedures in the former rely upon the mastery of skills learned in the latter. How can the two courses be combined into one in such a way that students understand how to use the material in later science courses?

Corinne Manogue and Tevian Dray, designers of the Vector Calculus Bridge Project (http://www.physics.oregonstate.edu/bridge), have found that using geometry, and emphasizing differentials and geometric visualization over algebraic manipulations, can help students "bridge the gap" between the presentation of vector calculus in mathematics and the application of those concepts to physics and engineering courses. Not surprisingly, this approach extends nicely to a multivariable calculus course. This talk will give an overview of the Vector Calculus Bridge Project and highlight the changes required to bring this approach to a typical multivariable calculus course. It will point out the unexpected traps, surprises, and nice consequences for students which occur as a result of this approach, and show how language can help students correctly use skills which they have not yet mastered.

**Magdalena Luca** (magdalena.luca@mcphs.edu ) Winona State University

### Massachusetts College of Pharmacy and Health Sciences

The K–12 mathematics curriculum, its delivery to and assimilation by the students, has a strong impact on teaching mathematics courses in colleges and universities. In my presentation I will talk about the effects of the K–12 mathematics education on the math curriculum in our college where pharmacy and health sciences degree programs are offered. Furthermore, I will discuss the implications of the newly released report by the National Mathematics Advisory Panel in March 2008 on this topic, and how the findings of the report could facilitate the process of improving the mathematics we teach in colleges.

**Stewart Ernest Brekke** (stewabruk@aol.com) International University for Graduate Studies

### The Elementary Mathematical Theory of Parallelism, Convergence and Divergence

Parallelism is often thought of as "similarity or sameness." 'The magnitude of parallelism is defined as the distance between two parallel lines,curves,planes or surfaces. The smaller the value of the distance between two parallel lines,curves, planes or surfaces, the greater the value of the magnitude of coincidentalism. The degree of parallelism is defined as the reciprocal of the magnitude of parallelism (distance between them). Therefore, the greater the degree of parallelism, the greater the amount of coincidentalness or sameness. Between three parallel lines, planes or surfaces,the greater value of the degree of parallelism or sameness, the greater the coincidentalness of the pairs of lines, planes or surfaces. In this manner the amount of parallelism (sameness) or coincidentalness of parallel lines, curves, planes or surfaces many be quantified Convergence and divergence of two lines, planes or surfaces may be quantified by the angle between them. The Law of Convergence states that the smaller the value of the angle of convergence, the greater the convergence. The Law of Divergence states that the greater the value of the angle of divergence, the greater the divergence.

**Xuhui Li** (xli2@csulb.edu ) California State University, Long Beach

### Strengthening and Assessing College Students' Learning Outcomes in General Education Mathematics Courses

California State University Long Beach (CSULB) is a highly diverse public university with nearly 37,000 students. Each year the average SAT Math score of the freshmen is below 500 and over 34% of them need remediation in mathematics. Through offering extensive pre-baccalaureate and lower-division general education mathematics courses, the university and the mathematics department have tried hard to ensure that all students acquire a certain breadth of knowledge and critical skills in mathematics during their studies.

This presentation reports some ongoing efforts faculty teams at CSULB are making in implementing a CSU system-wide initiative that aims to reinforce the goals, value, and effectiveness of general education in its four foundation areas, including mathematics. A specific focus of the presentation is a set of newly drafted benchmarks for assessing the effectiveness of general education mathematics courses. The benchmarks encompass six major expected learning outcomes: 1. Recalling and performing technical skills; 2. Recognizing mathematical structures for further study of mathematics; 3. Recognizing mathematical structures arising outside mathematics classrooms; 4. Organizing and analyzing data; 5. Synthesizing ideas and generating questions; and 6. Evaluating and critiquing information, statements, arguments, and reasoning.

The presenter will provide a detailed description for each learning outcome, propose ways in which the benchmarks could be utilized, and demonstrate a sample assessment item. Through discussion, the presenter hopes to elicit comments, thoughts, and experiences from colleagues at other college universities who have made or will be making similar efforts in promoting quantitative literacy and general education in mathematics.

**Herman Rubin** (hrubin@stat.purdue.edu) Purdue University

### Teach concepts and understanding, and do not concentrate on manipulations which hinder this.

The current teaching of mathematics proceeds from manipulations to theorems, with rarely any understanding. The same applies to other subjects as well. Now one can start with theorems and become a mathematician without anything more than a vague conceptual understanding, but even this can hinder the development of understanding. The situation is even worse from manipulations; try convincing a calculus student that the "fundamental theorem of calculus" is a statement that two apparently unrelated things are equal. Furthermore, the person who understands the concepts of calculus will be able to convert his "real world" problem to a pure mathematical problem, to which the powers of mathematics, including computation, can be applied.

So the user of mathematics needs to understand the concepts, and not even start out with the computational course, which is usually the terminal course. The mathematician who is going to participate in the process needs to understand the concepts as well, to be able to communicate.

Sometimes, the same mathematical object has more than one concept behind it. The counting numbers have two unrelated basic concepts; the ordinal and the cardinal. The original "new math" tried starting with the simple (to those who understand it) cardinal concept, but got bogged down, and the teachers could not understand what the children could. The ordinal would have been much easier, and to know what is a finite integer requires the ordinal concept at some point.

In summary, concepts can and should be taught directly, and whatever else is done should follow. This will increase the pace at which it can be done, and also improve retention, as concepts are not easily forgotten.

**Paul F. Stang** (paulstang@gmail.com ) non affiliated

### Angle Trisection in the Nine-Pointed Star

The golden mean; $\phi$, is one of the favorites of the recreational mathematician. Dividing the circle by 3, 4, 6, 8, and 12 with only a compass and ruler is easy. Using the $\phi$ ratio we can also divide the circle by 5 (and use the same process to create nearly a 100% accurate 7-pointed star).

Of the first ten factors of 360 only the number 9 is still elusive with regard to techniques of division using just compass and ruler. We have developed a very nearly 100% technique of generating 18-, and thus 9-, fold geometry but there is still the need to obtain exactly 40*, and also 20*. How do we do it without the availability of a "third"-angle identity?

While we have not found the solution to angle trisection, we have developed a new method of doing it, much more user friendly than Euclidean and other available techniques. We have further discovered a proportion contained within 9 and 18 point geometries which behaves tantalizingly like phi. I have named it $\psi$ for this reason. From the thorough studies available on $\phi$, we will extract certain topics for comparison with 9-fold division, seeing a possible fit between 3-, 5- and Gauss' 17-fold division. We will explore this proportion in detail, opening further possibilities.

**Gregory Gerard Wojnar** (gwojnar@frostburg.edu ) Frostburg State University

### Angle Exploring a Quartet of Triangle Theorems—Old and New Insights via Geometer's Sketchpad

Menelaus of Alexandria Egypt (1st Century C.E.), King Al-Mu'taman ibn Hud (11th Century, Spain), Giovanni Ceva (1648–1734, Italy), Jakob Steiner (1796–1796, Switzerland), John Routh (1831–1907, England). All studied triangles in related ways. Under what condition are 3 points on the sides of a triangle sure to be collinear? Under what condition are the lines (cevians) connecting those 3 points to the triangle vertices sure to be concurrent? If these 3 lines are not concurrent, then they intersect to make a smaller triangle—How does the area of this triangle compare to the area of the original triangle?

Using Geometer's Sketchpad we can visually discover new connections, including 3 ellipses naturally connected with any triangle, and closely connected to Steiner's circumscribing and inscribed ellipses. The quantities $\frac{(n-1)^3}{n^3-1}$ and $\frac{n^3+1}{(n+1)^3}$ prove important.

Many other questions come to mind that are good questions for student research. In general, in many cases where a geometric theorem mentions "for any... ", Geometer's Sketchpad can visually present many such cases in a unified presentation, thus prompting new conjectures. We shall also present a triangles representation space which further promotes deeper grasp of how different instances of a theorem are continuously related.

**Jacob Robert Heidenreich** (jacob.Heidenreich@loras.edu ) Loras College

### The Mathematics of Games as Experiential Learning

In Fall 2007 and January 2008 I taught an experiential learning course on the mathematics of games. "Experiential Learning" is a pedagogy in which the students are put into an experience-rich environment and then construct their own learning as a community. After teaching the students how to play each of five different games, they conjectured and tested their own analyses of each game. Due to the experiental nature of the pedagogy, one of the main goals of the course was to give the students as much freedom as possible in pursuing their analyses. This worked differently for the Fall course, which consisted of upper level math majors, and the January course, which consisted of Freshmen getting Gen Ed credit. I will report on the successes and challenges of this pedagogical method and of this particular approach. The games used were "Board of Pair-of-Dice," "Kyboi," "Lost Cities," "Balloon Cup," and "Magic: the Gathering."

**Sue Beck** (s.beck@moreheadstate.edu) Morehead State University

### Redesigning Developmental Math to More Efficient and Effective

Morehead State University, in Morehead, Kentucky, is at the foothills of the Appalachian Mountains, where we are especially challenged to meet the needs of the students we serve—to provide success at the college level. Data indicates that approximately half of the incoming freshmen are not prepared for college level mathematics. Also, the Kentucky Council on Postsecondary Education's Developmental Task Force has raised the bar in an effort to upgrade the standards of education in Kentucky. In 2009, all students entering college in Kentucky will have to be remediated if their Math ACT is below 19—this will increase the number of students in our developmental math program significantly. We will also be held accountable for helping with "early intervention" of the public high school students.

We have been using technology for assessment and evaluation in our developmental classes for 15 years in order to increase student involvement, understanding and comprehension. In more recent years this use of software has allowed us to provide a more diverse style of classes to meet the diverse needs of our students. I will be providing data from our recent Academic Audit and student surveys as to the success of our program. The different types of classes will also be discussed, along with a short illustration of how our software helps us cater to diverse teaching and learning styles.

**Janet Marie McShane; Phillip Mlsna, Jennifer Maynard, Chester Ismay, Sarah Brown** (janet.mcshane@nau.edu, phillip.ml sna@nau.edu, jennifer.maynard@nau.edu, chester.ismay@nau.edu, sarah.brown@nau.edu) Northern Arizona University

### Mathematics Skills Assessment and Training in Freshman Engineering Courses

In recent years, the professors who have taught freshman engineering courses at Northern Arizona University have expressed some disappointment regarding the level of students' abilities and their rates of academic success. A major cause, we believe, is the inadequately developed mathematical intuition and skills that students possess when they begin college. To address this issue, we have developed and deployed a pilot program called TIMES: Training Intuition in Math for Engineering Success. Once students are assessed to determine their skill levels in six chosen numeracy areas, guided practice and training is provided to each student who has exhibited difficulty. All students are required to reach a level of mastery as measured by a post-test instrument. The goals have been to increase retention and academic success for these engineering students and to measure the effectiveness of the TIMES approach. Four semesters have been completed and more than 1000 students have participated. The majority of the students have shown weakness in one or more of the targeted skill areas. In this presentation, we present both quantitative and qualitative results of the first four semesters of this ongoing project.

**Zsolt Lavicza** (zl221@cam.ac.uk ) University of Cambridge
**Laszlo Erdodi** (lerdodi@emich.edu ) Eastern Michigan University

### Mathematicians' uses of Computer Algebra Systems in mathematics teaching in the UK, US, and Hungary

The use of Computer Algebra Systems (CAS) is becoming increasingly important and widespread in mathematics research and teaching. In our talk, we will report on a questionnaire study enquiring about mathematicians' use of CAS in mathematics teaching in three countries; the United States, the United Kingdom, and Hungary. Based on the responses from 1103 mathematicians, we will give an overview of the current extent of CAS use in universities; uncover why or why not mathematicians choose to integrate CAS into their classroom teaching; describe what influences their decision for using CAS; offer some examples of their classroom uses of CAS; and explain how their envisage the role of technology in mathematics teaching in the future. We will also outline similarities and differences of my findings among the participating countries and different types of universities. In addition, we will present models, based on statistical modelling, on the influencing factors of CAS integration into university-level mathematics teaching and how this integration can be assisted.

**Richard Brazier** (brazier@geosc.psu.edu ) Penn State University DuBois Campus
**Shauna Knarr** (srk200@psu.edu ) Penn State University
**Andra Barraclough** (arb275@psu.edu ) Penn State University

### Limit Cycles from a Sum and Number of Digits Concatenated Sequence

Consider the sequence: 918, 183, 123, 63, 92, 112 What term comes next? This sequence is calculated by summing up the digits i.e. 9+1+8=18 and concatenating it with the number of digits in this case 3, giving 183. If the first term in the sequence has $k$ digits $n_1$ to $n_k$ it can be written as:

$$s_1 = 10^k n_1 + 10^{k-1} + \cdots + 100n_{k-2} + 10n_{k-1} + n_k$$

The affect of the concatenation is to multiplying the sum by a power of ten (depending order of magnitude of $k$) and then adding the number of digits. The mathematical formula for the second term in this sequence with $k < 10$ is:

$$s_2 = (n_1 + n_2 + \cdots n_{k-1} + n_k) * 10 + k$$

These types of sequences produce limit cycles of various lengths for different bases. For example in base ten there are two possible cycles of length four ranging from two to three digits:

918 183 123 63 92 112 43 72 92 $\cdots$ 112 43 72 92, and 919 193 133 73 102 33 62 82 102 $\cdots$ 33 62 82 102.

We can show in general that these sequences converge to limit cycles and there exists a unique number of cycles and length of cycle, and the product of length and number of cycles is less than the base, in base ten there product is 8.

**Deborah Gougeon** (gougeond1@scranton.edu ) University of Scranton

### Data Mining: An Emerging Topic in Mathematical/Statistical Education

With the proliferation of computers and computer technology in the last decade, decision makers in business and government have been inundated with massive amounts of data. Whether you shop at a supermarket where your items are scanned into a database, or mouse click on a web site that you are browsing, or use your credit card to purchase an item, or make a phone call to a friend overseas, more data are continually being collected. At a time when companies are attempting to determine why their customer base is rising or falling, or what new and promising drug will be effective in treating an illness, and when government intelligence agencies are trying to determine what combination of phone calls, e-mails, and foreign travel might signal terrorist activity, it seems imperative that we educate our college students regarding the statistical tools that are used in making these decisions. This paper focuses on the development of an undergraduate course in Data Mining, also known as Knowledge Discovery in Data (KDD), Exploratory Data Analysis (EDA), and Business Intelligence. Several technical procedures will be discussed that are used to summarize and interpret data, identify patterns and trends, and assist one in making the best decision.

**Jonathan Paul Lambright** (lambrij@savstate.edu ) Savannah State University

### The Next Phase In Developing Effective Teaching Strategies For Undergraduate College Algebra Courses

Since the summer of 2005, a core of mathematics faculty at Savannah State University have been involved in a pilot project that focuses on changing the was college algebra is taught. The project was initiated at The United States Military Academy in West Point, NY. under the direction of Dr. Don small. Several other institutions are involved in the effort and have been seeing positive results. Students continue to arrive in the college classroom under prepared in mathematics and they struggle one enrolled in college algebra and follow-on courses. This pilot project emphasized a refocus of course content, presenting problems from a real world perspective, and in class group exercises and board work. The pilot project at Savannah State University has been effect for four semesters and we have five sections being taught. This paper presents the structure of the project, results to date, challenges, and the future direction.

**Kazuko Ito West** (westc-k@post.harvard.edu ) Keio Academy of New York

### College Entrance Examinations You Need to Pass to Be a Middle School or High School Mathematics Teacher in Japan

To become a middle school or high school teacher in Japan, one must first pass an entrance examination and enroll in a college which grants teacher certificates. A mathematics certificate requires a mathematics major in a faculty of education or a mathematics, physics or engineering major in a faculty of arts and sciences or engineering. Different faculties have different examinations. After graduation, one takes the local government employment examination which includes college transcripts, an interview and a written test.

Some college faculties of education are just as difficult to get into as faculties of arts and sciences or medicine. For example, according to one ranking of private universities, acceptance to the education faculty of Waseda University, a top tier university, is equal to or more difficult than acceptance to six of twenty eight college medical faculties, all of which rank within the top ten percent of college faculties. Interestingly, although almost eighty percent of Japanese colleges grant teacher certificates, high school teachers—at least in the greater Tokyo area—are typically graduates of first tier colleges. Entrance examination problems of first tier colleges are often more complex and difficult than those on teacher employment examinations.

**Kien Hwa Lim** (kienlim@utep.edu ) University of Texas at El Paso

### Addressing the Hammer-and-Nail Phenomenon in Mathematics Classrooms

“For a person with a hammer, everything looks like a nail” (a proverb). This phenomenon is commonly observed in mathematics classrooms. For example, after learning the advantage of using a ratio of length to width, over using a difference between length and width, to compare ‘squareness’ of rectangles, 12 out of 22 pre-service teachers used either length-to-width ratio or length-and-width difference, instead of length-and-width product, to compare the palm-sizes of hands. Because mathematics is taught in a linear and compartmentalized manner, many students developed the habit of applying recently learned ideas to solve a mathematics problem instead of reasoning with the quantities and relationships involved in the problem. To address this phenomenon, we need a different classroom culture, one that requires students to think and be skeptical. One strategy is to pose problems for which a recently learned idea would not work, and thereby presents a need for students to investigate the principle(s) underlying the idea. A general pedagogical principle is NOT to make “learning” easy for students; once students are shown a procedure or a shortcut there is no need for them to grapple with the mathematics. Items used in quizzes and exams should not be straightforward, but instead require students to understand the problems and consider the applicability of the ideas learned in class. Since “what gets measured gets learned,” students must experience for themselves the harm of *diving into the first thing that comes to mind*. After using these strategies, the number of proportional solutions for non-proportional missing-value problems decreases from 45% in the pre-test to 21% in the post-test.

**Kumnit Nong** (knong@gmu.edu ) George Mason University

### Thin Film over Thin Porous Layers

We study the simulated models of the aqueous layer on the pre-corneal tear films of a human eye. These models describe the behavior of fluid films with and without the inclusion of the permeable porous medium that models a contact lens. A fluid dynamic model for the thin fluid film over thin porous layers is formulated by using a nonlinear fourth order partial differential equation with four boundary conditions and one initial condition. The evolution equations are solved numerically in Matlab in order to predict the effect of various parameters (at realistic values) on time of the thin films rupture. The results indicate that the presence of thin porous layers is a dominant effect and the different slip conditions at the liquid-lens boundary also have significant impact on thinning the thin aqueous layers. The computed numerical results allow us to predict film break up times for tear films on a contact lens.

**Ibtesam Bajunaid** (ibajunaid@hotmail.com) King Saud University

### Biharmonic-Extension Space

Defining biharmonic spaces in a general way in the axiomatic potential theory of Brelot, I obtain some extension properties of biharmonic functions defined outside a compact set and use them to study the removable singularity of bounded biharmonic functions.

**Raymond Stanley Puzio** (puzio1@excite.com ) PlanetMath

### On the Representation of Recursive Functions by Integrals

Integral representations are a powerful tool in the chest of the analyst. Oftentimes, studying an integral representation of a function turns out to be a most efficient and efficacious means for understanding asymptotic behavior, constructing analytic continuations, making approximations, finding zeros, and otherwise investigating the behavior of a function. In this paper, it will be shown that a surprisingly large class of functions—namely all computable functions of one or more real variables—admit integral representations and hence are potentially amenable to such analysis.

This result leads to some interesting consequences and suggests avenues for future research. It challenges the common wisdom that most problems cannot be solved in closed form and hence must be studied numerically and provides a rigorous basis for the assertion that integration is harder than differentiation. Knowing that they must exist, the natural impulse is to look for explicit integral representations of functions which were hitherto not known to have such representations and see of what use they might be. On a more abstract level, considering integral representations may prove useful in the study of constructive analysis.

**Raymond Stanley Puzio** (puzio1@excite.com ) PlanetMath

### On the Representation of Recursive Functions by Integrals

Integral representations are a powerful tool in the chest of the analyst. Oftentimes, studying an integral representation of a function turns out to be a most efficient and efficacious means for understanding asymptotic behavior, constructing analytic continuations, making approximations, finding zeros, and otherwise investigating the behavior of a function. In this paper, it will be shown that a surprisingly large class of functions — namely all computable functions of one or more real variables — admit integral representations and hence are potentially amenable to such analysis.

This result leads to some interesting consequences and suggests avenues for future research.. It challenges the common wisdom that most problems cannot be solved in closed form and hence must be studied numerically and provides a rigorous basis for the assertion that integration is harder than differentiation. Knowing that they must exist, the natural impulse is to look for explicit integral representations of functions which were hitherto not known to have such representations and see of what use they might be. On a more abstract level, considering integral representations may prove useful in the study of constructive analysis.

**Ralucca Gera** (rgera@nps.edu ) Naval Postgraduate School
**Linda Eroh** (eroh@uwosh.edu ) University of Wisconsin Oshkosh
**Steven J. Winters** (winters@uwosh.edu ) University of Wisconsin-Oshkosh
**Grady Bullington** (bullingt@uwosh.edu) University of Wisconsin-Oshkosh

### Pizza Delivery: 2-Stop-Return Distances in Graphs

Recall that the distance of a vertex $x$ is $d(x) = \sum_{\forall u \in V(G)} d(x, u)$. We define the 2-*stop-return* distance of $x$ with two stops at $y$ and $z$ $(y \neq z)$ to be

$$d_{2s}(x, \{y, z\}) = d(x, y) + d(y, z) + d(z, x).$$

For simplicity, we will write $d_{2s}(x, y, z)$ instead of $d_{2s}(x, \{y, z\})$.

The 2-*stop-return eccentricity* $e_{2s}(x)$ of a vertex $x$ in a graph $G$ is the maximum 2-stop-return distance from $x$, that is,

$$e_{2s}(x) = \max_{y,z \in V(G)} (d(x, y) + d(y, z) + d(z, x)).$$

The minimum 2-stop-return eccentricity among the vertices of $G$ is the 2-*stop-return radius* that is, $\mathrm{rad}_{2s}(G) = \min_{x \in V(G)} e_{2s}(x)$. The maximum 2-stop-return eccentricity among the vertices of $G$ is the 2-*stop-return diameter* that is, $\mathrm{diam}_{2s}(G) = \max_{x \in V(G)} e_{2s}(x)$.

This particular metric minimizes the distance that a pizza delivery guy would need to travel if he wants to make two deliveries in one trip. We present results about the 2-stop-return distance in graphs.

**Tabitha T. Mingus** (mingust@wmich.edu) Western Michigan University
**Richard M. Grassl** (richard.grassl@unco.edu) University of Northern Colorado

### Validating the Warrant $P(k) \to P(k+1)$: Does Any Means Justify the Ends?

For practicing and prospective secondary mathematics teachers, the role of mathematical reasoning and proof in their mathematical training is crucial for a number of reasons: (1) formal mathematical proof serves as a tool for teachers to develop a profound understanding of mathematics and as a means for communicating their understanding; (2) reasoning and proof are featured prominently in the secondary school curricula as envisioned in the NCTM Principles; and, (3) reasoning and proof can form the basis for the development and assessment of student learning and discourse. Following the scheme suggested by the Seldens (2003), we undertook a cross-sectional study of pre- and in-service secondary mathematics teachers' abilities to validate purported proofs by mathematical induction with an eye toward determining what facets they attended to when reading such an argument. Of the 82 validation attempts only 9 correctly identified that the attempt was flawed.

The reviews of our article submission were mixed, including one reviewer who insisted that the proposition $P(k+1)$ can be established by any means. This sparked an in-depth analysis of the mathematical induction argumentation style using Toulmin's

(1958) argumentation scheme. The concept of a warrant from Toulmin's scheme is the logical connection between what is given in a proof and what is concluded. In this talk we will discuss the flawed arguments presented to the participants, how Toulmin's scheme can be utilized to analyze students' attempts at proof by mathematical induction, and how the scheme can be used as a pedagogical tool in the undergraduate mathematics classroom.

**AbdelNaser Al-Hasan** (alhasan@msoe.edu) MSOE & An-Najah National University

### The Status of Mathematics Education in Palestine

This talk will provide an overview of the current educational system in Palestine. The primary focus will be on mathematics. We will discuss the non-political challenges that students and teachers in k10-k12, students and faculty at the undergraduate level are facing and proposed solutions to these challenges by faculty of the department of mathematics at An-Najah National University in Nablus, Palestine.

**Philip B. Yasskin** (yasskin@math.tamu.edu) Texas A&M University
**Douglas B. Meade** (meade@math.sc.edu) University of South Carolina

### Maplets for Calculus—Tutoring without the Tutor

Maplets for Calculus is a collection of 94 Maple applets for teaching single-variable calculus, including precalculus, limits, derivatives, integrals, differential equations, sequences, series, and polar coordinates. They can be viewed either on a computer or on any internet accessable device with a large enough display such as an iPhone. The Maplets are highly pedagogical, using either algorithmically-generated or user-entered problems. Most of the Maplets ask the student one or more questions and guide the student through the solution process. They help build intuition, guide students through the steps of simple proofs and/or provide routine computational practice. The Maplets for Calculus balance the development of understanding and technical skills. Students find the hints, immediate feedback, step-by-step checking of their responses and infinite patience as effective as a private tutor.

Instructors may also use the maplets as effective classroom demonstrations making use of the 2D and 3D graphics and animations and the ability to launch a Maplet with a specific example. Other instructors develop a project or a lab that utilizes a Maplet in a guided-discovery exercise.

This collection of Maplets are proving useful at both the college and high school level. An updated version was released in January 2008. A Table of Contents and sample videos may be seen at m4c.math.tamu.edu. This project was awarded the 2008 ICTCM Award for Excellence and Innovation using Technology in Collegiate Mathematics.

**Monika Vo** (monika.vo@saintleo.edu) Saint Leo University

### SPLN—What is it?

Technology has truly changed the way we teach, the way we are expected to teach and of course the way our students learn. In this talk, we will demonstrate an SPLN, a skeleton power point lecture note and how we can use Tablet PC's and Smartboards to enhance your mathematics teaching in all levels. By posting SPLNs on class websites for remedial, college algebra as well as Calculus classes, the students had an outline for each class period and had the opportunity to prepare for each class. By having the outline prior to the class and having the outline in the classroom projected, the students had opportunity to ask questions regarding the main definitions and theorems without having to spend time copying, there were ample opportunity in the classroom for more worked-out examples and for active learning exercises. Students came to class more prepared and had more time to work on problems in the classroom.

**Noel Heitmann** (heitmann@millersville.edu) Millersville University of Pennsylvania

### Fluid Dynamics in Numerical Analysis: Using the Convection Diffusion Problem for Multiple Topics

In this presentation I will investigate some of the many ways that the convection diffusion equation of fluid dynamics can be used as an application problem for the demonstration of many topics covered in a typical numerical analysis course. The convection diffusion equation is given by, find $u$ such that $u_t - \triangle u + \mathbf{b} \cdot u + gu = f$. It models a scalar quantity such as heat or contaminant level in a fluid flow and has a wide range of current applications such as pollution abatement, global climate studies, and oil recovery to name only a few. I will demonstrate how this problem can be used to investigate interpolation, numerical differentiation, initial and boundary value problems for differential equations, and solution methods for linear systems of equations.

**James E. Carpenter** (jcarpenter@iona.edu) Iona College

### A Proof and Generalization of the Ladder of Eudoxus

An approach to finding a sequence of rational numbers which quickly converges to the square root of 2 has been attributed to the Greek mathematician Eudoxus. In this paper, the Ladder of Eudoxus is proved and extended to any prime of the type $g^2 + 1$.

**Michael Dobranski** (m.dobranski@moreheadstate.edu) Morehead State University

**Online Mathematics Courses: The Good, the Bad, and the Ugly**

Many of us are hesitant to offer online mathematics. It's difficult to arrange group activities. The time commitment to read and respond to student work and questions is huge. Most students can't (or won't) read their mathematics texts - even after they've seen the material in a traditional class. On the other hand, students and administrators are demanding more and more online classes, especially with the high cost of driving to classes on campus. I will discuss some of what I've learned in several online courses I have taught. Most of these courses have been for in-service teachers. Some things have worked well, some things haven't worked at all, and many things have needed drastic "makeovers."

# Graduate Student Poster Session

**Zachary Alexander** (zachary_alexander@yahoo.com) Eastern Washington University

### A Self-contained Derivation of the Navier-Stokes Equations in Three Dimensions

I have given a derivation of the Navier-Stokes equations which makes only one unnecessary assumption. I have assumed Cauchy's Law which states that if the Cauchy stress tensor $\sigma$ depends continuously on $\nabla\mathbf{u}$ then the dependence is linear. In particular I have provided proofs that $\sigma$ is symmetric and that it depends only on the symmetric part of $\nabla\mathbf{u}$ . In general, these questions tend to be answered with the tools of modern differential geometry. The treatment of the problem that I have given is entirely accessible to students in a first course on multivariable calculus.

**Daniel Baczkowski** (baz@math.sc.edu) University of South Carolina

### Diophantine Equations, Multiplicative Functions, and Factorials!

Florian Luca established that for a fixed rational number $r$, there are a finite number of positive integers $n$ and $m$ for which $f(n!) = r \cdot m!$ where $f$ is one of the multiplicative functions $\tau$ (the number of divisors function), $\phi$ (Euler's $\phi$-function), or $\sigma$ (the sum of the divisors function). In this joint work with M. Filaseta, F. Luca, and O. Trifonov, we establish a generalization of these results, in particular a consequence of our work is the following: Let $k$ be a fixed positive integer. Then there are finitely many positive integers $n, m, a$ and $b$ such that $b \cdot f(n!) = a \cdot m!,\ \gcd(a, b) = 1$, and the number of distinct prime divisors of $a, b$ is $\leq k$.

**William Oversteegen Bond** (bondoverstee@gmail.com) University of Alabama at Birmingham

### Who Are We Teaching and How Do We Teach Them?

"The Wu Li master does not teach but the student learns," or so says Gary Zukav in his book The Dancing Wu Li Masters, a book on quantum physics. We use this maxim as a point of departure to challenge the traditional paradigm of the sagacious mathematician delivering knowledge to the eager (or not so eager) student. First, many of the undergraduates a beginning professor teaches are in fact not post-calculus, but pre-calculus. The corresponding courses are so-called service courses, and the beginning young professor may find he or she has several such courses in the first term (and if not then, soon). How should these courses be taught? Second, even in a post-calculus (or calculus) class, some of the students may be aiming at teaching as a career, but at the secondary level. What is the teaching role that one should model for them? Third, the future academic mathematician in a post-calculus class ought to be pondering the preceding questions in a few years. How do we encourage reflection upon one's role as a teacher of teachers? We will discuss an approach taken in response to these questions by the Greater Birmingham Mathematics Partnership.

Advisor John C. Mayer is professor and associate chair of mathematics; William O. Bond is a master's degree student in mathematics.

**Lyrial Marie Chism** (lmchism3@hotmail.com) University of Mississippi

### On Independence Polynomials and Independence Equivalence in Graphs

The independence polynomial of a graph $G$ is $\sum f_k(G)x^k$ where $f_k(G)$ is the number of $k$-element independent vertex sets in $G$. Graphs $G$ and $H$ are said to be independence-equivalent if they have the same independence polynomial. I present examples of infinite classes of independence-equivalent graphs and I also determine some previously-unknown independence polynomials using vertex-reduction, recursion relations, and generating functions.

**David Clark** (dcclark@mtu.edu) Michigan Technological University

### Symmetric Nets in Reed-Muller Codes

Reed-Muller codes are a class of error-correcting codes which admit fast and simple construction and decoding algorithms. We show that these codes contain highly structured block designs (symmetric nets) which provide counterexamples to a conjecture of Hamada. We demonstrate the extendability of these results, as well as applications to relative difference sets, quantum error-correcting codes, and majority-logic decoding.

**Leigh Cobbs** (cobbs@math.rutgers.edu) Rutgers University

### An Application of Covering Theory of Graphs of Groups: Constructing Lattice Subgroups of Rank-2 Kac-Moody Groups

Utilizing graphs as a visual encoding for groups, this research explores which graphs fold onto (cover) the graph of a Kac-Moody group and hence which graphs encode subgroups of Kac-Moody groups. Kac-Moody groups are the groups associated to a class of infinite-dimensional Lie algebras with important applications to physics. Rank 2 Kac-Moody groups admit an action on their Bruhat-Tits tree. The work of Bass and Serre showed that a group with a tree action is the fundamental group of its quotient graph, making it possible to study a group from its quotient graph. Any subgroup of a Kac-Moody group must have a quotient graph which covers the Kac-Moody quotient graph. Coverings of quotient graphs of groups were developed in the work of Bass and Kulkarni. The known constructions of Kac-Moody groups by Rémy-Ronan and Carbone-Garland are technical and complex. Using covering theory allows a more combinatorial concrete way method for constructing Kac-Moody subgroups. This work exhibits a number of examples of subgroups that can be constructed in this manner, including both finite and infinite graphs (compact and noncompact lattice subgroups). We give structural properties that any quotient graph which covers a Kac-Moody group must satisfy. We further give some necessary and some sufficient conditions for determining whether or not this covering exists in some special cases. This includes joint work with Lisa Carbone and Scott Murray.

**Michelle Craddock** (mcraddoc@olemiss.edu) University of Mississippi

### The Grothendieck Space Property for Fremlin and Wittstock Tensor Products of Banach Lattices

Let $X$ be a Banach lattice and let $1 < p, q < \infty$ such that $1/p + 1/q = 1$. Then $\ell_p \hat{\otimes}_F X$ (respectively, $\ell_p \tilde{\otimes}_i X$), the Fremlin projective (respectively, the Wittstock injective) tensor product of $\ell_p$ and $X$, is a Grothendieck space if and only if $X$ is a Grothendieck space and each positive operator from $\ell_p$ (respectively, from $\ell_q$) to $X^*$ (respectively, to $X^{**}$) is compact.

**Clinton Curry** (clintonc@uab.edu) University of Alabama at Birmingham

### Telling the future with a polynomial

Let $P$ be a polynomial. What happens to a point under iteration of $P$? In other words, for some $x_0$, what is the fate of the sequence $[x_0, P(x_0), P(P(x_0)), \ldots]$? Does it settle down into a predictable pattern, or is it unpredictable? If $x_1$ is very close to $x_0$, does its corresponding sequence behave similarly? Questions of this sort fall into the realm of dynamical systems, and we will concentrate on the case when the coefficients of $P$ are allowed to be complex numbers. We will explore the amazing amount of structure present in the answer, as evidenced by some of the most beautiful fractal pictures from all of mathematics.

**Yilin Dai** (ydai@mtu.edu) Michigan Tech University

### Using Wavelet-transforms to Improve Power for Linkage Disequilibrium

We develop a powerful novel statistical method to identify genetic variants related to disease. The new method uses wavelet-transforms on genotypes, with minimal degrees of freedom, to construct a weighted test statistic which captures significant information from multiple gene loci. Simulation is used to compare the power of the new procedure to existing, less general methods. The new statistic has significantly improved power.

**Melissa Desjarlais** (melissa.desjarlais@valpo.edu) Valparaiso University

### Gender Differences in Mathematics Performance on the AMC 8

This talk will describe both how the Mantel-Haenszel procedure is used to measure gender differences in performance on the American Mathematics Competition AMC 8 and a few subtest analyses based on NCTM mathematics content standards. During the past few decades, the mathematics education community has been interested in gender differences in mathematics performance. Many studies have been conducted, and while some have found gender differences, the type and magnitude of the differences vary. Each year more than 100,000 students, located throughout the United States and approximately 20 other countries, participate in the American Mathematics Competitions AMC 8 contest. Results from this contest create a large data set which can be analyzed to answer questions related to gender differences in performance on mathematics competitions. Using five years of AMC 8 data, the Mantel-Haenszel procedure is applied to assess differential item function on individual questions and subtest analyses are done using the NCTM mathematics content standards. Results from the statistical analyses will be presented and possible interpretations of the data will be given.

**Reginald Dorcely** (regmix@hotmail.com) LSAMP-NYC

### Observation and Assessment of a Teacher Development Program in NYC

The National Council of Teachers of Mathematics (NCTM) recommends that teachers rethink their objectives and practice in teaching mathematics. Prior research indicates that a lack of success in implementing standards-based mathematics learning environments may be due to the fact that teachers generally have not been prepared for this endeavor (c.f. Hiebert, 2003). For this purpose, a teacher development program facilitates teachers' thinking about mathematics and learning. Therefore, a teacher development program must be tailored to meet the needs of students in the classrooms. The purpose of this project is to observe and assess teachers' lessons based on pedagogy, use of context, and knowledge of mathematics. To carry out this research we have been observing teachers from two middle schools located in New York City. Our main focus is taking field notes and writing reflections on what we observing being taught in the classroom. At present, we have found that meaningful and interesting mathematics lessons supported students' critical thinking. Further investigations will be done to draw sound preliminary conclusions.

**Brittany Terese Fasy** (brittany@cs.duke.edu) Duke University

### A Problem in the Intersection of Group Theory and Topology

A basic problem in mathematics is the classification problem. In group theory we have the Classification Problem for Groups: Given a collection $\mathbb{G}$ of groups, classify the groups $G$ in $\mathbb{G}$ up to isomorphism. And in homotopy theory, we have the Homotopy Classification Problem for Spaces: Given a collection $\mathbb{T}$ of topological spaces, classify the spaces $X$ in $\mathbb{T}$ up to homotopy equivalence.

We will show that in some cases, these classification problems are actually equivalent. In addition, we will give some examples classifying groups by the centralizers and classifying the path components of function spaces.

**Charles Feldhaus** (feldhaus@math.ohiou.edu) Ohio University

### Students' Reflections on Learning in an Inquiry-based Number Theory Course

While traditional college mathematics courses are generally teacher-centered and tend to promote passive learning and non-engagement by students, inquiry-based mathematics courses are more exploratory and alter the instructional nature of the course from instructor-centered model to a student-centered model. Students are given a list of exercises, questions, and theorems which students are expected to have prepared before class. Employing formative assessment techniques the instructor expects students to present their solutions, answers and proofs to the class, and the class is in turn responsible to critically analyze the student's work and provide intelligent questions and suggestions to improve the presented work. The instructor ideally plays a minimal role, acting more as a facilitator and a course designer rather than as a lecturer. By shifting the responsibility for learning from the instructor to the student, this method promotes the students' independent mathematical thinking, proving skills, and reasoning ability. Also, inquiry-based learning gives the instructor multiple examples of formative assessment at both the class-level and the individual level, and also allows instructors enough flexibility to adjust the course to the needs of the class. Using qualitative methodology of personal interviews of students and participant observation in the class, this paper will examine students' views of an undergraduate number-theory course taught using an inquiry-based mathematics format. The focus of the study will emphasize how the students developed mathematically while in the course within the context of a non-traditional classroom environment.

**Shanzhen Gao** (sgao2@fau.edu) FAU

### Proof of a Lattice Paths Conjecture Connected to the Tennis Ball Problem

We will present a history of the so-called tennis ball problem, and discuss its relation to lattice path enumeration. We also prove a conjecture related to a solution of the symmetric case, namely when the number of balls removed each turn is exactly half the number inserted.

**Michelle Hackman** (mehackma@indiana.edu) Indiana University

### Screw-Motion Invariant Minimal Surfaces

The fence of catenoids is a known example of a singly periodic minimal surface. If we twist this surface, we get a new family of surfaces, each differing by the twist angle, which are invariant under a screw motion $\sigma$. Modulo the screw motion, each surface is a torus and thus can be parameterized by a map from a parallelogram torus into $\mathbb{R}^3/\sigma$. We scale the parallelogram domain so that it has periods 1 and $\tau$.

In my research, I have been studying several questions about this family of surfaces. First of all, does a minimal surface with these properties exist? If it does, it must satisfy period conditions, which in this particular case reduce to solving an equation that depends on the screw motion angle and the period $\tau$. What I have found is that for every choice of a period $\tau$, there exists at least one corresponding screw motion angle solving the period condition and defining a screw-motion invariant minimal surface. Other questions I have studied include: Is there such a minimal surface for any twist angle $0 < \xi < 2\pi$? For a given twist angle, is there a 1-parameter family of values $\tau$ which satisfy the period condition? Finally, what happens to the surfaces as the twist angle approaches 0? My poster will illustrate my results thus far, as well as conjectures about the above open questions.

**Shane Hubler** (hubler@math.wisc.edu) University of Wisconsin - Madison

### Combinatorial Analysis of Peptide Sequences

Mass spectrometry, when applied to the field of proteomics, involves identifying the linear sequence of amino acids that make up a protein or peptide. The most common approach used in identification involves the creation of databases of all possible peptides for a particular species. However, if the protein comes from a species whose genome is unknown or if we expect that it is mutated, this approach will fail. Thus, de novo techniques, those that do not use a reference protein database, are necessary in some areas of mass spectrometry. There are several combinatorial questions that arise using these techniques: How many (theoretical) peptides have a mass of M or less? How many amino acid compositions have a mass of M or less (unordered selection of amino acids, with replacement)? How many elemental compositions are there with a mass less than M (masses corresponding to an amino acid composition)? The last two questions have the same general solution while the first is a different class of problem. We show that the answer to the first question is exponential in M while the answers to the last two questions are polynomials of degree dependent on the size of the alphabet used (20th order for the 20 amino acids in the second question and 5th order for the elemental case, for the 5 elements found in amino acids). Furthermore, this allows us to solve similar problems for other alphabets, such as when we add modifications to the amino acids in the peptide.

**Lesa L. Kean** (keanles@iit.edu) Illinois Institute of Technology

### Teachers Beliefs about the Nature of Mathematics and Their Effects on Classroom Practice

Not every teacher means the same thing when they say, "mathematics." In fact, there is a surprising variety of ways that teachers answer the question, "What is Mathematics?" This research focuses on the case studies of two very different teachers and the choices they make in the classroom. The question researched is, 1) what are teachers' beliefs about the nature of mathematics, and 2) what effect do these beliefs have on their classroom practice? This preliminary study combines classroom observations, teacher interviews, and a teacher self-report survey to determine each teacher's personal view of the nature of mathematics. These views are compared to their actual classroom practice to look for patterns of consistency or inconsistency.

**Jennifer Susannah Gee Lopez** (jslopez@sfsu.edu) San Francisco State University

### TangleSolve

TangleSolve (bio.math.berkeley.edu/TangleSolve) is a java applet/application that implements the tangle method for site-specific recombination. The tangle method is a mathematical method based on knot theory, which is used to compute the topological mechanism of certain enzymes called site-specific recombinases. Site-specific recombinases catalyze the exchange of genetic material between specific sites on a DNA molecule. In general, substrates and products of recombination belong to a well characterized family of knots and links called 4-plats. Each site-specific recombination reaction gives rise to a system of tangle equations which can be solved, for a specific type of tangles called rational, using the tangle method. TangleSolve offers an interactive tool to compute solutions to such systems of tangle equations.

We present the TangleSolve tutorial, which we have updated to be more user-friendly. We illustrate the tangle method using the Xer recombination example. Xer recombination is catalyzed by a pair of tyrosine recombinases XerC and XerD, which display topological selectivity and specificity. Under certain biological and mathematical assumptions, the tangle method finds 3 solutions to the Xer equations.

Tools in low-dimensional topology can be used to determine whether there are non-rational solutions to a system of tangle equations. We take such tools into account in our software. Furthermore we are expanding our work beyond the tangle method to incorporate new results of E. Flapan and D. Buck into TangleSolve.

This is joint work with Yuki Saka (Florida State University), and Wenjing Zheng(U.C. Berkeley) and Mariel Vazquez (SFSU)

**Amy Mihnea** (amalyamy@yahoo.com) FAU

### Applications of Permutations Groups

I present some applications of permutations groups in image processing and give experimental results.

**David Millman** (dave@cs.unc.edu) UNC-Chapel Hill

### Lower Degree Predicates for the Additively Weighted Voronoi Diagram

This work considers the problem of incrementally constructing additively weighted Voronoi diagrams in $\mathbb{R}^2$. Incremental constructions assume a diagram of $k$ sites and considers the insertion of a new site $s$. In general, this type of construction consist of two steps: first, identify the the *conflict region*, the set of diagram components which $s$ destroys; second, repair the conflict region as to return the diagram to a valid state. In the paper *Dynamic Additively Weighted Voronoi Diagrams in 2d*, Karavelas and Yvinec describe such a procedure, determining the conflict region using predicates of algebraic degree 14. We propose a different set of predicates for determining this region, which achieves the same results, but has an algebraic degree of only 6. In addition, this method handles degeneracies in a manner which results in a diagram insensitive to insertion order. Finally, we show that implementing these lower degree predicates result in 39 to 66 percent less running time.

**Ray Molzon** (remolzon@mtu.edu) Michigan Technological University

### Illustration of l'Hopital-type Rules for Monotonicity Patterns

An introduction will be given to some l'Hôpital-type monotonicity rules, wherein the monotonicity pattern of a function $r = f/g$ on the interval $(a, b)$ may be determined by the monotonicity pattern of $\rho = f'/g'$. As an illustration of these rules, the monotonicity of a function from the realm of mathematical statistics (the asymptotic relative efficiency between Kendall's $\tau$ and Spearman's $\rho$ from a bivariate normal distribution) will be proven. The proof will require only the l'Hôpital-type rules and basic knowledge of continuity and differentiation taught in a Calculus I course.

**Kumnit Nong** (knong@gmu.edu) George Mason University

### Thin Film Evolution over Thin Porous Layers

We study the simulated models of the aqueous layer on the pre-corneal tear films of a human eye. These models describe the behavior of fluid films with and without the inclusion of the permeable porous medium that models a contact lens. A fluid dynamic model for the thin fluid film over thin porous layers is formulated by using a nonlinear fourth order partial differential equation with four boundary conditions and one initial condition. The evolution equations are solved numerically in Matlab in order to predict the effect of various parameters (at realistic values) on time of the thin films rupture. The results indicate that the presence of thin porous layers is a dominant effect and the different slip conditions at the liquid-lens boundary also have significant impact on thinning the thin aqueous layers. The computed numerical results allow us to predict film break up times for tear films on a contact lens.

**Juliet Marie Portillo** (joules09@yahoo.com ) San Francisco State University

### Exploring DNA Unknotting by Type II Topoisomerases

Our group studies DNA using topology, knot theory and various computational methods. In particular we are interested in the function of Type II Topoisomerases as a DNA unknotting mechanism. Type II topoisomerases perform strand passage to unknot DNA. It is known that this process is non-random. Our goal is to find a model for the enzymatic action that reproduces the experimental data. We model DNA as polygons in three dimensional space, and using different algorithms we sample the space of knotted conformations and organize these knots by knot type. We perform random strand passage using Dowker Thistlethwaite (DT) Codes to show that repeated strand passage results in unknotting. We explore different topological mechanisms for the enzymatic action and we study of the average writhe of the knotted conformations as well as the strand-passage metric (of Gordian distance) between knots. We have improved certain Gordian distance bounds and we propose that that the sign of the average writhe is an invariant of knots. This research is supported by a RISE fellowship (NIH 5-R25-GM59298-08), by NIH grants P20 MD000262 and 2S06 GM052588, by the UC Berkeley URAP program, and by the SFSU Center for Computing in the Life Sciences. This is joint work with , Trevor Blackstone, Reuben Brasher, Nathan Shayefar (UC Berkeley), Itamar Landau (UC Berkeley), Janella Slaga and Mariel Vazquez

**Tina Kathleen Rapke** (tkrapke@ucalgary.ca ) University of Washington

### Integrating Boundaries: A journey of learning and doing mathematics

The education literature suggests that there is an emerging interest in the phenomenon of mathematicians as learners and the possible role an understanding of this phenomenon may play in the classroom. This intent of this poster presentation is to put forth the ideas of the author's doctoral research. The research will add to this emerging genre of literature as it will be a first hand account of a mathematician reflecting on her own learning and doing of mathematics. The dissertation is being developed in the form of a fictionalised, conversation between two characters-a graduate student mathematician and her supervisor. Through this conversation, the complex and shifting roles of student/learner and supervisor/mathematician/educator will be explored, new mathematics will be revealed, and significant themes of learning and teaching mathematics will be examined.

**Sriyani Renuka Menike Rathugamage** (rsmenike@oakland.edu) Oakland University, Michigan

### A Damageable String

We describe a model for the vibrations of a string when the evolution of its material damage is taken into account. The damage is caused by opening and growth of microcracks and microcavities. The model consists of a hyperbolic equation of motion coupled with a parabolic differential inclusion which describes the evolution of the damage. The proof of the existence of the unique weak solution for the dynamic model with viscosity is described to a regularized problem. The motivation for this research comes from the need in structural engineering to predict fatigue failure of structures.

**Andrzej Sokolowski** (andrzejsoko@yahoo.com) Texas A&M University

### Mathematical Modeling Supported by Physics Simulations

Mathematical modeling is a territory between science and mathematics where science curricula focusing on science content, leave it to math, and mathematics, focusing on pure rather than applied topics, leaves it to science. This territory has been abounded due to tightly defined objectives in science and math curricula. Fulfilling this gap in a content rich environment where students will be given opportunity to apply mathematical tools in science seems to be beneficial for enhancing both; science and math, and more importantly it will built in students' minds a unified structure of the nature. In addition to providing opportunities for integrating math and sciences, there are also other advantages of utilizing science simulations in math classes:

- They are free from mathematical equations, symbols, and coordinate axes showing clear scientific scenarios.
- Measuring devices such as rulers and stopwatches are usually embedded in the simulations and can be used by students to take data or to manipulate on variables.
- The process of the experiment can be played back providing opportunity for multiple observations. and modifications of the environment.
- While applying math tools, students reinforce scientific theorems and laws.
- Students might be placed in roles of engineers who determine the best combination of scientific parameters to achieve a desired output.

During the presentation, I would like to share a few ideas on utilizing physics simulations (created by Physics Education Technology project at Colorado State University) to modeling functions studied in a high school curriculum.

**Erwin Suazo** (suazo@mathpost.asu.edu) Arizona State University

### An Integral Form of the Nonlinear Schrodinger Equation with Variable Coefficients

We propose an explicit construction of the fundamental solutions to the one-dimensional Schrödinger equation with a particular linear time-dependent Hamiltonian. For some special choice of coefficients of the Hamiltonian this system can be integrated and therefore the fundamental solution has an explicit form. Applications to physics are outlined.

**Zoe Nicole Talbot** (znt@sfsu.edu) SFSU

### Modeling circular DNA and its thermal fluctuations in solution

Computer simulation methods are useful tools in modeling DNA polymers of various topologies. We model circular DNA which occurs in bacterial chromosomes, naturally occurring plasmids, as well as human mitochondrial DNA. Type II topoisomerases are enzymes that change the topology of such DNA forms by inducing a transient strand passage. To gain an understanding of this topological mechanism, we model circular DNA as polygons in the simple cubic lattice, $\mathbb{Z}^3$ . We use the BFACF algorithm which is a dynamic Monte Carlo method to generate a Markov chain of polygons. However, polygons in $\mathbb{Z}^3$ are not good models for DNA due to their rigidity and bending properties. Furthermore, lattice chains manipulated by the BFACF algorithm do not take into account parameters such as the solution's ionic strength, which can alter DNA conformations. We propose to make our polygons into a better model for DNA by incorporating a potential energy function used by Tesi *et. al.* (1994) that takes into account ionic conditions. The potential energy function is dependent on the short range force between non-bonded monomers (edges) and a Coulomb potential which can be reduced to making adjustment on one parameter proportional to ionic concentration. We here present our preliminary results.

**Stephanie Lee Vance** (slvance@math.washington.edu) University of Washington

### Mordell's Inequality for Eisenstein and Hurwitz Lattices

Lattice sphere packings in $n$-dimensional Euclidean space are configurations of non-overlapping congruent spheres whose centers form a lattice. Their optimal packing density is known only for $n \leq 8$ and $n = 24$ and many of the densest known lattice packings have a lattice of sphere centers with an Eisenstein or Hurwitz structure. We prove a version of Mordell's inequality that gives an upper bound on the density of Eisenstein and Hurwitz lattice sphere packings. This inequality is used to prove that the 16-dimensional Barnes-Wall lattice sphere packing is optimal as a Hurwitz lattice sphere packing.

**Eric E Westlund** (eewestlu@mtu.edu) Michigan Technological University

### Cayley Graphs and Alspach's Conjecture

If $(A, +)$ is a finite group and $S \subset A \setminus \{0\}$ , then the Cayley graph $\textsc{cay}(A, S)$ is defined to be the graph with vertex set $A$ and edge set $\{xy : x - y \in S \text{ or } y - x \in S\}$. In 1984, Alspach conjectured that every $2k$-regular connected Cayley graph on a finite abelian group has a decomposition into $k$ edge-disjoint Hamilton cycles. We will discuss recent progress on this problem when $k = 3$.

# Graduate Student Paper Session

**Leigh Cobbs** (cobbs@math.rutgers.edu) Rutgers University

### And ... Action!

In mathematics what objects move things around, directing an inputted item to a new location? If you answered "functions", you are correct. You probably know that a bijective function is a 1-1, onto map between two sets. You might also know that a permutation is just a bijection between a set and itself. You may even know what a *group* is (if not, just think about the set of integers $\mathbb{Z}$). Groups like $\mathbb{Z}$ are the real "movers and shakers" in mathematics; they can actually be viewed as sets of permutations. This identification is called a *group action*. In this talk, we will take a look at what it means for a group to act on a graph (diagrams of vertices and edges, not $(x, y)$-coordinates). After we are comfortable moving around vertices and edges, we will "divide" by the action of the group to get a quotient graph (no remainder graphs, though). Correctly labeling this new graph will give us a *graph of groups*. Finally, we will learn how reverse this process, that is how to recover the original group from a graph of groups. And . . . cut!

**Benjamin Galluzzo** (bgalluzz@math.uiowa.edu ) University of Iowa

### Introduction to Groundwater Flow Modeling using Finite Difference Methods

In many parts of the world, the search for potable water is non-trivial. However the mathematics needed to model fluid flow are easily within reach of a calculus student. In this talk, we will introduce standard hydrogeologic principles and use algebraic manipulation and basic calculus to develop a working model. The talk will conclude with a brief discussion of current research in the area.

**Alvaro Guevara** (aguevara@math.lsu.edu) Louisiana State University

### Three reasons why convexity is important in mathematics

Convexity plays a key role in several areas of mathematics and its applications. Three reasons behind this will be highlighted: geometric connections, optimization problems, and efficiency of algorithms. All the basic definitions will be given and several examples illustrating these facts will be presented.

**Michelle Hackman** (mehackma@indiana.edu) Indiana University

### Minimal Surfaces: The Catenoid

In this talk I will introduce the catenoid and explain how it can be altered to create other minimal surfaces. The catenoid is a surface of revolution created by revolving the catenary curve about the $x$-axis. It is a minimal surface because it has mean curvature equal to 0 at every point. Intuitively, the mean curvature of a surface at a point measures on average how curved the surface is near the point.

Once we have established that the catenoid is a minimal surface, we can use it as a basic building block to create some other minimal surfaces. One idea is to add a handle to the catenoid, but a result of Schoen states that such a minimal surface does not exist. In fact, a parametrization that should give the catenoid with a handle instead defines a minimal surface called the fence of catenoids. Another idea then is to twist the fence of catenoids. This leads to a family of minimal surfaces, each differing by the angle of the twist. The talk will focus on the intuition behind minimal surfaces and will include images of the various surfaces discussed.

**Malgorzata Aneta Marciniak** (mammw3@mst.edu) Missouri University of Science and Technology

### On Special Surfaces in 3-Dimensional Space

During my talk I will introduce ruled surfaces (which allow a line to pass through every point) and minimal surfaces (with zero mean curvature or minimal area). I will then present some open problems in Differential Geometry that are accessible for undergraduate students of any level.

**Stanley Andrew Parkerson** (msu-sparkerson@student.mcneese.edu ) McNeese State University

### Triple systems

A *balanced incomplete block design*, or BIBD, is an ordered pair $(V, B)$, where $V$ is a finite set of elements, and $B$ is a collection of equally-sized proper subsets of $V$, or *blocks*, such that each unordered pair of elements in $V$ occurs exactly $\lambda$ times in $B$. A *triple system* is a BIBD with blocks of size 3. In this talk we will discuss the history of this field of combinatorial design, different types of triple systems, the research that has been done on them, and open problems.

# Index of Speakers