## Abstracts of Papers

 presented at MathFest 2007

# Abstracts of Papers Presented at MathFest 2007 

San Jose<br>August 3-5, 2007



Published and Distributed by
The Mathematical Association of America

## Contents

## Invited Addresses

AWM-MAA Etta Z. Falconer Lecture ..... 1
Comparing Evolutionary Trees
Katherine St. John
Sunday, August 5, 8:30 AM-9:20 AM
Earle Raymond Hedrick Lecture Series ..... 1
Jennifer Tour Chayes
Lecture 1: Models of the Internet and the World Wide Web
Friday, August 3, 10:30 AM-11:20 AM
Lecture 2: Mathematical Behavior of Random Scale-Invariant Networks
Saturday, August 4, 9:30 AM-10:20 AM
Sunday, August 5, 9:30 AM-10:20 AM
Euler Society Invited Address ..... 2
Euler in Three Acts
William Dunham
Sunday, August 5, 1:00 PM-1:50 PM
James R. Leitzel Lecture ..... 2On Being a Mathematical Citizen: The Natural NExT Step
Lynn Arthur Steen
Saturday, August 4, 10:30 AM-11:20 AM
Joint MAA-SMB Invited Address ..... 2On the Dynamics and Evolution of Emergent and Re-emergent Diseases: from Tuberculosis toSARS to the Flu
Carlos Castillo-ChavezFriday, August 3, 8:30 AM-9:20 AM
MAA Invited Addresses ..... 2
Revenge of the Twin Prime Conjecture
Saturday, August 4, 8:30 AM-9:20 AM
Why Did Lagrange "Prove" the Parallel Postulate?
Judith V. GrabinerSunday, August 5, 10:30 AM-11:20 AM
Managing Natural Resources: Mathematics Meets Politics, Greed, and the Army Corps of En-gineersLouis J. Gross
Friday, August 3, 9:30 AM-10:20 AM
MAA Student Lecture ..... 3
Splitting the Rent: Fairness Problems, Fixed Points, and Fragmented Polytopes
Francis Edward Su
Friday, August 3, 1:30 PM-1:50 PM
NAM David Blackwell Lecture ..... 3
Puzzling Probabilities Featuring the Street Game of Craps
Jack Alexander
Saturday, August 4, 1:00 PM-1:50 PM
PME J. Sutherland Frame Lecture ..... 4
Negafibonacci Numbers and the Hyperbolic Plane
Donald Knuth
Saturday, August 4, 8:00 PM-8:50 PM
MAA Alder Awards Session ..... 4
Saturday, August 4, 2:00 PM-3:30 PM
Engaging Students Mathematically—Pitch by Pitch
Timothy Chartier
Reclaiming Da Vinci: Visualization and Mathematics
Satyan Devadoss
Motivating Student Learning through Real—World Applications of Higher MathematicsDarren Narayan
Invited Paper Sessions
Euler Society Invited Paper Session ..... 5
Sunday, August 5, 1:00 PM-1:50 PM
Organizers: Rob Bradley, Adelphi University and
Ed Sandifer, Western Connecticut State University
Gems in Applied Mathematics ..... 7
Saturday, August 4, 8:30 AM-10:30 AM
Organizer: Kay Somers, Moravian University
Graph Theory Ideas for Undergraduate Research ..... 8
Sunday, August 5, 1:00 PM-4:00 PM
Organizer: Aparna Higgins, University of Dayton
Manifolds with Density and Partitioning Problems ..... 9
Friday, August 3, 1:00 PM-4:00 PM
Organizer: Frank Morgan, Williams College
Mathematical Questions in Bioinformatics ..... 10Friday, August 3, 1:00 PM-4:00 PMOrganizers: Jennifer Galovich, St. John's University and Laurie Heyer, Davidson College
Prime Numbers-New Developments on Ancient Problems ..... 11
Saturday, August 4, 1:00 PM-4:00 PM
Organizers: Dan Goldston, San Jose State University and Carl Pomerance, Dartmouth College
Research with Undergraduates ..... 12
Saturday, August 4, 1:00 PM-3:30 PM
Organizer: Mario Martelli, Claremont-McKenna College
Contributed Paper Sessions
Advances in Recreational Mathematics ..... 13
Sunday, August 5, 1:00 PM-3:45 PM
Organizers: Paul R. Coe, Dominican University and
Kristen Schemmerhorm, Dominican University
Attracting and Retaining Students to Mathematics Programs via Outreach ..... 15
Friday, August 3, 8:30 AM-10:30 AM
Organizer: Sangeeta Gad, University of Houston-Downtown
Biomathematics in the First Two Years ..... 16
Saturday, August 4, 8:30 AM-10:30 AM
Saturday, August 4, 1:00 PM $-3: 00 \mathrm{PM}$
Sunday, August 5, 8:30 AM-10:30 AM
Organizer: Timothy D. Comar, Benedictine University
Challenges and Successful Strategies in Teaching a Numerical Analysis Course ..... 21
Sunday, August 5, 8:30 AM-10:30 AM
Organizer: Olga Brezhneva, Miami University, Ohio
Current Issues in Mathematics Education ..... 23
Friday, August 3, 1:00 PM-5:30 PM
Organizers: Carol Vobach, University of Houston-Downtown andNancy Leveille, University of Houston-Downtown
Emerging Technologies for Mathematics Teaching ..... 28
Friday, August 3, 1:00 PM-3:00 PMOrganizers: Lila F. Roberts, Georgia College \& State University andAmy F. Kelley, Georgia College \& State University
Euler Society Contributed Paper Session ..... 30
Saturday, August 4, 1:00 PM-5:30 PM
Organizers: Rob Bradley, Adelphi University andEd Sandifer, Western Connecticut State University
Fun and Innovative Teaching Techniques for an Abstract Algebra Class ..... 31
Saturday, August 4, 3:15 PM-5:30 PM
Organizer: Sharon Clarke, Pepperdine University and Andrew Hetzel, Tennessee Tech University
General Contributed Paper Sessions ..... 33
\# 1 Friday, August 3, 8:30 AM-10:30 AM\# 2 Friday, August 3, 1:00 PM-3:00 PM
\# 3 Friday, August 3, 3:15 PM-5:15 PM
\# 4 Sunday, August 5, 8:30 AM-10:30 AM
\# 5 Sunday, August 5, 8:30 AM-10:30 AM
\# 6 Sunday, August 5, 1:00 PM-3:00 PM
\# 7 Sunday, August 5, 1:00 PM-3:00 PM
\# 8 Sunday, August 5, 1:00 PM-3:00 PM
\# 9 Sunday, August 5, 3:15 PM-5:15 PM
\#10 Sunday, August 5, 3:15 PM-5:15 PM
\#11 Sunday, August 5, 3:15 PM-5:15 PM
Organizer: Sarah J. Mabrouk, Framingham State College
Getting Students to Discuss and to Write about Mathematics ..... 46
Saturday, August 4, 1:00 PM-5:15 PM
Organizer: Murphy Waggoner, Simpson College
Graph Theory and Applications ..... 50
Saturday, August 4, 8:30 AM-10:30 AM
Sunday, August 5, 1:00 PM-3:00 PM
Organizers: Ralucca Michelle Gera, Naval Postgraduate School andRichard M. Low, San Jose State University
Innovative Ideas for Teaching Concepts in an Introductory Statistics Course ..... 52
Friday, August 3, 3:15 PM-6:15 PM
Sunday, August 5, 1:00 PM-3:00 PM
Organizer: Murray H. Siegel, Governor's School for Science \& Mathematics, South Carolina
Mathematics and the Arts ..... 56
Saturday, August 4, 1:00 PM-3:00 PM
Sunday, August 5, 1:00 PM-3:00 PM
Organizer: Douglas E. Norton, Villanova University
Mathematics of Sports and Games ..... 59
Friday, August 3, 8:30 AM-10:30 AM
Organizers: Howard Lewis Penn, United States Naval Academy andE. Lee May, Salisbury University
Student Research in Industrial Mathematics ..... 60Sunday, August 5, 8:30 AM-11:15 AMOrganizers: Bem Cayco, San Jose State University and Tim Hsu, San Jose State University
Teaching a History of Mathematics Course ..... 61
Sunday, August 5, 8:30 AM-10:30 AMSunday, August 5, 2:00 PM-5:00 PMOrganizers: Joel Haack, University of Northern Iowa andAmy Shell-Gellasch, Pacific Lutheran University
Teaching Calculus in High School: Ideas that Work ..... 65
Sunday, August 5, 8:30 AM-10:30 AM
Organizers: Dan Teague, North Carolina School of Science and Mathematics andSusan Schwartz Wildstrom, Walt Whitman High School
Graduate Student Poster Session ..... 66
Saturday, August 4, 1:00 PM-2:30 PM
Organizer: Jim Freeman, Cornell College
Index of Speakers69

# Invited Addresses 

# AWM-MAA Etta Z. Falconer Lecture 

Sunday, August 5, 8:30 am-9:20 am

Katherine St. John (stjohn@lehman.cuny.edu) City College of New York

## Comparing Evolutionary Trees

Evolutionary histories, or phylogenies, form an integral part of much work in biology. In addition to the intrinsic interest in the interrelationships between species, phylogenies are used for drug design, multiple sequence alignment, and even as evidence in a recent criminal trial. A simple representation for a phylogeny is a rooted, binary tree, where the leaves represent the species, and internal nodes represent their hypothetical ancestors. This talk will focus on some of the elegant, combinatorial questions that arise from assembling, summarizing, and visualizing phylogenetic trees.

## Earle Raymond Hedrick Lecture Series

## Jennifer Tour Chayes (jchayes@microsoft.com) Microsoft Corporation

## Lecture 1: Models of the Internet and the World Wide Web Friday, August 3, 10:30 am-11:20 am

Although the Internet and the World Wide Web have many distinct features, both have a self-organized structure, rather than the engineered architecture of previous networks, such as the phone or transportation systems. As a consequence of this self-organization, the Internet and the World Wide Web have a host of properties which differ from those encountered in engineered structures: a broad "power-law" distribution of connections (so-called "scale-invariance"), short paths between two given points (so-called "small world phenomena" like "six degrees of separation"), strong clustering (leading to so-called "communities and subcultures"), robustness to random errors, but vulnerability to malicious attack, etc. During this lecture, I will first review some of the distinguishing observed features of these networks, and then review the recent models which have been devised to explain these features. The basic models have their origins in graph theory and statistics.

## Lecture 2: Mathematical Behavior of Random Scale-Invariant Networks <br> Saturday, August 4, 9:30 am-10:20 am

This lecture will be devoted to a mathematical analysis of some of the standard models of random scaleinvariant networks, including models of the Internet, the World Wide Web, and social networks. I will show how these models can be rewritten in terms of a Polya urn representation, which will allow us to prove that the models exhibit some of the observed properties of real-world networks, including scale-invariance and vulnerability to attacks by viruses. Using these models, I will also examine various strategies for containment of viruses and epidemics on technological and social networks.

## Lecture 3: Convergent Sequences of Networks Sunday, August 5, 9:30 am-10:20 am

In the final lecture of this series, I will abstract some of the lessons of the previous lectures. Inspired by dynamically growing networks, I will ask how we can characterize general sequences of graphs in which the number of nodes grows without bound. In particular, I will define various natural notions of convergence for a sequence of graphs, and show that, in the case of dense graphs, many of these notions are equivalent. I will also give a construction for a function representing the limit of a sequence of graphs. I'll review examples of some simple growing network models, and illustrate the corresponding limit functions.

# Euler Society Invited Address 

Sunday, August 5, 1:00 pm-1:50 pm
William Dunham (wdunham@muhlenberg.edu) Muhlenberg College
Euler in Three Acts
Act I provides an overview of his work to suggest the depth and diversity of his life's achievement. Act II considers the 1737 derivation of the "Euler product-sum formula" and its use in proving the divergence of the reciprocals of the primes - arguably the birth of analytic number theory. Finally, in Act III we see Euler's evaluation of a non-trivial integral in which he cleverly enlisted the aid of two illustrious predecessors.

# James R. Leitzel Lecture 

Saturday, August 4, 10:30 am-11:20 am
Lynn Arthur Steen (steen@stolaf.edu) St. Olaf College

## On Being a Mathematical Citizen: The Natural NExT Step

As public concerns about education and competitiveness evolve, so too must the responsibilities of collegiate mathematicians, including especially the participants and alumni of Project NExT. No longer can we afford to focus only on our students, our department, our college, or our research. Mathematics at all levels and of all kinds is at the center of major challenges to the nation's education and economy. These issues challenge us all to be good mathematical citizens in this evolving national landscape.

## Joint MAA-SMB Invited Address

Friday, August 3, 8:30 am-9:20 am

Carlos Castillo-Chavez (chavez@math.asu.edu) Arizona State University

## On the Dynamics and Evolution of Emergent and Re-emergent Diseases: from Tuberculosis to SARS to the Flu

The role of mass transportation, immigration, tourism, demographic growth, and bioterrorism are but some of the engines behind disease dynamics and disease evolution. Examples using recent epidemic outbreaks will be used to highlight the role of mathematics in the evaluation of the impact of these epidemic drivers. Mathematics will also be used to highlight the relevance of "borderless" health policy perspectives.

## MAA Invited Addresses

## Dan Goldston (goldston@math.sjsu.edu) San Jose State University

## Revenge of the Twin Prime Conjecture

Saturday, August 4, 8:30 am-9:20 Am
Two years ago Pintz, Yildirim, and I proved that there always exist primes that are very close together - very close meaning much closer than the average distance between neighboring primes. Our method also proves that if the primes are well distributed in arithmetic progressions then one can obtain results not too far from the twin prime conjecture. For example, if the Elliott-Halberstam conjecture is true then there are infinitely many pairs of primes with difference 16 or less. With these successes I was hopeful that before too long our method could be pushed to unconditionally show that there are infinitely often pairs of primes closer than some fixed bounded distance, i. e. bounded gaps, a giant step towards the twin prime conjecture. In this talk I will discuss the method and why perhaps further progress towards bounded gaps and the twin prime conjecture is going to be difficult, although I will be delighted to be proved wrong.


#### Abstract

Judith V. Grabiner (jgrabiner@pitzer.edu) Pitzer College Why Did Lagrange "Prove" the Parallel Postulate? Sunday, August 5, 10:30 AM-11:20 AM In 1806, Joseph-Louis Lagrange read a memoir "proving" Euclid's parallel postulate to the Academie des Sciences in Paris, but stopped, saying, as the story goes, said, "I have to think about this some more." We'll look at Lagrange's (still unpublished) Paris manuscript on this subject, and place this activity in the context of his mathematical career. We also look at how the ideas in this manuscript are related to Lagrange's philosophy of mathematics, Newtonian mechanics, and Leibniz's Principle of Sufficient Reason. Finally, we reflect on what this episode tells us about eighteenth-century attitudes toward geometry and space.


Louis J. Gross (gross@tiem.utk.edu) University of Tennessee

## Managing Natural Resources: Mathematics Meets Politics, Greed, and the Army Corps of Engineers Friday, August 3, 9:30 AM-10:20 AM

The availability of satellite-based remote sensing, computers capable of handling large databases, rapid communication networks, and small radio sensors able to transmit details on individual animals has fostered the development of computational ecology. By combining mathematical and computer models of natural systems with geographically-explicit details of the biotic and abiotic components of the environment, we can compare alternative virtual futures to better plan sustainable ecosystems. Opportunities exist for mathematicians to develop and apply models for harvest regulation, control of invasive species, fire management, and disease and pest control. This optimistic view of the potential for computational methodologies to aid in managing natural systems is tempered by the reality that factors other than scientific best practices are involved. I will discuss a range of applications from relatively simple models for invasive plant control to models applied to long-term planning of an immense restoration effort in the Everglades of South Florida.

## MAA Student Lecture

## Friday, August 3, 1:30 Pm-1:50 PM

Francis Edward Su (su@math.hmc.edu) Harvey Mudd College

## Splitting the Rent: Fairness Problems, Fixed Points, and Fragmented Polytopes

How do you divide the rent among roommates fairly? My friend's dilemma was a question that mathematics could answer, both elegantly and constructively. We show how it and other "fair division" questions - the most famous of which is the problem of Steinhaus: how do you cut a cake fairly? - motivate a host of "combinatorial fixed point theorems" and problems about polytopes. They provide excellent examples of how mathematics can address an old class of problems in new ways, and conversely, how problems in the social sciences can motivate new mathematics - where topology, geometry, and combinatorics meet social applications, and where research by undergraduates has played a big role.

## NAM David Blackwell Lecture

## Saturday, August 4, 1:00 Pm-1:50 Pm

Jack Alexander (jalexan2@mdcc.edu) Miami Dade College

## Puzzling Probabilities Featuring the Street Game of Craps

The study of probability has, for some time now, been quite intriguing to me. Part of this fascination is fueled by the fact that some probability challenges require strategies that employ various aspects of mathematics to obtain a solution. This presentation uses calculus, algebra, geometry, graphing, as well as probability theory. To illustrate this contention, this presentation will give analytic solutions and computer simulations for three probability problems that I find quite interesting. These problems are: Count Buffon's Needle Problem; The Triangle from a Line Segment Problem; and The Street Game of Craps. The Street Game of Craps was
detailed in a problem from a book entitled Introduction to the Theory of Statistics, 3rd Edition, (1963). This text was written by Alexander M. Mood, Franklin A. Graybill, and Duane C. Boes. It was edited by David Blackwell and Herbert Solomon. The book was part of a series of probability and statistics texts published by McGraw-Hill.

## PME J. Sutherland Frame Lecture

## Saturday, August 4, 8:00 pm-8:50 pm

Donald Knuth Professor Emeritus of the Art of Computer Programming, Stanford University

## Negafibonacci Numbers and the Hyperbolic Plane

All integers can be represented uniquely as a sum of zero or more "negative" Fibonacci numbers $F_{-1}=1$, $F_{-2}=-1, F_{-3}=2, F_{-4}=-3$, provided that no two consecutive elements of this infinite sequence are used. The NegaFibonacci representation leads to an interesting coordinate system for a classic infinite tiling of the hyperbolic plane by triangles, where each triangle has one $90^{\circ}$ angle, one $45^{\circ}$ angle, and one $36^{\circ}$ angle.

## MAA Alder Awards Session

## Saturday, August 4, 2:00 pm-3:30 pm

Timothy Chartier (tichartier@davidson.edu) Davidson College

## Engaging Students Mathematically—Pitch by Pitch

Pitching can play an important role in engaging students - pitching ideas, that is. This talk will reflect on my experiences (both successful and challenging) in presenting students with ideas that engage them in and out of the classroom. We will discuss student work that includes scholarly research, expository writing and service to the community.

## Satyan Devadoss (satyan.devadoss@williams.edu) Williams College

## Reclaiming Da Vinci: Visualization and Mathematics

The Renaissance exemplified a natural unity of the sciences and the arts. Leonardo da Vinci epitomized this unity in his paintings, sculptures, inventions, and scientific study, bringing together artistic creativity and systematic rigor. For me, his mindset has been a driving force, motivating and directing my own work. This talk portrays my struggles and successes in visualization, most notably in designing new courses and fostering undergraduate research. The topics range from particle collisions and polyhedra in mathematics, to origami design and cartography in computer science, to manufacturing and modeling in studio art.

## Darren Narayan (darren.narayan@rit.edu) Rochester Institute of Technology

## Motivating Student Learning through Real—World Applications of Higher Mathematics

A good teacher inspires and motivates students to tap into the tremendous energy that can be derived from learning and doing mathematics, as well as prepare them for the world they will enter after graduation. Students perk up when they can apply the mathematics that they have learned. Traditional curricula seldom offer students concrete examples of cutting-edge, real-world applications of higher mathematics. As a result students finish their undergraduate mathematics career asking themselves the question, "What else can I do with a mathematics degree besides teach?" To address this need, we launched the STEM Real World Applications Modules Project funded by an NSF-CCLI grant. Topics included in this project include applications of graph theory to reconstruction of three-dimensional images (Microsoft Research), analysis of flight route maps for airlines (JetBlue Airways), mathematical analysis of telecommunication networks (Level 3 Communications), and identification of WWW cyber communities (Google). The goal of the STEM Real World Applications Modules Project is to better prepare faculty to answer the question, "What else can mathematics graduates do besides teach?" A student completing these modules will already know.

# Invited Paper Sessions 

Euler Society Invited Paper Session<br>Sunday, August 5, 1:00 pm-1:50 pm

Robert E. Bradley (bradley@adelphi.edu) Adelphi University

## Euler's Analytic Geometry

Leonhard Euler wrote his textbook Introductio in analysin infinitorum early in his Berlin period to serve as a bridge between elementary algebra and differential calculus. In it, he strove "to develop more adequately and clearly than is the usual case those things which are absolutely required for analysis." Book I of the Introductio is rich with innovation and is reasonably well-known even today as the place where, for example, trigonometric and logarithmic functions first received their modern treatment. The second volume, though less revolutionary and less well remembered than the first, represented an important step in the evolution of analytic geometry. As with much of Euler's work, it contributed to the gradual shift in the 18 th century from a geometric conception of analysis to an algebraic one. In this talk, we will examine the place of Book II of Euler's Introduction in the historical development of analytic geometry and provide illustrations of some of the innovations contained therein, including Euler's classification of cubic equations and his treatment of intersections of plane curves.

## Lawrence D'Antonio (ldant@ramapo.edu) Ramapo College

## How Euler Almost Did It

Much has been written and said about Euler's many magnificent triumphs. This talk will instead look at Euler's misses and near-misses. This includes results that Euler thought he had proven but did not, those which he almost proved but others published first, and those in which Euler had all of the necessary ideas but did not put them together. Examples include the $n=3$ case of Fermat's Last Theorem, the 4 -square Theorem for natural numbers, and the theory of elliptic functions.

## Brian Hopkins (bhopkins@spc.edu) Saint Peter’s College

## Partitions, Divisor Sums, and the Pentagonal Number Theorem

This talk will survey and connect Euler's work in two areas of number theory. Topics related to the sum of divisors of an integer have been studied since antiquity. Euler discovered several new amicable pairs and proved a conjecture about the form of even perfect numbers. Questions about partitions of integers were fairly new in Euler's day. He basically initiated the serious study of partitions with the application of generating functions. These two topics are connected by the Pentagonal Number Theorem, a result that took Euler ten years to prove to his satisfaction. We will see that both partitions and divisor sums have a similar recursion formula that follows from the Pentagonal Number Theorem, results that prompted d'Alembert to write that "your theorem on series seems very beautiful."

## Stacy Langton (langton@sandiego.edu) University of San Diego

## Euler on the Principles of Elasticity

During his second St. Petersburg period, Euler wrote the two great papers "Genuine principles of the theory of equilibrium and motion of perfectly flexible or elastic bodies" (E410), and "On two methods of determining the equilibrium or motion of flexible bodies" (E481), in which he reflected on the ideas underlying the work on elasticity that he had carried out over a period of more than forty years and tried to extract and formulate the basic principles of the subject, at least in the case of an elastic or flexible line (that is to say, a onedimensional body). In this talk I will describe what Euler does in these papers and show how it fits into his previous work and into the further development of the subject.

## Mark McKinzie (mmckinzie@sjfc.edu) St. John Fisher College

## Euler's New Theory of Music

Euler's "Tentamen novae theoriae musicae" (1739) was famously described by Nicolas Fuss as having little success, for it contained "too much geometry for musicians, and too much music for geometers". Recent trends in music composition and music theory have generated renewed interest in Euler's treatment of temperament and modes. In this talk we will give a brief overview of Euler's theory and recent work influenced by it.

## Shawnee McMurran, V. Frederick Rickey (smcmurra@csusb.edu) Calif State Univ, San Bernardino

## Euler Can't Resist Ballistics

In the first half of the 18th century, Benjamin Robins, a British mathematician and military engineer, invented the ballistic pendulum. This device allowed for fairly accurate estimates of the muzzle velocities of artillery. In 1742, Robins published New Principles of Gunnery, the first book to deal extensively with external ballistics. Robins' work motivated deeper mathematical analyses of projectile motion - a topic tackled by such mathematicians as Leonhard Euler and Daniel Bernoulli. Not surprisingly, Euler chose to "annotate" Robins' work, effectively tripling its length. This talk will describe some of Robins' conclusions and Euler's commentary, with a focus on Euler's novel approach to the equations of projectile motion in a vacuum.

## Maria Clara Nucci (nucci@unipg.it) Universita- di Perugia

## A Potpourri from Euler

Who was Euler and what did he do? Let his work speak for him. We present a potpourri made with excerpts from two of his books ("Methodus Inveniendi Lineas Curvas Maximi Minimive Proprietate Gaudentes", and "Theoria Motus Corporum Solidorum Seu Rigidorum"), and some of his papers on differential equations and celestial mechanics.

## Kim Plofker (Kim_Plofker@alumni.brown.edu) Brown University

## Euler, Lunar Theory, and the Calculus

It has often been claimed that mid-eighteenth-century breakthroughs in lunar theory were the first truly socially relevant application of contemporary differential and integral calculus. Recent research, however, suggests that these developments in astronomy were driven more by empirical pragmatism than by mathematical advances in theory. This paper explores Euler's contributions to the production of new lunar tables and the impact of his mathematical approach to the problem.

Ed Sandifer (esandifer@earthlink.net) Western Connecticut State University

## The End of his Triumph: Euler's Second St. Petersburg Years

Euler had three goals when he returned to St. Petersburg in 1766. He wanted to finish his comprehensive study of mechanics, to give the Academy an inventory of articles that could fill the pages of their journals for twenty years after his death, and to train a new generation of scholars as a legacy to sustain the Academy. He was successful in two out of these three goals. We describe his progress towards these goals against the backdrop of Russian history, Academy politics and episodes in Euler's life.

## Lee Stemkoski (stemkoski@adelphi.edu) Adelphi University

## Cataloging and Publishing Euler's Works: A History

Since Euler's death, there have been repeated efforts to compile a comprehensive index of his works (most notably by Fuss and Enestrom) with additional efforts to publish these works. We will survey these endeavors and comment on the current "total" number of Euler's works.

## Dieter Suisky (suisky5@aol.com) Humboldt University Berlin

## Geometry and Calculus in Euler's Mechanics

It will be demonstrated that Euler's program for mechanics presented in the treatise Mechanics or the analytical representation of the science of motion paved the way for a successful development of mechanics in the 18th
century. In contrast to Newton's geometry-related procedure in the Principia Euler formulated mechanical laws preferentially in terms of the differential calculus. Euler claimed that "those laws of motion which a body observes when left to itself in continuing either rest or motion pertain properly to infinitely small bodies". Geometrically, these bodies can be considered as points, but mechanically they are less than any extended body, but different from mathematical points due to their finite mass. As a consequence, mechanical quantities are subdivided into two classes, (i) in those which are only represented by finite quantities, like mass and force, and (ii) those which are represented both by finite and infinitesimal quantities, like translation, time and velocity. Necessarily, the general and fundamental equation of motion $m d v=K d t$ is formulated in terms of quantities of both types. Completing Newton's procedure, Euler made a significant progress by adjoining geometry to the calculus since, as it was demonstrated later in Institutiones calculi differentialis, the foundation of the calculus becomes independent of geometrical models and is based on the transfer of rules for finite differences to infinitesimal differences. Analytically, motion is described in terms of infinitesimal time intervals whereas, geometrically, it is related to straight lines and planes as basic elements. Later, Ehrenfest made use of the same procedure to demonstrate that trajectories are only stable in 3D space.

## Ruediger Thiele (Ruediger.Thiele@medizin.uni-leipzig.de) University of Leipzig

## The Rise and Evolution of the Function Concept in 18th Century Analysis

Can you imagine attending a mathematical lecture or reading a mathematical paper without coming across the function concept? Are you fully aware that this concept has changed through time? The roots of the analytical function concept are to be found at the end of the 17th century in Johann Bernoulli (the isoperimetric problems, 1697), and it was Leonhard Euler (1707-1783) as Bernoulli's disciple who picked up this calculating idea and formed the analytical expression representing a function by power series. Euler developed this technique to its limits, considering general power series, especially Puiseux and Laurent series, instead of Taylor series. However, during the controversy over the vibrating string, he noticed that the definition with which he begun had ceased to make sense. Consequently, he found it convenient to take trigonometric series to be the significant concept for the new problems of mathematical physics. This paper gives insight into Euler's openness in adapting the function concept to mathematical problems.

## Erik Tou (etou@dartmouth.edu) Carthage College

## Euler and the English

It is somewhat well-known that Euler corresponded with Caspar Wetstein, a former classmate of Euler's who for many years was Secretary to the Prince of Wales. It was this correspondence that gave rise to a handful of articles published in the Philosophical Transactions of the Royal Society. However, there is one article that appears in the Transactions that did not arise from this correspondence, but was actually initiated by an Englishman, John Dollond. Dollond was particularly interested in some of Euler's work regarding telescope design, and wrote to Euler of a flaw in the mathematics that he had found. Was Dollond right and Euler wrong? Or vice-versa? We will briefly survey Euler's connection to English science and scholarship, and look at a new translation of Euler's reply to Dollond.

## Gems in Applied Mathematics

## Saturday, August 4, 8:30 am-10:30 am

Annalisa Crannell (annalisa.crannell@fandm.edu) Franklin \& Marshall College

## Size Matters

We know that the area of a mathematical object increases with the square of its length, and the volume increases as the cube of its length. This principle has deep consequences in a variety of other fields. We present a panorama of fields that study the effects that size have on structure. These range from bio-mechanical (could a T. Rex run?) to economics (can we infer wealth from skeletal measurements?) to government (via Madison's Federalist 10) to the theory of evolution (are there constraints to evolution by selection?).

## Michael A. Jones (jonesm@mail.montclair.edu) Montclair State University

## A Voting Theory Approach to Golf Scoring

The Professional Golfer's Association (PGA) is the only professional sports league in which the rules describing how an event is scored, changes according to the event. Even without including match play or team play, PGA tournaments can be scored under stroke play or the modified Stableford scoring system; these two methods of scoring are equivalent to using voting vectors to tally an election. This equivalence is discussed and data from the Masters and International Tournaments are used to examine the effect of changing the scoring method on a tournament's results. With as few as 3 candidates, elementary linear algebra and convexity can be used to show that changing how votes are tallied by a voting vector can result in up to 7 different election outcomes (ranking all 3 candidates and including ties) even if all of the voters do not change the way they vote! Sometimes, regardless of the voting vector the same outcome would have occurred, as in the 1992 US Presidential election. I relate this to the question: Can we design a scoring method to defeat Tiger Woods? And answer it, retrospectively, for his record-breaking 1997 Masters performance.

## Nathan Bradley Shank (shank@math.moravian.edu) Moravian College

## Unsolved Gems in Random Graphs

One longstanding question is to prove a central limit theorem for the optimal traveling salesman tour on $n$ points uniformly distributed in $[0,1]^{2}$. A similar open question pertains to the minimal matching and Steiner tree problems. A second open problem of great interest to many different fields is to find the limiting constant in a law of large numbers for the longest common subsequence of two random sequences - also known as the longest common subsequence problem. This problem is very closely related to increasing sequences in the plane (Ulam's problem). The goal of this talk is to introduce the audience to these two specific open problems, explain recent advances, introduce new techniques, and show how to use the problems as motivation to students at different levels.

Jennifer Wilson (wilsonj@newschool.edu) Eugene Lang College the New School for Liberal Arts

## Algebraic Models of Kinship

The development of algebraic models of kinship coincided with the rise of structuralism in the social sciences. Anthropologists Claude Levi-Strauss, Edmund Leach and Harrison White, and mathematicians John Kemeny, J. Laurie Snell and Gerald Thompson, building on the work of Andre Weil, used permutation matrices and other algebraic tools to model marriage laws in indigenous societies. Interest in kinship and structural models waned in the 70 's and 80 's as the limitations of the approach became apparent. But there has been a resurgence of interest in the last decade, fueled by graph-theoretic techniques and social network theory. In this talk, we will review some of the earlier models and discuss recent work of Thomas Schweizer, Douglas White and others who combine formal modeling with empirical analyses.

# Graph Theory Ideas for Undergraduate Research Sunday, August 5, 1:00 pm-4:00 pm 

Joanna Ellis-Monaghan (jellis-monaghan@smcvt.edu) Saint Michaels College

## Minimum Tile Types For Self-Assembled DNA Graphs

There have been recent advances in DNA self-assembly of nanoscale mathematical constructs, in particular graphs such as a cube, truncated octahedra, and more recently a rigid octahedron. One construction method uses $k$-armed branched junction molecules, called tiles, whose arms are double strands of DNA with one strand extending beyond the other, forming a 'sticky end' at the end of the arm that can bond to any other sticky end with complementary Watson-Crick bases. A vertex of degree $k$ is formed from a $k$-armed branched molecule, and joined sticky ends form the edges of the target graph. A good undergraduate research project was determining the minimum number of tiles and edge types necessary to create a given graph under three different laboratory scenarios: 1 . Where the incidental creation of complexes of smaller size than the target graph is
acceptable; 2 . Where the incidental creation of complexes the same size as the target graph is acceptable, but not smaller complexes; 3 . Where no complexes the same size or smaller than the target graph are acceptable. In each of these cases, we found bounds for the minimum and maximum number of tile and edge types that must be designed and gave specific minimum values for common graph classes (complete, bipartite, trees, regular, etc.). For these classes of graphs, we provided either explicit descriptions of the set of tiles achieving the minimum number of tile and edge types, or efficient algorithms for generating the desired set.

## Stephen G. Hartke (hartke@math.uiuc.edu) University of Illinois

## Iterated Line Graphs

Given a graph $G$, the line graph $L(G)$ is the graph whose vertex set consists of the set of edges of $G$, and where two vertices of $L(G)$ are adjacent whenever the corresponding edges of $G$ are incident on a common vertex. The iterated line graph $L^{i}(G)$ is formed by successive applications of the line graph operator: $L^{i}(G)=L\left(L^{i-1}(G)\right)$ for $i \geq 1$, and $L^{0}(G)=G$. Regardless of the starting graph $G$ (with a few small exceptions), iterated line graphs grow exponentially and are locally very dense (as $i$ becomes large), but are globally very sparse. This phenomenon gives iterated line graphs some very nice properties. I will describe projects by undergraduates studying the degree growth, $k$-ordered hamiltonicity, and distinguishability of iterated line graphs.

## Pallavi Jayawant (pjayawan@bates.edu) Bates College

## Small-World Networks

A small-world network is a network in which one can travel from one vertex to another vertex using a relatively small number of edges compared to the size of the network. Many real world networks, such as the World Wide Web and the social network of acquaintances are observed to be small-world networks. Algorithms that generate small-world network models with certain characteristics help us understand the structure and properties of these networks. We look at some of the existing models. A modified model can be described that accounts for the age of the nodes as the network builds over time. The new model can be compared to the existing models using measures such as the clustering coefficient and the characteristic path length.

## Steven J Winters (winters@uwosh.edu) University of Wisconsin Oshkosh

## Distance Properties of Graphs

The distance from a vertex $u$ to a vertex $v$ in a connected graph $G$ is the length of a shortest $u-v$ path in $G$. The eccentricity of a vertex $v$ in a connected graph is the distance between $v$ and a vertex farthest from $v$. The center of a graph is the subgraph induced by those vertices having minimum eccentricity while the periphery is the subgraph induced by those vertices having maximum eccentricity. The distance of a vertex $v$ in $G$ is the sum of the distances from $v$ to the vertices of $G$. The median of a graph is the subgraph induced by those vertices having minimum distance. Other distance related subgraphs will also be defined during this talk. Graph theory problems related to these subgraphs are excellent for undergraduate research projects. For example, we could consider the relative location of subgraphs or the appendage number of a subgraph. We also investigate what happens when an edge is removed from the graph at random.

## Manifolds with Density and Partitioning Problems <br> Friday, August 3, 1:00 pm-4:00 pm

Frank Morgan (frank.morgan@williams.edu) Williams College

## Manifolds with Density and Partitioning Problems

Perelman's stunning 2006 proof of the million-dollar Poincaré Conjecture needed to consider not just manifolds but "manifolds with density" (like the density in physics you integrate to compute mass). Yet much of the basic geometry of such spaces remains unexplored. Partitioning problems provide a good place to start. Speakers: Frank Morgan (Williams College), Michael Hutchings (UC Berkeley), Neil Hoffman (UT Austin), members of the Williams College undergraduate research Geometry Group, Joseph Corneli (PlanetMath.org).

# Mathematical Questions in Bioinformatics 

## Friday, August 3, 1:00 pm-4:00 Pm

Stephen C. Billups (Stephen.Billups@cudenver.edu) University of Colorado at Denver and Health Sciences Center

## Microarray Data Analysis

Gene expression microarrays allow biologists to measure mRNA expression levels of thousands of genes simultaneously in a single experiment. This technology has greatly accelerating biological discovery, particularly with respect to understanding the roles various genes play in biological phenomenon such as disease responses, development cycles, and responses to drugs. But making sense of the vast amounts of data provided by the microarrays is a daunting task and provides numerous opportunities for mathematicians to contribute to biological understanding. This talk will give an overview of how microarrays work and will highlight some interesting mathematical challenges in analyzing the resulting data.

## Stephen G. Hartke (hartke@math.uiuc.edu) University of Illinois

## DNA Codewords and De Bruijn Sequences

In the design of DNA codewords for testing purposes, we want a sequence of nucleotides (thought of as a string over the alphabet $\{A, C, G, T\}$ where all of the substrings of $k$ consecutive letters are distinct. This corresponds exactly to the definition of a De Bruijn sequence, and it is well known how long such sequences can be and how to generate them. However, since a single strand of DNA can twist around and bind to itself, we wish to have a De Bruijn sequence where the $k$-substrings are also distinct under some more general equivalence relation-in this case, reverse complementation. We will present results on the maximum length of a De Bruijn sequence whose $k$-substrings are distinct under the relation of reverse complementation.

Laura Kubatko (lkubatko@stat.osu.edu) Department of Statistics, The Ohio State University
Phylogenetics Meets Genomics: Challenges in Inferring Evolutionary Relatedness Using Genome-Scale Data
Evolutionary relationships among organisms are typically represented by a phylogenetic tree, which is mathematically described as an acyclic connected graph in which each internal node is connected through exactly three edges. Phylogenetic trees are generally estimated on the basis of genetic data, such as DNA sequences. The rapid advance of DNA sequencing technologies has led to explosive growth in the amount of data available for phylogenetic studies, which has created challenges both in computation and in mathematical modeling based on such data. This talk will provide an overview of the primary issues involved in inferring phylogenies using data at several scales, ranging from individual genes to entire genome sequences. In particular, we highlight the interplay between appropriate modeling of evolutionary processes and the computational challenges presented by such models.

## Laurie Heyer (laheyer@davidson.edu) Davidson College

Living Hardware: Solving the Hamiltonian Path Problem with DNA
Following the seminal work of Len Adleman in 1994, the field of DNA computing has developed around the idea of operations that can be performed on a set of specially designed DNA sequences in vitro. Adleman proposed such a test-tube solution to the Hamiltonian Path Problem (HPP). We have taken a different approach by designing specialized computers inside living cells, including one to solve the HPP. We describe our work with students to design, model, build and test these living computers. This field is full of open problems and has incredible potential for interdisciplinary research by undergraduates.

Glenn Tesler (gptesler@math.ucsd.edu) University of California, San Diego
Distribution of Segment Lengths in Genome Rearrangements
The study of gene orders for constructing phylogenetic trees was introduced by Dobzhansky and Sturtevant in 1938. Different genomes may have homologous genes arranged in different orders. In the early 1990s, Sankoff
and colleagues modelled this as ordinary (unsigned) permutations on a set of numbered genes $1,2, \ldots, n$, with biological events such as inversions modelled as operations on the permutations. Signed permutations are used to indicate the relative strands of the genes, and circular permutations may be used for circular genomes. We use combinatorial methods (generating functions, asymptotics, and enumeration formulas) to study the distributions of the number and lengths of conserved segments of genes between multiple genomes, including signed and unsigned genomes, and circular and linear genomes. This generalizes classical work from the 1940s-60s by Wolfowitz, Kaplansky, Riordan, Abramson, and Moser, who studied decompositions of permutations into strips of ascending or descending consecutive numbers. In our setting, their work corresponds to comparison of two unsigned genomes (known gene orders, unknown gene orientations).

# Prime Numbers-New Developments on Ancient Problems 

Saturday, August 4, 1:00 pm-4:00 pm
Carl Pomerance (carl.pomerance@dartmouth.edu) Dartmouth College

## Primal Screens

Prime numbers are as old as Euclid yet we are still discovering new things about them. The world of computational number theory was stunned a few years ago when Agrawal, Kayal, and Saxena announced their "polynomial time" test for primality. We were stunned not only because this long-sought algorithm turned to be so simple, but because Kayal and Saxena had done their research as undergrads.

## Susan Landau Sun Microsystems

(susan.landau@sun.com)

## Primes, Research, Academic Freedom, and How the NSA Got What it Wanted

In the mid 1970's, academic researchers discovered how prime numbers could be used to design strong cryptographic systems. Cryptography had been the sole purview of governments and the National Security Agency fought to stop the research in the public sector; it lost. But from losing the battle, NSA has reaped mighty benefits. The fruits of public research are now fundamental to government systems.

## Kannan Soundararajan (ksound@math.stanford.edu) Stanford University

## Progressions of Primes

The prime numbers appear to be randomly sprinkled among the natural numbers subject only to their generally thinning out the higher you go. But certain patterns of primes seem to occur and then recur, and then again. It is easy to notice these things, but it is an entirely different issue to prove it. In 2004 Green and Tao proved the seemingly intractable conjecture that there are arbitrarily long arithmetic progressions of primes. Previously the best we knew in a result from about 70 years ago was that there are infinitely many progressions of 3 primes. It is interesting that the Green-Tao proof rests in part on a tool that Goldston and Yildirim had just developed to study the also thorny twin-prime problem.

## Brian Conrey American Institute of Mathematics

(conrey@aimath.org)

## The Riemann Hypothesis, Random Matrices, and Primes

Perhaps the most famous problem in mathematics is the Riemann Hypothesis. (Aside from fame, there is also a million dollar prize for a proof of this.) The conjecture is that the complex zeros of the zeta-function all lie on a vertical line, but it is also equivalent to the primes having a random but regular distribution. Starting with the initial discovery by Montgomery in the early 70 's, the zeros of the zeta function are apparently distributed according to the same laws as the eigenvalues of large random matrices, a subject of research by physicists. This is now an energetic field of research with input both from number theory and physics.

## Research with Undergraduates

Saturday, August 4, 1:00 pm-3:30 pm

## Estelle Basor (ebasor@calpoly.edu) California Polytechnic State University

## Eigenvalues of Random Matrices

This talk will describe results concerning the distribution of eigenvalues of some classes of random matrices, that is matrices whose entries are randomly generated. The focus will be on classes of matrices with particular structures, including some circulant matrices, block circulant matrices, and some perturbations of circulant matrices. Additional results concerning the singular values of the matrices will be described as well as a comparison to some of the classical ensembles of random matrices, such as the Gaussian Unitary Ensemble. The results were discovered by students working in senior or REU projects.

## Robin Leigh Blankenship (r.blankenshi@moreheadstate.edu) Morehead State University

## Exhilaration and Consternation: My First Year Supervising Undergraduate Research

As a new faculty member, conference talks inspired me to consider supervising undergraduate research for the first time. Where would I find good problems? Where would I find the time? Could I connect my dissertation research in topological graph theory with the abilities and interests of the undergraduates in my institution? The answer to all my questions became a resounding yes over the past year. Three capstone students, a grant involving undergraduate research, and a brilliant, precocious ninth grade high school student later, I reflect on what worked, what I will change, and why undergraduate research has become one of my most cherished endeavors.

## Frank Morgan (frank.morgan@williams.edu) Williams College

## The Double Bubble Theorem

The Double Bubble Theorem (Annals of Math, 2000) began as undergraduate research, appeared first as a conjecture in an undergraduate thesis, had an undergraduate research student as coauthor, and has been superseded by further undergraduate research.

Mike O'Neill (moneill@cmc.edu) Claremont Mckenna College

## An Inverse Theorem in Additive Number Theory

The Cauchy-Davenport inequality gives a lower bound for the size of sum sets in cyclic groups of prime order. The inverse theorem (due to Vosper) characterizes subsets A and B for which equality is attained. Pollard gave a generalization of the Cauchy-Davenport inequality which takes into account the number of times an element of the sum set is represented as a sum of something in A and something in B. In joint work with Claremont McKenna College undergraduates, Mike O'Brien, Eva Nazarewicz and Carolyn Staples we prove the inverse theorem for Pollard's inequality. I will also discuss some other research directions in this area which are appropriate for undergraduates and some connections between additive number theory and analysis.

Adolfo J Rumbos (arumbos@pomona.edu) Pomona College

## Solvability of Semi-linear, Two-point Boundary Value Problems

The goal of this project is to use elementary analysis techniques to prove existence and multiplicity results for a class of semi-linear two-point boundary value problems. The techniques used so far come from undergraduate courses on ordinary differential equations, real analysis and Fourier analysis. Students have had to learn a few less elementary techniques from the theory of differential equations (such as the "shooting method"), the calculus of variations and critical point theory. There are some open problems in the piece-linear case which are, perhaps, amenable to this elementary treatment. If these questions are answered, the results can shed some light for the case of semi-linear problems with nonlinearities that are either bounded or grow at most linearly at infinity.

# Contributed Paper Sessions 

## Advances in Recreational Mathematics

Sunday, August 5, 1:00 Pm-3:45 pm

## George Bell (gibell@comcast.net) Tech-X Corporation

## Solving Triangular Peg Solitaire

Peg solitaire on a triangular board of side 5 is a popular puzzle found in Cracker Barrel restaurants. We'll consider this game on a triangular board of arbitrary size. The basic game begins from a full board with one peg missing and finishes with one peg at a specified board location. We develop necessary and sufficient conditions for this game to be solvable. For all solvable problems, we give an explicit solution algorithm. On the Cracker Barrel board, we compare three simple solution strategies. We then consider the problem of finding solutions that minimize the number of moves (where a move is one or more jumps by the same peg), and (using computational search) find the shortest solution to the basic game on all triangular boards with up to 55 holes ( 10 holes on a side).

## Bruce Burdick (bburdick@rwu.edu) Roger Williams University

## Mathematical Problems from the Maine Farmer's Almanac

A recent paper by Albree and Brown discusses the presence of mathematical problems in The Ladies Diary (1704-1840), an almanac from Britain. We report on an American version of the same phenomenon that appeared a century later. In the nineteenth century, the Maine Farmer's Almanac made an annual feature out of posing various puzzles, including riddles, anagrams, and mathematical problems. Readers would have a year to work on the problem and then see the answers in the following issue. A few readers would mail in their solutions to the editor and be mentioned as solvers. Some of the mathematical problems were surprisingly sophisticated for a general readership. We will survey the most interesting examples of these problems and distribute to the audience a list of all the problems in the issues we have found so far.

Lisa A. Mantini (mantini@okstate.edu) Oklahoma State University

## Symmetry and Sudoku

For a certain class of block-symmetric sudoku solutions, we give a general method to construct an orthogonal Latin square which is also a sudoku solution. The construction generalizes to sudoku solutions of arbitrary size. In the 9 by 9 case, the sudoku solutions can be related to linear maps from the field of order 81 to the field of order 9 .

## Thomas Mattman (TMattman@CSUChico.edu) California State University, Chico

## Ribbonlength of Torus Knots

(Joint work with Kennedy, Raya, and Tating). A regular pentagon can be formed by tying a knot in a strip of paper. We generalise this construction and show that every regular polygon of at least seven sides can be formed from a single strip of paper. Using these constructions, we estimate the Ribbonlength, or length to width ratio, of certain torus knots. This allows us to give bounds on the constants $c_{1}$ and $c_{2}$ that relate Ribbonlength $R(K)$ and crossing number $C(K)$ in a conjecture of Kusner: $c_{1} C(K) \leq R(K) \leq c_{2} C(K)$.

Colm Mulcahy (colm@spelman.edu) Spelman College

## Circling the (Magic) Square

The perimeter of a magic square, say the standard three by three one, yields a magic circle (of size eight) with some interesting constant sum properties. We explore this along with some generalizations, including some surprising and delightful magic circles of size six.

## Colm Mulcahy (colm@spelman.edu) Spelman College

## Gibonacci Magic

Congruence classes of Gibonacci (generalized Fibonacci) numbers, reduced mod n, reveal unexpected periodicities and other patterns which allow for interesting prediction stunts in mathematical magic. We survey the original Stewart James discovery published in the early 1960s, and its application to 5 by 5 squares, and demonstrate some recent generalizations.

## Amber Rosin (arrosin@csupomona.edu) Cal Poly Pomona

## Equally Likely Standard Dice Sums

It is well known that a pair of 6 -sided dice cannot be labeled so that the sums $2,3,4, \ldots, 12$ are all equally likely. Edward J. Dudewicz and Ronald E. Dann showed that there is no way to label $m n$-sided dice with integers from the set $\{1,2,3, \ldots, n\}$ such that the resulting sums are all equally likely. We investigate a similar question, but we eliminate the condition that the dice only be labeled with integers from the set $\{1,2,3, \ldots, n\}$. Namely, when can $m n$-sided dice be labeled with any nonnegative integers such that the "standard sums" of $m, m+1, m+2, \ldots, m n$ are all equally likely? Any labeling of dice which yields the standard sums with equal probability will be called an equal labeling. Even if we eliminate Dudewicz and Dann's condition that the dice labels come from the set $\{1,2,3, \ldots, n\}$, pairs of dice will always fail to have an equal labeling, unless the dice are one-sided. However, there are infinitely many $m$ such that $m 6$-sided dice afford an equal labeling. We characterize all such $m$ and discuss the resulting dice labels.

## Jay Lawrence Schiffman (schiffman@rowan.edu) Rowan University

## Fortunate and Lesser Fortunate Numbers

Reo Franklin Fortune (1903-1979) was a social anthropologist at Cambridge University and in 1941 joined the faculty at The University of Toronto. From 1928-1935, Fortune was married to social scientist Margaret Mead. What is surprising to many anthropologists was Fortune's role in the study of prime numbers. In Euclid's classical proof on the infinitude of the primes, one is exposed to the quantity MATHEMATICA calls euclid ( $n$ ) which represents the product of the first n primes plus one. Hence euclid $(4)=2 * 3 * 5 * 7+1=211$. It is unknown if there are finitely or infinitely many prime outputs for this function. Fortune considered the mapping from the natural numbers $N$ into $N$ defined by $f(n)=\operatorname{NextPrime(euclid}(n))-\operatorname{euclid}(n)+1$. To cite an example, the next prime after $\operatorname{euclid}(4)=211$ is 223 and $223-211+1=13$ is prime. Fortune conjectured that all outputs for this mapping generated primes. In 1994, Richard Guy confirmed the truth of the Fortune Conjecture through $n=100$ and Eric Weisstein of Wolfram Research, Inc. on November 17,2003 verified the truth through $n=1000$. With the aid of MATHEMATICA, I have confirmed the conjecture through $n=1900$ and T.D. Noe through $\mathrm{n}=2000$. Analogous to the Fortune Numbers is a set dubbed as the Lesser Fortunate Numbers in which we consider the mapping from N into N defined by $f(n)=($ euclid $(n)-2)-$ PreviousPrime(euclid $(n)-2)+1$. To cite an example, euclid $(4)-2=209$ and PreviousPrime $(209)=199$. Thus the prime output of 11 occurs here. It has been conjectured that these numbers are likewise always prime. I have verified the truth of this conjecture through $n=1200$ and T.D. Noe through $\mathrm{n}=1900$. In this paper, a line plot for each mapping along with an analysis of the data will be furnished. While no new ground is broken, these numbers serve as an appealing excursion as well as extension to Euclid's genius over two thousand years ago.

Jonathan Stadler (jstadler@capital.edu) Capital University

## Let Us Teach Nim

Nim is a simple two-person game with an elegant winning strategy that was discovered by Charles Bouton at the turn of the 20th century. Unfortunately, Bouton's paper did not contain the impetus for his solution. During this talk, a plausible motivation will be presented in the context of presenting the winning strategy. Tips for incorporating Nim in the classroom as a means of introducing new material will also be discussed.

## Bruce F. Torrence (btorrenc@rmc.edu) Randolph-Macon College

## Arithmetic Combinations

Is it possible to write a given positive integer as an arithmetic expression in which each of the numbers 1 through $n$ appears exactly once? For instance, using the numbers 1 through 4 one may write $17=1^{3}+4^{2}$. What is the smallest number not obtainable as such an expression? How many ways can a given number be expressed as such an arithmetic combination? The British television show Countdown has a numbers game in which players try to construct a 3-digit target number as an arithmetic combination of some randomly chosen smaller numbers. What strategies are best? In this talk we will explore methods for investigating such matters.

## Attracting and Retaining Students to Mathematics Programs via Outreach <br> Friday, August 3, 8:30 am-10:30 am

## Crista Arangala (ccoles@elon.edu) Elon University

## Discovering Mathematics Through the Elon Traveling Science Center

The Elon Traveling Science Center is student driven and focuses on bringing interactive science and mathematics exhibits to local middle schools. This traveling science center is grounded in the general science and mathematics curriculum at Elon and is supported by classes which build and show their exhibits. A brief discussion about the foundation, partnerships, and support of the traveling science center as well as the curriculum of related courses will be presented. Statistical data supporting this program will also be presented.

## David Edward Boliver, Rocky Bargas (dboliver2@cox.net) University of Central Oklahoma

## Assessing a Collegiate Summer Academy for Students Rising to Grades 8-10

We have been running an academy for teenagers for over a decade. This academy has produced students who earn high grades and admission to prestigious institutions. Many of them are in STEM majors and we are able to identify several characteristics of the program which we think have contributed to their success. The presenter has made previous presentations to the MAA regarding the development of the curriculum and this will only be recapped briefly as a part of this presentation. Last year, a study was conducted by Mr. Rocky Bargas of the achievement of participants for the last 5 years and a data analysis of this study will be presented. We will briefly profile some of the more outstanding program alumni.

## Sangeeta Gad (gads@uhd.edu) University of Houston-Downtown

## Outreach—Planting Seeds for the Future and Retaining Students in Mathematics Through Mentoring and Tutoring

Women and disadvantaged minorities lag behind in the technology sector. The migration away from the science, technology, engineering and mathematics (STEM) fields starts in middle school and continues until the undergraduate years. We risk our nation's leadership role in the high technology society we have developed if the declining college enrollments in STEM subjects remain unchecked.
To arrest this trend and inculcate interest in STEM fields, University of Houston-Downtown offers two mathematics based enrichment programs for pre college students, Houston PREP and Saturday Academy, starting as early as 7th grade. These programs are producing results For retaining students in Mathematics once they enroll in UHD, we have developed several students' assistance programs such as open labs, one on one tutoring and computer labs. Faculty and student mentors are assigned to STEM majors.

## Cinnamon Hillyard (chillyard@uwb.edu) University of Washington Bothell

## Bringing Students to Mathematics with Art and Animation

Sprite, a 3-year NSF project, develops students' mathematics, science and computing skills by having them create their own animations with Microsoft Visual Basic. The interdisciplinary curriculum provide a visual-
ization tool that gives students the literal ability to "see" mathematics and science, and to receive immediate, clear, safe, and often humorous feedback to their efforts and errors. Students are willing to spend hours learning mathematical concepts in order to make their personal characters jump or twirl on the screen. In addition, the integration of art, sound, music, and group interaction broadens the appeal of the curriculum to many students who traditionally would show little interest in mathematics, science, and technology. This project is targeted towards high school and university students and their instructors. It is a collaboration between high schools, community colleges, universities, professional organizations, and industry. This presentation will provide key details of the project, samples of the embedded mathematics curriculum, examples of students' work, and opportunities and challenges we've discovered from doing this project.

Eva Marie Strawbridge (emstrawbridge@math.ucdavis.edu) University of California, Davis

## The University of California, Davis Explore Math Program

Explore Math is an outreach program for high school and undergraduate students that was created and is directed by graduate students in the Department of Mathematics at the University of California, Davis (UCD). The program reaches out to high school students in the Yolo and Sacramento County areas who are curious about mathematics. Participating students work with undergraduate and graduate students to learn about cutting-edge research and beautiful mathematics. The program encourages participation of all students, including underrepresented students who have limited mathematical exposure. Explore Math was founded by graduate students as a complement to its sister program, COSMOS (California Summer School for Mathematics and Science), so that students in the surrounding areas would have access to year-long academic activities in mathematics. The presentation will describe the Math Modeling Experience (offered in the Fall), Math Circle (offered in the Winter), and ARML training program (offered in the Spring). These three programs use math department graduate students to train UCD undergraduate students to work with the high school participants.

## Biomathematics in the First Two Years

Saturday, August 4, 8:30 am-10:30 am
Saturday, August 4, 1:00 pm-3:00 Pm
Sunday, August 5, 8:30 am-10:30 am

## Frederick Addison Adkins (fadkins@iup.edu) Indiana University of Pennsylvania

## A Calculus Module for Modeling Bioaccumulation, Biomagnification, and Elimination of Mercury

Bioaccumulation is the increase in concentration of a substance in organisms, as they take in contaminated air, water, or food more rapidly than can be eliminated by metabolization and excretion. As pollutants move from one link in the food chain to another they concentrate through the process of biomagnification. Through a module designed for use in an introductory calculus course, students explore these concepts using data derived from research on mercury levels in the environment and aquatic food chain. Students begin by tracking mercury across the food chain- from algae and bacteria, to insects, small fish, larger fish, and eventually to birds, mammals, and humans. Concentrations and bioacculumation factors are calculated and literature is reviewed for transfer ratios to fetuses and eggs. Students are asked to use their typical weekly consumption of fish to calculate their average daily dosage of mercury per kilogram of body weight. Based on models of absorption of mercury from food, a separable differential equation for mercury elimination in humans and the fraction of total body mercury in blood, a time dependent equation for blood mercury concentration is derived. Students then find their steady-state level of blood mercury concentration and compare this to the U.S. Environmental Protection Agency's "safe benchmark blood level."

## Pau Atela (patela@email.smith.edu) Smith College

## A New Approach to Phyllotaxis

Phyllotactic patterns of botanical elements, such as leaves and petals, are established at a microscopic scale at the shoot apical meristem, the growing tip of the plant. These patterns typically display two sets of spirals-
parastichies-winding in opposite directions. Strikingly, the number of parastichies in these two sets are most often two consecutive Fibonacci numbers. Our recent collaborative research between mathematicians and biologists in Phyllotaxis has led to material that is suitable for undergraduates with a minimum mathematical background. We will discuss approaches to Phyllotaxis that we have implemented in an undergraduate course.

## Kapila Rohan Attele, Dan Hrozencik (kattele@csu.edu) Chicago State University

## Mathematical Biology and Computational Algebra at the Sophomore Level

Changing the undergraduate mathematics curriculum to service the needs of the life sciences is a major challenge that faces the mathematical community. Non-infinitesimal algebraic techniques and computational algebra will play a significant role in this endeavor, but precisely what that is not yet quite clear. Our experience in teaching a new research oriented course in mathematical biology at Chicago State University, which, in this semester focuses on Matrix Modeling Applied to Population Dynamics (Math 251/Bio 251), will provide a frame work to explore some ideas and also discuss long-term research opportunities for math majors in the life sciences.

We also invite the audience to explore the opportunities provided by mathematical biology to reorganize the algebra curriculum into two parts: (a) computational algebra at the sophomore level (b) more rigorous proof-based abstract algebra courses built on that experience at junior/senior level

## Bruce Carpenter, Debra Woods (carpent@uiuc.edu) University of Illinois

## BIO 2020: Looking Ahead

Over ten years ago, University of Illinois faculty from the departments of Mathematics and Biological Sciences collaborated in the creation of a first semester calculus course (BioMath I) oriented toward students in the biological sciences. This course is based on Mathematica, a very powerful programming language as well as an integrated environment that combines calculations, text, and graphic visualizations. Responding to the call of BIO 2010: Transforming Undergraduate Education for Future Research Biologists (National Research Council, 2003), the two departments are again collaborating to create a follow-on course called BioMath II: Introduction to Biological Modeling. This course will debut at the University of Illinois in the 2007 Fall semester. This talk will briefly discuss the content of both courses, the experience of teaching BioCalc for more than a decade, and give the results of an independent evaluation of BioCalc.

## Vera Cherepinsky (vcherepinsky@mail.fairfield.edu) Fairfield University

## Mathematics in Genomic Analysis-A Module for Biology Students.

This talk describes a module on mathematics in computational biology, featuring a forensic exercise, presented as a guest lecture in an upper-level biology course at Fairfield University. The module started with an overview, which included a discussion of various places mathematics comes up in genomic analysis, such as in PCR, DNA fingerprinting, restriction mapping, and microarray analysis. The students then got really involved handson in a forensic "murder mystery" exercise (found on a SUNY Buffalo Biology department website), where they had to run simulated "gels" on DNA samples from suspects and the crime scene (sequences provided) and make conclusions based on their results as to the probable identity of the murderer. On the following day, I returned to class to go over the results students obtained and discuss the significance of their findings in the context of conditional probability. Modules similar to the one described here expose biology students to mathematical concepts, including topics in combinatorics, vector analysis, probability, and statistics, that they may not otherwise encounter in their coursework. Furthermore, such modules may help overcome the language barrier presently prevalent between mathematicians and biologists.

## Timothy Comar (tcomar@ben.edu) Benedictine University

## Biocalculus and Beyond at Benedictine University and College of DuPage: Reaching Biology Students

Benedictine University (BU) is a small university with about a quarter of its undergraduate population majoring in the biological and health sciences. College of DuPage (COD) is large community college near BU, and many BU transfer students come from COD. In this talk, we highlight several ways in which we expose
undergraduates to the integration of mathematics and biology at both institutions. These avenues include a rigorous two-semester biocalculus sequence for research oriented majors in the biological sciences, biologically oriented laboratory projects for all first semester calculus students at BU, and a biomathematics seminar that is run jointly by both institutions. We discuss how these activities introduce a wide range of range students to biomathematics and encourage students to become in involved with undergraduate biomathematical research.

## Gary De Young (gdeyoung@dordt.edu) Dordt College

## Introducing Modeling through Calculus labs

One way to introduced first year students to biological modeling is through calculus labs that are designed to demonstrate the basics of mathematical modeling. The labs that I will describe introduced biological modeling to first semester calculus students by having them collect data on an in class simulated flu epidemic, model a transmission mechanism with rate equations (mass action), and finally simulate an epidemic (with excel). The labs parallel the topics that are introduced in first semester calculus: limits, the definition of derivative, and approximations to derivatives. Students are able to better understand the nature modeling and appreciate how modeling (biological and other) may be applied in diverse areas of study.

## Susan Diesel (sdiesel@norwich.edu) Norwich University

## Fish Populations in the South San Francisco Bay, 1972-2002

The Marine Sciences Institute of Redwood City, California, was founded in 1970 to teach students about the bay environment. From 1972 to 2002, MSI conducted over 6000 trawls into the South San Francisco Bay, capturing and releasing fish and recording information about species collected, as well as measurements of water temperature, salinity, and other information from each trawl. This data is summarized in the report "Trends in South San Francisco Bay Fish Populations from 1972 to 2002 " published by MSI. In this paper we investigate some of the trends in the fish populations and make some suggestions for using the data in a calculus course case study.

## Sheldon Gordon (gordonsp@farmingdale.edu) Farmingdale State University

## Integrating College Algebra and Statistics to Meet Students' Mathematical Needs in Biology

The mathematical requirement for most introductory biology courses is College Algebra. Yet, in conversations between the mathematics community and leading biology educators, it is apparent that what most of their students really need is more exposure to statistics rather than manipulative algebra. In fact, the amount of mathematics that arises in biology classrooms is quite minimal; in reality, the one place that mathematics does enter into biology courses is in the laboratory when the students need to analyze their lab data. (The same is true of most other disciplines.) This presentation will focus on ways to integrate statistical ideas and methods throughout a College Algebra course to better meet the needs of the students and the other disciplines, particularly biology. The goal is to have the statistics arise in college algebra contexts, so that the course becomes a natural mesh of the two rather than a disconnected add-on to a college algebra course. This approach is also in the spirit of the MAA's initiative to change the focus of college algebra and related courses.

## David J Hunter (dhunter@westmont.edu) Westmont College

## Discretion is the Better Part of Valine: Why Biology Majors Should Take Discrete Mathematics

In the December 2004 issue of PLoS Biology, Joel Cohen writes,
In the coming century, biology will stimulate the creation of entirely new realms of mathematics. In this sense, biology is mathematics' next physics, only better.
As a possible corollary, discrete mathematics is biology's next calculus, only better. While discrete mathematics has always served a core audience of computer science students, this course can naturally meet the demands of new applications of mathematics in biology. Specific topics include pattern matching in DNA, phylogenetic trees, population growth, morphogenesis, and Monte Carlo methods. Perhaps more important, however, are the general concepts and skills that the study of discrete mathematics develops. To participate in the "new realms"
of mathematics inspired by modern biology, future biologists and mathematicians alike need to be familiar with logic, algorithms, enumeration, modeling with graphs and other discrete structures, and probability. This talk will discuss ways to integrate biomathematical ideas into existing discrete mathematics courses, and will consider strategies for recruiting biology students and cooperating with biology faculty to encourage participation in discrete mathematics.

Karl H Jopin (joplin@etsu.edu) ETSU Edith Seier (seier@etsu.edu) ETSU

## Integrating Mendelian Genetics with Probability and Statistics: A Teaching Module of SYMBIOSIS

As part of our HHMI funded grant to integrate the teaching of Mathematics with Biology at the freshman level, we are developing modules in a 3 semester course entitled SYMBIOSIS. Early in the first semester, which concentrates on statistics and biological themes, we are presenting a two week module on Mendelian genetics as a framework to introduce probability and statistics along with the biological concepts of phenotype and genotype, inheritance, alleles, meiosis and linkage. We do this by giving the students Mendel's raw dataset and exploring how to describe the data. Random experiments with coins are used to explain the passage of information from parents to offspring and probability trees are used to explore the possible combinations when several characteristics are involved. The notion of independent events and conditional probability as well as the binomial distribution are introduced. The frequencies obtained by Mendel are compared with those obtained from a probability model and the differences are summarized. Simulation is used to explore how Mendel data relates to the probability model. The exercise requires the students learning and using statistical and probability tools and software on a dataset and using the analysis in a biological context. Supported by HHMI grant \# 52005872.

## Istvan Karsai, Thomas Schmickl, and Jeff Knisley (ikarsai@hotmail.com) ETSU IQB

## Integrating Math and Biology through Storytelling: The Salmon Example

A traditional freshman biology textbook presents a laundry list of topics through typical examples, thus insuring that students will be familiar with results of classical studies related to each topic. A drawback of this approach, however, is that the curriculum can become a sequence of disconnected discussions of many different organisms and systems. Students can become confused, and they may miss the important overarching concepts of how science operates and how research is carried out, especially if results are presented by cherrypicking graphs and tables from classical papers. In order to integrate mathematics into such a course, great care must be taken to not only prevent further difficulties but also to hopefully alleviate some of the existing drawbacks. We have found that an engaging story not only allows us to develop a common context for mathematics and biology, but that it also illustrates the scientific method and the process of research. In our talk, we present an example of how mathematics can be combined with a well defined biological problem in order to provide such a story, one that is appropriate for integrating differential calculus into a freshman level biology course.

## Jeff Knisley and Istvan Karsai (knisleyjr@earthlink.net) East Tennessee State University

East Tennessee State University

## Integrating Mathematics and Biology Through Storytelling: Some Insights into the SYMBIOSIS project

Mathematics and biology courses individually face significant challenges already - too much content, difficult concepts, and the need to cover a diversity of seemingly unrelated topics. The integration of mathematics and biology can add to these challenges the difficulties of separate terminologies, different interpretations of concepts, and seemingly different techniques and methodologies. For example, biologists often define the notion of a "functional response" within a highly specified biological context, thus suggesting that different contexts may lead to different notions of functional response, whereas mathematicians tend to view the concept of functional response in a more abstract and universal manner. In this presentation, we show how many of these difficulties can be addressed through storytelling - that is, through case-study like canonical examples of key concepts. In particular, we will focus on how storytelling can produce a shared conceptualization and terminology that serves both fields. To illustrate, we will show how storytelling can be used to develop a
common notion of "functional response" that does not compromise either its mathematical or its biological character. Supported by HHMI grant \# 52005872.

## Myrtis Leigh Lunsford and Consuelo J. Alvarez (lunsfordm@@longwood.edu) Longwood University

## Creating and Analyzing cDNA Microarrays: Building Collaborative Undergraduate Research Projects

For the past year the authors have worked to create a collaborative undergraduate research program at Longwood University, a primarily undergraduate teaching-oriented institution. Dr. Alvarez had previously started working with biology and chemistry students to create cDNA microarrays via the NSF funded program GCAT (Genome Consortium for Active Teaching). DNA microarrays allow scientists to observe the entire genome when conducting research experiments. By monitoring the behavior of thousands of genes simultaneously, scientists can study how genes function together, especially under different conditions. Due to the vast quantities of data produced by DNA microarray experiments, data analysis has become more prominent in the life sciences. Thus, this past academic year Dr. Lunsford worked with a mathematics student to begin to analyze some of the microarrays Dr. Alvarez and her students created. This spring semester our students presented their work at different conferences. We will discuss what it takes to get a project like this going at a teaching-oriented institution; the learning curves for the biologist, the mathematician, and the students; peaks and pitfalls along the way; suggestions and resources for others who are interested in creating collaborative projects in this area; and our future plans.

## Mark MacLean (maclean@math.ubc.ca) University of British Columbia

## Science One: Integrating Mathematical Biology into a First-year Science Program

Founded in 1993, UBC's Science One Program takes an integrated approach to teaching biology, chemistry, mathematics, and physics to first-year students. A team of 8 faculty members representing these four areas works with 72 students to create a science learning community. The majority of these students intend to be life sciences majors and have a deep interest in biology, and so biological problems are an important focus for learning science and mathematics. One major goal of the program is to build students' skills in creating and analyzing mathematical models of biological systems. The basic mathematical tools are drawn from calculus and elementary differential equations. Initially, we present students with simple models and they learn to see how the model relates to the system they are studying. As they build experience, we ask them to modify the models to take into account things like perturbations to the system. Eventually, we ask them to create, analyze, and critique their own models for situations. We have done an analysis of the performance of the Science One students as they progress through their degrees, comparing them to comparable science students, and especially to life sciences students in biochemistry, microbiology, immunology, physiology, and pharmacology, amongst others. The comparisons are on overall performance, and on performance in core courses identified as requiring high-level learning and problem-solving skills. The results of these analyses show a measurable difference between students who have been in Science One and their peers.

## Mike Martin (mmartin@jccc.edu) Johnson County Community College

## Application Bases that Span the Curriculum: Pharmacology and Cardiology

The common practice of periodically ingesting drugs can result in dramatic changes in drug concentrations in the body. The mathematical modeling of the phenomena related to this practice range from linear functions, rates of change, geometric sequences, exponential decay, semi-log plots, and to much more. Likewise, the propagation of chemical signals within the heart provides its ability to beat and is modeled in a surprisingly similar fashion to concentration decays between beat (or dosing) periods. Both of these application areas readily pique student's interests and can be tied contemporary events and issues. This talk will exhibit several submodules within each of these two application areas appropriate for intermediate algebra, precalculus, trigonometry, discrete mathematics, calculus, differential equations, and/or probability theory. The modules are flexible in design, can be used for group work, include parameter exploration, require a written report, and are supported by some dynamic web pages for simulations. Additional media including video clips, including research video for pathologies of the beating heart, supports some of the submodules. Discussions of the breadth of resources that can be used will a theme to the talk. An overarching theme to both application areas
is that of rhythm. Normal rhythms are associated to normally beating hearts and normally oscillating drug concentration levels. What types of rhythms can be produced by each of these two models? Are they realistic occurrences? How does the idea of rhythm fit with precalculus and trigonometry?

## Raina Robeva (robeva@sbc.edu) Sweet Briar College

## Biomathematics: Desegregating Mathematics and Biology

As contemporary characterization of biological systems reaches unparalleled level of detail, virtually any advance in the life sciences requires sophisticated mathematical approaches. Modeling of biological systems is evolving into an important partner of experimental work, and there is a rapidly increasing demand for people with solid mathematics and statistics training in the field of biomathematics. According to the Department of Education, however, the percent of bachelor degrees in mathematics and statistics combined has dropped nationwide from 3\% in 1971 to less than $1 \%$ in 2004.

The problem is known to be multifaceted and the talk focuses on aspects rooted in the traditional segregation of mathematics and biology courses in the undergraduate curricula. Some ideas for addressing this problem in the first two years of college education will be presented. The speaker will also share thoughts and experiences from teaching an undergraduate course "Topics in Biomathematics" designed for students with minimal background in mathematics and biology and based on projects from ongoing medical research.

## Bruce F Torrence (btorrenc@rmc.edu) Randolph-Macon College

## Sequence Alignment

A module on sequence alignment combines the disciplines of mathematics, biology, and computer science. In a new year-long freshman seminar at Randolph-Macon College titled "The Human Genome", team taught by a biologist and a mathematician, the topic of sequence alignment provided the culmination of the mathematical portion of the course. Delannoy numbers were used to develop the Needleman-Wunsch and Smith-Waterman algorithms. The algorithms are effectively illustrated and explored with Mathematica.

## Challenges and Successful Strategies in Teaching a Numerical Analysis Course

## Sunday, August 5, 8:30 am-10:30 am

Gerard Awanou (awanou@math.niu.edu) Northern Illinois University

## Numerical Differential Equations at Northern Illinois University

The Numerical Differential course at Northern Illinois University is a one semester course where students are exposed to the finite difference and finite element methods for solving ordinary and partial differential equations. The class is usually small, 3 or 4 students and is required for students pursuing a master in applied mathematics. I will discuss my choice of topics and activities to meet the following goals, keeping in mind that students may have no computational experience, or may not have the background to follow some of the error analysis.

1. Cover the classical material to prepare the students to take the preliminary examinations
2. Give students computational experience
3. Meet the demands of potential advisors of the PhD students in providing experience with the finite element method
4. Provide students with an introduction to current topics of applied research through group projects

## Olga Brezhneva (brezhnoa@muohio.edu) Miami University, Ohio

## Designing a Numerical Analysis Course: Key Elements, Ideas and Strategies That Work

In this talk, methods of designing a Numerical Analysis course will be discussed. The key elements include course goals, strategies to achieve the goals and assessment techniques. In addition, our discussion will include
methods for preparing questionnaires that help find more information about interests and expectations of the students in the class. We will also talk about creating projects, course materials and additional workshops such as an introduction to the MATLAB. These techniques may be used in designing any undergraduate or graduate course.

## Michelle Ghrist (michelle.ghrist@usafa.edu) United States Air Force Academy

## A Very Applied Numerical Methods Project

My goals for the assigned project in my Numerical Methods course this spring were to (a) give students a chance to apply the methods discussed in class to real-life problems, (b) have students research journal articles, (c) show students the wide variety of applications of numerical methods, and (d) work on improving written and oral communication of technical material. For this project, students had to find a problem in an article from a scientific (but not mathematics) journal for which they could apply some of the techniques discussed in class: root finding, fixed point iteration, function approximation, numerical differentiation or numerical integration. They then applied the various appropriate methods discussed in class and analyzed the performance of these methods. Finally, they presented their results to the class and submitted a written paper detailing their findings. Overall, this was a very effective assignment. In this talk, I discuss my implementation of this project, including what worked well and what I would change.

## Mark Gruenwald (mg3@evansville.edu) University of Evansville

## The Perfect Numerical Computing Environment

One of the challenges in teaching a numerical analysis course is that of selecting and incorporating an appropriate computing environment that can be used to encode and execute numerical algorithms. This is especially the case when the course includes students with a wide range of programming backgrounds and abilities. Wouldn't it be convenient if there were a computing environment that most students already knew how to use for computational tasks that required no programming, but which was capable of handling programming tasks as well? Such an environment does in fact exist in the form of Microsoft Excel. Most students have some level of familiarity with Excel. Consequently, most students have little difficulty learning to use Excel to carry out the many numerical analysis procedures for which it is ideally suited. By gradually introducing the students to VBA programming, this same familiar Excel environment can be used to teach students how to translate pseudo-code into code that can actually be executed. Moreover, the skills students develop in writing VBA macros will serve them well not only in their numerical analysis course but in many different contexts as well.

## Richard Dean Neidinger (rineidinger@davidson.edu) Davidson College

## A Divided-Difference Algorithm for Multivariable Interpolation

Interpolating polynomials are a standard topic in numerical analysis but are usually restricted to one variable, perhaps because existence and uniqueness is much more complicated in the multivariable case. In one variable, divided-differences provide a sleek algorithm to compute the Newton form of the interpolating polynomial for any data with distinct abscissas. This algorithm generalizes to multivariable data and polynomials and handles a class of configurations of data points. This natural, efficient and practical generalization appears in some very old textbooks but has been almost lost in the computer-age textbooks. The basic idea is a nice contrast of theory between the one-variable and multivariable case. Simple examples can show how the algorithm generalizes, then the full algorithm is best presented using multi-index notation. Natural questions arise about other configurations of data points and about how data collections can be tested for the existence and uniqueness of interpolating polynomials.

Kyle Riley (Kyle.Riley@sdsmt.edu) SD School of Mines \& Technology

## Making Connections with Interpolation

Interpolation is a traditional topic in numerical analysis that is capable of delivering powerful estimates on error and convergence. However, the inclusion of interpolation in a numerical analysis course often results in low student morale since the corresponding calculations can be tedious and students typically find the utility of
cubic splines far more alluring. The use of technology can easily address the issue of tedious calculation and allow students more opportunities to explore the theory. In addition, the classic polynomials of interpolation provide a valuable foundation for a variety of numerical methods. With some careful illumination of the role interpolation can play in numerical analysis you can transform this "old" classic into a major theme that connects the topics you cover in your course.

Anthony Tongen (tongenal@jmu.edu) James Madison University

## Computer Error is Not a Contradiction in Terms

When first presented, machine representation of floating point numbers is a completely esoteric topic. In fact, most students today have such a high opinion of computers that this topic is often met with considerable skepticism. In this talk, we will present two projects that elucidate the importance of machine round-off error. The first application is a numerical implementation of Conway's Prime Producing Machine, which is an algorithmic approach to computing prime numbers. While students can completely understand the algorithm, a naive computer implementation will not even generate the first prime number due to round-off error. The second application is a piecewise fixed point iteration given by

$$
x_{n+1}=\left\{\begin{array}{cl}
2 x_{n}, & \text { if } 2 x_{n}<1 \\
2 x_{n}-1, & \text { if } 2 x_{n} \geq 1
\end{array}\right.
$$

This function is also referred to as a Bernoulli map or the dyadic transformation. Students can analytically calculate periodic orbits very easily; however, their computer programs are not as clever.

Ilie Ugarcovici (iugarcov@depaul.edu) DePaul University

## Assigning a "Toy Version" of the Google Page-Rank Matrix Problem as a Final Project

Based on a project description found in the book "Numerical Analysis" by T. Sauer and assigned to my numerical analysis class, I will present some of the conceptual and computational challenges, pitfalls and eventual satisfactions my students had while studying a toy version of the Google matrix. The $15 \times 15$ stochastic matrix is larger than those commonly found in textbooks, but definitely smaller than the real-world one. I will try to make the point that assigning problems of increased computational size might improve students' abilities in working with real-world problems and make them realize the importance of an efficient approach.

## Current Issues in Mathematics Education

Friday, August 3, 1:00 pm-5:30 pm

## Scott Beaver (beavers@wou.edu) Western Oregon University

## Probability and Statistics for Elementary Teachers at Western Oregon University

Elementary school students can study various topics in Probability and Statistics at all grade levels, but the topics can be subtle and counterintuitive. Our future elementary teachers should have a good general understanding of these subjects. At Western Oregon University, we offer a course in Probability and Statistics designed for pre-service elementary teachers. It combines basic instruction, group work, computer simulations using Fathom, examples of counterintuitive results in probability, real-world examples of misleading statistics, and lesson plan generation aligned with Oregon State standards. An overview of the topics covered in the course is presented.

## Satish C Bhatnagar (bhatnaga@unlv.nevada.edu) UNLV

## To Be, or Not To Be

The paper focuses on the issue of the rightful place of math education faculty in mathematics departments. The context is UNLV and comparable institutions. Highlights of a brief history of mathematics education as well as the hiring and retention of mathematics education faculty will be discussed. Reasons advocating for
the hiring of mathematics education faculty with a PhD in mathematics education include: bringing credibility to the program, improving instruction in mathematics education courses, and enhancing the opportunities of bringing funds into departments through competitive external mathematics education grants. Mathematics education can be as integral to the development of mathematics departments as other traditional areas.

## Robert Edward Buck (robert.buck@sru.edu) Slippery Rock University

## Mathematics Minors for Elementary Education Majors: Followup Report

Many recent reports suggest the need to increase the mathematical preparation of K-12 teachers. The Mathematics Department at Slippery Rock University has developed two mathematics minors to encourage elementary education majors to include more mathematics courses in their programs. These programs are intended to give prospective elementary and middle school teachers a better understanding of what mathematics really is, as well as showing them some of the different roles that mathematics plays in an increasingly technological society. These minors have become quite successful over the past eight years. This presentation will describe the initial requirements and subsequent evolution.

## Jenna Price Carpenter, Galen Turner (jenna@coes.latech.edu) Louisiana Tech University

## Integrating Math and Secondary Math Education Majors in a Freshman and Sophomore Level Curriculum

Because science and mathematics content are usually taught as discrete courses, students often fail to make connections between disciplines. To resolve this "mismatch", we have implemented a freshman and sophomore integrated science curriculum for all math, science, secondary mathematics education and science education majors. The curriculum is primarily composed of up to 6 mathematics courses (a review of algebra and trigonometry, along with calculus, units on statistics, and differential equations) and a 5 -course "backbone" laboratory/research series (introductory labs in biology, chemistry and physics). Students take those portions of the curriculum that are required by their major. The curriculum puts students in cooperative, technology supported learning environments where content is integrated among courses. The goals of the curriculum include improved teaming and communication skills; improved laboratory and research skills; improved mathematical thinking and problem-solving skills; early awareness of innovative teaching techniques in education majors; and development of learning communities of students. Existing courses were revised to incorporate examples and demonstrate relationships among concepts from multiple disciplines; relate student learning to real world examples and experiences; integrate teaming, critical thinking, problem solving and communication skills; and integrate effective use of technology for instruction. Here, we will provide an overview of the curriculum as it relates to education majors, outline the opportunities and challenges of the integrated curriculum, and discuss assessment results to date. We will also present strategies we have used to model innovative instructional strategies in science and mathematics within the curriculum.

## Joan Elizabeth DeBello (debelloj@stjohns.edu) St. John’s University

## The History and Influence of Women in Mathematics Education

This paper will discuss the importance and impact of women in mathematics education. It will also discuss the reasons why women choose mathematics education instead of pure mathematics if they continue on with post graduate work. It will also discuss the contributions of some famous women in mathematics education and the impact they have had over the years.

## Yvette d'Entremont (yvette@ualberta.ca) University of Alberta

## Ethnomatical Activities in the Post-Secondary Classroom: Linking Curriculum Standards to Practice

The Alberta Mathematics Program of Studies indicates that teachers should understand the diversity of cultures of their students. The introduction of multicultural perspectives into the mathematics curriculum helps students appreciate the contributions of other cultures as well as their own. The infusion into the curriculum of the cultural heritage of minority students encourages them to become more interested in mathematics. The term ethnomathematics has long been used to express the relationship between mathematics and culture. The Alberta Mathematics Program of Studies for Francophone minority students states that culture and identity
must be addressed in all subjects as part of the curriculum. However, it is not always evident how to integrate the culture and identity of minority students into specific content material. This session, and examples of activities suggested by teachers in the field, will address this issue as it relates to mathematics education programs for minority students.

## James Anthony Mendoza Epperson (epperson@uta.edu) The University of Texas at Arlington

## Mathematical Knowledge and Fluency Inservice Teachers Need to Create Technology-enhanced Mathematical Tasks

The Master of Arts program (MAMT) in the Department of Mathematics at The University of Texas at Arlington (UT-Arlington) is designed for inservice teachers who are interested in strengthening their understanding of mathematics and enriching their mathematics teaching. The courses in the MAMT program simultaneously integrate rigorous mathematics, effective pedagogy, and mathematics education research on student learning. Tasks designed for the course "Mathematics-specific Technologies" - a core requirement for the MAMT program - will be presented. The course includes the study of many mathematics-teachingrelated freeware programs, graphing calculators, Mathematica, and Sketchpad ${ }^{\circledR}$. The author will focus upon the mathematical knowledge inservice teachers must use with fluency to complete these tasks and the power of using mathematics-specific technology as a vehicle for gaining insight into inservice teachers' conceptual understanding of the mathematics they teach.

## Elana Epstein (eepstein@sjcny.edu) St. Joseph’s College

## Global Experiences in Mathematics Education

I developed a mathematics education course which encouraged global awareness. During the semester we met to compare education in the United States with education in England. At the end of the semester we traveled to England. The heart of the trip was in classroom observations and visits to Cambridge and Oxford. We observed at a private school and a public school. We got a good sense of what went on in the classroom. The teachers told us about the exams students take and other basic information about the schools. Looking through textbooks also gave us an idea of the sequence of the lessons and what topics are taught. At Oxford we chatted with a professor who explained how the University worked both academically and socially. We also sat in on a lecture and met with four Oxford students who answered all of our questions about attending the school. We dialogued about the differences in our educational systems and decided that both ways had their advantages. My students now have some understanding as to how the educational system in England works. Through this experience they developed a teaching philosophy that incorporates a global consideration. Most teacher training programs just concentrate on how classrooms are run in the United States, but these students now have glimpsed education in another country. This global view will, hopefully, make them better teachers.

## Russell Goodman (goodmanr@central.edu)Central College

## Using Oral Exams to Help Prepare Future Elementary Mathematics Teachers

The ability to effectively communicate mathematics is a priority for future elementary mathematics teachers. An oral examination, if used appropriately, is an excellent tool for assessing such skills. Moreover, an oral exam is a useful pedagogical tool for helping future elementary mathematics teachers improve their skills in communicating mathematical concepts. The speaker has used oral exams in his department's mathematics course for future elementary mathematics teachers and will share the logistical details for how he includes an oral exam component to these courses. Additionally, he will share anecdotal evidence for the pedagogical effectiveness of the oral exams, along with some student feedback.

Hong Yuan, Annie Han (ahan@bmcc.cuny.edu) The City University of New York, BMCC

## Mathematics in Pre-service Elementary School Teacher Programs in China and the US

Many American teachers convey the message that mathematics is too difficult to understand and to teach (Kenschaft, 2005). Compared to their Chinese counterparts, American elementary school teachers lack a
"profound understanding of fundamental mathematics" (Ma, 1999). Teacher Education Curricular differences exist in the kinds and depth of mathematics content courses and method courses required in each country. The curriculums themselves have varied over the past decade. The researchers will share their preliminary examination of mathematics components in past and current elementary school pre-service teacher programs in Shanghai and New York.

Meg B. Huddleston (mbhuddleston@schreiner.edu) Schreiner University

## A Student Exploration of the Texas State Standards for Mathematics Education

The Texas Essential Knowledge and Skills (TEKS) is the curriculum framework for Texas public schools. These represent the state standards for what children should know and be able to do at each grade level. The Mathematics TEKS are divided into six strands: Number, operation, and quantitative reasoning; Patterns, relationships, and algebraic thinking; Geometry and special reasoning; Measurement; Probability and statistics; and Underlying processes and mathematical tools. It is not surprising, then, that it is often difficult for students in pre-service education courses to grasp the complexities of these TEKS. This paper deals with the exploration of the TEKS in a pre-service mathematics education course for prospective elementary and middle school teachers. Rather than studying the TEKS either by grade-level or by strand, students are expected to consider them by topic. Following the developmental structure of the course itself, the students search the TEKS for ideas that complement what they are learning in class. They then provide examples of the TEKS for each grade level (as appropriate), along with a discussion of the evolution of each concept throughout the grades. Topics considered include: Problem Solving, Attributes, Multiplication and division, Fractions, Percent, "Shape" words, and a final Overview. This process encourages the students to explore the development of concepts within and across grade levels, to appreciate the structure and interdependence of mathematical concepts, and to better understand their responsibilities as teachers. This paper presents the structure of these assignments, samples of student work, and assessment strategies for evaluation of student learning outcomes.

## Gulden Karakok, Tina L. Johnston, Maggie Niess, Rachel Harrington, and Tevian Dray <br> (gkarakok@science.oregonstate.edu) Oregon State University

## Evaluation of a MSP-Professional Development Project

To increase K-8 teachers' content knowledge and pedagogical content knowledge of mathematics, a partnership was formed between Oregon State University and four rural school districts in Central Oregon that was funded by a Title II MSP grant. A two-week summer institute followed by a year-long professional development program was designed at Oregon State University and has been delivered three times in central Oregon. School districts recruited teachers of grades $\mathrm{K}-8$ to be placed in control and experimental groups each year. Teachers in the experimental group worked collaboratively to learn rigorous mathematics content in Probability and Statistics, Geometry and Algebra, and to plan implementation of the activities in their classrooms. Experimental group teachers were also required to participate in online discussions posted on the project website. The impact of the professional development was evaluated using multiple data sources. State test scores (TESA) assessed mathematics knowledge of students in both control and experimental teachers' classes. Mathematics content knowledge was collected from both groups using Praxis middle level mathematics exam scores. Changes in experimental group teachers' content knowledge and pedagogical content knowledge were gathered through pre and post DTAMS exams and RTOP observations. Mathematics and mathematics education faculty assessed the mathematics content of the professional development curriculum. The overall impact of the professional development program was assessed through interviews of the former participants in the project. Ongoing results from the data analysis and the plans for sustainability will be discussed.

## Robert McGrail (mcgrail@bard.edu) Bard College

## Mathematical (Pro)Logic: More is Less is More

This talk considers the challenge of retrofitting an undergraduate mathematical logic course originally designed for 16 -week semesters to a 10 -week MAT course. The basic approach is to introduce students to the logic programming language Prolog. This reduces many of the more technical hurdles on the road to Codel's Incompleteness Theorem to programming exercises. Moreover, logic programming provides a concrete context
for decidability, avoiding much of the usual handwaving. The speaker will share his experiences from a section of this course offered to Bard College MAT students during the fall semester of 2006.

## Sandra Richardson (sandra.richardson@lamar.edu) Lamar University

## Integrating the Tl-84 Plus Silver Edition in a Calculus Concept Course for Teachers

Much of the reform in mathematics education advocates approaches that require students to use technology as an aid in developing and explaining their mathematical ideas. This session will outline teaching techniques coupled with the use of the TI-84 Plus Silver Edition calculator in a Calculus Concepts course for teacher education candidates seeking 4-8 certification. Examples of how the graphing calculator can be used to augment the content and delivery of such courses will be provided.

## Sharon Siegel (sbsiegel@sc.rr.com) Francis Marion University

## The Use of Alternate Base Systems in the Preparation of Pre-Service Elementary Teachers

It has been said that an elementary school teacher must have a profound understanding of arithmetic to effectively teach mathematics. Most teacher preparation programs have specific mathematics courses designated for pre-service elementary teachers. A significant number of texts used in these courses have an investigation of alternate bases. Typically this material is focused on converting from one base to another. Instead of using them solely in that manner, alternate base systems can also be used to develop an understanding of whole number operations. By requiring a student to learn to add, subtract, multiply and divide in various bases, he or she is able to actually develop an understanding of place value. The idea that " 10 " can be five sticks in base five and eight sticks in base eight is something that students have a difficult time grasping. However, once a pre-service teacher finally understands how an alternate system "works", he or she will be better able to deal with their future students' frustration as those students attempt to master arithmetic concepts.

## Chen-Han Sung (drsung@tamiu.edu) Texas A\&M International University

## How to Improve Middle School Mathematics Teacher Quality: Lessons Learned from 2005-2007 Texas TQGP Grants

During the period of 2005-2007, our TQGP grants helped build up partnerships between the Texas A\&M International University and the high-needs independent school districts in the Laredo region by the Rio Grande. The objective was to improve student achievement in this southern border region in the state of Texas by means of improving middle school mathematics teacher quality. The main outcome measure was student performance on the state mandated tests. Most of student and teacher populations in the region are Hispanic/Latino. A teacher professional development model that focuses on improving the content knowledge and pedagogy among 4th-8th grade mathematics teachers through activity-based, hands-on, experiential learning was developed. Concepts of numbers (whole, fractions, integers, and rationals) and their operations, statistics, probability, geometry, and measurement are covered through dynamic and connected ways of learning. Both summer institute and academic year workshops have been offered through the 2005-2007 period. Pre-service and in-service, out-of-field and in-field, new and veteran teachers were included in the study through our TQGP grants. The type of assessments and the results of these assessments applied to those teachers and their students will be presented and discussed. Some current issues in middle school mathematics education and special concerns regarding such in this southern Texas region will be reported and analyzed with recommendations.

* This work was partially supported by the 2005-2006 and 2006-2007 TQGP grants on Middle School Mathematics Part I and II, under the Teacher Quality Grant Program, Texas Higher Education Coordinating Board.


## Kazuko Ito West (westc-k@post.harvard.edu) Keio Academy of New York

## How High School Mathematics Teachers are Produced in Japan—an Insider's View

Japan constantly tries to secure high quality teachers by keeping teacher salaries competitive, not only with the salaries of other professionals in government but with those in private industry as well. Many Japanese high school mathematics teachers are first tier university graduates who initially trained for careers as mathemati-
cians, physicists or engineers. College graduates with the education courses necessary for teacher certification are screened by a highly competitive employment selection procedure, which includes evaluation of college transcripts, written tests and an interview. The mathematics test covers all high school mathematics, discrete mathematics, and college calculus, but usually not areas such as multivariable calculus or abstract algebra. Applicants are also tested on general knowledge, natural and social sciences, English language and Japanese literature, and knowledge of teaching as a profession, such as the laws regarding education. Once employed, a new teacher is mentored and supported by experienced teachers in every aspect of school work. This is facilitated by the presence in each school of the teachers' room, a common room where all teachers work and meet with students except for the time when they are in a class, which is three class periods a day on average. Here teachers form an intellectual community which provides the collegial environment in which each can learn from the other. The focus of the talk will be on actual college entrance examination and teacher employment examination problems, and on some of the experiences of a veteran mathematics teacher in the public high schools of Japan.

## Emerging Technologies for Mathematics Teaching

Friday, August 3, 1:00 pm-3:00 pm

## Patricia B Humphrey (phumphre@georgiasouthern.edu) Georgia Southern University

## Podcasts, Video "Tutors" and More in Introductory Statistics

How many times have you heard a student complain "I understood it perfectly in class, but when I got home, I was lost!" This type of comment may become a relic of the past. For many years, we who teach Introductory Statistics thought we were "high tech" because we were using a computer package or a graphing calculator. This is no longer state-of-the-art. Many publishers are now going "all out" with electronic ancillaries through their websites. I will discuss my (and my students') experiences with StatsPortal, a website companion to David Moore's text "The Basic Practice of Statistics." This new website includes not only an interactive complete text, quizzing and homework capabilities, but the "Stat Tutor" which can be called up at many points in perusing the e-book (or by itself if students "forget part of a lecture"), and podcasts of the material for each chapter. One advantage for the instructor is that the site is easy to use, and you don't have to invent everything yourself.

## Adam Lucas (arl3@stmarys-ca.edu) Saint Mary's College of California

## Using iclickers to Enhance Student Engagement in Calculus

The iclicker, a radio frequency classroom response system, is an important emerging technology in math instruction enabling students to be actively involved in their own learning in a fun and anonymous way. I spent a third of class time in two different small-sized Calculus classes this year using iclickers. I wanted to determine what was the effect of this technology on student engagement and student learning. To collect data I conducted student surveys, videotaped my classes, recorded student exit interviews, and analyzed student's iclicker score. Testing student's understanding of course material with the iclicker has positive learning benefits as well as challenges. Students get the opportunity to hear diverse viewpoints on how to solve a problem from their peers. Also, alternating between lecturing and the iclicker keeps students focused. Rewarding students with points using iclicker software keeps students motivated and encourages attendance. Iclicker scores appear to be a better indicator of test performance than homework grades. Also, initial studies indicate that the iclicker increases student's appreciation of student-student interaction compared to student-instructor interactions. The challenges of using iclickers include reduced lecture time, getting students to explain the reasoning behind their answers, and developing good iclicker questions.

## Mike Martin (mmartin@jccc.edu) Johnson County Community College

## Storytelling, Mathematics, and the Digital Tradition: A Historical Context for Conveying Mathematics

The roots of mathematics have been relayed through the generations via a variety of modes and traditions. From verbal to written and now electronic form, what is presented to students as representing mathematics has evolved, in a neutral sense, over the years. This talk will highlight some of these changes, paying particular
attention to the emergence of electronic resources in the form of applets and video. In that context, dynamic web tools and concept videos developed by the author will be used to analyze the trends in resources. The dynamic web tools utilize webMathematica and allow users to run Mathematica over the web without doing any coding and getting results over the same web page. The dynamic web tools that were developed received the 2004 ICTCM Award for Excellence and Innovation with the Use of Technology in Collegiate Mathematics. The concept videos for the mathematics curriculum cover a mathematics topic by engaging the learner in an application context, relevant to the learner, that motivates the instructional unit and, in the end, returns back to the application setting. The videos received an Innovation of the Year Award from the League of Innovation in the Community College and was also a Finalist for the Community College Future's Assembly's Bellwether Award. While these two initiatives will be briefly highlighted, the trail of how we got here, what might be next, and ideas of resource integration will be pursued in the talk.

## Lila F. Roberts (lila.roberts@gmail.com) Georgia College \& State University

## A Pocket Full of Mathematics

Handheld calculators and computer algebra systems have had a tremendous positive impact in mathematics classrooms as powerful tools for visualization. Other handheld devices such as iPod, Zune and PDAs have found their way into many of our students' backpacks. These recent innovations have the potential to enhance the mathematics educational experience for our students. This presentation will be an overview of several handheld devices, including examples of content and descriptions of implementations. In addition, student perceptions from an experiment to implement iPod and Podcasting technology into a first course in calculus will be discussed.

## Chen-Han Sung (drsung@tamiu.edu) Texas A\&M International University

## Strategies on Teaching Mathematics to Students of the e-Generation

In the established learning environment, knowledge is treated as being static and presented in a hierarchical style with subjects being divided. The behaviorism and cognitivism are the major theories of learning. It is instructional focused and managed by the teacher. Therefore content delivery and coverage in a one-to-many communication style are the pedagogical focuses. For teachers with students growing up browsing the Web in the e-generation today, the above face challenges as never before. Knowledge is viewed as being dynamic and gathered in an adaptive style with subjects being connected. The connectivism and constructivism emerge as the major theories of learning. It is learner centric and managed by the learner. Therefore learner participation and collaboration in a many-to-many communication style are the pedagogical focuses. Incorporating emerging technologies into mathematics instruction can help such students to learn mathematics their way. For example, mathematical concepts, such as those in calculus, can be visualized dynamically and animatedly using handheld video iPods, versus illustrated by static graphics in a textbook. Augmented with audio through iPods' notes function, it can guide student inquiry. Using its captions/subtitles function, hearing-impaired students can also access those. It not only keeps students interest, but also helps students grasp mathematics concepts and problem solving processes. Students can be taught to show their mathematical thinking and explain the meaning behind their solutions. From their own discovery, students increase their understanding of mathematics The pros and cons of implementing this technology will be presented and discussed more in detail.

## Murphy Waggoner (murphy.waggoner@simpson.edu) Simpson College

## Using a Wiki to Encourage Collaborative Learning in Linear Algebra

The students of my Linear Algebra class participated in two Wiki assignments during the spring semester 2007. The first assignment required them to create Frayer models for definitions of linear algebra terms and then to collaboratively revise the definitions. In the second assignment the students wrote the answers to True/False questions and, again, collaborated in the revision of the answers. The Wiki assignments were motivated by a desire to help students understand the process of collaborative work, to allow the students to benefit from the work of others by making that work accessible and to improve the engagement of off campus students. In this talk, I will describe the assignments, including an explanation of Frayer models, explain the grading process and the rubrics used, and give a summary of the students' and my evaluation of the experience.

# Euler Society Contributed Paper Session 

## Saturday, August 4, 1:00 pm-5:30 pm

Jordan Bell (jbel13@connect.carleton.ca) Carleton University

## Euler and Analytic Number Theory

I will discuss some of Euler's results in analytic number theory and how he proved them. In particular, I will talk about $\sum_{p} \frac{1}{p}=\log \sum_{n} \frac{1}{n}$.

Robert E Bradley (bradley@adelphi.edu) Adelphi University

## Teaching a Liberal Arts Seminar on Leonhard Euler

We will examine a senior seminar course on the life, work of Leonhard Euler and his role in the European Enlightenment, as actually taught in Spring 2007. Challenges and successes will be discussed. We will also consider the possibility of constructing similar courses on the life and times of other great mathematicians.

## John Bukowski (bukowski@juniata.edu) Juniata College

Classroom Activities from Elementa Doctrinae Solidorum (E230)
Euler's work Elements of the Doctrine of Solids has been widely discussed because of the appearance of the formula $V-E+F=2$ for polyhedra. Parts of this paper can be used to develop some interesting activities for students, even those in a liberal arts mathematics class. We will discuss some possible classroom activities dealing with discovery and classification of particular solids.

Thomas Drucker (druckert@uww.edu) Department of Mathematical and Computer Sciences,University of Wisconsin-Whitewater

## Why Wasn't There an Eighth Bridge?

Euler's paper on the seven bridges of Koenigsberg was the progenitor of the field of graph theory and perhaps even of topology. It did not, however, get the fields started promptly. While the ideas behind the paper are easy to express to an undergraduate audience, it can take a little more context to come up with answers for questions about why mathematics had not been brought to bear on the problem before and why the success of the paper did not generate application to other problems, even in a recreational setting. A contrast will be drawn between the response to Euler's paper and the speed with which the Four Colour Conjecture attracted attention in the next century.

Axel Mainzer Koenig (dspace21@aol.com) Koenig \& Associates, Inc.

## Euler's Contribution to Rational Fluid Mechanics and Naval Science

Euler's contributions to Rational or Mathematical Fluid Mechanics are presented with the emphasis on the General Theory of the Motion of Fluids. Also Euler's generalization of the stream function concept to a pair of stream functions or stream surfaces is discussed. Euler's Potential was formulated about 100 years before a similar detailed mathematical exposure was formulated by Jacobi and Clebsch around 1844. Furthermore Euler's work initiated the establishment of Naval Science in Russia and influenced the art of building Naval Ships in Russia in particular in the 18th century. An overview about Euler's accomplishments in Naval Architecture and Ship Hydrodynamics at Russia's St. Petersburg Academy of Sciences is also included.

Johan Ernest Mebius (jemebius@xs4all.nl) Delft University of Technology

## On Euler's 1770 paper "Problema Algebraicum"' (E407)

The full title is "Problema algebraicum ob affectiones prorsus singulares memorabile" which means "An algebraic problem which is memorable because of its rather noteworthy impressions." The problem is to find all linear transformations of $N$ independent variables which preserve the sum of their squares. Euler solves
the problem for $N=2,3,4,5$ and touches upon a solution for arbitrary $N$. Euler's treatment is purely algebraic; angles solely serve to parametrize the transformations. The three angles needed to parametrize the transformations in the case $N=3$ became later well-known as "Euler angles". Euler angles are widely applied in solid geometry, physics, aeronautics and astronautics, and computer graphics. Presumably Euler did not have geometrical applications in mind when writing this paper. This conjecture is supported by three places in E407 where the transformation is a rotoreflection rather than a rotation; rotoreflections cannot be parametrized by angles in the way Euler uses them. The last four chapters (Ch. 33-36) treat rational parametrizations for the cases $N=3$ and $N=4$ and a certain class of magic squares of order four.

Ed Sandifer (esandifer@earthlink.net) Western Connecticut State University
Please pass the $\pi$ : Euler and the Digit Race
We describe Euler's role in calculating $\pi$ to 140 decimal places without actually having to do the work himself. The mysterious and romantic Slovenian hero Jurij Vega plays a key role in this drama.

James Sellers (sellersj@math.psu.edu) Penn State University

## On Euler's Partition Theorem Relating Odd-Part Partitions and Distinct-Part Partitions

Euler's classic partition identity, which states that the number of partitions of the positive integer $n$ into odd parts equals the number of partitions of $n$ into distinct parts, has been generalized and extended by many. In this talk, we will briefly consider the historical background behind Euler's identity and then discuss two results proven in the last few years which generalize this identity. Some of these results were developed in collaboration with Gary Mullen and Drew Sills.

## Fun and Innovative Teaching Techniques for an Abstract Algebra Class <br> Saturday, August 4, 3:15 Pm-5:30 pm

## Ezra Brown (brown@math.vt.edu) Virginia Tech

You CAN Hold a Group in Your Hand
A remark made by an unhappy student in an abstract algebra course led to a teachable moment - and how! This is a story of how using rod-and-ball models of crystals having different kinds of symmetry groups completely changed that course, that student, the rest of the class, and its teacher.

## John Jones (jj@asu.edu) Arizona State University

## More Group Tables and Subgroup Diagrams

We will present a series of worksheets for students studying beginning group theory. Each worksheet tries to enhance students' understanding of a group theory concept through investigations with group tables and/or subgroup diagrams and goes with a free tool available through the web.

## Jerry Morris, Brigitte Lahme (morrisj@sonoma.edu) Sonoma State University

## Activities in Abstract Algebra: Advantages and Challenges

We know exactly what we are teaching our students; after all, we are preparing very careful lectures and lessons. But what are the students actually learning? Introducing activities in Abstract Algebra made our students learning more visible. For example, students used marshmallow and toothpick tetrahedra to investigate properties of a group. Peer discussions and hands-on explorations revealed their confusion in identifying group elements and operations. This allowed us to identify students' misconceptions before they became major roadblocks. Listening to students' group discussions also gave us a glimpse into their thinking and helped us learn about a variety of mathematical issues we would never have expected. In this talk, we will look at some of our activities and share what we learned from them.

## Colm Mulcahy (colm@spelman.edu) Spelman College

## Exploring the Composition Laws of Small Groups with a Pack of Cards

There are simple games one can play with a pack of ordinary playing cards, which illustrate the laws of composition in various small groups. Each adjacent pair of cards is replaced in turn with a single card, and the goal is to predict the last card remaining. We'll show how this can be done for any group of size less than ten, with a focus on the cyclic groups of order two and three, the Klein four group, and the quaternions. The roles of the associative and abelian properties are particularly interesting.

Nancy Rodgers, Doug Anewalt, Paul Lee (rodgers@hanover.edu) Hanover College

## Groups Unlimited: A New Software Program

Three years ago we started developing a software program called Groups Unlimited (GU) to enhance the learning of group theory. It is similar to the DOS program Exploring Small Groups (ESG), which was developed in 1989 by Ladnor Geissinger, but GU is written in Visual Basic for Windows with a graphical interface that is extremely user-friendly. Tables can be easily created via entries in a blank table or opened from the group library and displayed side-by-side for comparison purposes. The user can find subgroups generated by selected elements, or request a listing of all subgroups with an option to see a colorful graphical display of the lattice of subgroups. Other features include tests for normal subgroups with group tables arranged by colored cosets, and tests for homomorphisms via 2-column tables for defining a function. In this talk, we will demonstrate some of the unique features of the program.

## Amber Rosin (arrosin@csupomona.edu) Cal Poly Pomona

## A Colorful Introduction to Group Theory

In order to ease students into thinking abstractly and using formal proof, it helps to give them a hands-on, intuitive overview of the topics they will encounter. We will investigate a series of hands-on models for groups such as hexaflexagons, color wheels, overlapping cards, pinwheels, and the more standard symmetries of regular polygons. Many of these have the advantage of possessing non-obvious identities and inverses. We will also discuss how comparing, contrasting, and coloring the Cayley tables for these models allows students to intuitively discover and explore concepts such as group isomorphisms, orders of elements, abelian groups, cyclic groups, and subgroups before working with formal definitions and proofs.

## Dorothy Zeiser (zeiser.d@gmc.edu) Gwynedd-Mercy College

## Maple and Flash Animations for Abstract Algebra

This presentation will demonstrate the use of one or both of two Maple worksheets developed during and after the 2005 MAA PREP Workshop: Exploring Linear Algebra using Maple. I have used these worksheets in a beginning abstract algebra course for undergraduate students majoring in mathematics and graduate education students seeking certification in secondary mathematics. The animations in the worksheets explore the group of symmetries of an equilateral triangle and the group of symmetries of a square. Maple is used as the tool for producing the animated visualizations of each of the rotations and reflections, as well as sequences of two such transformations. The second part of the presentation will include one or more Flash animation tools to be developed during the 2007 MAA PREP Workshop: Flash at the Beach: Creating Mathlets with Adobe Flash. These tools are intended for use with a beginning undergraduate class in abstract algebra. One intended tool is a visualization of rearranging rows and corresponding columns of a given Cayley table of a group to show that the group is isomorphic to another group which is given by its Cayley table. This is based on the visualization technique used in the 1996 video "Isomorphism" produced by Films for the Humanities and Sciences.

General Contributed Paper Sessions<br>\#1 Friday, August 3, 8:30 am-10:30 am<br>\#2 Friday, August 3, 1:00 Pm-3:00 pm<br>\#3 Friday, August 3, 3:15 Pm-5:15 pm<br>\#4 Sunday, August 5, 8:30 am-10:30 am<br>\#5 Sunday, August 5, 8:30 am-10:30 am<br>\#6 Sunday, August 5, 1:00 pm-3:00 pm<br>\#7 Sunday, August 5, 1:00 pm-3:00 pm<br>\#8 Sunday, August 5, 1:00 Pm-3:00 PM<br>\#9 Sunday, August 5, 3:15 pm-5:15 pm<br>\#10 Sunday, August 5, 3:15 Pm-5:15 pm<br>\#11 Sunday, August 5, 3:15 Pm-5:15 pm

## Sayel Ali (alis@mnstate.edu) Minnusota State Univ. Moorhead \& The Petroleum Institute/Abu Dhabi

## The $m$ th Ratio Test: New Convergence Tests For Series

The famous ratio test of d'Alembert for convergence of series fails if the limit of the ratio is 1 . Several tests, like Raabe's and Gauss tests were devised to cover these cases where the ratio test fails. We will give a new ratio test and several examples of series where this new ratio test succeeds but the ordinary ratio test fails. This test has a wide range of applications that it covers, with few calculations, many series that appear in a calculus or advanced calculus book.

## Roger Baker (baker@math.byu.edu) BYU

## Numbers with a Large Prime Factor

We describe recent progress with G.Harman in a problem of K.Ramachandra. When an interval $I=[x, x+y]$ ( $x$ large), is too short for the Riemann hypothesis to yield a prime in $I$, we may show (as a substitute) that some integer in $I$ has a large prime factor. Others (besides the presenter, Harman and Ramachandra) who have obtained results of this kind are S.W. Graham, C.H. Jia, H.Q.Liu and Jie Wu.

Lalgudi J Balasundaram (jagbala@comcast.net) Harvard Institute for Learning in Retirement

## On a Combinametric Approach to Goldbach Conjecture

The Goldbach Conjecture - any even number > 2 can be expressed as sum of two primes - enunciated in 1742 has been difficult to prove though its correctness has been verified for even numbers in the range of $10^{17}$. Combining $6 * n-1 / 6 * n+1$ prime distribution patterns with combinatorial probability, it is possible to compute prime pairs for all even numbers $>2$ supporting the Goldbach Conjecture.

## Linda Becerra, Ron Barnes (becerral@uhd.edu) University of Houston-Downtown

## Mathematical Curiosities

This paper considers a number of mathematical curiosities, puzzles and visual oddities. A series of vanishing pictures including the vanishing astronaut, vanishing leprechaun, disappearing rabbit and beer glass puzzle, among others, will be examined. A brief explanation of why such objects disappear will be illustrated. Relationships among magic squares, Latin squares, sudoku puzzles, and n-Queen problems will be noted. Formulations of a few famous mathematical paradoxes will be given: including Cantor's paradox, Russell's paradox, and the Banach-Tarski paradox. Implications of the independence and consistency of the continuum hypothesis with respect to Zermelo-Frankel set theory will also be mentioned. Some curious games and puzzles, such as toe-tac-tic, the two-key problem, a shortest route problem and a disk puzzle will also be considered.

A handout including a few challenging puzzles and paradoxes will be distributed to the audience for their entertainment. The problems range from a simple high school geometry proof to the Monte Hall problem and the final sudoku puzzle given at the most recent world championship sudoku contest. (None of the finalists could solve the puzzle. Can you?)

## Satish C Bhatnagar (bhatnaga@unlv.nevada.edu) UNLV

## Rationale for History of Mathematics course

This paper is an outcome of author's experience of teaching an experimental undergraduate course and a graduate course in history of mathematics. Currently, at UNLV, history of mathematics course in not required at the undergraduate level. The paper provides a background and rationale for such a required course. The only prerequisite for this course is class standing at senior level. The course can be used as a capstone course integrating several mathematical topics, mathematicians and cultures. It can be used as technical writing course since writing short and long papers are natural. Assessment and diversity are the buzz words in academe. It can be used as an assessment tool for a program. The course provides a cross section of diversity on the role and achievements of women and minorities in mathematics. There are PR merit points in it. Currently, history of math courses are available only for students planning to go into school teaching.

## Daniel Birmajer (abirmaj6@naz.edu) Nazareth College

## Arithmetic in the Ring of Integer Formal Power Series

We study the arithmetic (units, irreducible elements, unique factorization, etc.) in the ring $\mathbb{Z}[[x]]$ of formal power series in the indeterminate $x$ with integer coefficients, and discuss some irreducibility criteria. We will examine in some detail whether or not a quadratic polynomial is irreducible as a power series.

## David Lee Blackman (gribear@mac.com) Retired UC Berkeley

## Mathematical Model for Myocyte Function

Employing common physical principles, Fermi distribution, Chemical capacitive discharge and thermodynamic principles, a theoretical model for the three principle currents involved in the T-wave interval is developed. The model explains the relationship between three currents. Sodium/Calcium exchanger ties polarization to cell relaxation. The equation for the passive current T -wave is a bit more complicated than the HodgkinHuxley equation but nonetheless explains T-wave alternans. Thermodynamic elements of the slow step for Sodium/Potassium-ATPase predicts that slightly elevated serum potassium can correct the problem behind the T-wave Alternans. Theory promises a treatment for cardiac arrhythmia.Talk will emphasize derivation of equations. Present standard model involves 11 currents, 91 differential equations with 101 initial conditions and constants.

## Vera Cherepinsky (vcherepinsky@mail.fairfield.edu) Fairfield University

## Getting Students to Learn from their Mistakes.

Students rarely use their graded exams to study. An alternative method of grading exams (proposed by Chris Black, CWU) encourages students to use their mistakes as a learning resource. In this grading method, the test is returned with each question simply marked as "right" or "wrong". Students can then go over their exams and resubmit them with separate corrections, where for each wrong question they must identify the error, decide how serious it was, and describe how to fix it. The exam and corrections together are graded, and for each error correctly identified, classified, and fixed, the student gets back half the points lost. I implemented this in two applied calculus classes for non-majors in spring 2006 as well as in a year-long calculus sequence for engineers in 2006-2007. It was very well received, as indicated on the surveys the students filled out after each in-class exam. Most students indicated that they spent more time going over their exam and felt they learned a lot more from their mistakes, compared to traditionally graded exams. In this talk, I will discuss the details of this grading method, the logistics involved in administering it, and the results of student surveys. Insofar as this method encourages students to actively learn from their mistakes, it is well worth the time investment on the part of the instructor.

## Kelly Cline (kcline@carroll.edu) Carroll College

## Classroom Voting in Linear Algebra and Differential Equations

Classroom voting is a teaching technique where the instructor poses a multiple-choice question to class, gives them a few minutes to discuss the question with their peers, and then has each student vote on the correct answer using a hand-held "clicker." The results are immediately tabulated and displayed on a computer, giving the instructor real-time feedback. The instructor then guides a Socratic discussion of the question, asking different students to explain what they voted for and why. This pedagogy can have a dramatically positive effect on the classroom, creating a very student-centered learning environment: Strong majorities of students agree that this teaching method is fun, it helps them engage in the material, and they believe that it helps them learn. At Carroll College, we have integrated several voting questions into almost every period of our classes, finding that if used carefully, we can cover exactly the same material, teaching the same syllabus, and giving the same exams: Nothing has to be removed. Classroom voting is simply a different way of presenting the material. Large libraries of classroom voting questions or ConcepTests exist for calculus, and so to build on this success, over the past year, we have developed and tested libraries of classroom voting questions for our linear algebra and differential equations classes. We are currently looking for faculty at other institutions who would be willing to collaborate with us as we study the effects of these questions and this pedagogy in linear algebra and differential equations classes.

## Helen Cloherty (cloherth@stjohns.edu) St. John’s University

## Insights and Observations: Mathematics Education in Russia

In order to study the way that Russia teaches mathematics and trains mathematics teachers, I went to St. Petersburg and Moscow with a group of graduate students from Teachers College, Columbia University. I will talk about the classes that we observed in both specialized academies and in regular schools on various grade levels. I will also discuss meeting with government officials, educators and teacher trainers, and conclude with concepts and practices and their implications for mathematics instruction.

## Kumer Pial Das (kumer.das@lamar.edu) Lamar University

## ELL Students and Their Counterparts: An Analysis of TAKS Data

The Texas State University System (TSUS) Math for English Language Learners (MELL) initiative is a multiyear effort funded by Texas Education Agency (TEA) to develop instructional resources designed to increase the effectiveness of math instruction for ELL students in PK-12 schools. As a part of this study, the statistical analysis of Mathematics TAKS (Texas Assessment of Knowledge and Skills) data is needed to identify and develop instructional tools. Moreover, this analysis will be used to guide policymakers in the state of Texas. The TAKS is divided into objectives. For example, for the grade 11 exit-level there are 10 objectives. Within the categories of these objectives, there is one further grouping of the objectives themselves of Algebra I, geometry and measurement, percents and proportional relationships, probability and statistics, and mathematical process. In this study, the performance of ELL students as well as non-ELL students has been analyzed. It's important to identify the categories where student tend to perform poor. Statistical analysis has been employed to identify those categories (objectives).

## Tevian Dray (tevian@math.oregonstate.edu) Oregon State University

## Putting Differentials back into Differential (and Integral) Calculus

For the last 40 years, differentials have been taboo in calculus, having been turned into something else entirely: a tool for linear approximations. While mathematicians can be justly proud of the rigor introduced in the 1800 s, we seem to have forgotten that calculus was used successfully before then, let alone that those early methods were rigorously justified in the 1960s. Many scientists and engineers apply calculus by manipulating differentials, and for good reason: it works. We argue that the differentials of Leibniz - not the weak imitation found in modern texts - naturally capture the essence of calculus, and should form the core of introductory calculus courses.

## Genghmun Eng (genghmun.eng@aero.org) The Aerospace Corporation

## 2X More Littlewood Oscillations Compared to MathFest 2005

In MathFest 2005 and AMS 2006 we showed how to use sets of $\delta$-functions to model the Eratosthenes Sieve process. Let $\rho_{S}(N)=$ Prime Density be a smoothed fraction of primes near $N$. Let $\rho_{D}(N)=$ Divisibility Density be a smoothed fraction of all integers not divisible by any prime up to $N$. Since the primes up to $N$ determine all new primes in the interval $X=\left\{N, N^{2}\right\}$, then $\rho_{D}(N)$ gives information about $\left[\rho_{S}(X) ; X=\right.$ $\left.\left\{N, N^{2}\right\}\right]$. But $\rho_{D}(N)$ is a point estimate, so a key question is which $X=\left\{N^{1}, N^{2}\right\}$ best gives $\rho_{S}(X) \equiv$ $\rho_{D}(N)$ in the large- $N$ limit. Define $\rho_{S}\left(X=N^{+\Lambda}\right)=\rho_{D}(N)$, or $\rho_{S}(X)=\rho_{D}\left(X^{+1 / \Lambda}\right)$ as a general model, which requires only that $\Lambda$ approach a nonzero constant at large $N$. We then derived:

$$
\begin{equation*}
1 \frac{d \rho_{D}(Q)}{d Q}+\rho_{D}(Q) \cdot \frac{\rho_{D}\left(Q^{1 / \Lambda}\right)}{Q}+\mathcal{O}\left(\frac{1}{Q^{2}}\right) \equiv 0 \tag{1}
\end{equation*}
$$

as the fundamental equation for $\rho_{D}(Q)$, having a primary solution of $\rho_{D}(Q)=\mathbf{1} /\{\Lambda \ln Q\}$ and $\rho_{S}(Q)=$ $1 / \ln Q$, giving the Prime Number Theorem (PNT). Mertens' Theorem sets $\Lambda=\Lambda_{F} \equiv e^{\gamma_{0}} \approx 1.781072418 \ldots$, independent of the PNT, where $\gamma_{0}$ is Euler's constant. Let $Z(Q)$ be the first correction to the PNT: $\rho_{D}(Q)=\mathbf{1} /[\{1+Z(Q)\} \Lambda \ln Q]$; it gives for $Z(Q) \ll 1$ :

$$
\begin{equation*}
\frac{d Z(Q)}{d Q}+\frac{Z\left(Q^{1 / \Lambda}\right)}{Q \ln Q} \approx 0 \tag{2}
\end{equation*}
$$

having solution $Z(Q)=\sum_{m=0}^{\infty} A_{m} /(\ln Q)^{+\beta m}$, with a side constraint $\beta_{m} \equiv \Lambda^{+\beta_{m}}$. For $\Lambda>\exp (+1 / e) \approx$ $1.44466786 \ldots$, an infinity of complex $\beta_{m}$ values arises as Littlewood oscillation corrections to the PNT, which are all nearly periodic in $[\ln \ln Q]$. Let $\beta_{m}=U_{m}+i V_{m}=R_{m} \exp \left(+i \Omega_{m}\right)$ with $R_{m}^{2}=U_{m}^{2}+V_{m}^{2}$, $\tan \left(\Omega_{m}\right)=V_{m} / U_{m}$, then $R_{m}=\left|U_{m} / \cos \Omega_{m}\right|$ and:

$$
\begin{equation*}
\left|\gamma_{0} \cos \Omega_{m}\right|=\left|\Omega_{m} \cot \Omega_{m}\right| \exp \left(-\Omega_{m} \cot \Omega_{m}\right) . \tag{3}
\end{equation*}
$$

The absolute value gives twice as many solutions as previously reported, where the first few roots now are: $\beta_{0} \approx(1.2069+i 1.6036), \beta_{1} \approx(3.48252+i 7.40216), \beta_{3} \approx(4.54656+i 13.0248)$.

Katie Evans, Belinda Batten (kevans@latech.edu) Louisiana Tech University

## Theoretical Considerations of Control Design for the Klein-Gordon Relativistic Wave Equation

When designing controllers for physical systems governed by partial differential equations (PDEs), there are several areas of theoretical detail required to guarantee a finite error bound for the difference between the infinite-dimensional PDE system and a finite-dimensional approximation to it, i.e., an ordinary differential equation (ODE) system. In this talk, we will consider the linear Klein-Gordon relativistic wave equation to demonstrate the necessary theoretical considerations. We will pose the PDE as an abstract Cauchy problem and define a suitable domain for the state dynamics operator. We will use this information to show that the state dynamics operator generates a strongly continuous semigroup, and we will provide a Lyapunov argument that will guarantee exponential stability of the semigroup. This will be sufficient to allow us to conclude that the truncation error resulting from approximating the Klein-Gordon PDE by an ODE is finite.

## Martin E. Flashman (flashman@humboldt.edu) Humboldt State University

## Visualizing Mappings with Technology (Preliminary Report)

Mappings are discussed and are valuable tools for understanding transformations from the beginning of calculus through courses in linear algebra and complex variables. The author will demonstrate a variety of visualizations of these mappings using Winplot, freeware from Peanut Software.

William P. Fox (wpfox@nps.edu) Naval Postgraduate School

## Lanchester Equations as a Discrete Dynamical System

In our mathematical modeling for decision-making course we cover a block in discreet dynamical systems (DDS). We cover an introduction to first order affine, nonlinear, and system of dynamical systems. We end that
block with examples of Lanchester Equations for combat models as a dynamical system. We look at historical examples such as the battle of Trafalgar, Custer's Last Stand, and the Battle of Iwo Jima. We cover guerilla warfare as well as parity issues. We finish the block with models for insurgency and counter-insurgency battles. Our technology platform is Excel.

Jim Fulmer and Tom McMillan (jrfulmer@ualr.edu) University of Arkansas at Little Rock

## Patterns on Pascal's Triangle

Pascal's Triangle was known by the Chinese some 500 years before Pascal ever lived. This session will exhibit that Chinese Triangle. Patterns continue to be discovered. Topics to be explored include: rows, columns, diagonals, powers of eleven and two, palindromes, binomial expansions with positive integer and negative integer exponents, Fibonacci numbers, probability, hexagons, cubic numbers, squares, and triangular numbers. Patterns will be connected to Arithmetic, Set Theory, Algebra, Geometry, Probability, Trigonometry, Calculus, Linear Algebra. A three-dimensional model of Pascal's Pyramid will be exhibited.

## Ralucca Gera and Linda Eroh (RGera@nps.edu) Naval Postgraduate School

## Global Alliance Partitions in Graphs

"Let $G$ be a connected graph with vertex set $V(G)$ and edge set $E(G)$. A (defensive) alliance in $G$ is a subset $S$ of $V(G)$ such that for every vertex $v \in S,|N(v) \cap S| \geq|N(v) \cap(V(G)-S)|$. The alliance partition number, $\psi_{a}(G)$, is the maximum number of sets in a partition of $V(G)$ such that each set is a (defensive) alliance. Similarly, $\psi_{g}(G)$ is the maximum number of sets in a partition of $V(G)$ such that each set is a global alliance, i.e., each set is an alliance and a dominating set. We present bounds for the global alliance partition number in terms of the minimum degree, which gives exactly two values for $\psi_{g}(G)$ in trees, and we present conditions that classify trees to have $\psi_{g}(G)=i(i=1,2)$, presenting a characterization for binary trees."

## Berit Nilsen Givens and Amber Rosin (bngivens@csupomona.edu) Cal Poly Pomona

## The Interassociates of the Semigroup of Injective Functions on the Natural Numbers

Two semigroups ( $S, \cdot$ ) and $(S, *)$ (with the same underlying set) are said to interassociate if and only if $a \cdot(b * c)=(a \cdot b) * c$ and $a *(b \cdot c)=(a * b) \cdot c$ for all $a, b, c \in S$. We consider the semigroup of all injective functions on the natural numbers and classify its interassociates. In particular, we find necessary and sufficient conditions that any two interassociates are isomorphic.

## Susan Goldstine (sgoldstine@smcm.edu) St. Mary’s College of Maryland

## Fortunatus's Purse: a Many-Colored Story

"You have heard of Fortunatus's Purse, Miladi? Ah, so! Would you be surprised to hear that, with three of these leetle handkerchiefs, you shall make the Purse of Fortunatus, quite soon, quite easily?" - Lewis Carroll

In the wonderland of topology, we can hold the entire world in a bottle or in a purse. We will assemble this purse, dissect it, rearrange it, and see what it has to tell us about how our world and other worlds fit together. This talk features many colored (and many-colored) fabric models.

Thomas Hagedorn (hagedorn@tcnj.edu) The College of New Jersey
Computation of the Jacobsthal function for $n \leq 50$.
Consider the set of units $\mathbf{Z}_{n}^{*}$ in $\mathbf{Z}_{n}$. Let $j(n)$ denote the largest difference $u_{i+1}-u_{i}$ between consecutive units in $\mathbf{Z}_{n}$ (including $u_{1}-u_{\phi(n)}$ ). Jacobsthal defined and studied the function $j(n)$ and its companion function $J(n)=j\left(P_{n}\right)$, where $P_{n}$ is the product of the first $n$ primes. We have been able to calculate the values of $J(n)$ for $n \leq 50$. Previously, $J(n)$ was known only for $n \leq 24$.

## Mike Hall (mhall@astate.edu) Arkansas State University

A New Day in Algebra Instruction: Implementation of a Computer-Assisted Learning Program in Algebra
The purpose of this paper is to present the processes and procedures used by one university in the implementation of a computer-assisted learning environment in introductory, intermediate, and college algebra courses.

The preliminary results of a research project that aims to determine the effectiveness of the program on student success in the courses is the primary focus of the project. Results include a year-long longitudinal study of student attitudes, student success rates (DWF rate), and instructor attitude. In addition, a discussion of future directions of the program and advice for others seeking to implement such a program are also included.

## Annie Han, Michael Sunderland, Hong Yuan, Chunxia Qi

(ahan@bmcc.cuny.edu) The City University of New York, BMCC

## Doctorate of Mathematics Education Programs in China-A Bright Future

This presentation focuses on doctoral programs in mathematics education in China. China offers a rare opportunity to witness the evolution of doctoral programs in mathematics education from the ground up. The researchers will discuss the nature of programs from the beginning of the last decade to the present. Specifically, what courses traditionally taught in a mathematics department are offered to or required of prospective doctoral candidates? How are such courses coordinated with the schools of education or departments of pedagogy? What, how many, and how are these pure and applied mathematics content courses distributed across education programs? How has this changed over time? These programs are compared and contrasted with existing programs here in the United States. The first American doctoral programs in mathematics education were established at Teachers College, Columbia University and at the University of Chicago in the early 1900s. Today more than 100 universities are offering such degrees in the United States. These doctoral programs all have well established curriculums and award 50-115 doctorates per year. Many of these mathematics education doctoral programs are located in university departments of mathematics, though the majority of the programs are housed within the schools of education or curriculum teaching departments (Reys \& Kilpatrick, 2001). Initially and importantly, the earliest such American program, as created by David Eugene Smith at Teachers College-Columbia University, emphasized the history of mathematics, the examination of programs in other countries, and the expectation that teachers actively contribute to the furthering of the field. In America's early mathematics education programs, a doctoral candidate might choose pure or applied mathematics, as well as pedagogy, philosophy, or history of mathematics for a dissertation topic. The role of the such programs was to prepare faculty to teach college mathematics and to conduct research related to mathematics education. In sharp contrast, today's mathematics education doctorate programs in China are still in their developmental stages. The first such doctorate degree was awarded just a decade ago at the East China Normal University in Shanghai. As of today there are only 5 institutions in all of China that award similar degrees. By reviewing such graduate programs in China the field of mathematics education is furthered.

## Kevin Hartshorn (hartshorn@moravian.edu) Moravian College

## Culture Points: Engaging Students Outside the Classrrom

In 2005, we instituted a "Culture Points" component in several of our mathematics courses. This concept has fostered a culture of mathematical discussion and exploration both among students and between students and faculty. Activities have ranged from reflection papers on colloquia to oral presentations of mathematical topics to artistic displays of mathematical concepts. In addition to increased participation in mathematical activities outside the classroom, we have seen more engagement from our students in class. I will share some insights into the kinds of activities that have fallen under the "culture points" umbrella and some lessons learned.

## Jeff Johannes (johannes@member.ams.org) SUNY Geneseo

## Four-Dimensional Tic-Tac-Toe on a Torus-The Game of Set

Geometry, topology and combinatorics all come together in the deceptively simple world of the card game Set. We will see how Set is built up from tic-tac-toe and explore a few of the many counting questions that the game inspires.

Rahim G Karimpour (rkarimpour@pmu.edu.sa) Prince Mohammad Bin Fahd University An Alternative Evaluation of $\lim _{x \rightarrow \infty} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!}(\ln x)^{n}$ by an Alternative Technique to Integration by Parts One of the most troublesome topics in calculus is the technique of integration by parts. The usual approach to this technique is $\int u d v=u v-\int v d u$. This method is a general one applicable to a wide variety of integrals. Selecting the right function for $u$ is the most difficult task for many students of calculus. In this article, we explain a method that, once mastered, may facilitate the understanding of how the technique works.

## Azar Khosravani (akhosravani@colum.edu) Columbia College Chicago

Exploring Binary Expansions With a Number Game
A guessing game using cards, each containing 32 positive integers, is used to demonstrate properties of binary expansions of natural numbers.

## Azar Khosravani (akhosravani@colum.edu) Columbia College Chicago

## A Brief History of the Genus Concept

An overview of the historical development of the genus concept and its role in the theory of binary quadratic forms will be presented.

## Patricia Kiihne (pkiihne@ic.edu) Illinois College

## Multiplication and Division Algorithms from a Historical Context

A surprising number of alternate computational algorithms are studied both in mathematics content courses aimed at future teachers and in history of mathematics texts. This talk will explore several of these algorithms not only as a part of math history but also in terms of their usefulness in understanding computational processes. Algorithms examined will include multiplication and division by duplation, the "ship" or galley method of division, and division by differences, as well as algorithms using Chinese counting rods.

## Kendra Killpatrick (Kendra.Killpatrick@pepperdine.edu) Pepperdine University

## Multi-Ordered Differential Posets

Richard Stanley first introduced the idea of an $r$-differential poset in 1988. There are two well-known types of differential posets: the first is the 1-differential poset of partitions, Young's lattice, and its generalization to the $k$-differential $k$-ribbon poset and the second is the 1-differential Fibonacci lattice $Z(1)$ and its generalization to the $r$-differential Fibonacci lattice $Z(r)$. I will discuss a new kind of poset called a multi-ordered poset and define what it means for a multi-ordered poset to be differential. The standard example of a differential multi-ordered poset is an analogue to Young's lattice. I will also discuss several open questions about these multi-ordered posets related to the number of chains in the poset.

## Anand Kumar (anandkumar1234@yahoo.com) Ramanujan School of Mathematics

## Maxima and Minima Through Geometry

The main goal of all prestigious undergraduate level mathematics competitions around the world is to present a good mathematical challenge to brilliant young minds. Problems based on concepts of inequalities are vital source of this challenge. From the beginning of our school days, we are gradually trained to learn about basic concepts of inequalities. But it is often said, "A picture is worth more than a thousand words". Therefore, it will be better to understand, as well as explain, some of these inequalities with the aid of drawing pictures. Several times, we can be able to easily solve the inequality problems via geometrical methods. In this talk I would like to discuss some examples of this nature.

## Jonathan Paul Lambright (lambrij@savstate.edu) Savannah State University

## Contemporary College Algebra: A Pilot Project For New Teaching Techniques in College Algebra

The methods for teaching college algebra have been the subject of many research topics, discussions and debates. The reasoning has been that students continue to struggle through college algebra and in particular
they struggle in follow-on math classes such as Pre-Calculus or Calculus. The College Algebra Reform Project was instituted to change the way college algebra is presented in text as well as the way that it is taught in the classroom. In this pilot project emphasis is placed upon 1) a refocus of course content; data, functions, and modeling comprise the primary major otp level topics, 2) presenting material and problems from a real world perspective and teaching students to model and solve such problems and 3 ) in-class group exercises and board work. This paper presents the results of the pilot project for the past year at Savannah State University. The work shows that students benefited from shift in the way the course was taught and gained from the in-class group assignments. Three contemporary college algebra pilot sections have been taught thus far and the number of sections offered will increase to five in the next semester.

## Andrew S Leahy (aleahy@knox.edu) Knox College

## Pappus, Guldin, and James Gregory too

A primary goal of many history of mathematics courses is to provide a deeper understanding of and context for the mathematics that students have already learned. The Theorem of Pappus is a result featured in many calculus sequences that provides an almost magical way of evaluating volumes of revolution in terms of areas and centroids. It is also a surprisingly ancient result, and its rediscovery by Paul Guldin in the 17th Century touched off a contentious mathematical exchange between Jesuit mathematicians and the mathematical disciples of Bonaventura Cavalieri. This talk will look at the history of the Pappus-Guldin Theorem, focusing in particular on a later proof by the Scottish mathematician James Gregory and describing its use in a history of mathematics course which emphasizes the history of calculus.

## Mulatu B. Lemma (lemmam@savstate.edu) Savannah State University

## The Fascinating Perfect Numbers

Mathematicians and non-mathematicians have been fascinated for centuries by the properties and patterns of numbers. They have noticed that some numbers are equal to the sum of all of their factors (not including the number itself). Such numbers are called perfect numbers. Thus a positive integer is called a perfect number if it is equal to the sum of its proper positive divisors. The search for perfect numbers began in ancient times. The four perfect numbers $6,28,496$, and 8128 seem to have been known from ancient times. The purpose of this paper is to investigate properties of perfect numbers and try to make the concept of perfect numbers easy to undergraduate mathematics students. We will give simple and easy alternative proof of the well-known Euclid's Theorem (Theorem I).

## Weiping Li (wli@walsh.edu) Walsh University

## Hecke Algebras and Support Varieties for Algebraic Groups

Support varieties associated to the modules over a restricted Lie algebra are subvarieties of its cohomological spectrum. The Lie algebra of a reductive algebraic group over a field of positive characteristic has a structure of a restricted Lie algebra and the modules of a reductive algebraic group are naturally modules for its restricted Lie algebra. There are many studies on the computations of support varieties for modules of algebraic groups. Here we use a result by Nakano, Parshall and Vella and results from Hecke algebras to determine explicitly the support varieties for some induced modules which are closures of some nilpotent orbits. We also find the representatives for these orbits.

## Richard M. Low (low@math.sjsu.edu) San Jose State University

## On the Integer-magic Spectra of Tessellation Graphs

Let $A$ be an abelian group with non-identity elements $A^{*}$. A graph is $A$-magic if there exists an edge-labeling using elements of $A^{*}$, which induces a constant vertex labeling of the graph. In this paper, we determine, for certain classes of triominoes, honeycombs and polyominoes, the values $k \geq 2$ for which the graphs are $Z_{k}$-magic.

## Behailu Mammo (matbzm@hofstra.edu) Hofstra University

## A Mean Value Theorem for a Number Field

Let $k$ be an algebraic number field and let $N\left(k, C_{\ell}, m\right)$ denote the number of abelian extensions $K$ of $k$ with $G(K / k) \cong C_{\ell}$, the cyclic group of prime order $\ell$, and the relative discriminant $D(K / k)$ of norm equal to $m$. In this talk, we derive an asymptotic formula for $\sum_{m \leq X} N\left(k, C_{\ell}, m\right)$.

## Vincent J. Matsko (vince.matsko@gmail.com) Illinois Mathematics and Science Academy

## Edge Nets of the Cube

When making a paper cube, it is convenient to design a net; that is, a connected set of squares which may be cut out and folded together to form a cube. When the squares are folded together, pairs of edges from the squares come together to form edges of the cube. If one edge from each pair is removed so that the edges form a connected graph, the remaining twelve edges are called an edge net of the cube. In other words, an edge net is a connected graph with twelve edges of the same length which may be folded to make a cube. A complete enumeration of the edge nets of the cube will be given. The enumeration involves an interesting interplay of algebra, geometry, and graph theory.

Alex Meadows (ammeadows@smcm.edu) St. Mary's College of Maryland

## Using Tornadoes to Prove Regularity for PDEs

In the theory of Partial Differential Equations, a big question is to decide when solutions to the PDEs must be continuous or differentiable or many times differentiable. We use a new idea called Tornado Solutions to show solutions of a certain class of PDEs must be continuous.

## John G Mersch (merschj@nsula.edu) Northwestern State University of Louisiana

## Six Pretabular Relevance Logics

A logic, $\mathcal{L}$, which has no finite characteristic algebra is said to be pretabular if all the proper extensions of $\mathcal{L}$ have a finite characteristic algebra. That is, a pretabular logic is a logic that lies on the boundary between those logics that have finite characteristic algebras and those that can only be characterized by infinite algebras. In 1972 Maksimova proved that there are exactly three pretabular extensions of the intuitionistic propositional calculus. In 1975 Maksimova went on to prove that there are exactly five pretabular extensions of the modal logic S4. In this paper we present six pretabular extensions of Anderson and Belnap's relevance logic R. At this time the exact number of pretabular extensions of $\mathbf{R}$ is unknown. We end the paper with a discussion of the problem of completely characterizing all the pretabular extensions of $\mathbf{R}$.

## Will Miles, Daniel R Plante, Matt Deyo-Svendsen (wmiles@stetson.edu) Stetson University

## Using a Genetic Algorithm to Improve Finite Element Solutions of Differential Equations via Mesh Rearrangement

This work investigates the use of a genetic algorithm to arrange the points within a computational mesh so that the resulting finite element solution is improved. The Galerkin FEM technique is applied to a second order differential equation using piecewise continuous quadratics as the approximating space. Error estimates are analyzed and formulated so that the method may be applied without knowledge of the true solution, with error estimates used as the fitness measure for the genetic algorithm. The GA iteratively improves the mesh until specified convergence criteria (or a maximum number of generations) are met. It is shown that the use of the GA to select the computational mesh allows one to use far fewer "well-placed" points to achieve the same level of accuracy as a much larger number of uniformly spaced points in the grid. Approximation errors are analyzed for problems with known solutions and conclusions are made.

Ellen Mir (emir@elon.edu) Elon University

## Leximorphic Spaces

A leximorphic space is a linearly ordered topological space (LOTS) with first and last points that is isomorphic to its Cartesian product given the lexicographic ordering. Examples of leximorphic spaces will be detailed,
and cardinality and consistency conditions characterizing leximorphic spaces will be described. Questions concerning the connectivity of leximorphic spaces will also be discussed.

## Jason Joseph Molitierno (molitiernoj@sacredheart.edu) Sacred Heart University

## Capstone Projects for Senior Mathematics Majors

Many colleges and universities are now requiring that their senior mathematics majors take a capstone or senior seminar course near the end of the major sequence. This course usually requires that the students do independent research. In this talk, I give ideas for senior capstone projects that I have used when I have taught the course. I will discuss topics that my students have researched such as chaos, alternate geometires, applications of statistical methods, random walks, and proofs of irrationality - just to name a few.

Jack Y. Narayan, Robert Schell (narayan@oswego.edu) Department of Mathematics SUNY at Oswego

## A Spreadsheet Learning Environment (SLE)

The authors have designed a course where over 100 students in a Finite Mathematics and Applied Calculus course are engaged in a "Spreadsheet Learning Environment (SLE)" in which they solve mathematical problems for decision making and write up their results in a clear and understandable manner. The spreadsheet, with its text box feature, allows students to do calculations and explain their steps using their laptop computers. In class, students are encouraged to work cooperatively in pairs or small groups as they learn how to: read a problem, propose a solution, choose a model, and code the information in a spreadsheet. The SLE is rich enough to enable students to use analytical, algebraic, and graphical techniques with little effort. Homework and special projects are created in the SLE and submitted electronically within a software management system. Students receive electronic as well as face-to-face feedback and are allowed more that one submission. Samples of student's work are readily available for assessment purposes during and after the semester. Preliminary results indicate that students with poor algebraic skills learn enough of the "spreadsheet algebra" to access mathematical techniques for decision making that were previously considered beyond their capabilities. In short, students who have been away from mathematics for an extended period, or who never mastered algebraic fluency, benefit greatly from this approach. Further work is needed to test whether learning with this approach is sustainable and could serve as a foundation for growth.

## Robert Booth and Hieu Nguyen (nguyen@rowan.edu) Rowan University

## Pascal's Square: Determinants, Bernoulli Polynomials, and the Arithmetical Triangle

In this talk we present a less well known but explicit formula for Bernoulli polynomials expressed as determinants of square matrices constructed from Pascal's triangle. We demonstrate how this formula extends to hypergeometric Bernoulli polynomials, a class of polynomials generated from the confluent hypergeometric series and containing the classical Bernoulli polynomials as a special case.

## Joanna Papakonstantinou (jpapa@rice.edu) Rice University

## The Historical Development of the Secant Method in 1-D

Many people believe that the Secant Method arose out of the use of a finite difference approximation of the derivative in Newton's Method. However, historical evidence reveals that the Secant Method predated Newton's Method by over 3000 years. It was used to solve problems that could be represented as linear equations and was most commonly referred to as the Rule of Double False Position. In this paper, I trace the development of the Secant Method in 1-D and discuss how it evolved through many centuries and civilizations.

## Marcus Pendergrass (marcus.pendergrass@gmail.com) Hampden-Sydney College

## Lying Oracles and Misguided Tourists

The Lying Oracle Game is a two-person competitive game in which the first player wagers on the truth or falsity of statements that are made by the second player. Typically, the game is played for a finite number of turns, and the second player's ability to lie is subject to some constraint. Here I consider a generalization of the Lying Oracle Game, called the Tourist Game. In the Tourist Game, the second player is leading the first
player through a strongly connected directed graph, and on each turn the first player wagers on which node they will visit next. Optimal strategies are identified, both for the finite-duration game and the infinite-duration game. I also consider the case in which the directed graph is not strongly connected, which leads to some interesting variants of the original Lying Oracle Game.

## Maijian Qian (mqian@fullerton.edu) CSU, Fullerton

## Financial Planning Using Spreadsheets: A Discrete Application of Calculus

Using spreadsheet applications can yield powerful financial and mathematical information. In this paper we discuss different models for bank accounts and their applications of these models to daily life. We also explore the relationship between continuous models based on the calculus and discrete models using spreadsheet applications.

## Marc Renault (msrenault@ship.edu) Shippensburg University

## Lost in Translation: A Reflection on the Ballot Problem and Andrés Original Method

Suppose that Alice and Bob are in an election. Alice receives $a$ votes, Bob receives $b$ votes, $a>b$, and Alice wins. The ballot problem asks, "how many ways can the $a+b$ votes be ordered so that Alice maintains a lead over Bob throughout the counting of the ballots?" Désire Andre is widely credited with creating the clever and elegant "reflection method" to solve the ballot problem. However, it turns out that Andf́never actually employed this method. We will consider Andre's original solution from 1887 and its relation to the reflection method.

## David Richeson (richesod@dickinson.edu) Dickinson College

## Symbolic dynamics and Irrational Rotations on the Circle

Let $f_{\alpha}: S^{1}=\mathbb{R} / \mathbb{Z} \rightarrow S^{1}$ be the rigid rotation given by $f_{\alpha}(x)=x+\alpha(\bmod 1)$ and let $J \subset S^{1}$ be an interval or finite collection of intervals. The point $0 \in S$ has an associated itinerary ( $a_{0}, a_{1}, a_{2}, \ldots$ ), where $a_{i}=1$ if $f_{\alpha}^{i}(0) \in I$ and $a_{i}=0$, otherwise. In this talk we discuss the uniqueness of these itineraries. In particular, given only an itinerary, is it possible to determine $\alpha$ and $J$ ? If so, how do we recover them?

## Jay Lawrence Schiffman (schiffman@rowan.edu) Rowan University

## Exploring Prime Distances With MATHEMATICA and The VOYAGE 200 CAS

The fascinating dichotomy with regards to the prime numbers in the sense that their infinitude on the one hand is offset by the ability to create prime gaps as large as one pleases in no small measure renders their appeal. An intriguing aspect associated with the primes is the distance between a positive integer and the nearest prime, called the prime distance function. The prime distance $p d(n)$ for a nonnegative integer $n$ is the absolute difference between n and the nearest prime. One may round up or down in case of ties. To cite an example, $p d(93)=4$; for the nearest prime to 93 is 97 if we round up or 89 if we round down. I choose to round up in such cases. Associated with the prime distance function are prime gaps of length $n$ which constitutes a run of $n-1$ consecutive composite integers between two successive primes. In the above example, one has a gap of 8 between the primes 89 and 97 and hence a run of 7 consecutive composites. In this paper, we will construct a scatter plot for the prime distance function and examine several associated integer sequences. Our culminating activity will include simple programs for The VOYAGE 200 to obtain the nearest prime and the previous prime. A program already exists in the manual to obtain the Next Prime. MATHEMATICA 5.2 in its Number Theory Add-On Package has commands for the Next Prime and the Previous Prime. Please join us to witness CAS technology furnishing new and exciting explorations related to the remarkable prime numbers.

## Morgan Sherman (morgan.sherman@csuci.edu) California State University, Channel Islands

## Dynamical Systems on the Riemann Sphere

We will describe some dynamical systems on the Riemann sphere and explain how they relate to a version of Einstein's equations.

## Thomas Q Sibley (tsibley@csbsju.edu) St. John's University <br> Sublimital Analysis

Limits of subsequences have played a supporting role in some important theorems and definitions of analysis. However, it appears that these "sublimits" haven't ever received star billing. This talk redresses this oversight by taking a closer look at subsequences and their sublimits. While sublimits may well be hidden in the original sequence, they aren't placed there to send subliminal (subconscious) messages. They are an enticement to mathematical exploration.

## Murray H. Siegel (mhsiegel@sc.rr.com) SC Governor’s School for Science \& Mathematics

## Writing in a Pre-Calculus: 'Why Does an Exponential Model Make Sense?"

In a typical pre-calculus class, students work with linear, polynomial, exponential, logarithmic, and piece-wise defined functions. A student is asked, in a project assignment, to obtain paired data on a topic of interest to him/her and examine the use of various types of algebraic functions to model that data. The student must assess each model and select the best overall model. A final part of the project report (worth $20 \%$ of the project grade) requires the student to describe why it makes sense that the chosen model truly describes the relationship between the two variables. For this reason, it is important that the student's data are drawn form an area of interest to the student. Two recent examples include a quadratic model to describe the relationship between hits and runs scored by Major League Baseball players and the exponential decay model describing the relationship between a player's height and the average number of digs per game for NCAA Division I volleyball players.

## John L Simons (j.1.simons@rug.nl) University of Groningen, Netherlands

## Recent Developments in Solving the Generalized Collatz Problem

Consider a sequence of integers given by $x_{n+1}=\frac{x_{n}}{2}$ if $n$ is even and by $x_{n+1}=\frac{3 x_{n}+1}{2}$ if $n$ is odd. The famous Collatz or $3 x+1$ conjecture states that for every $x_{n}>0$ finally the cycle $(1,2)$ occurs. Define an $m$-cycle as a hypothetical cycle with $m$ local minima. Steiner proved in 1978 that (1,2) is the only 1-cycle. Recently Simons and de Weger have proved that if an $m$-cycle exists, then $m>77$. Their approach can be generalized to $p x+q$ sequences. For each $p, q$ all $m$-cycles up to a given upper bound for $m$ can be found. There exist exotic Collatz sequences with $p>3$ with arbitrarily many cycles.

## Edwin Tecarro (tecarroe@uhd.edu) University of Houston-Downtown

## Computational Studies of a Model of Signaling Pathways in Embryonic Xenopus Iaevis

An existing mathematical model of biochemical signaling pathways in the embryonic development of Xenopus laevis is modified to incorporate other biological findings. The model consists of a system of coupled, nonlinear ordinary differential equations. Numerical computations include one-parameter and two-parameter bifurcation studies, which may be helpful in elucidating the interactions within the biochemical signaling network in embryonic Xenopus laevis.

## Vincent Jozef van Joolen (vanjoole@usna.edu) United States Navy Academy

## From Taylor to Cramer to Vandermonde to Schur: <br> Finite Difference Approximations of Derivatives Random Grids

As nano-engineering emerges as a viable technology, it will soon be possible to randomly scatter thousands, if not millions, of tiny sensors over a landscape. If the purpose of these sensors is to determine rate-of-change data, there must be a capability to approximate derivatives using data from random sensor locations. This paper proposes a powerful, but simple algorithm to generate a finite difference approximation equation for a derivative using a random, asymmetric, one-dimensional grid schemes. In a classic approach, Taylor series polynomials are used to obtain the weights used in the approximation. Creamer's rule provides a vehicle to solve the system. Patterns in the system invoke Vandermonde matrix properties and Schur polynomials to produce a closed form solution that is easily coded. The resulting equations are well suited for the scenario suggested and could rapidly adjust to an emergent grid resulting from the loss of one or more sensors.

## Monika Vo (monika.vo@saintleo.edu) Saint Leo University

## To MyMathlab or Not in College Algebra

I may be old fashioned, but I still like to see my students know how to sketch graphs of functions without the use of calculators. However, in my search for new ways to get my students to learn and appreciate functions and their graphs and for them to have immediate access to the help they need, I have, for the last two years, used in my College algebra class the Prentice Hall software MyMathlab. In this presentation, I would like to share with you my findings, successes and what I would not do again with my different approaches to the use of the software.

Tingxiu Wang, Gloria Liu, Joe Kotowski, and Bob Sompolski<br>(tingxiu@oakton.edu) Oakton Community College

## Mentoring Students for Success in STEM

Students face many obstacles for success in Science, Technology, Engineering, or Mathematics (STEM). Research indicates that students gradually lose their interests in studying a course and majoring in a field in STEM from high school to college. To increase the number of students studying and pursuing a degree in STEM, we provide mentoring for our STEM students. This presentation will discuss and analyze the obstacles that prevent students from success in STEM, and share the successful experience and difficulties of mentoring STEM students. This presentation is based upon work supported by the National Science Foundation under Grant No. DUE-0622329.

## Derek Webb (dwebb@bemidjistate.edu) Bemidji State University

## Enabling Student Success Across the Disciplines Through a Lab-Based Math Class

There is a growing need across America for a mathematical sciences class that is interesting to both high school seniors and first or second year college students. Research is demonstrating that juniors and seniors in high school are losing interest in mathematics classes as they progress and are doing poorly or not taking mathematics classes at all. A strong argument can also be given that the typical mathematics track that culminates in calculus is not appropriate or valuable to a majority of college students. We have developed a liberal arts mathematics class that is a combination of three disciplines: algebra, computer science, and statistics. The class presents algebra topics in the context of computing and statistics topics through the use of Microsoft Excel and is taught in a computer laboratory environment. The class is specifically designed to help students be successful in future mathematics and statistics based classes in their chosen discipline of study. An overview of research indicating the need for and leading to the development of the class and specific examples from the class will be presented.

## Amy Wheeler (amy.wheeler@uc.edu) University of Cincinnati

## Active Learning to Improve Student Preformance in Remedial Mathematics

The use of active learning has been proven effective in many areas of curriculum. The development of critical thinking skills and higher order thinking are goals that many developmental mathematics courses address. Through active learning, students have the opportunity to develop the much needed abilities. This presentation will cover activates that align with the active learning model. An emphasis will be placed on inquiry based learning. Material will be available. This material presented is appropriate for elementary through college algebra.

## Amy Wheeler and Bella Zamansky (amy.wheeler@uc.edu) University of Cincinnati

## Training Effective Tutors within Remedial Mathematics

Training tutor in a developmental mathematics program has challenges that traditional tutor training does not experience. This presentation will discuss the development of a training manual, session implementation, challenges faced and successes achieved within the University of Cincinnati's Center for Access and Transition. Materials will be available and questions can be addressed.

## Carol Ann Whyzmuzis (whyzmuzc@stjohns.edu) St. John's University

## How to Incorporate Mathematics into a General Biology Course

When students sign up for courses such as chemistry and physics, they are not surprised when mathematics plays an important role in the concepts and applications. In contrast, when students sign up for a general biology course, they do not expect to be exposed to any mathematical concepts. When asked to do simple calculations in biology laboratory, I have had students respond "I can't do math," or "This is why I didn't take chemistry or physics." In addition to teaching the metric system, mathematical concepts can be incorporated into lectures as well as laboratory exercises. For example, mathematics can be used to show order and structure in living organisms (Fibonacci numbers), for statistical analysis of data they collect, and for probability problems in genetics.

## Aklilu Zeleke, Robert Russell Molina (zeleke@msu.edu) Michigan State University

## Some Remarks on Convergence of Maximum Roots of a Fibonacci-type Polynomial Sequence

Consider a polynomial sequence defined by $G_{0}(x)=-1, G_{1}(x)=x-1$, and for $n \geq 2, G_{n}(x)=$ $x^{k} G_{n-1}(x)+G_{n-2}(x)$. Here $k$ is a fixed positive integer. Denote by $g_{n}$ the maximum root of $G_{n}$. It is known that for $k=1$ the sequence of the odd-indexed maximum roots converges to $3 / 2$ from below and that of the even-indexed roots converge to $3 / 2$ from above. We will show that for $k=2$ the odd-indexed and even-indexed roots behave in similar form with limit $\sqrt{2}$. Directions for future research will also be discussed.

Kathy Zhong (zhongk@udmercy.edu) University of Detroit Mercy

## An Experience for On-line Quizzes Using Blackboard $®$

Blackboard ${ }^{\circledR}$ is Education Queensland's flexible e-learning and online communities system for faculty and students. In this paper, we discuss some experience for on-line quizzes using Blackboard. We will present the reasons and formats of the quizzes. Sample quizzes are given in the appendix. Pros and cons of using on-line quizzes are discussed. Finally, a statistical analysis was performed on the grades of students taught in traditional class and that of the students in this hybrid format. No significance difference in grades was found.

# Getting Students to Discuss and to Write about Mathematics 

## Saturday, August 4, 1:00 pm-5:15 pm

Feryal Alayont (alayontf@gvsu.edu) Grand Valley State University

## Using an Inquiry-based Approach to Improve Student Proofs

In this talk I will discuss the implementation and results of an inquiry based approach in a proofs course. Before each class students were assigned problems to work outside of class. During the class period, students presented their solutions on the board and the whole class discussed the correctness and the completeness of the solutions. I will demonstrate, using examples of actual student work, how the inquiry based approach helped students to make sense of proof methods and to improve their proof writing skills. The approach also helped create a more collaborative and interactive classroom environment.

## Scott Beaver (beavers@wou.edu) Western Oregon University

Seeing the Forest Through the Trees: How to Stop Students From Memorizing Advanced Calculus Proofs
Memorizing proofs is a poor way to learn Advanced Calculus. For problems of more than minimal complexity, I ask that students write a (scored) strategy, containing no computational or algebraic details, and with explicit indication of where hypotheses are used, above the problem solution. This helps students to recognize
similarities and differences between problem solutions and hence gain a broader perspective on the material. With strategies already written out, students can study for exams without having to read every detail of every problem solution. Examples, suggested strategy parameters, and student testimonials will be given, as well as suggestions for application in other courses.

## Scott Beaver (beavers@wou.edu) Western Oregon University

## Assessing Student Presentations of New Material in Advanced Calculus

Students in Advanced Calculus at Western Oregon University are required to present a class-length lecture of new material once each term. They are assessed on the effectiveness of their introduction, logical clarity, ability to answer questions, blackboard technique, delivery, pace, and creativity, among other things. Different weights are assigned to each subscore, according to (my perception of) the relative importance of the evaluation criteria. I present the rubric I use to evaluate their presentations.

## Beth Burroughs, Sharon Brown (burroughs@humboldt.edu) Humboldt State University

## Poster Projects: Mathematics in Context

The purpose of this paper is to describe a general procedure for implementing group poster projects in lower and upper division mathematics courses. The authors have found that students who complete projects gain a deeper understanding of the mathematics they study, refine their ability to communicate mathematically, and engage in an activity that requires initiative and persistence. The paper gives examples of successful projects and includes grading rubrics.

## Kelly Cline (kcline@carroll.edu) Carroll College

## Using Classroom Voting to Promote Discussions

The classroom voting pedagogy can be a quite effective technique for teaching students to discuss mathematics. This method involves posing a multiple-choice question to the class, giving the students 2 or 3 minutes to discuss the question, and then having the students vote on the right answer with a set of hand-held "clickers." Their votes are instantly tabulated on a computer giving the instructor immediate feedback as to the state of the class. The vote is then used as a springboard for a Socratic discussion, as the instructor asks different students to explain their votes. At Carroll College, we have integrated several of these voting questions into almost every period of our calculus, multivariable calculus, linear algebra, and differential equations classes. Because this process requires every student to form an opinion about the question, it is very effective at motivating the students to discuss each question in small groups before each person makes up their own mind. The practice of doing these votes in every class period helps the students develop their skills in talking about mathematical concepts. The types of questions that promote discussions most effectively often deliberately provoke common misconceptions, or are questions that are designed so that many different options will be selected by substantial percentages of the class. We have found that the classroom voting statistics from previous classes are a very powerful tool that aids the instructor in selecting the questions that are most likely to promote fruitful discussions of the key mathematical concepts.

## Richard Bedient and Sally Cockburn (scockbur@hamilton.edu) Hamilton College

## Thinking, Speaking, Writing: The Hamilton Senior Seminar

The senior seminar at Hamilton College requires our concentrators ( 40 per year) to work collaboratively in small sections to produce joint documents in advanced areas of mathematics. The professor takes a back seat in the classroom as students take turns presenting examples, counterexamples and proofs; students discuss and refine their colleagues' work in class, and when a consensus emerges that the material is correct, a 'scribe' writes it up in TeX ® and adds it to the growing document. Before the end of the semester, the group edits their joint project for accuracy and consistency in style. Students are graded on the quality of the final product, both in terms of content and form, as well as the quality of their contributions to class as both presenters and critics. This format has been successfully adapted to covers such areas as finite group theory, topology, graph symmetries and advanced set theory. Comments from student evaluations include: "Amazing
discussions; learned proving and problem-solving skills." "I was inspired to challenge myself just so I could enjoy the class discussion more." "Although the instructor didn't actually teach during class, I felt that I learned more in this math class than any other.", "It was the most thought provoking class I've ever taken."

## Anna Davis (davisa@ohiodominican.edu) Ohio Dominican University

## Writing as an Effective Teaching and Assessment Tool

Recently great efforts have been made to promote writing across the curriculum. The primary motivating factor behind these efforts is improving the quality of student writing. But writing in a specific discipline can result in much more than improved grammar. Writing about mathematics gives students another tool for internalizing the concepts, and it gives the instructor a chance to evaluate student knowledge from a different perspective. This presentation will concentrate on two types of writing assignments. An "auxiliary" assignment expands a student's knowledge about the discipline and its contributors. An example of such an assignment may be a biographical essay or an interview with a mathematician. An "integrated" assignment is designed to help students internalize a specific concept. Speaking and writing assignments designed to facilitate the process of internalization will be discussed.

## Tevian Dray (tevian@math.oregonstate.edu) Oregon State University

## Non-Euclidean Geometry: A Writing Intensive Course

OSU requires a writing intensive course (WIC) in the major for all students. The most successful WIC in mathematics is a course on non-Euclidean geometry, originally intended primarily for future teachers, but now taken by most math majors. This course requires substantial written work, including both short and long assignments, some graded and others ungraded, and has recently begun to require peer review. Some of the methods and content developed for this course have also been successfully adapted for use in professional development courses for $\mathrm{K}-12$ math teachers. We report here on the successes and failures of 10 years of the evolution of this course.

## Sommer Gentry (gentry@usna.edu) United States Naval Academy

## Two-sentence Summaries in an Ungraded Lecture Series Course: Focus on the Main Idea

Our entering math majors take an ungraded lecture series course which meets one hour per week to hear from various math faculty. Still, I hope to: motivate students to become better writers, model the actions of an experienced mathematical writer in revising prose, and ensure that the presentation content has been understood. Motivation: class begins with a discussion of the best and the not-best of the previous week's summaries, so that students compete for inclusion in the best category. Modeling: I revise the students' writing on the overhead, moving toward less wordy and more precise statements. Content: The students and I collaboratively correct the authors who have missed the main idea of the previous week's talk. After modeling sentence revision for a few weeks, I ask students to peer-revise. One page essays summarizing each lecture often failed to present a clear main idea and were too long to rewrite interactively with the class. Instead, we write two-sentence summaries in the following format: the first sentence defines the mathematical object, and the second sentence tells who uses that mathematics to do what and why. Two-sentence summaries are my extension of the one-sentence summary format from Classroom Assessment Techniques: A handbook for college teachers (Angelo and Cross). An example: Algebraic Geometry is the study of shapes that are defined by polynomial equations. By understanding the properties of polynomials, Bezout created a theorem that relates the product of the degree of two polynomials with the possible number of intersections between two shapes.

## David Hartz (dhartz@csbsju.edu) College of St Benedict

## Writing About Applications in Linear Algebra

One of the problems teaching a linear algebra class is that the students are always complaining about how they don't see any use for the material. To address this problem I assign semester projects to pairs of students where each pair will examine a different application of linear algebra. The students will study this application, create some examples of their own illustrating this use of their topic, write a 4 to 7 page paper on this topic
and do a classroom presentation of their work. In this talk I will look at these assigned applications and my expectations for the project, in addition to my assessment of the value of these projects in linear algebra.

Jeff Johannes (johannes@member.ams.org) SUNY Geneseo
Where Are We From?-An Entire Class Project
In our secondary mathematics capstone course, our students completed a class project to find the population center of the permanent residences of the college. In this talk, I will share the assignment, the results, and the impact on classroom community that the project had.

Sarah L Mabrouk (Smabrouk@frc.mass.edu) Framingham State College
Let's Discuss it—Online!
Students frequent chat rooms, post on discussion boards, and communicate (sometimes even during class, Oh My!) using AIM and email. These online pleasures of the students are great tools for both traditional and online mathematics courses, lending themselves nicely to analysis and exploration for course exercises as well as for projects. Each mode enables the instructor to view the transcript of the interaction of the students as well as to monitor their progress and to contribute ideas. In this presentation, I will discuss some of the ways in which I have used these modes of communication and interaction as learning tools in my Introductory Statistics, both traditional and online, and Calculus III courses.

## Tracey McGrail (tracey.mcgrail@marist.edu) Marist College

## My Talk is Better Than Yours

Increased emphasis on student-centered research on the undergraduate level has led to more venues for undergraduates to present the results of their research projects. However, the average student often lacks the confidence, motivation, and skills to properly prepare a research talk. The speaker will describe a capstone course designed to help the student bridge this gap. It includes a gradation of assignments that build mathematical confidence and maturity as well as hone presentation skills. Students actively engage in many aspects of the course including constructive feedback and criticism. This leads to healthy competition with polished presentations as the end result.

## Margaret Morrow (mmmlmorrow@aol.com) SUNY Plattsburgh

## Discussing and Writing in An Online Course

An online course has more potential than ever for students to simply not engage with the mathematics. In this presentation I will describe some strategies I have devised to get around this in our online offering of Calculus for Life, Management and Social Sciences. Specifically I will discuss graded team discussions and applications papers I have assigned in this course. I will present some of the assignments, selected student work on these, and student comments about them. The assignments could easily be adapted for a conventional classroom setting.

## Darlene Olsen (dolsen1@norwich.edu) Norwich University

## Preparing Students for a Senior Seminar Research Project

Mathematics majors at Norwich University have a capstone requirement met by students working on an undergraduate research project during their senior year with a faculty member. In order to ease the department's majors into undergraduate research, the department began a new course called "Communications in Mathematics" in Spring 2007. The intent of the course is to illustrate "the organization of the mathematical literature, the efficient search of the literature and a formal introduction to writing mathematics". We will discuss the design of the course and the lessons learned from teaching the course.

## Thomas Read (read@cc.wwu.edu) Western Washington University

Oral and Written Communication of Mathematics in an Introduction to Analysis Course
For many years I have taught our junior level introduction to analysis course by a version of the "Moore method." There is no text-students work independently on a list of problems that I distribute and present
their solutions (mostly proofs) to the class for class discussion and acceptance (or not). These presentations count for most of the course grade. For the past several years I have also required students to hand in written versions of solutions to all problems on a weekly basis. In general the method has worked very well for this often troublesome course. I will discuss some of the issues and benefits.

## Stephanie Salomone (salomone@up.edu) University of Portland

## How Do You Know What You Know? (And Other Big Questions)

Students expect to be taught content in their math courses. What would happen if, in our quest to train mathematical thinkers, we also expected them to ask some big questions of themselves and of their classmates. Is this proof complete, correct, and rigorous? Is math a science? What is the role of discovery in mathematics? In this talk, I will discuss student reactions to these questions at the beginning and at the end of a course in real analysis, and demonstrate the positive impact that asking such questions has on classroom environment and student learning.

## Sarah Ann Stewart (stewarts@mail.belmont.edu) Belmont University

## Reflecting, Writing, and Presenting Mathematics in a Senior Capstone Course

Senior capstone courses vary from school to school. Some mainly focus on undergraduate research while others delve into a topic (such as number theory) that is not normally covered in an undergraduate curriculum. At Belmont University, we have a one-hour senior seminar course in which the students discuss and write reflections on what they have learned in their math classes and then do an oral presentation on a topic that was previously unknown to them. I will discuss the reflection pieces submitted by the students and how the students measure their learning against our seven departmental goals. These reflection pieces make up a part of the senior portfolio for each student, and I will discuss how our department uses these portfolios for assessment. The students are able to use the skills and knowledge gleaned from previous math classes to research a new topic and then present their findings in an oral presentation. I will discuss presentation topics and the results of these oral presentations.

## Graph Theory and Applications

## Saturday, August 4, 8:30 am-10:30 am <br> Sunday, August 5, 1:00 PM-3:00 PM

Ji Young Choi (jychoi@ship.edu) Shippensburg University of PA

## Minimum $k$-total Weights of Graphs

The sum of weights for each path of length $k$ in a graph with edges of weight 1 or -1 is called the $k$-total weight of the graph. In this talk, the minimum of the $k$-total weights of paths, cycles, and a simple type of trees are presented.

## Suzanne Ingrid Doree (doree@augsburg.edu) Augsburg College, Minneapolis

## Coloring the Generalized Tower of Hanoi Graphs

The state graphs of the multi-peg version of the famous Towers of Hanoi puzzle are complex, recursively constructed (self-similar) graphs that are highly non-planar. In this talk I will introduce these beautiful graphs and show a natural minimal vertex coloring. (The result on the chromatic number is new.) There are no prerequisites to this talk, and I particularly invite undergraduates to attend.

Angela Harris, Michael Ferrara (michael.ferrara@cudenver.edu) University of Colorado at Denver

## Hamiltonian Cycles Avoiding Sets of Edges in a Graph

Let $H$ be a subgraph of a graph $G$. If there is a hamiltonian cycle $C$ in $G$ such that $E(C) \cap E(H)=\emptyset$ then we will call $C$ an $H$-avoiding hamiltonian cycle and say that $G$ is $H$-avoiding hamiltonian. In this talk, we will give minimum degree and degree-sum conditions, based on $H$, that imply a graph is $H$-avoiding
hamiltonian. In particular, we will consider the cases where $H$ is a family of edge-disjoint hamiltonian cycles or a family of edge-disjoint perfect matchings. In either case, an $H$-avoiding hamiltonian cycle serves to extend these families.

## Sommer Gentry, T. S. Michael (gentry@usna.edu) United States Naval Academy

## Optimal Weighted and Stable Matchings on Graphs for Increasing Live Donor Kidney Transplantation

Kidney paired donation matches one patient and his incompatible donor with another patient and donor in the same situation for an organ exchange. Let patient-donor pairs be the vertices of a graph, with an edge between two vertices if a paired donation is possible. Graph theory finds an essential medical application here. Stable matchings may not exist in these non-bipartite graphs, but they are perhaps unnecessary. The simplest optimization problem finds the matching of maximum cardinality in G. However, some matches are geographically undesirable. Also, the expected survival of a transplanted kidney depends on immunologic compatibility between the donor and the recipient. Thus G becomes an edge-weighted graph on which a maximum weighted matching should be found. Regrettably, these weighted matchings might not have the maximum number of edges, but with some restrictions on the edge weights it is possible to maximize edgeweight among matchings of maximum cardinality. Setting edge weights within a range of $(b, B)$ where $\frac{B}{B-b}>\left\lfloor\frac{n}{2}\right\rfloor$ guarantees that a maximum edge weight matching will also have maximum cardinality. Similarly, if there is a favored subgroup of patients, say, those whose transplants are medically urgent, a different edge weighting scheme can guarantee that the maximum number of subgroup patients are matched, without penalizing the cardinality of the matching and without ignoring the edge properties. Finally, kidney recipients will have slight preferences among the available kidney donors. This provides incentives for kidney recipients to misrepresent their needs, possibly compromising optimal matchings.

## Stephen G. Hartke (hartke@math.uiuc.edu) University of Illinois

## Graph Classes Characterized both by Forbidden Subgraphs and Degree Sequences

Given a set $\mathcal{F}$ of graphs, a graph $G$ is $\mathcal{F}$-free if $G$ does not contain any member of $\mathcal{F}$ as an induced subgraph. We say that $\mathcal{F}$ is a degree-sequence-forcing set if, for each graph $G$ in the class $\mathcal{C}$ of $\mathcal{F}$-free graphs, every realization of the degree sequence of $G$ is also in $\mathcal{C}$. This definition is motivated by the well-studied class of split graphs. We prove that for any $k$ there are finitely many minimal degree-sequence-forcing sets with cardinality $k$. We also give a complete characterization of the degree-sequence-forcing sets $\mathcal{F}$ when $\mathcal{F}$ has cardinality at most two, and partial results when $\mathcal{F}$ has cardinality three.

## Steve Horton, Jean Blair, Ralucca Gera (steve.horton@usma.edu) United States Military Academy Dynamic Domination in Graphs

In this paper we introduce and examine the topic of dynamic domination in graphs. A dynamic dominating set is a dominating set $S \subseteq V(G)$ such that for every $v \in S$, either $S-\{v\}$ is a dominating set, or there exists a vertex $u \in(V(G)-S) \cap N(v)$ such that $(S-\{v\}) \cup\{u\}$ is a dominating set. We present computational complexity results and bounds on the size of dynamic dominating sets in arbitrary graphs. We also give a polynomial time algorithm to find minimum dynamic dominating sets for trees.

Doug Bauer, Nathan Kahl, Louis Hakimi, and Edward Schmeichel (kahlnath@shu.edu) Seton Hall University

## Strongest Monotone Lower Bound for the Independence Number of a Graph

We present a lower bound for the independence number of a graph in terms of its vertex degrees, and prove it is optimal among a large and natural class of such bounds.

## Lew Ludwig (ludwigl@denison.edu) Denison University

Linking in Straight-edge Embeddings of Complete Graphs
In 1983, Conway and Gordon and Sachs proved that the complete graph on six vertices, $K_{6}$ is intrinsically linked (every embedding of the graph contains a non-trivial link of two or more components). This result has spawned a significant amount of work, including the complete classification of minor minimal examples for intrinsically linked graphs by Robertson, Seymour, and Thomas. After this classification, work has turned
to finding graphs in which every embedding has a more complex structure. One particular type of complex structure is the complete graphs composed of straight edges. Although every embedding of $K_{6}$ has ten disjoint triangle pairs that may be linked, in 2004 Hughes and Caragiu independently proved that every straight-edge embedding of $K_{6}$ has only one or three pairs of linked components. We expand this work by classifying and enumerating all 2-component links in the family of graphs generated by straight-edge embeddings of $K_{7}$ that form convex polyhedra with seven vertices. Beyond the interesting connection between graph theory and knot theory, this work may be of interest to molecular chemists who are trying to synthesize topologically complex molecules.

John P. Georges, David Whittlesey Mauro, Yan Wang (david.mauro@trincoll.edu) Trinity College On Graphs with Optimal Non-Surjective L(2,1) Labelings
For positive integers $j \geq k$, an $L(j, k)$-labeling of graph $G$ is an assignment $L$ of nonnegative integers to the vertices of $G$ such that the smallest assigned integer is 0 and $|L(x)-L(y)| \geq k$ (resp. $j$ ) if $d(x, y)=2$ (resp. 1). An optimal $L(j, k)$-labeling of $G$ is an $L(j, k)$-labeling $\lambda$ with minimum span, and a hole of $\lambda$ is an integer between 0 and $\max \{\lambda(v)\}$ that is assigned to no vertex. Recent work has involved the nature of graphs for which every optimal $L(2,1)$-labeling has alternating holes $1,3,5,7, \ldots, 2 r-1$ for some $r$. We review some of these results and present new classes of graphs with this property. Generalizing this work, we also consider graphs for which every optimal $L(j, k)$-labeling has image $0, j, 2 j, \ldots, j r$ for some $r$.

Robert Russell Molina, Myles McNally, and Ken Smith (molina@alma.edu) Alma College
Characterization of Randomly $P_{k}$-Decomposable Graphs, $2 \leq k \leq 11$
A graph G is H -decomposable if it can be expressed as an edge disjoint union of H -subgraphs. We say that G is randomly H -decomposable if every collection of edge disjoint H -subgraphs of G can be extended to an H -decomposition of G. In this paper we characterize randomly $P_{k}$-decomposable graphs for $2 \leq k \leq 11$. We also prove several structural results related to these graphs. Finally, suggestions for further research in this area are given.

## Wasin So (so@math.sjsu.edu) San Jose State University <br> Graph Energy Change Due to Edge Deletion

The energy of a graph is defined as the sum of the absolute values of eigenvalues of its adjacency matrix. In this talk, we discuss how the graph energy changes when an edge is deleted. Examples show that all cases are possible: increased, decreased, unchanged. Our goal is to find a graph theoretical characterization for each case.

Kathryn Weld and Richard Goldstone (kathryn.weld@manhattan.edu) Manhattan College

## A Note on Graphically Abelian Groups

A group $G$ is said to be graphically abelian if the function $g \mapsto g^{-1}$ induces an isomorophism of every Cayley graph of $G$. We give some equivalent characterizations of graphically abelian groups and show that a nonabelian group $G$ is graphically abelian if and only if $G=Z \times Q$, where $Z$ is an elementary abelian 2-group and $Q$ is a quaternion group.

## Innovative Ideas for Teaching Concepts in an Introductory Statistics Course

Friday, August 3, 3:15 pm-6:15 pm
Sunday, August 5, 1:00 Pm-3:00 pm
Christopher E. Barat (f-barat@mail.vjc.edu) Villa Julie College
Manual Labor: Developing a Laboratory Manual for an Introductory Statistics Course
Many introductory statistics courses now include a dedicated laboratory session, involving the use of one or more software packages. How should such a course component be structured? At Villa Julie College,
we have attempted to formalize the course's lab component (and create a consistent template for outcomes assessment)by creating a formal lab manual for our Statistics and Probability course. The perils, successes, and pitfalls we have encountered along the way will be discussed in this presentation.

## Kenneth Brown, Cheryl P. Gregory (brownkm@smccd.edu) College of San Mateo

## Feasible Inferential Statistics Projects for Introductory Statistics

Authentic projects on which students collaborate, apply their statistical and critical thinking skills, and employ statistical software to produce a finished written product are an essential part of an Introductory Statistics course and in line with the GAISE Project recommendations. This presentation will describe a project designed to give students the opportunity to clearly communicate statistical analyses which use descriptive and inferential techniques. Such projects using inferential techniques are difficult to construct because of the lack of availability of good random samples. This project uses a sufficiently large real estate data file rich in quantitative data (but not so rich in categorical data) from which random samples of size 300 are provided to each student team. The description of the project will include the project assignment, the scaffolding of preparatory and interim assignments that support student success, a grading rubric, student reflection on the project, extracts of student papers, and instructor reflection on "lessons learned" in the classroom. The instructors are "Writing Across the Curriculum" faculty who believe in student writing projects and, in collaboration with writing faculty, are honing their pedagogical skills in the area of integrative learning. The statistical package used in the project is Fathom.

## Connie Campbell (campbcm@millsaps.edu) Millsaps College

## Using Newspaper, Web, and Journal Articles in an Introductory Statistics Class

One of the objectives of an elementary statistics class is to help the students learn to think critically about claims which invoke data. Particularly in a society where data is often used to market products and ideas, responsible citizens simply must know to ask critical questions and be equipped to evaluate evidence.

Over the years we have tried to encourage classroom discussions by invoking current newspaper, journal, and web articles pertaining to concepts covered in the classroom. For this presentation, we discuss some of those materials that have piqued student interest and consequently have generated good classroom discussions as to how we can be better citizens of data as well as more informed consumers.

## Kumer Pial Das (kumer.das@lamar.edu) Lamar University

## Misconceptions About Statistics and Probability

Professional in the field of industry, business, medical science, and government recommend that their employees be skilled in the areas of probability and statistics. An introductory statistics course is popular than ever before. But it has been found that the students of this course often fail to make intra-disciplinary, as well as interdisciplinary connections. Another problem that leads to very serious learning difficulties in this course is the set of "misconceptions" student may have from previous inadequate teaching, informal thinking, or poor remembrance. It's not only important to deconstruct those misconceptions but also teachers must help students reconstruct correct conceptions. In this study these two major concerns and their remedies have been discussed.

## Ayesha Delpish (adelpish@elon.edu) Elon University

## Is this an English Course? A Case Study Approach to Statistics for Non-Majors

Answer-no; neither is this a typical mathematics course. While case studies have been used extensively as a teaching tool in fields such as medicine and related professional education fields for many years, this approach is not as widely used to teach undergraduate courses in the mathematical sciences. Instead of "another mathematics course" where students perform rote calculations or are lectured to, they are exposed to a series of real-world case studies that require them to work in groups and apply course content to solve problems. In this setting, they not only develop critical thinking skills, but also acquire useful skills in collaboration, communication, and real-world problem-solving. This forum will serve as a platform for the
exchange of methods, ideas, successes and difficulties encountered while using case studies in a statistics course for non-majors.

Penelope Dunham (pdunham@muhlenberg.edu) Muhlenberg College

## Developing Statistical Concepts: Simulating Type I and Type II Errors

How do we motivate the concept of error in hypothesis testing? How can we help statistics students understand the meaning of Type I and Type II errors, the likelihood of each error for a given test, and the connection between that likelihood and computing p -values or selecting decision criteria in classical tests? Often the probabilities $\alpha$ and $\beta$ are simply mysterious Greek symbols whose definitions aren't meaningful because students haven't considered how errors can occur during testing or how they might be controlled. The challenge of teaching about error is now easier because of ready access to probability simulations that take advantage of improved random number features on graphing calculators. In this talk, I shall demonstrate how I develop the concept of error in an applied statistics course via a calculator exploration that uses repeated simulations of a test with different decision criteria and both true and false null hypotheses. I'll provide a worksheet that helps students discover the types of errors, assess the probability associated with each, and improve their understanding of this critical feature of hypothesis testing.

## Florence Gordon and Sheldon Gordon (flogo@optonline.net) NYIT (retired)

## Sampling + Simulation = Statistical Understanding: Dynamic Graphical Simulations in Excel

This presentation will illustrate the use of interactive graphical simulations in Excel for investigating a wide variety of random processes and sampling distributions that arise in introductory statistics courses including simulations of: $\rightarrow$ various probabilistic processes such as coin flipping and dice rolling; $\rightarrow$ the normal, binomial and other distributions; $\rightarrow$ the Central Limit Theorem; $\rightarrow$ confidence intervals and hypothesis testing; $\rightarrow$ regression and correlation analysis. The talk will emphasize a variety of ways that such graphical simulations in Excel can be used effectively to promote student understanding of two of the key themes in statistics - randomness and variability between samples and the long-term patterns that arise as sample size increases. The speakers will describe their use, particularly as classroom demonstrations, to increase student understanding of otherwise difficult concepts and methods, to provide tools by which the students can personally discover the underlying statistical concepts, and to "justify" or validate the statistical theory.

## Erin M. Hodgess (hodgesse@uhd.edu) University of Houston - Downtown

## Using Captivate to Produce Tutorials for Statistics Packages

In many undergraduate business courses, students are expected to be able to work with statistics packages. But course time is often limited in terms of computer instruction. Using the Captivate package, I have constructed Flash movies for students to watch at their leisure. The audio portion of the movies is my description of the computer actions, while the video shows the actual demonstration. Students can then replicate the activities until they are comfortable. These movies remove some of student reluctance toward statistics packages.

## Patricia B Humphrey (phumphre@georgiasouthern.edu) Georgia Southern University

## Some More Lab Experiences in Introductory Statistics

For several years, I have used "lab" experiences in teaching Introduction to Statistics I, in which students collect or generate the data themselves. At Georgia Southern, we also have Introduction to Statistics II, which is taken by a few majors such as Information Technology, Political Science, and International Studies. We have traditionally focused our "lab" experiences in this course to exercises using some computer package, such as Minitab or SPSS. This past Spring semester, I taught the second course for the first time in several years. In an attempt to introduce more active data collection and analysis, I devised four active lab experiences in the spirit of what I do in the first course. We guessed jelly beans in jars for linear regression (and inference), measured the strength of facial tissues (two-sample $t$-test), measured absorbency of paper towels (one-way analysis of variance), and flew model airplanes (two-way analysis of variance). This presentation will discuss the projects, how they were received by the students, and the successes and failures with my approach.

Inasmuch as these topics are generally taught in AP Statistics, the presentation will be suitable for instructors at all levels.

## Magdalena Luca (magdalena.luca@mcphs.edu) Massachusetts College of Pharmacy and Health Sciences

## Our Students: Calculators, Thinkers or Both?

The topic of my presentation will address the evolution of teaching an Introductory Statistics course to students enrolled in pharmacy and health sciences degree programs. I will show how, since I have begun teaching this course in the Spring of 2004, my goal has been to advance my students' understanding of statistical concepts, in particular concepts needed to interpret and validate statistical analyses in medical research papers. To this end, my presentation will discuss ideas and methods I have explored in the classroom with various degrees of success, such as: raising awareness about misleading statistics used in the mass-media, different approaches to using hand-held calculators, incorporating data and examples from the real world, finding a balance between teaching students the basics - by requiring them to do calculations and to memorize theory - and encouraging them to engage in analytical thinking so that they could gain a deeper understanding of statistics and its applications.

## Sarah L. Mabrouk (Smabrouk@frc.mass.edu) Framingham State College

## Interactive Tools for Exploring Statistics

Since 2002, I have developed a collection of interactive tools using MS Excel that enable the user to explore concepts in courses such as College Algebra, Precalculus, Calculus, and Introductory Statistics. These tools allow user-controlled active exploration and enable the user to examine concepts symbolically, numerically, and graphically while providing opportunities for making connections among these representations as well as for concept discussions. Some of the currently available interactive tools for Introductory Statistics accessible on my web site, www.framingham.edu/faculty/smabrouk/Interactive/, are useful for exploring distribution types, the mean, the median, outliers, linear regression, the (Standard) Normal distribution, and confidence intervals. In this presentation, I will discuss how I use these tools in my Introductory Statistics classes, both traditional and online, student reaction to these tools, and computer security settings that allow the user to enable the macros used in some of these tools.

## Teri Jo Murphy (tjmurphy@ou.edu) University of Oklahoma

## Clicker Questions for Conceptual Understanding

Classroom response system (CRS) technology can facilitate active learning strategies, and therefore increase conceptual understanding, by enabling immediate feedback to students and instructors. The technology includes a handset (a.k.a., a clicker) that allows a student to respond, anonymously, to a multiple choice question; software records the student's response. The software offers a bar graph display of the response distribution, which the instructor can use to determine subsequent instructional actions. One challenge of using CRSs is writing good multiple choice questions. As part of two-year grant from the National Science Foundation (award DUE-0535894), we are producing a starter-set of multiple choice questions for use with CRSs in introductory statistics courses. Topics covered by our project include: descriptive statistics, probability, probability distributions, sampling, confidence intervals, hypothesis testing, and measurement. We have piloted items in an algebra-based statistics course (housed in Psychology) and two calculus-based statistics courses (housed in Mathematics, and Meteorology). We have discussed issues such as: the balance between questions that involve computation vs. questions that involve no computation; questions that have one clear answer to indicate mastery vs. questions that are more ambiguous to generate discussion; the differences that arise from the timing of the question (e.g., at the beginning of a topic vs. at the end of the topic; at the very end of one class period vs. at the very beginning of the next one) This presentation will offer examples of our items and raise related issues for discussion.

## Darlene Olsen (dolsen1@norwich.edu) Norwich University

## Stimulating Statistics: Engaging Students in an Introductory Statistics Course

There is a negative correlation between the number of years I have taught and the average number of students sleeping in my Elementary Statistics course. Perhaps it can be explained by activity-based learning using
real-life data. We will discuss some of the classic data sets along with some pop culture data sets that are relevant to students. We will also look at some classroom activities using Minitab and the TI-83 calculator.

Murray H. Siegel (mhsiegel@sc.rr.com) SC Governor’s School for Science \& Mathematics

## Using Simulations to Discover the Truth About Sampling Distributions

Students in introductory statistics courses are taught the importance of describing a distribution using center, spread and shape. When it comes to learning about sampling distributions it has been found that students gain a better understanding if they actually discover the "truths" rather than have them absorbed during a lecture. A simulation activity is used where students each draw random samples from a population of the digits zero through nine. Each student is assigned a specific sample size. The students then report the mean and standard deviation for their particular sample size distribution. The class discovers that the mean of the sample means is the population mean, the standard error of a distribution is the population standard deviation divided by the square root of the sample size and that as sample size increases, the shape of the sampling distribution approaches normality.

## Pat Touhey College Misericordia

(ptouhey@misericordia.edu)

## Basic Statistics via Elementary Geometry

Basic algebraic formulas for some standard descriptive statistics are often beyond the comprehension of beginning students. We use some simple geometric algebra to help clarify their understanding of some elementary concepts, e.g., variance, standard deviation. This then makes a proof of Chebyshev's Theorem almost transparently obvious.

## Mathematics and the Arts

Saturday, August 4, 1:00 Pm-3:00 Pm

Sunday, August 5, 1:00 PM-3:00 PM
Pau Atela (patela@email.smith.edu) Smith College

## A New Class of Fibonacci Tilings

We present a beautiful family of planar tilings arising from our research in Phyllotaxis. These tilings consist of equilateral triangles and squares with sides of same length. They propagate to infinity from a center forming trapezoids and larger squares, all with Fibonacci dimensions.

## Gwen Laura Fisher and Blake Mellor (glfisher@calpoly.edu) California Polytechnic State University

## Three-dimensional Finite Point Groups and the Symmetry of Beaded Beads

Beaded beads are clusters of beads woven together (usually around one or more large holes). Their groups of symmetries are classified by the three-dimensional finite point groups, i.e. the finite subgroups of the orthogonal group of degree three, $O(3)$. The question we answer is whether every finite subgroup of $O(3)$ can be realized as the group of symmetries of a beaded bead. We show that this is possible, and we describe general weaving techniques we used to accomplish this feat, as well as examples of a beaded bead realizing each finite subgroup of $O(3)$ or, in the case of the seven infinite classes of finite subgroups, at least one representative beaded bead for each class.

Martin E. Flashman (flashman@humboldt.edu) Humboldt State University

## Mathematics, Music, and the Guitar (Preliminary Report)

A presentation on the mathematical problems of tonality - scales and proportionality in theory and in practice as applied to the guitar. Why are there twelve tones in conventional western music? Why do individual key
scales sound differently? Using technology to create scales. How does the construction of a guitar effect its tonality for scales? What is the connection between the frets on a guitar and logarithms?

Brian Hollenbeck (bhollenb@emporia.edu) Emporia State University

## 4-Square Challenge

The simplest way to separate the unit square into four equal areas is to partition it into four smaller squares. What other "interesting" ways can this be done? This question was posed to a group of students ranging from freshmen to seniors. With the help of Maple, PhotoStudio, and a color printer, a collage of mathematicallyinspired art was the result. This activity is appropriate for students of any mathematical level, as it allows for a variety of approaches. Techniques can range from using symmetry to integration to Green's Theorem. Curves can be generated using polar, implicit, or parametric equations. Ideas such as pursuit curves, infinite series, and rotation of axes can inspire solutions. The activity can be extended to ask existence and uniqueness questions and make conjectures. It gives each student the opportunity to be creative in his or her solution.

## Carla Martin (carlam@math.jmu.edu) James Madison University

## Linear Recurrence Relations in Music

As a mathematician and violinist, I have always been intrigued by patterns in melody lines. No one can dispute the themes that various composers such as Beethoven and Brahms place throughout their symphonies. In mathematics, linear recurrence relations provide a means of generating sequences based on previous elements of the sequence. Inspired by a desire to quantify certain melodies, this talk will include a discussion of linear recurrence relations in music. In particular, whether or not we can find exact or approximate linear recurrence relations in classical works, independent of the key signature of a piece. We will also use such linear recurrence relations to generate melodic sequences and see how closely they resemble the original work. This also provides a way to introduce linear recurrence relations to students by listening to the relationships. Analyses are performed in Matlab using standard Musical Instrument Digital Interface (MIDI) file format. We conclude by letting the audience "compose" music based on a recurrence relation of their choice!

## Blake Mellor (bmellor@lmu.edu) Loyola Marymount University

## How Does a Course in the Mathematics of Symmetry Affect Students in the Liberal Arts?

Most universities require every student to take a course in mathematics to graduate. There is substantial debate in the mathematics community about how to best serve these students - is it more important to train students in basic mathematical skills, to show them how mathematics relates to other disciplines or to everyday life, or to give them an appreciation of mathematics as its own discipline and way of thinking? This project studied a non-traditional course on the Mathematics of Symmetry, taught at Loyola Marymount University in Spring 2006, that emphasized mathematical thinking and connections with the visual arts. We collected both qualitative and quantitative data in the form of anonymous surveys on students' attitudes towards mathematics as well as samples of students' work in the class. We try to answer several questions: How did students' attitudes towards mathematics change? How did their conception of what mathematics is, and its role in a liberal arts curriculum change? Did they demonstrate an ability to think mathematically? The results will be compared to data collected for a fairly standard quantitative reasoning course taught to the same audience at LMU. We will discuss implications for developing and teaching general education mathematics courses.

## Annalisa Crannell (annalisa.crannell@fandm.edu) Franklin \& Marshall College

## Reflections on Spheres

We describe some simple, geometric methods that can recover information about the original objects whose images appear as reflections in spheres. We apply these general findings to two specific etchings by Escher: "Hand with Reflecting Globe" and "Three spheres II". We will provide modest justifications to support these claims. In general, the reflected images of some lines have one vanishing point, but the images of other lines have two. It is the distance between the artist and the sphere that determines which lines are which. Escher was 15.5 inches away from the sphere in "Hand with Reflecting Globe". In "Three spheres II", Escher drew his head (but not his body) about $25 \%$ too large.

## Karl Schaffer (karl_schaffer@yahoo.com) De Anza College, Dr. Schaffer and Mr. Stern Dance Ensemble

## Making Mathematics Dance

The presenter co-directs a dance company which has included mathematically-inspired dance concerts in its repertory since 1990. Mathematical and dance areas that we will glimpse through slides, short video clips, and brief demonstrations include the use of polyhedra and giant tangrams in creating dances. We use linked sections of PVC pipe, loops of rope, or limbs to create metamorphising polyhedral structures onstage, and also utilize variations on dissection puzzles such as tangrams to tell stories or provide versatile dance props. Often the choreographic process leads to mathematical investigations as well, and we will discuss several mathematical questions that have arisen in this way. For example, we will examine several sequences of transformations which produce a variety of polyhedra. We will also discuss more general linkages between the science of dance and the art of mathematics.

## Mary D. Shepherd (msheprd@nwmissouri.edu) Northwest Missouri State University

## Symmetry Groups, Subgroups and Cosets in Counted Cross Stitch

Symmetry groups on the plane are concrete examples combining algebraic concepts with geometry and visualization. The symmetry group D4 (symmetries of a square) is easily realizable in counted cross stitch and with some identification, it is possible to "see" all the possible subgroups of D4. In addition, for each subgroup, colors can be used to indicate the sets of left and right cosets. A design using these ideas will be shown. In addition, we will explore similar ideas for the infinite symmetry groups represented in friezes and wallpapers.

## Scott Allen Taylor (scotchtaylor@hotmail.com) UCSB

## Reading Flatland

Edwin Abbott's novel Flatland does more than wrap a biting social satire in the husk of a higher dimensional handbook. It also reflects several of the most captivating debates of Victorian intellectual society: the role of imagination in science, the role of Euclid's Elements in mathematics education, and the relationship of religion and science. In the past twenty-five years, scholars, guided by Abbott's other writings, have situated Flatland in the context of these debates. I will survey the work of these critics and use similarities between Flatland and portions of Plato's Republic to expand their arguments. In particular, these similarities suggest a resolution to differences over the meaning of Flatland's conclusion. I will also discuss the inclusion of these ideas into a multivariable calculus class that I taught in the Fall of 2006.

## Murphy Waggoner (murphy.waggoner@simpson.edu) Simpson College

## Symmetry, Sewing and Service: Quilt Design in a Symmetry Course

Nine students in my May Term class designed and sewed baby quilts based on the symmetry concepts they had leaned. After the quilts were completed, they were donated to Caring Connections, an organization which distributes quilts to all newborn babies in Warren County, Iowa. After studying rigid motions, the students developed their own classification schemes for finite figures, border patterns and wallpaper patterns. They also studied various schemes for creating tessellations. Simultaneously, the students learned about the mathematics of piecing quilts: geometry, the four color theorem, sequences and series, measurement and estimation. The students designed their own quilt tops, some based on traditional designs and others from original designs. They then cut and pieced the tops. Volunteers from the Simpson College community tied the quilts and applied bindings. I will talk about the project in general, the designs of the quilts, and some mathematics covered in the course.

## Steven B. Zides (zidessb@wofford.edu) Wofford College

## Physical Aesthetics: Scientific Metaphors in the Visual Arts

In the hope of creating a meaningful liberal arts physics experience, I have created an introductory course which examines how physical metaphors creep into our society through the visual arts. Covering everything from particles to Picasso, this odd mix of traditional physics and art history tends to reach out to even the most science-phobic student. In this talk I will discuss the overall structure, academic materials, and assessment
strategies used throughout the course. To avoid superficiality, the talk will center around one or two specific examples. By looking in depth at these example topics, the audience should get a good sense of the inner workings of the entire course. Emphasis will be placed on the more mathematical topics that could be adapted for use in a standard liberal arts mathematics course.

## Mathematics of Sports and Games

## Friday, August 3, 8:30 am-10:30 am

Morley Davidson and Joseph Miller (davidson@math.kent.edu) Kent State University

## Statistical Analysis of Popular Rubik's Cube Solution Systems

Rubik's Cube speedsolving competitions are increasingly popular these days, with 15 -second solutions a common occurrence. A separate competition category is that of solving for fewest moves, given one hour; top competitors can typically save $20-25$ moves from the roughly $55-60$ moves required by speedsolvers. But is it possible to accomplish such savings quickly, say within one minute, for a typical starting permutation? In this talk I describe joint work with Joseph Miller where we used GAP to simulate and analyze various known and newly found solution algorithms. Our work shows that indeed, there are solution systems with a median face-turn count around 35 moves and which require only a humanly-memorizable "algorithm library" of move sequences. Although memorizing such algorithm libraries presents a severe challenge in comparison to algorithm libraries in popular use, one may sacrifice moves at key steps in order to reduce the memory load. In this way we propose a method that puts within reach, to the highly motivated cuber, the long-standing dream of being able to speedsolve a typical scrambling in at most 40 moves.

Robert Franzosa (franzosa@math.umaine.edu) University of Maine

## The Baseball Simulator: Accurately Simulating Major League Baseball Games with a Minimum Number of Statistics

We will present a model (The Baseball Simulator) for simulating major league baseball games and seasons based on team (rather than individual) statistics. The model was motivated by the desire to create an accurate simulation model that employs as few statistics as possible. We will present a statistical overview comparing Baseball Simulator results and actual major league results for the seasons 1901-1993. We will also present results of a Baseball Simulator Ultimate Baseball League season where every major league from 1901 to 1993 plays every other team from the same timespan to determine an overall best Baseball Simulator major league team. Finally, we will share a related classroom activity where students employ probability concepts in the design of their own teams that compete in dice-roll baseball games.

## Jacob Robert Heidenreich (Jacob.Heidenreich@loras.edu) Loras College

## Experiential Learning in the Mathematics of Games

In this talk I will discuss an experiential learning course I've developed in the mathematics of board, card, and miniatures games. 'Experiential Learning' is a pedagogy in which students are placed in an 'experience-rich environment' and are left to draw their own conclusions from these experiences. The paradigmatic example of experiential learning is sending a group of students on a service trip and having them write a paper reflecting on what they've learned. On the face of it, this pedagogy might not be a good fit with a traditional mathematical syllabus. In my class, the students will play various games and then use statistical tools to explore these games and develop strategies. Thus, the students will be exposed to standard material from an entry level statistics class, yet must determine for themselves how to use these materials to improve their play. I will discuss the difficulties of using this pedagogical approach in a math class, and some strategies I've developed to overcome these difficulties.

Brian Hollenbeck (bhollenb@emporia.edu) Emporia State University
When the "Best" Strategy Fails: Variance Trumps the Mean
For many games, a winning strategy is pursued by maximizing the expected score per turn. But does this strategy actually maximize winning? We introduce the simple dice game, Kyboi, which allows us to analyze
this question. The goal of Kyboi is to reach a predetermined score in as few turns as possible. However, in some situations optimal play coincided with the strategy that had greater variance in score per turn, rather than expected value. We will discuss how the optimal strategy is a function of the score needed to win, as well as the number of players. The game was analyzed in the context of a project for a Probability and Statistics class. This project motivated classroom discussions on expected value, sampling distributions, and confidence intervals. It gave students the opportunity to reinforce their understanding of hypothesis testing, counting techniques, and statistical methods involving proportions and means. Full analysis of the game required the use of Maple and Excel.

## T. S. Michael (tsm@usna.edu) United States Naval Academy

## Mathematics and Collegiate Wrestling Tournaments

Collegiate wrestling leagues face a challenging scheduling problem at their championship tournaments: What is a fair way to distribute the first-round byes among the individual wrestlers? It turns out that the standard method sometimes leads to unsatisfactory brackets. We discuss the recent successful implementation of the speaker's method, which relies on a least-squares calculation to guarantee fairness.

Charles F. Rocca (roccac@wcsu.edu) Western Connecticut State University
Monopoly in the Classroom
Trying to instill in students an appreciation, an interest, or even an understanding of probability is at best a challenge and at worst an impossibility. Introducing games into the classroom offers an opportunity to put students at ease in familiar and comfortable territory from which to begin to learn the basic principles of probability. Monopoly offers a particularly nice opportunity given how common it is, the simplicity of the basic rules, and the number of different ways the rules of probability can be brought to bear.

## Student Research in Industrial Mathematics <br> Sunday, August 5, 8:30 am-11:15 am

Bem Cayco (cayco@math.sjsu.edu) San Jose State University
CAMCOS, Its Challenges and Rewards
The Center for Applied Mathematics, Computation and Statistics is a student research program in the Math Dept of San Jose State University. Student teams, under the supervision of a faculty member, work on a problem supplied by an industry sponsor. I will discuss the challenges and rewards of our program and share some of my experiences as a faculty supervisor.

Marian Hofer (marianhofer@yahoo.com) San Jose State University

## Maximizing Information from Accumulating Data through Bayesian Adaptive Sampling

This presentation discusses a project sponsored by NASA Ames Research Center (Moffett Field, CA). The project involved developing software for efficient mapping of the spectral surface of the ground using a spectrometer mounted on a robotic helicopter with onboard computing capability. As a team leader of a research group of five seniors, I worked on developing an algorithm that utilized "adaptive Bayesian sampling." In this sampling technique, the information available at any given point was used to direct future data collection from locations that were likely to provide the most useful observations in terms of gaining the most accuracy in the estimation of quantities of interest. The implementation of this technique allowed for gaining maximum information about the ground surface while minimizing the number of required observations to achieve the desired reduction in estimation error. This project introduced me to the world of scientific research and inspired me to purse applied mathematics beyond my undergraduate degree.

Susan Martonosi (martonosi@math.hmc.edu) Harvey Mudd College
Senior Clinic Projects: Capstone Experience for Students, Exercise in Self-Restraint for Faculty
Harvey Mudd College mathematics majors must participate in a capstone experience during their senior year, typically either an individual senior thesis or an industry-sponsored team "clinic" project. Clinic projects
offer students the opportunity not only to develop their mathematical research skills but also to develop professional communication and project management skills. Faculty advisors walk a tightwire to balance guiding the students both mathematically and interpersonally and giving the students the freedom to take ownership of the project, make mistakes and work together to correct them. In this talk, I will draw on my experience advising a clinic project this past year and on the experiences of my colleagues to discuss the clinic process from an advisor's standpoint.

## Thomas Ratliff, Michael Kahn (tratliff@wheatoncollege.edu) Wheaton College

## Mathematical and Statistical Consulting at Wheaton College

Wheaton College (MA) offered a project-based, team-taught course in Mathematical and Statistical Consulting for the first time in Spring 2007. Two teams of four undergraduates each worked on problems solicited by the faculty from local businesses. One project was related to long term weather forecasting for the National Weather Service, and the other concerned monitoring levels of coliform in the Peconic Bay area of Long Island for Battelle. The semester culminated with the students giving formal presentations to the clients at their place of business and preparing a white paper for the clients. In this talk, we will discuss the structure and logistics of the course, the many aspects of the course that worked well, and the minor modifications we will make for the course in Spring 2008.

## Slobodan Simic (simic@math.sjsu.edu) San Jose State University

## Student Research in Applied Mathematics at San Jose State

The Center for Applied Mathematics, Computation, and Statistics (CAMCOS) at San Jose State University is a student research program whose goal is to give students the experience of working on applied math problems in collaboration with local companies and research labs. I will describe my experience supervising two CAMCOS projects with NASA Ames Research Center.

## Teaching a History of Mathematics Course

## Sunday, August 5, 8:30 am-10:30 am <br> Sunday, August 5, 2:00 PM-5:00 PM

## John Bukowski (bukowski@juniata.edu) Juniata College

## A Course in the History of Mathematics with Student Presentations and Some Original Sources

I will discuss a first attempt at a history of mathematics course taught as a special topics offering for a small class, allowing for great flexibility. The course began with mathematics of antiquity and ended with half of the semester in the 17th and 18th centuries. Student presentations played a large role in the course, while the use of several original sources was a particularly appealing component.

## Kathleen Clark (kclark@coe.fsu.edu) Florida State University

Using History in the Teaching of Mathematics: A Course for Pre-Service Secondary Mathematics Teachers The NCATE/NCTM Program Standards for Programs for Initial Preparation of Mathematics Teachers (2003) state that teachers of mathematics should "demonstrate knowledge of the historical development of [topics] including contributions from diverse cultures" (p. 4). Several manifestations of a history of mathematics course can be designed to provide opportunities for prospective teachers to gain such knowledge and consequently plan for its use in teaching. In this session I will describe the context and foundation for the history of mathematics course, "Using History in the Teaching of Mathematics," required of the secondary mathematics education majors at Florida State University. The course is currently designed to provide pre-service mathematics teachers with opportunities to: (1) work with mathematical topics as they evolved over time, with respect to the mathematics found in secondary school curricula today; (2) study and discuss the historical and cultural influences on and because of the mathematics being developed; and (3) develop the pedagogical knowledge needed to integrate an historical perspective in the teaching of school mathematics. Five course tasks are
used to create the opportunities for mathematical, historical, and pedagogical learning: key topic explorations; library assignments; a journal assignment; a "book club" experience; and a model lesson assignment. The session will focus on the five course tasks and pre-service mathematics teachers' perceptions of the role of the history of mathematics in their learning and the learning of their future students gained via their experience with the course.

## Pam Crawford (pcrawfo@ju.edu) Jacksonville University

## Student Engagement in History of Mathematics

Students majoring in mathematics at Jacksonville University are required to take an upperlevel History of Mathematics course. Most students are very apprehensive about this course due to the nature of the course. Most mathematics courses involve class discussion of mathematics problems and of the mathematical techniques necessary to solve such problems. These abilities are not necessarily those that best serve students in History of Mathematics. Here classroom discussion centers on important episodes, problems and discoveries in mathematics, with emphasis on the historical and social contexts in which they occurred. With a grant I received from the Jacksonville University Center for Teaching and Learning, I investigated whether the guided discovery teaching technique of giving students an historical event to serve as a seed for their exploration as to a possible link to mathematics better engaged my History of Mathematics students in the course material than my previous teaching methods. This talk will discuss the results of my study, including students' reactions to the assignments and their performances in the course. I will discuss using this guided discovery technique in a graduate-level History of Mathematics course, also.

## William Roger Fuller (w-fuller@onu.edu) Ohio Northern University

## Paradigms and Myths: A New Approach to Teaching the History of Mathematics

What patterns do we find in how mathematicians have developed mathematical knowledge over recorded history? What patterns do we find in how historical accounts of mathematics treat their "data?" Such patterns may indicate the presence of paradigms and myths which can serve as an exciting basis for a course in the history of mathematics. This talk will outline the foundations of such a course and will illustrate how these ideas might shape a unit on Hypatia, the last mathematician of note in antiquity.

## Jim Fulmer, Tom McMillan (jrfulmer@ualr.edu) University of Arkansas at Little Rock

## Integrating Ancient Numeral Systems into a History of Mathematics Course

Ancient Numeral Systems are a common topic in a History of Mathematics course. This session will reflect on a History of Mathematics course at the speakers' university and how we strive to develop a more in-depth understanding of numeral systems than just writing numerals in that system. Examples of ancient numeral systems in the course will include: Egyptian, Babylonian, Greek, Roman, Mayan, Hindu-Arabic, Traditional Chinese, East African, and Chinese Stick/Rods. In our course, we require students to develop a detailed overview of each numeral system consisting of: name of system, base, symbols, symbol for zero, place value, and principles needed to write numerals with examples. This serves to provide an in-depth look at each of the ancient numeral systems included in the course and an appreciation of the concept of number vs numeral.

## Daniel Campbell Kemp (dan.kemp@sdstate.edu) South Dakota State University

## A History of Mathematics Course That Emphasizes the History of Calculus

When the person who had been teaching our History of Mathematics class retired, I volunteered to teach the class. I decided to emphasize the history of calculus because that was an area in which I thought I was knowledgeable. This turned out to be false. The resulting course that did emphasize calculus history will be described.

## Lisa A. Mantini (mantini@okstate.edu) Oklahoma State University

## Problem Solving and the History of Mathematics

The problems given on the OSU High School Math Contest from its inception in 1990 until 2003 were written primarily by the late Abraham Hillman, with assistance from M. Newton and several others. Pre-
service teachers find these problems interesting but often extremely challenging. We describe their use as a supplement to a standard History of Mathematics course for pre-service secondary teachers, as one way of linking historical mathematical concepts to their continued use and applicability in the current day.

## Kimber Tysdal, Betty Mayfield (mayfield@hood.edu) Hood College

## A Locally Compact REU in the History of Mathematics

We received a small grant from our institution to conduct some research with four undergraduate students this summer. We chose a topic in the history of mathematics - specifically, Women and Mathematics in the Time of Euler - and decided to run our project like a mini-REU, with guest speakers and weekly field trips as well as library research. Our students are here with us at MathFest, where they will report on the results of their work, in the student paper sessions, and we will describe our experience from the faculty point of view.

## Maria Clara Nucci (nucci@unipg.it) Universita di Perugia

## A Mathematical Journey through Space and Time

I am a mathematical physicist, who loves history, and has been teaching a course on History of Mathematics to Math senior students (mainly Italian, but also English, German, Polish, Turkish) at University of Perugia in Italy in the last 2 years, quite a challenge for somebody who usually teaches Classical Mechanics to Math/Phys major students. Indeed, the three guidelines of my Mathematical Physics courses have always been:

1. the historical development;
2. the continuous update on the current research by using the web;
3. the use of technology, which indeed resulted in a 2000 MAPLE prize for my students:
www.maplesoft.com/company/casestudies/perugiauniversity.aspx
4. the "salt and pepper" of life, viz. I never forget to add fun and wittiness [e.g., M.C. Nucci, Tales of Gods and Heroes: The nectar of the Gods, Notices of the AMS 39, May/June 1992, p. 427].
My course on the History of Mathematics is based on original works (whenever is feasible), excerpts from scholarly (and amateur) books and papers, and the beauty (and ugliness) of innumerable web-pages. A PowerPoint presentation is used to link them all. I don't forget some "salt and pepper". Samples: why should a virologist read Plato? what has Ptolemy got in common with Lewis Carroll? why does Dante strongly disagree with Fibonacci?

## Sharon A. O'Donnell (sodonnell@csu.edu) Chicago State University

## Teaching a History of Mathematics Course?-Enjoy!

"I am not an historian, I am a mathematician. We do mathematics in this course." There are always a few students who are dismayed by this opening statement in my first class session of History of Mathematics. They thought this was going to be a course in memorization of dates, names, and places. However, a history of mathematics course is much more than this. It is a vehicle to teach mathematics that students may not have encountered in previous mathematics courses. It is also a means to explore the effects of history on the evolution of mathematics as well as a study in diversity. I have taught the History of Mathematics course required of secondary education mathematics students and elective for mathematics major for many years, and developed a second course, History of Mathematics for Middle School Teachers, taken by pre-service and in-service middle school teachers. Both of these courses serve as a capstone course taken near the end of the student's study of mathematics. This paper will focus on preparing to teach such a course, topics to include, methods of assessment, and resources for the instructor.

## Michael Reynolds (reynoldm@mcpherson.com) McPherson College

## Teaching a 17-Day History of Mathematics Class—An Exercise in Prioritization

I recently had the opportunity to teach a regular, 3 credit hour history of mathematics course during an accelerated semester containing only 17 class meetings. In this talk I will discuss how I addressed the challenge of fitting over 3000 years of mathematics history into this short time period, including what worked
and what did not work. This presentation will also include the feedback I received from the students, what I plan on doing differently if I have the chance to teach the course again, and a general discussion of the feasibility of fitting this (or any) mathematics course into such a schedule.

Charles F. Rocca (roccac@wcsu.edu) Western Connecticut State University
History for the Masses
At Western Connecticut State University, as at most universities, students are required to take a math course in order to satisfy their general education requirements. This is often viewed by the students as an onerous task and so they condemn the course before it has begun. In an attempt to attract students into a course that they could relate to I introduced a general ed non-majors course in the history of mathematics. Since mathematics has a long and rich history, knowledge of this history enhances understanding and appreciation for the subject as well as giving some insight into the cultures and times in which the discoveries were made. Thus this course has the potential to attract students from varied backgrounds with the promise of a course they can relate to and hopefully allow them to leave with an understanding of some new mathematics and its relevance to their lives.

Charlie Smith (charlie.smith@park.edu) Park University
Math History Without Prerequisites
Imagine a liberal learnings course on the history of mathematics which is open to all students. How can this be done with such a seemingly fatal handicap? This talk presents one person's methods, experience, and observations.

## Stan Schmidt and Robert Francis Vivona (Robert.Vivona@marist.edu) Marist College

## History of Mathematics For The Non-Mathematician

This paper will discuss the experience of teaching a pair of history of mathematics courses at the Marist College Center for Lifetime Study (CLS), a learning centered program for senior adults. The first course, Genesis of the Calculus, covered the time period from the Babylonians to the twentieth century. The second course, Solving the "Unsolved": An Historical Approach, presented the drama of proving four major mathematical results: Fermat's Last Theorem, The Four Color Problem, Gödel's Incompleteness Theorem, and The Insolubility of Certain Polynomial Equations in Radicals We will address the differences in CLS curriculum, course objectives, methods of delivery, and source material with the typical academic setting. Furthermore we will discuss the backgrounds of the co-presenters, both with over forty years of mathematical experience (but with no training in teaching the history of mathematics), one in academia, the other in industry, and how they approached the course curriculum and delivery. The sequence of proposing a curriculum and having it accepted at CLS will be described. Using the course outline and the set of course objectives as a basis, the course content and requirements will be reviewed. The backgrounds, motivation and expectations of the students contrast strongly with those of a degree program class, especially college students in general mathematics courses to meet distribution requirements. Student backgrounds and their expectations and reactions to the course will be discussed. Finally we will provide our assessment of what worked, what didn't, and what improvements were recommended.

## Bostwick F. Wyman, Daniel W Dotson (wyman@math.ohio-state.edu) Ohio State University

## History of Mathematics at Ohio State: Web Resources

The audience for the History of Mathematics course at Ohio State (Math 504) consists mostly of students intending to teach high school mathematics. This course (or a similar experience) is required for the secondary teaching license in the state of Ohio. Wyman teaches this course primarily as a writing course, with assignments including a book review, a biography of mathematician, a research paper, and an oral presentation. The Ohio State Libraries are promoting the use of electronic resources. Dotson maintains a web site, library.osu.edu/sites/sel/math/mathhis.php, with links to print and electronic resources, including videos. Wyman uses extensive Powerpoint files with web links, and these files will be available to the students through
the Ohio State internal course management system. These files will be available on the web by mid-April. Wyman and Dotson acknowledge with thanks a Course Enhancement Grant from the Ohio State University Libraries.

## Teaching Calculus in High School: Ideas that Work Sunday, August 5, 8:30 am - 10:30 am

Brad Huff (huffhaus@pacbell.net) NCTM/PDK/AAPT/Advanced Placement

## California Gold from an Old Timer

Nuggets from three decades of teaching calculus including his first day binomial expansion activity (why Newton had it put on his tomb), the "mirror method" of finding instantaneous rates of change, using data from a real car trip to introduce differentiation and integration early in the course, using calculators to find a base for the exponential function so the derivative is the same function, and, best of all, his "progressive quiz" technique for maximizing learning while testing.

## Doug Kuhlmann (dkuhlmann@andover.edu) Phillips Academy

## Discover the FTC via Numerical Integration

Starting with intuitive ideas of area, we introduce left-hand, right-hand, midpoint and trapezoid rules for estimating areas. Using a TI-84 program that students write themselves, they can then discover the fundamental theorem of calculus with some group explorations. Euler's rule is also developed using local linearization. Together they lead to a traditional proof of the FTC. Bonus: Simpson's rule can also be introduced early this way.

Dan Lotesto (lotesto@uwm.edu) Milwaukee Public Schools

## Using Average Velocity to Illustrate FTC

In the process of applying formulas in particle motion problems, students often forget the power of how these formulas are connected. By exploring the concept of average velocity via both a position function's secant line slope on an interval and the average value of it's corresponding velocity function on that interval, students can come to a better appreciation of the power of the FTC.

## Dave Lawrence Renfro (dave.renfro@act.org) ACT, Inc.

## Unmasking Implicitly Defined Functions in Calculus

The topic of implicitly defined functions is often marginalized in the teaching of calculus, despite its importance in more advanced theoretical and applied fields that use calculus. It is also a theme that can be used to unify, and make connections between, several other topics in a beginning calculus course, it is a subject in which it is easy to pose problems that have multiple methods of solution, and it provides the means by which one can formulate calculator-resistant problems in a variety of settings. I will describe a number of specific examples of how the topic of implicitly defined functions can be used to energize the teaching of elementary calculus, including some that I have used in my own teaching.

Bradley Stoll (bradleys@harker.org) The Harker School

## Teaching Mathematics Majors vs. Users

Let's face it, the vast majority of students who take AP Calculus will be using, not studying mathematics. The users will be engineers, physicists, biologists, statisticians, etc. Those who study, well herein lies the Mathematician or maybe the Computer Science major. These two different groups are motivated differently. I will demonstrate my approaches, which (can) rely heavily on the use of technology, specifically Mathematica.

Qibo Jing, Dan Teague (teague@ncssm.edu) North Carolina School of Science and Mathematics
Mental Substitution: A Powerful Tool in Doing Integration and Integration by Parts
When students do a complicated integration, they usually attempt the substitution method. However, when they do substitution, they have to use another variable by trial and error and then switch back to original
variable. This makes the job of doing integration by substitution very tedious. Since integration is essentially the inverse operation of differentiation, most of time we can use mental substitution to do the integration. From my calculus learning, sharing and teaching experience, I believe mental substitution can really release the burden and simplify the job of doing integration and integration by parts.

## Susan Schwartz Wildstrom (susan@wildstrom.com) Montgomery County Public Schools

Discovering Derivatives and Derivative Rules Including Product, Quotient and Chain Rule
Exploration activities using technology and experimentation in calculus will be demonstrated and the usefulness of such activities will be explained. Within available time one or more of the activities will be modeled. The materials for use in all of the activities which include developing the derivatives of the basic trigonometric, exponential and logarithmic functions and activities to help students develop product, quotient, and chain rule processes will be provided as handouts.

## Graduate Student Poster Session

## Saturday, August 4, 1:00 pm-2:30 pm

## Andrea Bruder Baylor University

It is well known that, for $-\alpha,-\beta,-\alpha-\beta-1 \notin \mathbb{N}$, the Jacobi polynomials $\left\{P_{n}^{(\alpha, \beta)}(x)\right\}_{n=0}^{\infty}$ are orthogonal on $\mathbb{R}$ with respect to a bilinear form of the type

$$
(f, g)_{\mu}=\int_{\mathbb{R}} f \bar{g} d \mu,
$$

for some measure $\mu$. However, for $\alpha=\beta=-1$, from Favard's theorem, the Jacobi polynomials cannot be orthogonal with respect to a bilinear form of this type for any measure. Are they orthogonal with respect to some "natural" inner product? Indeed, they are orthogonal with respect to a Sobolev inner product. We discuss this Sobolev orthogonality and construct a self-adjoint operator in this Sobolev space that has the Jacobi polynomials as a complete set of eigenfunctions.

## Donghui Chen Wake Forest University

## An Intuitive Interpretation of Moment Generating Functions

In this poster, we present an intuitive interpretation of moment generating functions (mgfs). Suppose $Y$ has mean zero and standard deviation one and that the mgf of $Y, m_{Y}(t)$, is defined for $t \in S(Y)$ (the support set of $Y$ ). We show that the ratio of $m_{Y}$ and $m_{Z}$, the mgf of a standard normal distribution, can be interpreted as a limit arising in the study of the nature of convergence for independent identically distributed (i.i.d.) sums for the distribution of $Y$. Several examples and inequalities for particular distributions are also given.

## Jennifer Froelich University of Iowa

Universal Deformation Rings and 2-modular Representations Related to the Symmetric Group $S_{5}$
Deformation theory has to do with the behavior of mathematical objects, such as group representations, under small perturbations. This theory is useful in both pure and applied mathematics and has led to the solution of many long-standing problems. In particular, in number theory, Wiles and Taylor used universal deformation rings of Galois representations to prove Fermat's Last Theorem. In this poster, we will discuss calculations which will lead to the determination of the universal deformation rings for certain 2-modular representations of the symmetric group $S_{5}$ and its double cover ${ }^{\sim} S_{5}$ whose Sylow 2-subgroups are quaternion. Since group representations of finite groups have a well-defined universal deformation ring when their stable endomorphism ring is given by scalars, we will discuss which 2-modular representations of $S_{5}$ have this property after inflating them to representations of ${ }^{\sim} \mathcal{F}$. Furthermore, the number of generators and relations of
the universal deformation rings for these representations is given (or bounded) by the dimension of certain Ext groups. We will determine these dimensions and show what conclusions can be drawn from them concerning the ring structure of the universal deformation rings. Our main tools come from the representation theory of finite groups and of finite-dimensional algebras in positive characteristic and include special biserial algebras and Erdmann's description of tame blocks of group rings.

## Gulden Karakok Oregon State University

## Transfer of Learning in Linear Algebra

One of the main problems in college education is stated to be students lacking ability to transfer of learning. Researchers designed a study to investigate if the same problem apparent at higher level undergraduate courses such as linear algebra. The concepts from linear algebra courses are used in many different courses in mathematics, science and engineering departments. Researchers first wanted to investigate how students understand some typical linear algebra concepts such as vectors, matrices, linear transformations, eigenvalues and eigenvectors, etc. Then, students' ways of understanding these concepts was explored further. Students' conceptual connections between and among different concepts in linear algebra may or may not be determined by the type of activities they do during linear algebra courses they take. Thus we invited students from different majors who have taken some sort of linear algebra courses. Our preliminary finding shows that certain activities help students make stronger connections among concepts. We wish to share the preliminary results and hope the findings will help educators improve the Linear Algebra Curriculum.

## Daniel Kearns Penn State University - Capital College

## Rediscovering Rithmomachia

For many centuries in the Medieval period, Rithmomachia reigned as the most intellectually stimulating and satisfying board game. Played in the university system and amongst the clergy and highly educated, this 'philosophers' game' is in a very crude sense a mathematical form of chess.

The intricate numerical relationships nested within the game play of Rithmomachia accent the honor, wonder and homage paid to numbers by the scholars before us. For the educated classes in Greek and Medieval culture, the mystery of numbers and their relationships took on a divine character.

Rithmomachia also illustrates the educational and recreational mindset of the Medievals; much different from our own. Education and recreation were intertwined, both necessitating rigorous mental exertion. Furthermore, the classical approach to education respected exhaustive effort simply for effort's sake; Rithmomachia did not prepare an individual to pass an examination or "real life" problems. Instead, it built mental and spiritual character.

A thorough analysis of Rithmomachia lends keen insight into the history of mathematics, from the Greeks' initial amazement at numerical relationships to its reemergence in the Medieval period. Furthermore, in comparing this Medieval board game to the games enjoyed in 21st century America, Rithmomachia exposes the lack of rigor, logic and mathematics in our current educational model. Perhaps by rediscovering Rithmomachia, we can also rediscover a more classical model of education bent upon producing well-rounded Renaissance men and women whose broad education has built intellectual, mental and spiritual character, rather than focused, narrow specialists.

## Usha Kotelawala Columbia University

## Factors Affecting the Role of Proving in Secondary Mathematics Classrooms

A study of 78 secondary mathematics teachers was conducted to examine factors affecting a teacher's attitude and beliefs about the role of proving in secondary mathematics classrooms. The results connected to the quantity of undergraduate and graduate mathematics courses taken were surprising. College mathematics coursework arose as a significant factor affecting proving in the classroom. More coursework led to less proving ( $p<.05$ ). More college mathematics coursework also led to teachers to a style of teaching in which students play a smaller role in the classroom in proving ( $p<.05$ ). Results were based on self-reported survey responses.

## Drew Pruett Baylor University

## Rationally Smooth Schubert Varieties Associated with $A_{n}$

This poster will demonstrate a poset isomorphism between the Pieri order on the set of Ferrers' diagrams whose row length plus column length is smaller than $n+1$ and a subset of ${ }^{\sim} A_{n} / A_{n}$ under the Bruhat order. The construction will guarantee a length condition on $w \in^{\sim} A_{n} / A_{n}$ such that $X(w)$ is not rationally smooth, and a necessary condition for rational smoothness for each of the (finitely many) other elements in the group.

## Katie Reif University of Iowa

## Hyperbolicity of Arborescent Tangles and Arborescent Links

This poster demonstrates part of a study of the hyperbolicity of arborescent tangles and arborescent links. We show that given an arborescent tangle $T$, the complement $X(T)$ is hyperbolic if and only if $T$ is a rational tangle, $T=Q m * T^{\prime}$ for some $m$ greater than or equal to 1 , or $T$ contains $Q_{n}$ for some $n$ greater than or equal to 2 . We use this result to prove a theorem of Bonahon and Seibenmann which says that a large arborescent link L is non-hyperbolic if and only if it contains $Q_{2}$.

## Teresa Selee North Carolina State University

## Computing PARAFAC on Large-scale Tensors with Special Structure

The matrix Singular Value Decomposition (SVD), also known as Principal Component Analysis (PCA), is very well known. We are considering a higher-order analogue of this decompositions called PARAFAC (a.k.a., CANDECOMP). Specifically, we consider the problem of computing PARAFAC for third-order tensors where the two-dimensional slices have a special structure. In particular, each slice is square and obtained from the product of a sparse matrix with its transpose. Though the resulting slice may be dense, we store it implicitly by storing only the sparse factors. We exploit the structure of the slices to greatly reduce the storage for the tensor and obtain a speed-up in the computation of the PARAFAC decomposition. We demonstrate the applicability of our decomposition on an application in multi-similarity clustering analysis where the tensors are roughly of size $10000 \times 10000 \times 7$.

## Scott Taylor University of California, Santa Barbara

## Split Links obtained by Refilling Meridians of Genus Two Handlebodies

One of the most basic knot invariants is the genus of a minimal genus Seifert surface. Schubert (1949) showed that the genus of a composite knot is the sum of the genera of its factors, thus proving that the sum of two nontrivial knots is not the unknot. Scharlemann (1985) generalized this last statement to the band sum of two knots and Eudave-Munoz (1988) generalized it to knots which differ from a split link by a rational tangle. In this poster, I outline a proof of a result which generalizes Scharlemann's and Eudave-Munoz's results to knots or links which are obtained by "refilling meridians" of genus two handlebodies. As an application, I am able to give a lower bound on the genus of a knot which differs (in a non-trivial way) from a split link by a rational tangle.

## Index of Speakers

Adkins, Frederick, 16
Alayont, Feryal, 46
Alexander, Jack, 3
Ali, Sayel, 33
Alvarez, Consuelo, 20
Anewalt, Doug, 32
Arangala, Crista, 15
Atela, Pau, 16, 56
Attele, Kapila, 17
Awanou, Gerard, 21

Baker, Roger, 33
Balasundaram, Lalgudi, 33
Barat, Christopher, 52
Bargas,Rocky, 15
Barnes, Ron, 33
Basor, Estelle, 12
Batten, Belinda, 36
Bauer, Doug, 51
Beaver, Scott, 23, 46, 47
Becerra, Linda, 33
Bedient, Richard, 47
Bell, George, 13
Bell, Jordan, 30
Bhatnagar, Satish, 23, 34
Billups, Stephen, 10
Birmajer, Daniel, 34
Blackman, David, 34
Blair, Jean, 51
Blankenship, Robin, 12
Boliver,David, 15
Booth, Robert, 42
Bradley, Robert, 5, 30
Brezhneva, Olga, 21
Brown, Ezra, 31
Brown, Kenneth, 53
Brown, Sharon, 47
Bruder, Andrea, 66
Buck, Robert, 24
Bukowski, John, 30, 61
Burdick, Bruce, 13
Burroughs, Beth, 47

Campbell, Connie, 53
Carpenter, Bruce, 17
Carpenter, Jenna, 24
Castillo-Chavez, Carlos, 2
Cayco, Bem, 60
Chartier, Timothy, 4
Chayes, Jennifer, 1
Chen, Donghui, 66
Cherepinsky, Vera, 17, 34
Choi, Ji Young, 50
Clark, Kathleen, 61
Cline, Kelly, 35, 47
Cloherty, Helen, 35
Cockburn,Sally, 47
Comar, Timothy, 17
Conrey, Brian, 11
Crannell, Annalisa, 7, 57
Crawford, Pam, 62

D'Antonio, Lawrence, 5
d'Entremont, Yvette, 24
Das, Kumer, 35, 53
Davidson, Morley, 59
Davis, Anna, 48
De Young, Gary, 18
DeBello, Joan, 24
Delpish, Ayesha, 53
Devadoss, Satyan, 4
Deyo-Svendsen, Matt, 41
Diesel, Susan, 18
Doree, Suzanne, 50
Dotson, Daniel, 64
Dray, Tevian, 26, 35, 48
Drucker, Thomas, 30
Dunham, Penelope, 54
Dunham, William, 2
Ellis-Monaghan, Joanna, 8
Eng, Genghmun, 36
Epperson, James, 25
Epstein, Elana, 25
Eroh, Linda, 37
Evans, Katie, 36

Ferrara, Michael, 50
Fisher, Gwen, 56
Flashman, Martin, 36, 56
Fox, William, 36
Franzosa, Robert, 59
Froelich, Jennifer, 66
Fuller, William, 62
Fulmer, Jim, 37, 62
Gad, Sangeeta, 15
Gentry, Sommer, 48, 51
Gera, Ralucca, 37, 51
Ghrist, Michelle, 22
Givens, Berit, 37
Goldstine, Susan, 37
Goldston, Dan, 2
Goodman, Russell, 25
Gordon, Florence, 54
Gordon, Sheldon, 18, 54
Grabiner, Judith, 3
Gross, Louis, 3
Gruenwald, Mark, 22
Hagedorn, Thomas, 37
Hakimi, Louis, 51
Hall, Mike, 37
Han, Annie, 25, 38
Harrington, Rachel, 26
Harris, Angela, 50
Hartke, Stephen, 9, 10, 51
Hartshorn, Kevin, 38
Hartz, David, 48
Heidenreich, Jacob, 59
Heyer, Laurie, 10
Hillyard, Cinnamon, 15
Hodgess, Erin, 54
Hofer, Marian, 60
Hollenbeck, Brian, 57, 59
Hong, Yuan, 38
Hopkins, Brian, 5
Horton, Steve, 51
Hrozencik, Dan, 17
Huddleston, Meg, 26
Huff, Brad, 65
Humphrey, Patricia, 28, 54
Hunter, David, 18
Jayawant, Pallavi, 9
Jing, Qibo, 65
Johannes, Jeff, 38, 49
Johnston, Tina, 26
Jones, John, 31
Jones, Michael, 8
Jopin, Karl, 19

Kahl, Nathan, 51
Kahn, Michael, 61
Karakok, Gulden, 26, 67
Karimpour, Rahim, 39
Karsai, Istvan, 19
Kearns, Daniel, 67
Kemp, Daniel, 62
Khosravani, Azar, 39
Kiihne, Patricia, 39
Killpatrick, Kendra, 39
Knisley, Jeff, 19
Knuth, Donald, 4
Koenig, Axel Mainzer, 30
Kotelawala, Usha, 67
Kotowski, Joe, 45
Kubatko, Laura, 10
Kuhlmann, Doug, 65
Kumar, Anand, 39
Lahme, Brigitte, 31
Lambright, Jonathan, 39
Landau, Susan, 11
Langton, Stacy, 5
Leahy, Andrew, 40
Lee, Paul, 32
Lemma, Mulatu, 40
Li, Weiping, 40
Liu, Gloria, 45
Lotesto, Dan, 65
Low, Richard, 40
Luca, Magdalena, 55
Lucas, Adam, 28
Ludwig, Lew, 51
Lunsford, Myrtis, 20
Mabrouk, Sarah, 49, 55
MacLean, Mark, 20
Mammo, Behailu, 41
Mantini, Lisa, 13, 62
Martin, Carla, 57
Martin, Mike, 20, 28
Martonosi, Susan, 60
Matsko, Vincent, 41
Mattman, Thomas, 13
Mauro, David, 52
Mayfield, Betty, 63
McGrail, Robert, 26
McGrail, Tracey, 49
McKinzie, Mark, 6
McMillan, Tom, 37, 62
McMurran, Shawnee, 6
McNally, Myles, 52
Meadows, Alex, 41

Mebius, Johan, 30
Mellor, Blake, 56, 57
Mersch, John, 41
Michael, T., 60
Michael, T.S., 51
Miles, Will, 41
Miller, Joseph, 59
Mir, Ellen, 41
Molina, Robert, 46, 52
Molitierno, Jason, 42
Morgan, Frank, 9, 12
Morris, Jerry, 31
Morrow, Margaret, 49
Mulcahy, Colm, 13, 14, 32
Murphy, Teri, 55
Narayan, Darren, 4
Narayan, Jack, 42
Neidinger, Richard, 22
Nguyen, Hieu, 42
Niess, Maggie, 26
Nucci, Maria Clara, 6, 63
O'Donnell, Sharon, 63
O'Neill, Mike, 12
Olsen, Darlene, 49, 55
Papakonstantinou, Joanna, 42
Pendergrass, Marcus, 42
Plante, Daniel, 41
Plofker, Kim, 6
Pomerance, Carl, 11
Pruett, Dres, 68
Qi, Chunxia, 38
Qian, Maijian, 43
Ratliff, Thomas, 61
Read, Thomas, 49
Reif, Katie, 68
Renault, Marc, 43
Renfro, Dave, 65
Reynolds, Michael, 63
Richardson, Sandra, 27
Richeson, David, 43
Rickey, V. Frederick, 6
Riley, Kyle, 22
Roberts, Lila, 29
Robeva, Raina, 21
Rocca, Charles, 60, 64
Rodgers, Nancy, 32
Rosin, Amber, 14, 32, 37
Rumbos, Adolfo, 12

Salomone, Stephanie, 50
Sandifer, Ed, 6, 31
Schaffer, Karl, 58
Schell, Robert, 42
Schiffman, Jay, 14, 43
Schmeichel, Edward, 51
Schmickl, Thomas, 19
Schmidt, Stan, 64
Seier, Edith, 19
Selee, Teresa, 68
Sellers, James, 31
Shank, Nathan, 8
Shepherd, Mary, 58
Sherman, Morgan, 43
Sibley, Thomas, 44
Siegel, Murray, 44, 56
Siegel, Sharon, 27
Simic, Slobodan, 61
Simons, John, 44
Smith, Charlie, 64
Smith, Ken, 52
So, Wasin, 52
Sompolski, Bob, 45
Soundararajan, Kannan, 11
St. John, Katherine, 1
Stadler, Jonathan, 14
Steen, Lynn, 2
Stemkoski, Lee, 6
Stewart, Sarah Ann, 50
Stoll, Bradley, 65
Strawbridge, Eva, 16
Su, Francis, 3
Suisky, Dieter, 6
Sunderland, Michael, 38
Sung, Chen-Han, 27, 29

Taylor, Scott, 58, 68
Teague, Dan, 65
Tecarro, Edwin, 44
Tesler, Glenn, 10
Thiele, Ruediger, 7
Tongen, Anthony, 23
Torrence, Bruce, 15, 21
Tou, Erik, 7
Touhey, Pat, 56
Turner, Galen, 24

Ugarcovici, Ilie, 23
van Joolen, Vincent, 44
Vivona, Robert, 64
Vo, Monika, 45

Waggoner, Murphy, 29, 58
Wang, Tingxiu, 45
Wang, Yan, 52
Webb, Derek, 45
Weld, Kathryn, 52
West, Kazuko, 27
Wheeler, Amy, 45
Whyzmuzis, Carol, 46
Wildstrom, Susan, 66
Wilson, Jennifer, 8
Winters, Steven, 9
Woods, Debra, 17
Wyman, Bostwick, 64
Zamansky, Bella, 45
Zeiser, Dorothy, 32
Zeleke, Aklilu, 46
Zhong, Kathy, 46
Zides, Steven, 58

