



# Abstracts of Papers Presented at Portland, Oregon



**Abstracts of Papers** 

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## **Invited Addresses**

## **Earle Raymond Hedrick Lecture Series**

Bjorn Poonen Massachusetts Institute of Technology (poonen@math.mit.edu)

Lecture 1: Undecidability in Number Theory Thursday, August 7, 10:30–11:20 AM, Ballroom Level, Grand Ballroom Lecture 2: Undecidability in Analysis and Topology

Friday, August 8, 9:30-10:20 AM, Ballroom Level, Grand Ballroom

Lecture 3: Undecidability Everywhere

Saturday, August 9, 9:30-10:20 AM, Ballroom Level, Grand Ballroom

Hilbert's Tenth Problem asked for an algorithm that, given a multivariable polynomial equation with integer coefficients, would decide whether there exists a solution in integers. Around 1970, Matiyasevich, building on earlier work of Davis, Putnam, and Robinson, showed that no such algorithm exists. But the answer to the analogous question with integers replaced by rational numbers is still unknown, and there is not even agreement among experts as to what the answer should be. Meanwhile, over the past decades, many problems in areas of mathematics other than number theory have also been found to be undecidable. I will present a sampling of these, and discuss a few problems whose undecidability status is not yet known.

## **AMS-MAA Joint Invited Address**

### Friday, August 8, 10:30–11:20 AM, Ballroom Level, Grand Ballroom

Sara Billey University of Washington (billey@math.washington.edu)

#### What is the Value of a Computer Proof in Research and Teaching?

In this talk, we will introduce some of the history of computer assisted proofs, modern applications, and how you can incorporate this technique into your every day life. Beyond the famous 4-Color Theorem, computer assisted proofs are found in hypergeometric series, geometry of Kepler's conjecture, and algebraic geometry related to Schubert varieties. Each new computer assisted proof adds to our collective repertoire with this relatively new technique. We will talk about some specific easy problems that can be verified by computer and some results in the literature for which no human only proof is known. We will address the important question "What does a computer proof add to our mathematical intuition?" We also will give several suggestions for where computer assisted proofs can be added in the undergraduate curriculum. In the long run, we speculate that computer assisted proofs will be taught right along side the techniques of induction and proof by contradiction.

## **MAA Invited Addresses**

### Thursday, August 7, 9:30–10:20 AM, Ballroom Level, Grand Ballroom

**Ricardo Cortez** Tulane University (rcortez@tulane.edu)

#### Understanding Microorganism Swimming using Mathematics

Biological fluid flows, like those surrounding moving bacteria and spermatozoa, are generated by viscous forces, which completely dominate inertial effects, so that their dynamics may be modeled as a sequence of steady-state snapshots. Microorganism motility has been an active area of research for the last 60 years motivated by questions like: What are effective locomotion strategies of microorganisms? How do they interact with the surrounding environment? How do microorganisms combine to create patterns of collective motion? What force-generating mechanisms do the organisms use to propel themselves? The only way to answer these questions is through a combination of theory, experiments, mathematical modeling and simulation. We will present recent collaborative mathematical work, some of it done with undergraduate students, that sheds light on these biological systems and challenges ahead.

### Saturday, August 9, 10:30–11:20 AM, Ballroom Level, Grand Ballroom

**Erika Camacho** Massachusetts Institute of Technology and Arizona State University (erika.camacho@asu.edu)

#### Mathematical Models of the Retina and In Silico Experiments: Shedding Light on Vision Loss

Mathematical modeling has been used to study diverse biological topics ranging from protein folding to cell interactions to interacting populations of humans but has only recently been used to study the physiology of the eye. In recent years, computer (in silico) experiments

have given researchers invaluable insights and in some cases have re-directed experimental research and theory. In this talk I will give a brief overview of the relevant physiology of the eye as it pertains to Retinitis pigmentosa (RP), a group of inherited degenerative eye diseases that characterized by the premature death of both rod and cone photoreceptors often resulting in total blindness. With mathematics and in silico experiments, we explore the experimentally observed results highlighting the delicate balance between the availability of nutrients and the rates of shedding and renewal of photoreceptors needed for a normal functioning retina. This work provides a framework for future physiological investigations potentially leading to long-term targeted multi-faceted interventions and therapies dependent on the particular stage and subtype of RP under consideration. The mathematics presented will be accessible to an undergraduate math audience and the biology will be at the level of a novice (and with a little help from Dr. Seuss).

#### Thursday, August 7, 8:30–9:20 AM, Ballroom Level, Grand Ballroom

#### Keith Devlin Stanford University (devlin@stanford.edu)

#### First Person Solvers—Using Video Games to Learn Mathematics and Solve Real Math Problems

The design of a good interface to an activity can have a significant impact on learning and use. The piano provides a more intuitive and direct interface to music than symbolic musical notation, the Hindu-Arabic numerals revolutionized arithmetic (and with it, trade and commerce), and symbolic algebraic notation was so successful that most people today think the interface is algebra, rather than the mathematical processes the notation represents. Devlin has spent the past several years developing casual games that provide representations of mathematics that enable children (and adults) to learn basic mathematics by "playing", the same way we can learn music by learning to play the piano.

### James R.C. Leitzel Lecture

#### Saturday, August 9, 8:30–9:20 AM, Ballroom Level, Grand Ballroom

**Joseph Gallian** University of Minnesota Duluth (jgallian@d.umn.edu)

#### Research in Mathematics by Undergraduates: Past, Present, and Future

Although involving undergraduates in research has been a long standing practice in the experimental sciences, it has only been recently that undergraduates have been involved in research in mathematics in significant numbers. In this talk I will trace the evolution of research by undergraduates over the past 25 years and the reasons for it. I will give my opinion on what lies ahead over the next ten years.

## AWM-MAA Etta. Z. Falconer Lecture

#### Friday, August 3, 8:30–9:20 AM, Grand Ballroom

#### Marie A. Vitulli University of Oregon (vitulli@uoregon.edu)

#### From Algebraic to Weak Subintegral Extensions in Algebra and Geometry

As students of algebra we quickly learn that for the purpose of solving polynomial equations the field of rational numbers is inadequate. We soon become acquainted with algebraic extensions of the rationals and later in our studies meet the fields of algebraic numbers, real numbers, and complex numbers, the latter as the algebraic closure of the real field.

As students of commutative algebra we learn about integral extensions of rings and their properties and consequences in the study of algebraic varieties and schemes. Again, for some purposes, integral extensions do not accomplish all that we had hoped for. Much more recently geometers and algebraists introduced the twin theories of weak normality and seminormality for commutative rings and algebraic varieties to address some of these deficiencies.

In this talk we outline the history of the twin theories with an emphasis on the recent developments in the area over the past fifteen years. For clarity of exposition we will focus our attention on the characteristic 0 case where the theories merge into one.

## Pi Mu Epsilon J. Sutherland Frame Lecture

#### Friday, August 8, 8:00–8:50 PM, Ballroom Level, Grand Ballroom

#### **Keith Devlin** Stanford University (devlin@stanford.edu)

#### Fibonacci and the First Personal Computing Revolution

The first personal computing revolution took place not in Silicon Valley in the 1980s but in Pisa in the 13th Century. The medieval counterpart to Steve Jobs was a young Italian called Leonardo, better known today by the nickname Fibonacci. Thanks to a recently discovered manuscript

in a library in Florence, the story of how this genius, about whom we know so little, came to launch the modern commercial world can now be told.

Based on Devlin's book *The Man of Numbers: Fibonacci's Arithmetical Revolution* (Walker & Co, July 2011) and his co-published companion e-book *Leonardo and Steve: The Young Genius Who Beat Apple to Market by 800 Years.* 

### The Jean Bee Chan and Peter Stanek Lecture for Students

#### Thursday, August 7, 1:00–1:50 PM, Ballroom Level, Grand Ballroom I

Jack Graver Syracuse University (jegraver@syr.edu)

#### The Founding of Pi Mu Epsilon 100 Years Ago

On December 8, 1913 the Mathematical Club of Syracuse University met on the 10th anniversary of it's founding. "Miss Florence Richert presented the paper 'The Evaluation and Transcendence of Pi.' Discussion followed the reading of the paper." There was a short business meeting before refreshments at which "Dr. Roe gave the report of committee appointed to consider changing the Club into a Mathematical Fraternity." The decision was made to proceed with this project and during the Spring semester of 1914 the details were worked out and PME was incorporated in New York State.

What did they want to accomplish by taking this action? What was background against which the decision to found PME was made? Using the Archives of Syracuse University and the detailed notes left by the founders, I hope to answer these questions and to develop an understanding of the historical context in which PME was born.

### NAM David Harold Blackwell Lecture

#### Friday, August 8, 1:00–1:50 PM, Ballroom Level, Grand Ballroom I

#### Mark Lewis Cornell University (mark.lewis@cornell.edu)

#### Markov Decision Processes, Turnpike Horizons and Blackwell Optimality

A Markov decision process (MDP) is defined by the 5-tuple: the time horizon, a state space, a transition matrix (of a Markov chain), a control or action space, and a cost or reward function. Together, this constitutes a mathematical construct for optimizing dynamic decision-making under uncertainty. In this talk, we outline the components of an MDP, discuss classic results of Blackwell, then show how Blackwell's optimality criterion can be used in controlled queueing systems and for the development of turnpike horizons.

## Martin Gardner Centennial Lecture

#### Saturday, August 9, 2:30–3:20 PM, Ballroom Level, Grand Ballroom I

Persi Diaconis Stanford University (diaconis@math.stanford.edu)

#### The Magic of Martin Gardner

Martin Gardner somehow managed to make mathematical ideas come alive to the broadest spectrum, from runaway teenagers to homemakers to professional mathematicians. The question is "How did he do it?" Along the way he exposed fake mediums, elucidated Alice in Wonderland, wrote awful poetry, and explained magic tricks. One recurrent theme in his writing: He thought that tricks, riddles, and jokes should be woven into our teaching. Neither the education establishment nor the upper crust of mathematicians seems to agree. I'll try to bring all of this to life. But, beware, as someone once wrote: "Warning: Martin Gardner has caused dozens of innocent youngsters to become professional mathematicians and thousands of professional mathematicians to become innocent youngsters."

## **Alder Awards**

## Alder Award Session

#### Friday, August 8, 2:00–2:50 PM, Ballroom Level, Grand Ballroom I & II

In January 2003 the MAA established the Henry L. Alder Award for Distinguished Teaching by a Beginning College or University Mathematics Faculty Member to honor beginning college or university faculty whose teaching has been extra-ordinarily successful and whose effectiveness in teaching undergraduate mathematics is shown to have influence beyond their own class-rooms. Each year, at most three college or university teachers are honored with this national award. The awardees are invited to make a presentation at MathFest on their work.

This year's honorees are:

Lara PudwellValparaiso UniversityDominic KlyveCentral Washington University

#### Lara Pudwell Valparaiso University

#### The Joy of Discovery

2:00-2:20 РМ

Arguably, one of the highlights of mathematics research is the joy of finally completing the proof of a new result after a long period of work. On the other hand, this feeling of happiness in mathematics is foreign to many students who view mathematics as rigid computation or who give up on a problem if it cannot be solved within 5 minutes. In this talk, I will discuss my endeavors to share the joy of discovery with undergraduates at a variety of levels, including in calculus classes, an experimental mathematics course, and various undergraduate research programs.

#### **Dominic Klyve** Central Washington University

#### There's Treasure Everywhere: When Student Work Matters

2:30-2:50 РМ

No one wants to believe that the work they do is unappreciated or unnoticed. All of us want our efforts to matter—and this includes our students. Traditional grading of homework and tests helps their work matter (since it determines their grade), but perhaps we can help students become more invested in their coursework by making their work important to other people as well. In this talk, we describe some efforts by the author and others to guide students to invest deeply in their work, by helping them build real connections between the classroom and the larger world.

## **Invited Paper Sessions**

## **Mathematical Epidemiology**

#### Thursday, August 7, 1:00–3:50 PM, Plaza Level, Pavillion East

#### Organizer: Ricardo Cortez Tulane University

Mathematical Epidemiology has grown at an accelerated pace over the last two decades through the integration of mathematical models, available data, computational methods and fieldwork. Successful epidemiological models are validated using parameters from particular epidemics, can predict likely outcomes of an epidemic, and can be used to propose specific interventions strategies.

Modern epidemiological models involve temporal and spatial features, age structure, transmission across networks or patches, deterministic and stochastic elements, seasonality, ecological factors, and more. The inclusion of these features also calls for new mathematical analysis of the models. This session features expository presentations covering a variety of aspects of modern Mathematical Epidemiology.

#### **Carrie Manore** Tulane University (cmanore@tulane.edu)

#### Comparing Risk for Chikungunya and Dengue Emergence using Mathematical Models

1:00–1:20 рм

Chikungunya is a re-emerging mosquito-borne infectious disease native to Africa that is currently spreading rapidly across the Caribbean. Two common mosquito species, Aedes aegypti and Aedes albopictus, which are found all over the world, are competent vectors for chikungunya virus. We design and analyze a nonlinear coupled system of ordinary differential equations with mosquito dynamics for the spread of chikungunya. The spread of chikungunya is then compared to that of another common mosquito borne virus, dengue. We use sensitivity analysis to indicate where future research and mitigation efforts can focus for greatest effect in controlling the spread of chikungunya.

#### Paul Hurtado Mathematical Biosciences Institute (hurtado.10@mbi.osu.edu)

## How Are Fish Population Dynamics Shaped by a Changing Environment? Insights from a Mathematical Model Driven by Temperature and Dissolved Oxygen Data from Lake Erie

1:30–1:50 рм

In this talk, I will first introduce and discuss a spatially explicit model of fish growth and survival that captures their movement through a heterogeneous physical environment here with strong gradients in water temperature and levels of dissolved oxygen [DO]. I'll focus on the population consequences of seasonal hypoxia (low DO) by using a species-specific fish bioenergetics model and high-resolution spatiotemporal data from Lake Erie to drive this model. I'll then discuss some hypotheses about how seasonal hypoxia might affect disease dynamics in pelagic fish species, using an extension of this modeling framework that includes infectious disease transmission.

#### **Bree Cummins** Montana State University (cummins@math.montana.edu)

## Determining Causal Networks in Nonlinear Dynamical Systems: Ecosystem Applications 2:00–2:20 PM

Given a set of time series of environmental variables, such as species populations, disease incidence, or climate, we would like to be able to deduce the relationships between them strictly from data analysis. In particular, we would like to know if one variable partially causes another, if there is mutual feedback, or if two variables are unrelated. There are many techniques that attempt to do this, and I will discuss one technique that is appropriate for analyzing deterministic nonlinear systems that are weakly coupled and that exist on an attractor. By this I mean that the variables are not completely synchronized - they maintain some independent dynamics - and they have an invariant character over time, but may be unpredictable due to the existence of a chaotic attractor. This work is in collaboration with Tomas Gedeon, Kelly Spendlove.

#### Kyle Hickmann Los Alamos and Tulane University (khickma@tulane.edu)

#### Epidemic Forecasting and Monitoring using Modern Data Assimilation Methods

#### 2:30-2:50 РМ

When was the last time you checked a weather report to see if you need to bring a coat or an umbrella with you to work? Imagine being able to check a report on the likelihood of contracting influenza to determine if you should wear a facemask or be extra vigilant about hand sanitizing. In this talk we will detail how the combination of three research areas in mathematics can be brought together to create such a report. As methods of modeling disease spread have grown more accurate, and new data sources for monitoring disease spread have become available, predicting the future course of seasonal disease outbreaks accurately has become more feasible.

#### **Stephen Wirkus** Arizona State University (swirkus@asu.edu)

## Qualitative Inverse Problems using Bifurcation Analysis in the Recurrent Neural Network Model 3:00–3:20 PM

We develop a framework for determining when there is bistability and multistability in the expression states of systems described by this network model. Our results show that, although bistability can be generated with autoregulation, it is also the case that both autorepression or no autoregulation can yield bistability as long as a sigmoidal behavior is present. Additionally, our results suggest that allowing only a single connection when inferring a network may be a reason why parameter values in the inferred gene networks in the literature are not realistic.

#### Abdul-Aziz Yakubu Howard University (ayakubu@howard.edu)

### Mathematics of Planet Earth 2013+: Management of Natural Resources

3:30-3:50 рм

The workshop on management of natural resources at Howard University in DC from June 4-6, 2015 will be investigating challenges for the mathematical sciences using models that describe processes affecting water, forests and food supplies. In this talk, we will use water contaminants and associated waterborne diseases to introduce some of the mathematical challenges.

## **Connections between Logic and Arithmetic Geometry**

#### Thursday, August 7, 1:45–3:45 PM, Ballroom Level, Grand Ballroom II

#### Organizer: Bjorn Poonen Massachusetts Institute of Technology (poonen@math.mit.edu)

In the past few years, ideas from model theory and computability theory, branches of logic, have led to proofs of new results in arithmetic geometry. Sometimes these ideas from logic serve as inspiration by analogy; other times they are directly used in the proofs. The proposed session will consist of survey talks by experts, suitable for a broad audience.

**Russell Miller** Queens College, City University of New York (russell.miller@qc.cuny.edu)

#### Computability Theory at Work: Factoring Polynomials and Finding Roots

#### 1:45-2:15 РМ

Given a field F, we consider two fundamental questions about polynomials p in F[X]. First, which ones have proper factorizations in F[X]? And second, which ones have roots in F? Clearly these questions are related, and initially it may seem that the first one is easier: when p has degree > 3, finding a factorization appears easier than finding a root. However, by the same token, it should then be harder to show that p has no factorization than it is to show that p has no root. So the basic intuitions do not suggest how to compare the difficulty of these questions.

The point of this talk is to show how computability theory (a.k.a. recursion theory) allows us to address the problem of deciding which of these two questions is more difficult. Working in a computable field F (that is, a countable field in which the field operations can be computed by a Turing machine), we first describe *Turing reductions*, using the above two questions as an illustration. Each question turns out to be Turing-reducible to the other, meaning that, by this measure, they have the same level of difficulty. However, we also describe the very natural notion of an *m-reduction*. All *m*-reductions are Turing reductions, but not vice versa, and we will see that *m*-reducibility definitively establishes which of the two questions is more difficult. To find out which it is, come to the talk!

#### Alice Medvedev University of California - Berkeley (alice@math.berkeley.edu)

## The Zilber Trichotomy Principle for Algebraic Dynamics: Hands-On Examples of Deep Notions from Model Theory 2:30–3:00 PM

Algebraic dynamics, the study of discrete dynamical systems given by polynomial equations, can be seen as a branch of arithmetic geometry. Some conjectures from arithmetic geometry, such as the Manin Mumford Conjecture and the Mordell-Lang Conjecture, admit natural generalizations in terms of invariant varieties of algebraic dynamical systems. Ideas from model theory, a branch of mathematical logic, have shed some light on these questions. This talk will not assume any familiarity with any of these.

I will focus on the following concrete question: given several one-variable polynomials  $f_i$ , what polynomials  $P(x_1, x_2, ..., x_n)$  are invariant under the function  $F(x_1, ..., x_n) := (f_1(x_1), ..., f_n(x_n))$ ? Many simple examples will be used to illustrate deep model-theoretic notions such as "disintegrated geometry" and the Zilber Trichotomy Principle.

#### Florian Pop The Pennsylvania State University (pop@math.upenn.edu)

#### On the Elementary Theory of Finitely Generated Fields

#### 3:15-3:45 РМ

The aim of this talk is to give an introduction to the so called "elementary equivalence vs isomorphisms problem" concerning fields K which are finitely generated (over their prime fields). Part of that problem is to give describe by a first order sentence the fact that K has characteristic zero, and/or that K has transcendence degree d. It is actually conjectured that for every such K there exists a first order sentence which characterizes the isomorphism class of K. This is the main open problem in the elementary theory of finitely generated fields.

## **Computational Aspects of Algebra, Geometry, and Combinatorics**

### Friday, August 8, 1:00–5:15 PM, Plaza Level, Pavillion West

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Sara BilleyUniversity of Washington (billey@math.washington.edu)Benjamin YoungUniversity of Oregon (bjy@uoregon.edu)
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This session will highlight recent advances in mathematics inspired by experimental and computational aspects of research. The talks will be in areas of combinatorics and probability related to algebra and geometry. This is a highly active area of research, which often lends itself to interesting talks accessible to a wide audience.

Federico Ardila San Francisco State University (federico@sfsu.edu)

#### The Combinatorics of CAT(0) Cubical Complexes and Robotic Motion Planning

#### 1:00–1:30 рм

A cubical complex is CAT(0) if it has global non-positive curvature; informally, "all its triangles are thin". These complexes play an important role in pure mathematics (group theory) and in applications (phylogenetics, robot motion planning, etc.). In particular, as Abrams and Ghrist observed, when one studies the possible states of a discrete robot, one often finds that they naturally form a CAT(0) cube complex. Gromov gave a remarkable topological/combinatorial characterization of CAT(0) cube complexes. We give an alternative, purely combinatorial description of them, allowing a number of applications. In particular, for many robots, we can use these tools to find the fastest way to move from one position to another one. The talk will describe joint work with Tia Baker, Megan Owen, Seth Sullivant, and Rika Yatchak. It will require no previous knowledge of the subject.

Kathryn Nyman Willamette University (knyman@willamette.edu)

#### A Borsuk-Ulam Equivalent that Directly Implies Sperner's Lemma

#### 1:45–2:15 рм

Sperner's Lemma is an elegant combinatorial analog to Brower's Fixed Point Theorem and both are equivalent to the KKM Lemma. Additionally, these three cozy theorems are implied by another triple of related results: Tucker's Lemma, the Borsuk-Ulam Theorem, and the LSB Theorem. In each of these triples, the first result is combinatorial, the second topological, and the third is a set covering result. Although direct proofs have been shown between the topological and set covering pairs of these theorems, a direct link showing Tucker's Lemma implies Sperner's Lemma was missing. We show that another combinatorial result, Fan's N + 1 Lemma, is also equivalent to the Borsuk-Ulam Theorem and directly implies Sperner's Lemma. Consequently, we might think of Fan's N + 1 Lemma as a more natural combinatorial analogue to the Borsuk-Ulam Theorem. Joint work with Francis Edward Su.

Dan Romik University of California - Davis (romik@math.ucdavis.edu)

#### The Combinatorics of Fully Packed Loops and Razumov-Stroganov Conjectures

#### 2:30-3:00 рм

Fully packed loops are certain interesting arrangements of lines on an  $N \times N$  square lattice that form connections between different points along the boundary. In 2001, physicists Razumov and Stroganov discovered using computer experimentation that these objects encode the answer to a seemingly unrelated question on the probability distribution of a certain random walk on "handshake patterns", which are ways for an even number of people standing around a table to shake hands without different hands having to cross over each other. This empirical discovery was finally given a beautiful proof in 2010 by Cantini and Sportiello. I will explore the story of this remarkable result and tell about some mysterious variations of the same problem that remain poorly understood.

#### David Perkinson Reed College (daviddp@reed.edu)

#### **Parking Functions and Tree Inversions**

#### 3:15-3:45 рм

We describe a bijection between the parking functions and the spanning trees of a labeled graph which relates the degree of a parking function with the number of inversions of its associated spanning tree. The special case of the complete graph solves a problem posed by Richard Stanley. This is joint work with Qiaoyu Yang and Kuai Yu.

#### Austin Roberts University of Washington (austinis@uw.edu)

#### Expanding Hall-Littlewood Polynomials into Schur Functions

#### 4:00-4:30 рм

The Hall-Littlewood polynomials and the Schur functions are important in the fields of symmetric functions, representation theory, and gauge theory, amongst others. The Hall-Littlewood polynomials have a simple expansion in terms of the Fundamental quasisymmetric functions.

By thinking of each term of these expansions as the node of a graph—called a dual equivalence graph—we demonstrate how to associate each component with a single Schur function, providing a new expansion of Hall-Littlewood polynomials in terms of Schur functions. We will also point out other topics in algebraic combinatorics where graph structures may lead the way to simpler transitions between bases.

#### Alexander E. Holroyd Microsoft Research (holroyd@microsoft.com)

#### Self-Organizing Cellular Automata

#### 4:45-5:15 рм

Cellular automata display an extraordinary range of behavior, ranging from very simple to apparently chaotic, with many cases in between. Perhaps the most interesting rules are those that yield multiple behavior types from different initial conditions—this is common even for one-dimensional rules started from finitely-supported seeds. If a rule yields chaos from some initial condition, it is tempting to conclude by analogy with the second law of thermodynamics that chaos should be prevalent from almost all initial conditions. For a certain natural class of rules, we prove that the opposite holds: typical (i.e., random) initial seeds self-organize into predictable (but non-trivial) evolution, while exceptional seeds generate more complicated behavior, including apparent chaos.

Deterministic evolution from random initial conditions is a combination that strikes fear into the hearts of probabilists—some randomness in the evolution typically makes analysis much easier. However, we were able to side-step this issue by taking advantage of simple linear algebra together with fractal-like properties of certain cellular automata.

See http://research.microsoft.com/~holroyd/ca/webxor.png for a picture. No prior knowledge assumed. Joint work with Janko Gravner.

## The Mathematics of Biological Fluid Dynamics

#### Friday, August 8, 2:00–4:50 PM, Plaza Level, Pavillion East

#### **Ricardo Cortez** Tulane University (rcortez@tulane.edu)

One exciting area of mathematical research within Mathematical Biology is "biological fluid dynamics," which consists of explaining and understanding the interaction of fluids and living organisms. This includes the motion of microorganisms such as bacteria and algae, cell motion, the fluid flow in the respiratory and cardiovascular systems, flying and swimming, and much more. The research problems are inspired by the need to understand basic functions of life, such as reproduction, growth, feeding, and locomotion.

The mathematics of biological fluid dynamics involves developing theory, creating models, and designing computational methods for numerical simulations of the systems being investigated. This is typically done in collaboration with experimentalists and other scientists. This expository session highlights a variety of applications of the mathematics behind biological fluid dynamics and identifies current research questions in this area.

#### Lisa Fauci Tulane University (fauci@tulane.edu)

#### Neuromechanics and Fluid Dynamics of an Undulatory Swimmer

#### 2:00-2:20 рм

The swimming of a simple vertebrate, the lamprey, can shed light on the coupling of neural signals to muscle mechanics and passive body dynamics in animal locomotion. We will present recent progress in the development of a multiscale computational model of the lamprey that examines the emergent swimming behavior of the coupled fluid-muscle-body system.

#### Robert Dillon Washington State University (dillon@math.wsu.edu)

#### Mathematical Modeling of Sperm Motility and Mucociliary Transport

#### 2:30-2:50 РМ

The motility of sperm flagella and cilia are based on a common physiological structure capable of generating a wide range of dynamical behavior. We describe a fluid-mechanical model for sperm and cilia coupling the internal force generation of dynein molecular motors through the passive elastic axonemal structure with the external fluid mechanics. As shown in numerical simulations for motile sperm, the model's flagellar waveform depends strongly on viscosity as well as dynein strength. We will also show numerical simulations of multiciliary interaction.

#### **Hoa Nguyen** Trinity University (hnguyen5@trinity.edu)

#### Modeling E. Coli Aspartate Chemotaxis in a Stokes Flow

#### 3:00-3:20 рм

For certain bacteria such as Escherichia coli, bidirectional propeller-like rotary motion of flagellar filaments results in the net movement of the cell through gradients of chemoattractant molecules toward areas of higher attractant concentrations. Directional switching of the flagellar

motor is governed by a phosphorelay circuit that transfers phosphoryl groups from donor to acceptor proteins; and protein phosphorylation state is controlled by binding of chemoattractants to specific receptors. The three-dimensional hydrodynamics of cell motility is modelled by coupling the chemotaxis equations of a simplified phosphorylation cascade with the method of regularized Stokeslets of the fluid motion through cell transport and external forcing from the flagellar motor. The results demonstrate how the phosphorylation affects the run and tumble mechanism of swimming bacteria. This work is in collaboration with Robert Bierman (Biology student), Jordan Bush (Math student), and Frank Healy (Biology faculty).

**Katarzyna A. Rejniak** H. Lee Moffitt Cancer Center & Research Institute and University of South Florida (Kasia.Rejniak@moffitt.org)

#### Modeling Interactions between Tumor Cells, Interstitial Fluid and Drug Particles

3:30-3:50 рм

The interactions between tumor cells and their microenvironment are complex, and this complexity is leveraged when both tumor and stromal cells are exposed to anticancer therapeutic agents. We will present a model based on fluid-structure interaction techniques that allows us to explore the role of the interstitial fluid flow and tumor tissue architecture on the extent of drug penetration into the tissue. We show how this model can be parameterized using available data from laboratory experiments, and how it can be applied to test various properties of anti-cancer agents.

#### Julie Simons Tulane University (jsimons@tulane.edu)

#### Sperm Motility and Cooperativity in Epithelial Detachment

4:00-4:20 рм

On their way to fertilize the oocyte, mammalian sperm must undergo drastic changes in their behavior and biochemistry. Along this journey, sperm have been shown to bind to cilia lining the oviductal epithelium and break these bonds near the time of ovulation. It has been hypothesized that changes in motility enable sperm to break free of the surface and progress through the oviduct. Using the method of regularized Stokeslets, we explore the separate roles that surfaces and binding have upon swimming in the viscous environment that sperm encounter. We also investigate whether nearby sperm will effectively cooperate to escape a surface.

Jacek Wrobel Tulane University (jwrobel@tulane.edu)

#### Swimming through Heterogeneous Viscoelastic Media

4:30-4:50 РМ

Elastic polymers and filamentous networks immersed in a fluid environment are common in biology. Those biological fluids are often inhabited by microorganisms whose motility depends on the surroundings. We consider a simple model of a free microswimmer in a highly heterogeneous, viscoelastic medium. An effect of viscoelasticity is modeled by immersing viscoelastic structures in viscous environment. Varying complexity of those structures allows medium to exhibit different viscoelastic properties. Regions of higher structural density can significantly change a swimming pattern of a microorganism.

## The Eyes Have It: Mathematical Modeling of the Retina

#### Saturday, August 9, 1:00–2:50 PM, Plaza Level, Pavillion West

Organizer: Erika Camacho Massachusetts Institute of Technology and Arizona State University (erika.camacho@asu.edu)

Models of the retina are crucial in understanding various retinal diseases and abnormalities that contribute to blindness such as myopia, glaucoma, retinitis pigmentosa, and others. In this session speakers will present mathematical models of retinal detachment, retinal blood flow, and melanopsin activation and inactivation. Utilizing a diverse set of mathematical techniques, analysis, and computer simulations from dynamical systems, numerical analysis, and stochastic processes these models investigate complex retinal process including elevated ocular pressure and forces from retinal adhesion, retinal pigment epithelium pumps, and retinal elasticity leading to retinal detachment, alterations in ocular curvature caused by a reduction retinal blood flow, and the chemical reaction associated with non-image forming process in the retina.

#### Thomas Chou UCLA Department of Biomathematics (tomchou@ucla.edu)

#### Mechanical Models for Exudative Retinal Detachments

#### 1:00-1:20 рм

We present a model of the mechanical and fluid forces associated with exudative retinal detachments where the retinal photoreceptor cells separate typically from the underlying retinal pigment epithelium (RPE). By computing the total fluid volume flow arising from transretinal, vascular, and retinal pigment epithelium (RPE) pump currents, we determine the conditions under which the subretinal fluid pressure exceeds the maximum yield stress holding the retina and RPE together, giving rise to an irreversible, extended retinal delamination. We also investigate localized, blister-like retinal detachments by balancing mechanical tension in the retina with both the retina-RPE adhesion energy and the

hydraulic pressure jump across the retina. For detachments induced by traction forces, we find a critical radius beyond which the blister is unstable to growth. Growth of a detached blister can also be driven by inflamed tissue within which, e.g., the hydraulic conductivities of the retina or choroid increase, the RPE pumps fail, or the adhesion properties change.

#### Andrea Dziubek SUNY Institute of Technology (dziubea@sunyit.edu)

#### New Paradigms in Retinal Blood Flow Simulation

#### 1:30-1:50 рм

Clinical studies show that ocular hemodynamics play an important role in many ocular diseases, such as glaucoma and myopia. Blood flow in the retina is a multi-scale phenomena; on the largest scale blood flows through arterioles and venules and on the smallest scale single red blood cells move through the capillaries. We developed a model of the retinal vasculature as a layered porous medium on a curved surface where the geometry of the larger vessels is described by network and fractal models. The solution of the resulting system of partial differential equations on a curved surface is a challenging problem beyond the particular application to ophthalmology. To be able to conduct clinically relevant studies with patient specific and population-based data we mathematical modelers work together with eye specialists. We will also give a short overview of current research on retinal blood flow modeling.

#### William J. Bottega Rutgers University Department of Mechanical and Aerospace Engineering (bottega@rci.rutgers.edu)

#### Analytical Mechanics and Evolution of a Detaching Retina

#### 2:00-2:20 рм

A mechanics based mathematical model for retinal detachment is developed, incorporating an energy based criterion for propagation. Retinas with and without central tears are considered and contraction of the vitreous and extension of its fibrils, along with a pressure difference across the retina, are taken as the stimuli for detachment propagation. Treatment for the affliction by mechanical banding is also modeled. In addition to the equations of motion, boundary and matching conditions, the variational formulation yields the self consistent energy release rate that governs detachment, and formulae for critical stress and critical deflections that provide a rational basis for measuring critical parameters. Exact analytical solutions are established for axisymmetric detachment of retinas with and without tears, and numerical simulations are performed based on these solutions. The results yield characteristic behavior, including threshold levels and stability of detachment, "dimpling" of the detaching retina, the effects of changes in material and geometric parameters, and the influence of the presence and size of the retinal tear on detachment propagation. The model predicts that once detachment ensues it does so in an unstable manner and is extensive in scope. This is in agreement with clinical observation. The predicted presence of "dimples" in the detaching retina appear to correspond to clinically observed phenomena as well. Results also suggest that, under appropriate conditions, the presence and size of a retinal tear or hole can have a "stabilizing" effect with regard to detachment propagation. Results for banding as a mechanism to close the detached area suggest the influence of the parameters of the band on its effectiveness with regard to closure and minimizing induced ocular pressure.

#### Christina Hamlet Tulane University Center for Computational Science (chamlet@tulane.edu)

#### Stochastic Modeling of Melanopsin Activation and Deactivation

#### 2:30-2:50 рм

Vertebrate melanopsin is a photo-pigment recently identified to be involved in retinal non-image forming processes such as circadian rhythm coordination and light/dark sensing. A stochastic model using Gillespie's Algorithm was constructed based on invertebrate rhodopsin cascades and is presented as a proposed mechanism for the vertebrate melanopsin cascade. Systems involving single light flashes and multiple light flashes were simulated and compared to experimental data. Results from the constructed model capture many of the key characteristics of the activation/deactivation process and sensitivity analysis indicates several reactions that are thought to strongly govern the kinematics of the cascade.

## Fast Algorithms on Large Graphs (and Matroids)

#### Saturday, August 9, 1:00–3:45 PM, Plaza Level, Pavillion East

#### Organizers: Brigitte Servatius Worcester Polytechnic Institute (bservat@math.wpi.edu) Martin Milanič University of Primorska

Very large graphs, such as the internet, have become part of our daily routine. Quite naturally they pose new challenges for the mathematician. What are the methods and tools to find out something about a structure so large that we cannot know all of it? Being greedy seems a successful real life strategy familiar to most of us.

Matroids are the most general structures on which the greedy algorithm finds a basis. Communications networks, such as the internet, organic molecules, quasicrystals, etc. are modeled by large graphs. The coarsest analysis uses the matroid structure only. However, in a general geometric setting many problems become hard. For example connectivity augmentation can be solved efficiently on matroids, but becomes NP-hard for geometric planar graphs, even on trees. The purpose of this session is to identify graph properties relevant to current applications

and their complexity behaviour as the setting is changed from matroid to graphs and geometric graphs. Speakers will direct their talks on this rapidly developing topic to a general audience.

#### Gary Gordon Lafayette College (gordong@lafayette.edu)

#### Pick a Tree, Any Tree

1:00–1:30 рм

Trees are an extremely important and useful topic in graph theory and network design. I'll talk about some of the motivation and history of the subject, including Cayley's famous formula that counts the number of spanning trees of a complete graph. Then we'll use that formula to figure out the probability that a randomly chosen subtree of a complete graph is a spanning tree. This is joint work with Alex Chin, Kelly MacPhee and Charles Vincent, three undergraduates in Lafayette College's REU program last summer.

#### Andrzej Proskurowski University of Oregon (andrzej@cs.uoregon.edu)

#### **Multi-Source Spanning Trees of Graphs**

#### 1:45-2:15 рм

Given a combinatorial graph with edge weights and a subset of nodes defined as sources, we want to find a spanning tree of the graph that minimizes some distance related cost metric. This problem can be used to model multicasting in a network where messages are sent from a fixed collection of senders and communication takes place along the edges of a single spanning tree. For a limited set of possible cost metrics of such a spanning tree, we either prove the problem is NP-hard or demonstrate the existence of an efficient algorithm to find an optimal tree.

#### Randy Paffenroth Numerica Corporation (randy.paffenroth@numerica.us)

#### Large Graphs in Internet Tomography and Cyber Defense

2:30-3:00 рм

Large graphs arise naturally in many network analysis problems, such as Internet tomography and cyber defense. For example, the Cooperative Association for Internet Data Analysis topology data set already consists of well over a petabyte of data and is growing on a daily basis. In this talk, rather than focusing on the network connectivity itself, we will instead consider graphs which arise from the dependence and independence of time-series measurements of network properties. Our key methodologies will include modern ideas in matrix completion and robust principle component analysis which efficiently scale to millions of sensors. In addition, we will discuss how properties of these large graphs are relevant to several current applications.

#### Brigitte Servatius Worcester Polytechnic Institute (bservat@math.wpi.edu)

#### Large and Sparse Graphs

3:15-3:45 РМ

In his survey article "Very Large Graphs," in *Current Developments in Mathematics*, 2008, pp. 67–128, Int. Press, Somerville, MA, László Lovász points out that "in the last decade it became apparent that a large number of the most interesting structures and phenomena of the world can be described by networks."

The best known and most studied is the *internet*, see Rany Paffenroth's talk in this session. We will concentrate on some techniques for sparse graph sequences as outlined by Lovász and touch on their applications in statistical physics and biology concerning flexibility analysis in biomolecules and networks.

## **Themed Contributed Paper Sessions**

# Undergraduate Research Activities in Mathematical and Computational Biology

### Session 1: Thursday, August 7, 8:30–10:25 AM, Plaza Level, Broadway I & II

#### Organizer: Timothy D. Comar Benedictine University (tcomar@ben.edu)

This session is dedicated to aspects of undergraduate research in mathematical and computational biology. First and foremost, this session would like to highlight research results of projects that either were conducted by undergraduates or were collaborations between undergraduates and their faculty mentors. Of particular interest are those collaborations that involve students and faculty from both mathematics and biology. Secondly, as many institutions have started undergraduate research programs in this area, frequently with the help of initial external funding, the session is interested in the process and logistics of starting a program and maintaining a program even after the initial funding expires. Important issues include faculty development and interdisciplinary collaboration, student preparation and selection, the structure of research programs, the acquisition of resources to support the program, and the subsequent achievements of students who participate in undergraduate research in mathematical and computational biology.

#### Anne Elizabeth Yust Birmingham-Southern College (ayust@bsc.edu)

#### A New Technological Paradigm for an Undergraduate Research Experience in Agent Based Modeling

Agent based modeling is an accessible field to undergraduates where they can make significant and novel contributions. The process gives students the opportunity to participate in literature reviews on a biological species, experience the mathematization of the species characteristics, creating an agent based model that replicates the known species behavior, and finally assess the results to determine the validity of the model. I will give an overview of the current biomathematics research project I am involved in with a student on my campus and an intercollegiate student/faculty duo at Rhodes College, a campus located 250 miles from Birmingham. I will also discuss the new paradigm for student/faculty research that we implemented to expose our students to the emerging methods of collaborative communication through video chat and other forms of technology. Finally, I will touch on the small grants and support we have received externally and internally to implement the program in the summer of 2013 and how we are continuing to support the student research experience in 2014.

#### **Timothy D. Comar** Benedictine University (tcomar@ben.edu)

#### Impulsive Models with Stochastic Behavior in Pest Management and Epidemiology

For several years, we have been mentoring undergraduate research projects studying the dynamics of impulsive differential equation models for pest management and epidemiology. Impulsive behavior in pest management models may appear as birth pulses, the application of pesticide, or the periodic introduction of predators or diseased pests into the ecosystem. In epidemiological models, impulsive behavior may appear as an impulsive vaccination strategy or drug therapy. We are now considering the introduction of stochastic behavior in the models such as random birth pulses and disease transmission rates in conjunction with mixed models. We present the models and results obtained in this research work with students.

#### **David R. Dorman** Middlebury College (dorman@middlebury.edu)

#### Getting into the Game: First Steps Into Math-Bio Research

Middlebury College is a small liberal arts institute. Mathematics and Biology have long been studied at the College but mathematical biology is relatively new. Nevertheless, several of our students have used the senior thesis program as an opportunity to combine both areas. Some recent theses include:

- (1) Dynamics of the Discrete Logistic Equation
- (2) Bifurcation Theory, the Allee Effect, and Infectious Disease Modeling
- (3) Transmission Dynamics of Hepatitis B.

We describe the senior thesis program and the trials, tribulations, failures and successes students have had while tackling these projects.

#### Dan Hrozencik Chicago State University (dhro@att.net)

#### A Course in Mathematical Biology Using Algebra and Discrete Mathematics

This talk will focus on the author's efforts to develop a course with minimal prerequisites that would introduce the topic of mathematical biology and stimulate undergraduate research at the author's institution. The talk will discuss the content of the course, prerequisites, recruiting students, plans for continuing research after the course, and lessons learned.

#### Irina Seceleanu Bridgewater State University (iseceleanu@bridgew.edu)

#### Mentoring an Undergraduate Research Project: Simulating the Effects of Plaque Aggregation on the Neuronal Network

In this talk we present the results of an undergraduate research project about the effects of plaque aggregation on the neuronal network in the human brain, and discuss aspects of the logistics of the project, the mentoring experience, the interdisciplinary collaboration with Biology faculty, as well as the subsequent achievements of the student. The talk will highlight the algorithm used to simulate the complex biological system of the neuronal network, and the probabilistic process employed to simulate the formation of plaque deposits in clusters. To study the effects of the plaque granules on the neuronal network, we integrate the two models and measure the number of neuronal connections before and after the depositing of plaque. Moreover, we will discuss many aspects of the very rewarding, but challenging experience of mentoring an interdisciplinary undergraduate research project.

Hannah BiegelUniversity of Portland (biegel15@up.edu)Alex QuackenbushUniversity of Portland (quackenb15@up.edu)Hannah CallenderUniversity of Portland (callende@up.edu)

#### Sensitivity Analysis of Stochastic Models of Integrin Signaling in Cellular Motility

A cell's ability to move to the correct location at the correct time is vital for maintenance of homeostasis; improper movement is often indicative of a pathogenic phenotype. As such, it is critical to understand the molecular phenomena of motility. A key step in the process of cell motility is the development of focal adhesions, which are protein complexes involving cytoskeletal elements, membrane bound proteins, and extracellular matrix components. A fundamental part of focal adhesions is integrin, the transmembrane receptor protein that links the actin cytoskeleton to extracellular matrix proteins. Here we develop and analyze a stochastic model of a nascent focal adhesion. The model captures the dynamics of the rate reactions over time between extracellular ligand molecules, intracellular adhesion proteins called talin, and integrins. We discuss results from sensitivity analysis screening using an adapted version of the Morris Method to account for the stochasticity of the model. Such analysis is useful for improving the model and for developing theories about the underlying biological process of focal adhesion creation and of cell motility in general.

#### Session 2: Friday, August 8, 8:30–10:45 AM, Plaza Level, Broadway I & II

#### Sheldon Lee Viterbo University (shlee@viterbo.edu)

#### Mathematical Biology as a Capstone Option for Science Majors

All biology, chemistry, and physics majors at our institution are required to take a three-semester series of courses related to independent research. Each student is asked to find a research mentor in the area of their interest, and go through the process of writing a research proposal, carrying on a research study, and writing up and presenting their results. In this talk, I will discuss my own experiences in mentoring a pre-veterinary major and a chemistry major in studying an infectious disease model. In particular, we modeled the spread of *Anaplasmosis marginale* on a cattle population, via the *Dermacentor* tick. I will discuss the model, results, and also the challenges of fitting a mathematical modeling problem into a traditional science capstone program.

#### Nathan Robert LaFerney Texas A&M University (nathanlaferney@tamu.edu)

#### An Optimization Method for the Spent Fuel Pool Storage at Nuclear Power Plants

The storage and disposal of spent nuclear fuel is one of the most serious issues facing the nuclear industry. Nuclear power plants will often store their spent fuel in storage pools for a number of years until the fuel can be stored in a permanent setting. These fuel units are categorized into discrete groups and storage arrangements are then prescribed. Thus how to achieve an arrangement that maximizes space in the pool becomes an interesting combinatorics problem.

This presentation outlines a combinatoric method of optimizing the arrangement of the spent fuel pool located at the South Texas Project nuclear power plant. The goal of the method is to maximize usable empty space in the storage pool. This includes finding methods to check if an optimal arrangement exists and finding the optimal arrangement that is most similar to the current arrangement. This method when completed could be potentially ported to other nuclear power plants that use a similar method of storage as well as any application that requires making arrangements based on prescribed patterns.

### Chad Topaz Macalester College (ctopaz@macalester.edu)

Andrew Bernoff Harvey Mudd College (ajb@hmc.edu)

#### Social Aggregation in Pea Aphids: Experimental Measurement and Stochastic Modeling

From bird flocks to fish schools and ungulate herds to insect swarms, social biological aggregations are found across the natural world. An ongoing challenge in the mathematical modeling of aggregations is to strengthen the connection between models and biological data by quantifying the rules that individuals follow. We present experimental and modeling results on aggregation of the pea aphid, Acyrthosiphon pisum, obtained as part of a faculty-student collaboration. Specifically, we conduct experiments to track the motion of aphids walking in a featureless circular arena in order to deduce individual-level rules. We observe that each aphid transitions stochastically between a moving and a stationary state. Moving aphids follow a correlated random walk. The probabilities of motion state transitions, as well as the random

walk parameters, depend strongly on distance to an aphid's nearest neighbor. For large nearest neighbor distances, when an aphid is essentially isolated, its motion is ballistic with aphids moving faster, turning less, and being less likely to stop. In contrast, for short nearest neighbor distances, aphids move more slowly, turn more, and are more likely to become stationary; this behavior constitutes an aggregation mechanism. From the experimental data, we estimate the state transition probabilities and correlated random walk parameters as a function of nearest neighbor distance. With the individual-level model established, we assess whether it reproduces the macroscopic patterns of movement at the group level.

#### Timothy A. Lucas Pepperdine University (timothy.lucas@pepperdine.edu)

#### Spatial Simulations of Chaparral Vegetation Response to Frequent Wildfires

This talk will focus on an ongoing collaboration with an ecologist that has led to an undergraduate modeling project. For the past several years we have been studying how the recent increase in fire frequency in the Santa Monica Mountains (SMM) has drastically impacted the surrounding vegetation. Chaparral, the dominant vegetation in the SMM, can be divided into three life history types according to their response to wildfires. Nonsprouters are killed by fire, but reproduce by seeds that germinate in response to fire cues. Obligate spouters survive by resprouting because their seeds are destroyed by fire. Facultative sprouters both resprout and reproduce by seeds post-fire. The undergraduate students developed discrete-time models of species survivorship under varying fire frequencies as well as a spatial simulation that models the growth, reproduction and resprouting behavior of individual shrubs that interact in an environment similar to our study site adjacent to Pepperdine University. The students have validated these models using 29 years of local data on species survivorship, density, growth, rainfall and wildfires. Besides the research itself, I will discuss the recruitment and training of mathematics undergraduates and their contributions to the data collection, statistics, modeling and programming involved in the project.

#### Ruijun Zhao Minnesota State University - Mankato (ruijun.zhao@mnsu.edu)

#### Studying Imperfect Vaccine of Malaria Using Mathematical Models

Malaria is the most prevalent parasitic disease in tropical and subtropical areas worldwide. Recently, a global effort to eradicate the disease came back to table after about half century. Among these efforts, vaccine development for malaria has proven to be successful and the first effective vaccine against malaria, called RTS,S, has gone through all human trial stages and is about to be available for public. However, the protection of vaccine is not perfect in the sense that it is only partially and temporarily effective. In this project, we propose a mathematical model studying the efficacy of this imperfect vaccine using an age-structured model. The project involves undergraduate students research in terms of collecting data, analyzing data, and actively improving understanding the problem more.

#### Katie Marie Sipes James Madison University (sipeskm@dukes.jmu.edu)

#### Understanding the Scales of Locomotion for Caenorhabditis elegans in a Viscous Fluid

Viscosity is a measurement of a fluid's resistance to the rate of deformation. An example of a liquid with high viscosity is tree sap, or a homogeneous mixture of mud. Both of these liquids run very slowly when acted upon by gravity. In contrast, water has a very low viscosity and flows readily. So how does a liquid's characteristics effect the locomotion of a swimming organism? Do higher viscosities change the dynamics that an organism implements in order to move in a solution? The Reynolds number is defined as the ratio of inertial to viscious forces and is given by  $\frac{\rho VL}{\mu}$  where  $\rho$  is the fluid density, V is characteristic velocity, L is the characteristic length of the system, and  $\mu$  is kinematic viscosity. In particular, I am interested in measuring these different scales in a system where the worm *C.elegans* is swimming in fluids of different viscosities. We will compare these measurements to different modes of locomotion.

#### Michael E. Martin www.biomathdynamics.com (michael.e.martin@gmail.com)

#### Simulating Action Potentials Along Non-Uniform Axon

This talk will focus on simulations for the Fitzhugh-Nagumo and Morris-Lecar models for signal propagation along axons with abrupt changes in the axonal diameter. The talk presumes students have had ordinary and at least some partial differential equations. The talk stems from research with undergraduate students at Johnson County Community College and undergraduate research with participants from across the nation at the Mathematical Biosciences Institute at Ohio State University. In the simulations, traveling waves can be propagated, delayed, reflected, and split depending on the parameters of the axonal discontinuity. Simulations correspond to varicosities in the axon, demyelinated axon, and various associated disease states.

## Undergraduate Research in Mathematics: How, When, Why

#### Session 1: Thursday, August 7, 1:00–3:55 PM, Ballroom Level, Galleria II

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Organizers: Emek KoseSt. Mary's College of MarylandCasey DouglasSt. Mary's College of MarylandAngela GallegosLoyola Marymount University
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Opportunities for undergraduate research have increased dramatically in recent years. There are many benefits of doing and guiding undergraduate research. We invite talks on a range of topics including, but not limited to: involving students in mathematics research, reports on successful programs, how to set up programs, and research results. We are especially interested in presentations from mentors and program directors about how programs are run and evidence of their effectiveness. We also welcome presentations from students focused on their experience and learning outcomes (talks about their research results should be submitted to other sessions). This session seeks to expand the network of undergraduate researchers and facilitators, exchange new ideas, and help make undergraduate research more accessible.

 Ryan Brown
 Georgia College (ryan.brown@gcsu.edu)

 Marcela Chiorescu
 Georgia College (marcela.chiorescu@gcsu.edu)

 Darin Mohr
 Georgia College (darin.mohr@gcsu.edu)

#### Building Capacity for a Research Rich Curriculum in Mathematics at Georgia College

The Department of Mathematics has recently revised its curriculum to make the undergraduate research a prominent feature of the major; we require all undergraduate students to complete a year-long research project, submit a written report, and give a presentation at our department's annual capstone day. Before we could implement a robust research experience we first had to build institutional capacity to support our students and faculty, and to ensure its sustainability. This presentation will provide the roadmap we developed, the lessons learned and an outline of our plans moving forward as we implement the next phase of our curriculum building.

Jane FriedmanUniversity of San Diego (janef@sandiego.edu)Lynn Carole McGrathUniversity of San Diego (lmcgrath@sandiego.edu)Perla MyersUniversity of San Diego (pmyers@sandiego.edu)

#### Creative UG Research Collaborations: Clash of the Critters; Statistical Analysis of SIDS and More

The general public often views mathematics research as a solitary, even lonely endeavor. In fact much of mathematics research is collaborative in nature. Collaborative undergraduate research provides an environment where students can explain their mathematical reasoning to others and hear feedback on how to improve, clarify and further their ideas. Collaboration also invites different approaches to solving a problem, which can potentially deepen mathematical learning. Through collaborative undergraduate research students develop communication and other skills which are valuable in industry and in graduate school. At the University of San Diego, we have successfully conducted collaborative undergraduate research projects. We will discuss a number of different examples of successful collaborative undergraduate research projects at USD, and provide information about how we structured the projects. The student perspective will also be reported.

Michael Dorff Brigham Young University (mdorff@math.byu.edu)

#### CURM: What it is and What are its Results

The Center for Undergraduate Research in Mathematics (CURM) is a national program that promotes undergraduate research in mathematics and is funded by NSF grants for over \$2.5 million. CURM (a) trains professors as mentors for undergraduate students doing research, (b) provides funds to professors to establish undergraduate student research groups, (c) advises professors on how to continue operating undergraduate research groups at their institutions, and (d) prepares undergraduate students to succeed in graduate studies and careers in the mathematical sciences.

CURM has had tremendous success. Since its inception, CURM has provided financial support to 231 undergraduate students (54% female, 29% minority, and 2% with disability) mentored by 78 professors (41% female, 19% minority) from 71 different universities and colleges nationwide. During that time, CURM students have written over 100 joint research papers, and have given about 200 presentations. In addition, data shows that at institutions participating in the CURM program, 63% of the CURM students go on to graduate school while only about 18% of the total math majors at these schools do so.

#### **HRUMC: The First Twenty Years**

In 1993, a small group of individuals from Siena, Skidmore, Union, and Williams Colleges, four geographically close liberal arts colleges in the Northeast, met to brainstorm details of a novel approach to welcoming students to the mathematics community. From this effort, a vision crystallized of a new kind of conference, one in which students and faculty investigating the same topic would speak about their results in the same session. The following year, the first annual Hudson River Undergraduate Mathematics Conference, inspired in part by the Joint Mathematics Meetings, was held. Since then, HRUMC has hosted thousands of students and faculty from scores of colleges and universities and has convened at twelve different institutions, with a recent event held jointly with the Spuyten Duyvil UMC perhaps the first joint mathematics meeting of regional undergraduate mathematics conferences. Here, we offer comments about organization, costs, financing, continuity, interaction with other mathematics organizations, quality control, timing, assessment issues, and student and faculty involvement at all levels of planning and participation.

#### David Housman Goshen College (dhousman@goshen.edu)

#### Maple Scholars Program

The author has mentored undergraduate research projects at five institutions funded by a variety of internal and external sources both during the summer and during the academic year. Unique to his experience, the Goshen College Maple Scholars Program places an emphasis on communication across disciplines. During the eight-week summer program funded by the college, 12-20 students share a common residential space and work on scholarly projects mentored by individual faculty in any discipline. This much is typical for summer research programs. What is different is the encouragement for students to visit each others' research groups: literary critics see how biologists are trying to efficiently produce algae for biofuels while chemists try playing musical instruments being designed by physicists. During their weekly talks, students try to make their scholarly progress clear to students and faculty in other disciplines. Mathematicians suggest logistics that may make an inside-outside prison exchange possible while theologians ask how a fair division characterization theorem might apply to a local dilemma. A former math major can be involved in collecting local women's stories that are integrated into a dramatic script, and a computer science major can help create a web based data base housing stories of the Mara region of Tanzania. This talk will describe the Maple Scholars Program and compare its goals and outcomes to other undergraduate research programs in which the author has participated.

#### Eric Kostelich Arizona State University (kostelich@asu.edu)

#### The CSUMS/MCTP Program at Arizona State University

This talk will describe an undergraduate research program in computational mathematics that has run at Arizona State since 2008. Approximately 100 undergraduates, mostly sophomores and juniors ranging in age from 15 to 31, have participated in 8-week, full-time summer research projects plus an academic-year research seminar. The mentoring goals of the program, as well as some of the scientific components, will be outlined. Student alumni have won 3 NSF Graduate Research Fellowships (plus 6 honorable mentions), 3 Goldwater Scholarships, 2 NIH graduate traineeships, and numerous other awards.

#### **Donna Beers** Simmons College (donna.beers@simmons.edu)

#### The Summer 2014 SURPASs Program and My Role as Faculty Mentor

This summer my institution initiated the Summer Undergraduate Research Program At Simmons (SURPASs), an intensive research experience for undergraduates. Twenty student research associates were selected through a competitive process whereby they submitted a research proposal for a faculty-mentored project. The goal of the program is for students to "attain a fuller understanding of what it means to engage in research in their chosen disciplines." In this talk we will describe the program mechanics including the student eligibility criteria, program duration, compensation for the student research associate, course credit and tuition waiver, and the student application guidelines. We will especially focus on the structure of the summer program; the expected scholarly outcomes; our mentee's research project in graph theory; and our role as faculty mentor. We will conclude with a discussion of program assessments.

#### Joni Jane Schneider Texas State University (js1824@txstate.edu)

#### Talk Math 2 Me: A Seminar for Students by Students

"Talk Math 2 Me" is a weekly seminar that provides students with the opportunity to present research of mathematics, mathematics history, and/or interesting math ideas to an audience of their peers. The seminar is set in a casual environment for student to share how they view mathematics. This is great way for students to practice speaking in front of an audience, which is a vital skill in any field. In this presentation, we will share how this seminar program was developed and how it is changing the culture of mathematics for undergraduate students.

Brianna Donaldson American Institute of Mathematics (brianna@aimath.org)

**Leslie Hogben** American Institute of Mathematics and Iowa State University (hogben@aimath.org)

**Ulrica Wilson** Institute for Computational and Experimental Research in Mathematics and Morehouse College (uwilson@morehouse.edu)

**Roselyn Williams** Florida A&M University (roselyn.williams@famu.edu)

#### Research Experiences for Undergraduate Faculty:

#### Supporting Undergraduate Faculty in Mentoring Undergraduate Research

Research Experiences for Undergraduate Faculty (REUF) encourages and supports involvement in research with undergraduates by faculty at colleges and universities that emphasize undergraduate education. REUF is funded by the National Science Foundation and co-sponsored by the American Institute of Mathematics and the Institute for Computational and Experimental Research in Mathematics. The program actively recruits faculty from colleges and universities that serve large numbers of underrepresented minority students, students with disabilities, and/or first generation college students, as well as faculty who are members of underrepresented minority groups.

Each summer, a REUF workshop introduces a new group of faculty participants to research projects that are suitable for investigation by undergraduates. The faculty split into research groups to work on their chosen project, with each group mentored by senior faculty with extensive experience mentoring undergraduate research. Throughout the workshop, faculty also participate in sessions aimed at helping them develop a plan for mentoring undergraduate research at their home institutions. After the workshop, faculty participants can apply for REUF funding to support continuation of their own research collaborations begun at the workshop. They are also invited to become a part of the REUF community, which maintains a discussion group and a website (http://reuf.aimath.org/), and hosts alumni gatherings at national meetings.

We present details of the REUF program's organization, describe overall program results in terms of increased faculty engagement and research with students, and provide examples of successful REUF research projects. Information on how to participate will also be provided.

#### Session 2: Saturday, August 9, 8:30–11:25 AM, Ballroom Level, Galleria II

**Therese Shelton** Southwestern University (shelton@southwestern.edu)

#### **Ensuring Engagement in Math Research**

Every student in our capstone course decides on and develops a mathematical model — a daunting task for them, supported by the instructor as a consultant, guide, and evaluator. Topics have included musical tuning systems, federal and state minimum wages, distracted driving, effects of the summer heat on municipal water use, wind energy, and much more. Student presentation at a regional conference creates a common bond among the cohort, boosts student confidence and independence, and aids their writing skills. Challenges include variation in course experience, previous performance levels (struggling C- students mixed with A+ students), and varied experiences (some continue summer research while others start from scratch.) For some students, the experience has led them to consider graduate study or a career in applied mathematics — perhaps the ultimate measure of success. Project guidelines, grading rubrics, and the timeline will be shared, as well as what the mentor has learned from this evolving research experience format.

**Tom Edgar** Pacific Lutheran University (edgartj@plu.edu)

#### 6959 Open Problems for Undergraduates

We discuss a simple method of involving students in undergraduate research: mining the Online Encyclopedia of Integer Sequences for open problems. In particular, we outline a project involving two undergraduates (so far) that has recently begun this year. Using minimal departmental funds, we paid students to write software that crawls the OEIS in order to grab interesting data. Our focus has been on searching for sequences with integral generalized binomial coefficients. So far, our search of the first 100k sequences in the OEIS has yielded a possible 6569 sequences to investigate. In turn, we have initiated the process of classifying these different sequences as well as searching for proofs of the integrality of the associated coefficients. This engaging method of undergraduate research works for students of varying abilities, and introduces students to many important aspects of mathematics research including experimentation, conjecture, literature searches, computer programming, and verification.

**William Gryc** Muhlenberg College (wgryc@muhlenberg.edu)

#### **Exploring Auction Theory in Undergraduate Research**

Auction theory lies in the intersection of game theory and mathematical economics, and is especially well-suited to undergraduate research. Indeed, it is accessible to advanced undergraduates, can be explored theoretically, computationally, and experimentally, and has obvious applications to bidding and selling behavior on online auction sites including eBay and so-called penny auction sites such as Beezid.com and QuiBids.com. In this talk we will discuss in the basics and prerequisites of auction theory and discuss several student projects that the author has advised in the discipline.

#### Byungik Kahng University of North Texas - Dallas (byungik.kahng@untdallas.edu)

#### Singularities of 2-Dimensional Invertible Piecewise Isometric Dynamics

Applications of invertible piecewise isometric dynamical systems include, kicked oscillators in non-linear physics and digital signal processing in electric engineering. The purpose of this talk is to introduce some research problems regarding the 2-dimensional cases of such systems. Partial results from previous undergraduate research will be discussed as well. Sound background on linear algebra and vector geometry is required.

#### Chad Awtrey Elon University (cawtrey@elon.edu)

#### One Approach to Undergraduate Research in Computational Galois Theory

The goal of this talk is to describe the speaker's recent collaborations with undergraduates on projects related to computational Galois theory. Included are discussions of general questions which have guided our work, sample research results, a summary of dissemination outcomes, and the subsequent impact on students' post-baccalaureate plans.

#### **David W. Lyons** Lebanon Valley College (lyons@lvc.edu)

#### **Undergraduate Research in Quantum Information Science**

Quantum Information Science (QIS) is an interdisciplinary field involving mathematics, computer science, and physics. Appealing aspects include an abundance of accessible open problems, good government and private funding, and an energetic, open and collaborative international research culture. We describe our student-faculty joint work in QIS that has led to many student co-authored papers in research journals. We address program structure and funding, and describe the work that our students do.

#### **Maria Zack** Point Loma Nazarene University (mzack@pointloma.edu)

#### Effective Undergraduate Research Using Questions Derived from Institutional Research and Computational Science

Where can one find student research projects that are simultaneously motivating to students and worthwhile in a broader sense? The answer may be right under our noses. Your own university probably struggles with a lack of resources to model data for strategic institutional research. For instance, have you ever wondered how to detect which students are likely to leave your major or, even worse, the school? How should the university's fundraising arm deploy their human resources to maximize the dollars raised? Why is accreditation data so hard to assemble and analyze? What should we do with the piles of data from all of the national surveys (HERI, NSSE, etc.)? How can assessment data be interpreted in meaningful ways for non-quantitative departments? Similarly, your colleagues in the sciences are often dealing with large amounts of data that they don't know how to manage/store, interpret and visualize quickly and effectively (they are using hand methods and old technology). Do your biologists know that a computer can process images more accurately than a human eye? Do the chemists know what a relational database is and how they might use it to store and manage their data? Do your geneticists know about different techniques for automated pattern matching? Both institutional research and computational science produce many interesting questions that lead immediately to research projects that will be intellectually stimulating to students, provide a talking point for their job interviews, and are of real service to their clients.

#### **David Strong** Pepperdine University (david.strong@pepperdine.edu)

#### Undergraduate Research Projects with a Dozen or so Math, Physics and CS Students over the Past Decade

While I certainly don't consider myself an expert, I've had some good experiences with working with students in research projects, some deeper and more difficult, some less so, but all interesting, fun and good experiences for my students and me. I'll share my experiences, both the good and the bad, as well as suggestions for both faculty and students. My projects have ranged from an interesting combinatorial problem that arose in the lottery for "Wicked" tickets to creating a collage of digital images to educational software development.

#### Will Traves United States Naval Academy (traves@usna.edu)

#### Undergraduate Math Research at the US Naval Academy

I will describe the undergraduate math research program at the United States Naval Academy. I'll provide examples drawn from our capstone, internship, and research programs. As well, I'll describe how these programs help us interact with our DoD partners and industry.

#### Session 3: Saturday, August 9, 1:00–3:55 PM, Ballroom Level, Galleria II

#### **Stephan Ramon Garcia** Pomona College (stephan.garcia@pomona.edu)

#### Four Steps to Undergraduate Research Success!

You've recruited an eager batch of undergraduate research students. Now what? We suggest four key elements that are crucial in getting undergraduate research projects off the ground and into peer-reviewed journals.

#### Hannah Callender University of Portland (callende@up.edu)

#### Strategies for Mentoring Undergraduate Research Teams: Lessons Learned from the CURM Model

Here I will discuss mentoring strategies I learned from receiving a Center for Undergraduate Research in Mathematics (CURM) grant, including methods for choosing students and preparing them for research, encouraging and training them throughout the research process, and maintaining momentum throughout the academic year.

#### Alessandra Pantano University of California - Irvine (apantano@uci.edu)

#### Research Communities as a Vehicle to Boost Students' Interest in Mathematical Research

The University of California Leadership Excellence through Advanced Degrees (UC LEADS) program at UC Irvine offers educationally or economically disadvantaged sophomore students in STEM an opportunity to begin their research training right before their junior year. Their unusual motivation can be "exploited" to pivot the involvement of a large number of young mathematics majors in undergraduate research through (group) independent study courses. The creation of little research communities makes the involvement in undergraduate research more desirable and less intimidating for students at an early stage of their careers. The "social component" of undergraduate research is then strengthened through carefully planned activities of the Mathematics Club (such as trips to local conferences, talks by faculty members to discuss in- and off-campus opportunities for undergraduate research, and talks by students to present their results). In this talk, I will describe my successful experience with this process and draw general conclusions, which can be applied to a variety of different contexts.

#### Heather Gronewald Southwestern University (gronewah@southwestern.edu)

#### A Student's Perspective on Undergraduate Research

Applications to summer research programs and graduate schools heavily emphasize previous engagement with undergraduate research: students are pressed to write about research experiences and faculty are encouraged to comment on students' research experiences and aptitudes. When I began the application process, I received mixed messages from the mathematics community about which of my experiences qualified as real undergraduate research. I will discuss merits and limitations of three diverse research experiences: a classroom-based semester of applied modeling, participation in the Mathematical Contest in Modeling, and an intensive summer REU program. Emphasis will be on a student perspective of overarching learning outcomes across a range of pure and applied mathematics. I will highlight elements important to my own growth in researching and articulating mathematics, noting the role of mentors and the importance of offering gateways to student research within undergraduate mathematics departments.

#### Violeta Vasilevska Utah Valley University (Violeta.Vasilevska@uvu.edu)

#### **Engaging Students as Math Researchers**

At the beginning of fall semester 2013, I started an undergraduate research group at Utah Valley Univeristy. Since then, I have been working with three undergraduate math major students on an undergraduate research project on Geometric Group Theory/Graph Theory topics. In this talk, I will discuss a few aspects of this work: how this research has been conducted, the role of being a mentor (the challenges and how to overcome them), the impact of the undergraduate research on student learning, as well as the effect on the students (the successes and failures, and how to deal with them). In addition, I will talk about the support received by CURM (Center for Undergraduate Research in Mathematics) – in terms of financial support as well as preparation for undertaking this task.

Furthermore, I will talk about my efforts not just to engage students in undergraduate research projects but also to encourage them to present their work at regional and national conferences.

#### **Aprillya Lanz** Norfolk State University (alanz@nsu.edu)

#### Mentoring Minority Undergraduate Students in Mathematics at Norfolk State University

Undergraduate research facilitates the enrichment of students' academic growth by providing these students with substantial experience to prepare them for their future academic and professional careers. The successful participation of deserving students will enhance their undergraduate experience and it will also inspire their desire to pursue graduate education. In this presentation, we will discuss the effort in initiating mathematics undergraduate research activities at Norfolk State University, which adapted the model of Mathematical and Theoretical Biology Institute at Arizona State University.

#### Zsuzsanna Szaniszlo Valparaiso University (Zsuzsanna.Szaniszlo@valpo.edu)

#### Year Long Undergraduate Research at Minimal Cost

The Mathematics and Computer Science Department of Valparaiso University has conducted year-long undergraduate research programs for over twenty years. We describe the philosophy, the evolution and the detailed workings of the program. Among other things we discuss

student recruitment, activity, outcome, faculty compensation, etc. We think this model is easy to replicate and is very rewarding for both students and faculty.

Saad El-Zanati Illinois State University (saad@ilstu.edu)

#### **Undergraduate Research with Future Teachers**

We report on two Illinois State University (ISU) programs that are designed to engage secondary mathematics majors in research. The first program is a course that has been taught at ISU every spring since 2004. The second program is an REU Site for pre-service and in-service teachers. The REU Site has been run every year since 2007. This presentation will describe the successes and challenges of these programs, sample research topics, components designed to help teachers' translate their research experience to the classroom, and suggestions for implementation.

**Britney Hopkins** University of Central Oklahoma (bhopkins3@uco.edu) **Kristi Karber** University of Central Oklahoma (kkarber1@uco.edu)

#### **Balancing Undergraduate Research While Teaching Four Courses**

Conducting undergraduate research does not come without its challenges, especially while teaching four courses each semester. In this talk we discuss how we navigate around the time constraints of teaching and the obstacle of providing accessible research topics. We offer a creative solution which empowers our students and encourages comradery within our department.

### **Recreational Mathematics: New Problems and New Solutions**

#### Session 1: Thursday, August 7, 1:00-4:55 PM, Plaza Level, Pavillion West

Organizers: Paul Coe Dominican University

Sara Quinn Dominican University

Kristen Schemmerhorn Dominican University

As with all mathematics, recreational mathematics continues to expand through the solution of new problems and the development of novel solutions to old problems. For the purposes of this session, the definition of recreational mathematics will be a broad one. The primary guideline used to determine the suitability of a paper will be the understandability of the mathematics. Papers submitted to this session should be accessible to undergraduate students. Novel applications as well as new approaches to old problems are welcome. Examples of use of the material in the undergraduate classroom are encouraged.

#### Colm Mulcahy Spelman College (colm@spelman.edu)

#### The Mathematics, Magic and Mystery of Martin Gardner

October 2014 marks the centennial of the birth of Martin Gardner (1914–2010), the most well-known writer on recreational mathematics of the twentieth century.

Martin was without a doubt the best friend mathematics ever had, and this year has seen numerous efforts to leverage his extensive written legacy—over 100 books—to turn new generations on to the magic and mystery of mathematics.

The theme of Mathematics Awareness Month 2014 was "Mathematics, Magic, and Mystery," prompted by the title and spirit of a classic book of his from 1956, and MathFest sees his deep influence celebrated in the Martin Gardner Centennial Lecture.

The goal is not only to inspire many "Aha!" moments, and add to Martin's record of turning innocent youngsters into mathematics professors (and mathematics professors into innocent youngsters), but also to ensure that all teachers are aware of the incredible range of fun material he wrote about, and its potential to engage students in and out of the classroom.

Twitter users may enjoy following @WWMGT (What Would Martin Gardner Tweet?) and @MGardner100th (Martin Gardner Centennial).

Brittany SheltonAlbright College (bshelton@alb.edu)Breeanne Baker SwartThe Citadel (bbaker2@citadel.edu)

#### **Generalization of the Nine Card Problem**

The nine card problem is a card trick credited to magician Jim Steinmeyer. In this talk, the nine card problem will be demonstrated. The trick will be explained using permutations. A generalization of the nine card problem will be discussed and explained in terms of permutations.

#### Brian J. Birgen Wartburg College (brian.birgen@wartburg.edu)

#### The Uniqueness of Rock-Paper-Scissors-Lizard-Spock

Rock-Paper-Scissors-Lizard-Spock is an extension of Rock-Paper-Scissors recently popularized by the TV show "The Big Bang Theory." I will show that this is the unique five move fair game (up to isomorphism). I will show there is not a unique seven move fair game and I will discuss creating four and six move fair games.

#### **Dana Rowland** Merrimack College (rowlandd@merrimack.edu)

#### **Candy Crush Combinatorics**

In the game of Candy Crush, differently colored candies are arranged in a grid. A valid starting configuration of a game of Candy Crush cannot have 3 consecutive candies of the same color in a row or column. Also, it must be possible to swap two adjacent candies to obtain at least 3 consecutive candies of the same color. In this talk, we determine the number of valid Candy Crush lines (you cannot not have three in a row, but you can get three in a row by swapping adjacent candies) and discuss extensions to an  $m \times n$  grid with q colors.

#### **Doug Ensley** Shippensburg University (deensley@ship.edu)

#### **Exploring Sliding Tile Puzzles on your Smartphone**

The classical Fifteen Puzzle consists of a  $4 \times 4$  grid containing fifteen sliding tiles and one empty spot with the goal of returning the scrambled tiles to their home positions. A 1974 paper of Richard Wilson characterizes the generalization of this puzzle to tiles sliding along edges between nodes of any simple graph. In this talk we will demonstrate a tool that allows one to set up any simple graph with any initial configuration of tiles in order to explore questions such as existence or minimum-length solutions to any puzzle of this type. To demonstrate the tool we will give a complete characterization of all minimum-length solutions to puzzles on graphs with five or fewer nodes.

Brant JonesJames Madison University (jones3bc@jmu.edu)Laura TaalmanJames Madison University (taalmala@jmu.edu)Anthony TongenJames Madison University (tongenal@jmu.edu)

#### Solitaire Mancala Games and the Chinese Remainder Theorem

Mancala is a generic name for a popular family of games that are played by sowing stones among positions on a game board. Many two-player variations allow a player to move again if their move ends in the store, so one of the strategic goals for a player is to set up a single chain of moves that allows the capture of all available stones. In this talk, we study a simple solitaire game called Tchoukaillon that facilitates the analysis of these "sweep" moves, where all of the stones are captured.

We represent a Tchoukaillon game by a vector, where each component records the number of stones in a position on the board. It turns out that for each positive integer n, there is a unique winning board having n total stones. Moreover, the vectors corresponding to winning games can be given non-recursively in terms of the vector ( $n \mod 2, n \mod 3, n \mod 4, \ldots$ ) that arises in the study of the Chinese Remainder Theorem.

We apply these results to study a board reconstruction problem. For example, suppose we want to find a winning vector where some of the board positions are prescribed. Does such a board exist? If so, how do we produce it? Using the Chinese Remainder Theorem, we give a criterion and an algorithm to answer such questions.

Our paper containing a survey of these results appears in the October 2013 American Mathematical Monthly.

Alex MeadowsSt. Mary's College of Maryland (ammeadows@smcm.edu)Bradley PutmanSt. Mary's College of Maryland (bwputman@smcm.edu)

#### A New Twist on Wythoff's Game

Wythoff's Game is a classic mathematical game played with two piles of sticks, in which players take turns either removing sticks from one pile or removing an equal number of sticks from both piles. The last player to remove sticks wins the game. The mathematical theory, in which the Fibonacci numbers and the golden ratio arise, is still an active field of research. We propose a new way to extend Wythoff to more than two piles, using ideas from knot theory. The new variant "Wythoff Twyst" is an ongoing topic of research with undergraduate collaborators. The talk will include some ideas of how one analyzes combinatorial games, and some recent results and open problems related to this new game.

Paul Cull Oregon State University (pc@cs.orst.edu)

#### **Graphs and Puzzles**

Graphs and puzzles are intimately related. The graph's vertices represent the configurations of the puzzle and the graph's edges represent the allowed puzzle moves. We present a class of graphs, the iterated complete graphs  $K_d^n$ , which generalize Spin-Out (d = 2) and Towers of Hanoi (d = 3) each with *n* pieces. The corresponding generalized puzzles have *q* towers and each piece has  $2^r$  possible spins where  $d = q 2^r$  with *q* odd. The vertices can be labeled with the puzzle configurations which we can convert to barycentric coordinates using a finite state machine. Then we can compute the distance (number of moves) between two arbitrary configurations in O(n) time, even though the distance is  $O(2^n)$ . We can also find a minimal sequence of moves between one configuration and another in  $O(2^n)$  time.

#### Robert W. Vallin Lamar University (rvallin425@gmail.com)

#### When You Cross Latin and Gilbreath

An  $n \times n$  Latin Square is an array where each row and each column contains the numbers 1, 2, ..., n in some order. A Gilbreath Permutation is a particular permutation of  $\{1, 2, ..., n\}$  which Martin Gardner first brought to the general public in his *Mathematical Games* column. So the questions are:

What do you get when you cross a Latin Square and a Gilbreath Permutation? Is it possible to create a Latin Square where each row/column is a Gilbreath Permutation? Can we generalize this idea from a square to a cylinder? What if we wrap the permutation around?

#### **Doy Ott Hollman** Lipscomb University (hollmando@lipscomb.edu)

#### Mathematics, Magic Squares, and Mirth (Humor)

A brief history of the Chinese Magic Squares will be given. Also, the mathematics of the construction of a selected number of them will be investigated. Humor and magic in mathematics are possible and will be demonstrated. Humor is a much-needed component in the classroom today at any level of instruction. Magic square worksheets will be distributed for completion, for hands-on experience. Cultural appearances throughout world history will be examined, especially in art and popular superstition. These conventional geometric configurations (as well as geomagic squares, alphamagic squares, etc.) have been a source of recreation through the centuries.

**Donna Flint** South Dakota State University (donna.flint@sdstate.edu)

#### A Magic Square Equation

At South Dakota State University, students are required to complete a 1-credit research experience course. The research can be original or simply original to the student. Recently, a student discovered a formula on Wikipedia for generating a Magic Square of odd order. This formula has input (i, j) and output k, the number to place in the Magic Square matrix in the i, j position. A general search yielded no information about the origins or validity of the formula. The student project: Verify that this formula generates a Magic Square of odd order. This talk will describe the formula, the proof, and the journey for this challenging, yet fun student project.

Max Alekseyev George Washington University (maxal@gwu.edu)

#### An Efficient Backtracking Method for Solving a System of Linear Equations over a Finite Set with Application for Construction of Magic Squares

We consider a problem of solving a system of linear equations over a given finite set S of integers. Under the generic settings (not using any specific properties of S), there seems to be no better way to solve the problem than a backtracking search that assigns values from S to the variables and tests if they can satisfy the given system for early detection of deadends. We argue however that the order, in which such search processes the variables, may have a significant impact on the overall search performance. We further propose an algorithm for computing an optimal order of the variables for the given system and illustrate it with the problem of construction of magic squares composed of prime numbers.

#### Session 2: Friday, August 8, 1:00–4:35 PM, Ballroom Level, Galleria I

Susan GoldstineSt. Mary's College of Maryland (sgoldstine@smcm.edu)Ellie Baker(ellie.baker@post.harvard.edu)

#### The Elusive Möbius and the Intractable Hexagon: Geometric Cross Sections in Bead Crochet

A traditional bead crochet rope bracelet consists of a narrow tube of crocheted seed beads sewn together to form an uninterrupted, hollow, flexible band. When these beads are all the same size, the result is a smooth, round beaded torus. By mixing beads of different sizes, we can create more interesting geometric effects, including bead tubes with polygonal cross sections.

In this talk, we will present some original bracelet designs with geometric cross sections and explain the general theory behind how to achieve different regular polygons with some simple geometry and arithmetic. In the process, we will see that even polygons are odder than odd ones, that we can move from geometry to topology with a tiny twist, and that in the realm of bead crochet, the number six is far from perfect.

Mike Krebs California State University - Los Angeles (mkrebs@calstatela.edu)

#### **Coloring the Plane with Rainbow Squares**

Assign every point in the plane a color so that no two points of distance 1 from each other have the same color. What is the smallest number of colors needed? Approximately half a century ago, it was shown that the answer is either 4, 5, 6, or 7. It remains an open problem to determine which of those is in fact the minimum number of colors needed. In this talk, we propose a slight variation on this "chromatic number of the

plane" question. Given a coloring of the plane, we say that a square is a "rainbow square" if its vertices have four different colors. What is the smallest number of colors needed to color the plane so that every square of side length 1 is a rainbow square? We will prove that the answer is at least 5 and no more than 13. Along the way, we will state several unanswered questions that may be amenable to the methods shown and therefore appropriate for REUs, senior theses, or other undergraduate research projects. This talk will be quite accessible to undergraduates; indeed, we use only high school algebra and geometry.

#### David Molnar (molnar.math@gmail.com)

#### Dividing the Plane: Variations on a Theme

The simple idea of cutting the plane with lines leads to an unending array of interesting problems. I will present a small cross-section of problems centered around this theme. These will vary in scope (counting problems, games,  $\dots$ ) and in difficulty. No background beyond high school mathematics is necessary.

#### **Russ Gordon** Whitman College (gordon@whitman.edu)

#### Integer-Sided Triangles with Trisectible Angles

We obtain a general method for finding integer-sided triangles for which all three angles in the triangle can be trisected with a compass and unmarked straightedge. Since some angles (such as  $60^{\circ}$ ) cannot be trisected using only these tools, some care is required to find triangles with these properties. By the law of cosines (since the sides of the triangles are integers), the cosines of the angles are all rational. In order for these angles to be trisectible, the rational cosine values must meet certain conditions. Consequently, we use some elementary aspects of the theory of constructible numbers to determine integer-sided triangles with trisectible angles.

#### Andrew Richard Reiter (arr@watson.org)

**Robin Young** University of Massachusetts - Amherst (young@math.umass.edu)

#### On (mod n) Spirals

This talk is intended to introduce the process of constructing (mod n) spirals and the idea of a complete spiral. The construction of a (mod n) spiral is similar in nature to Ulam's Spiral. Examples of various complete spirals will be shared along with grayscale visualizations of them to further illuminate interest in these objects. A theorem related to patterns seen in side lengths, iteration counts, and ending corners of these objects will be discussed and proved. Further, it will be shown that the construction process may be used as a deterministic means for discovering the greatest square divisor of integers  $n \ge 2$ . Generalizations to higher dimensions and spirals of other shapes will be given. The authors are unaware of other work or literature on this topic and want to share as it is a candidate for use in classroom learning.

Grant Barnes Lu	ther College (barngr01@luther.edu)
Michael Johnson	Luther College (johnmi10@luther.edu)
Cadence Sawyer	Luther College (sawyca02@luther.edu)

#### Finding the Catalan Numbers in the Sandpile Model

The study of avalanches in the one dimensional sandpile model is deeply connected with number theory. This dynamic model is characterized by repeatedly adding a grain of sand randomly to any of *n* consecutive locations in the sandpile. Avalanches occur when the slope of the sandpile becomes too large. Analysis of the number of recurrent configurations reveals many interesting connections with number theory. We present a short description of the sandpile model and use recurrence relations to help count all possible recurrent stable sandpile configurations. We then present an alternate proof showing that the number of recurrent stable states is always a Catalan number of the form C(n) =

 $\frac{(2n)!}{n!(n+1)!}$ . The Catalan numbers are an important sequence that appears when counting objects in many seemingly different settings.

Angeline Rao Clements High School (angie.rao@gmail.com)

Alexander Yang Clements High School (aypanda42@yahoo.com)

Vinciane Chen Westwood High School (smileyface12963@gmail.com)

#### A Characterization of Balance in Oriented Hypernetworks via Generalized Signed Walks

Hypergraphs serve as an ideal framework to model higher-dimensional networks, or hypernetworks, which are pervasive in the real world. An oriented hypergraph *G* consists of vertices and signed hyperedges. *G* is balanced if all of its hypergraphic cycles are positive. In this paper, we characterize the concept of balance, which has major implications in networking applications. An *incidence* is a triple (v, e, k) such that vertex *v* and edge *e* are incident and  $1 \le k \le n(v, e)$ , where n(v, e) denotes the number of such incidences. An incidence occurs m(i; W) times in walk *W*. We prove that an oriented hypergraph *G* is balanced if and only if for paths *P* and *P'* that share  $\alpha_P(P')$  incidences,  $sgn(P) = (-1)^{\alpha_P(P')} \cdot sgn(P')$ . We further generalize this result and prove that *G* is balanced if and only if for any closed weak-walk  $\tilde{W}$ ,

 $sgn(\tilde{W}) = (-1)^{\gamma}\tilde{W}$ , where  $\gamma_{\tilde{W}} = \sum_{i \in \tilde{W}} \left\lfloor \frac{m(i;\tilde{W})}{2} \right\rfloor$ . Additionally, we show that for the Laplacian matrix L of G,  $[(-1)^k L^k]_{ij}$  counts the difference between the positive and exactly k much matrix L of C. If

difference between the positive and negative k-weak-walks between vertices i and j. Our discoveries help optimize power consumption in today's mobile devices and predict the progression of relationships in a social network.

#### Shenglan Yuan LaGuardia Community College, CUNY (syuan@lagcc.cuny.edu)

#### Revisiting 12 Marbles, an Old-Fashioned Scale Puzzle

In this talk, we'll revisit a popular puzzle, the 12-Marble Riddle: There are 12 identical looking marbles. One of them, though, is a different weight than the others (you don't know, at the beginning, whether it's heavier or lighter). How can you identify the odd ball using nothing more than an old-fashioned balance scale and weighing balls just three times? With these restrictions can you tell if it's heavier or lighter than the others? We'll discuss the solution as well as generalizations of the puzzle. We'll also give some ideas about how puzzles like this one can connect to math topics and be used in the classroom.

#### **Frank Lynch** Eastern Washington University (lynch@ewu.edu)

#### The Car Talk Trip

The Car Talk Trip is a puzzler from the weekly radio show Car Talk. The puzzler describes an automobile that is traveling 75 mph and is 75 miles from its destination. The trip is completed by traveling one mile at 75 mph, one mile at 74 mph, etc., until the last mile is traveled at 1 mph. The goal of the puzzler is to determine how long it takes to arrive at the destination. We describe an exact method and two approximations for solving the puzzler. The approximations incorporate rich concepts from calculus and ultimately lead to the discovery of Euler's constant.

Christopher N. B.	Hammond	Connecticut College (cnham@conncoll.edu)
Warren Johnson	Connecticut	College (wpjoh@conncoll.edu)
Steven J. Miller	Williams Coll	ege (sjm1@williams.edu)

#### The James Function

We investigate the properties of the James function, associated with Bill James's so-called "log5 method," which assigns a probability to the result of a game between two teams based on their respective winning percentages. We also introduce and study a class of functions, which we call *Jamesian*, that satisfy the same *a priori* conditions that were originally used to describe the James function.

#### Jay Lawrence Schiffman Rowan University (schiffman@rowan.edu)

#### Exploring Five Integer Sequences Related to the Collatz Problem

More computer time has been devoted to The Collatz problem (also known as the 3X + 1 problem) than any other in number theoretic lore. This paper will focus on five integer sequences related to the problem. Among these include identifying where the maximum trajectory is reached for integers in the range from one to five hundred, furnishing the initial integer requiring between one and five hundred steps for the sequence to converge to one as well as determining the total number of steps needed for each of the initial five hundred integers to reach one. This conjecture has been open since 1937 and verified for all integers  $\leq 1.9 \times 2^{59}$ . In addition, we will study sequences related to the modified Collatz problem attributed to Jeffrey Lagarias (where the tripling and adding one for odd integers and halving all even integers and repeating the process is replaced by tripling, adding one and dividing by two for odd integers and halving even integers and repeating the process) and furnish related numerical data. The use of MATHEMATICA, a CAS graphing calculator and MathWorld will aid in our investigations into a easily posed problem that has fascinated yet eluded and frustrated mathematicians for more than seventy-five years.

## **Embodied Activities in the Teaching and Learning of Mathematics**

#### Thursday, August 7, 1:00–4:55 РМ, Parlor AB

#### Organizer: Hortensia Soto-Johnson University of Northern Colorado

In laymans terms we might describe embodied activities as events that connect cognition with action. In other words, these are tasks, where a student is physically and mentally engaged in a cognitive task designed to result in learning. These tasks are created so that students are the mathematics. Many hypothesize that manipulatives work because they provide an atmosphere where students are engaged in actions that assist in constructing mathematical concepts. Similarly, incorporating embodied activities into the classroom has proved fruitful not only with prospective teachers but with undergraduate mathematics majors who are learning related rates, geometric concepts, and proof constructions. Furthermore, they can serve as an entry point to inquiry-based learning because embodied activities go beyond communicating, writing, reading, and reflecting.

The purpose of this session is to share activities that require students to be physically engaged in learning all levels of mathematics, particularly undergraduate mathematics. Submitted abstracts should include the goals of the activity, description of the activity with details connecting the mathematics with the actions, and strengths and weaknesses of the activity. We encourage presentations that are audience-interactive, so that they may experience the activity in action. Talks that focus on general active learning strategies with little or no connections between cognition and physical action should submit talk proposals to the Active Learning in Mathematics contributed session.

## Brent HancockUniversity of Northern Colorado (brent.hancock@unco.edu)Marki DittmanUniversity of Northern Colorado (marki.dittman@unco.edu)

#### Pre-service Elementary Teachers' Perceptions of Geometric Translations in Embodied Activities

In this brief presentation, we will explicate eight pre-service elementary teachers' perceptions of embodied activities regarding geometric transformations. In particular, we will discuss findings related to a task-based interview wherein participants completed transformation tasks in pairs on a whiteboard, and subsequently discussed general perceptions of previously conducted embodied activities during their class. This presentation will focus specifically on participants' responses during the translation task, wherein students conceived of translations as both point-wise and rigid transformations. We will additionally highlight parallels between students' responses to the translation task during these interviews and their analogous behavior during the embodied activities we observed during a prior class.

## Marki Dittman University of Northern Colorado (marki.dittman@unco.edu)

Brent Hancock University of Northern Colorado (brent.hancock@unco.edu)

#### Pre-service Elementary Teachers' Conception of Perpendicular Bisector in an Embodied Reflection Task

In considering reflections of two-dimensional figures across a line of reflection, an understanding of the term "perpendicular bisector" is crucial. Pre-service elementary teachers in a required geometry course worked in groups to perform a series of embodied tasks designed to explore the concept of reflections. In particular, these tasks forced students to solidify and, at times, reevaluate their understanding and interpretation of a perpendicular bisector and its role in determining the location of reflections of a given figure. We will demonstrate these embodied activities as well as discuss video-recorded classroom observations of students' completion of the tasks. Additionally, we will examine eight pre-service teachers' conceptions of perpendicular bisector as they work in pairs to answer a question about reflections during a task-based interview that took place during the same semester as the course.

#### Sandra Fital-Akelbek Weber State University (sfitalakelbek@weber.edu)

#### Hands-on Activities to Enrich Basic Geometry Proofs: Angles in a Triangle and Parallelogram.

Cutting angles and folding parallelograms are examples of hands-on activities that will help students discover theorems about angles in a triangle and parallelogram. Presented activities will lead students through the proofs of basic geometric theorems, such as: the sum of angles in a triangle, exterior angle theorem and properties of angles in a parallelogram. Participants are encourage to bring scissors.

#### **Susan Jeannine Durst** University of Arizona (sdurst@math.arizona.edu)

#### Measuring Around The Unit Circle

Most students' first exposure to trigonometric functions involves right triangles and ratios between their edge lengths. Some students have trouble transitioning to thinking of sine and cosine as functions on the real numbers. What does it mean to take the sine of a number greater than  $\pi$ ? How can I estimate the cosine of 4 radians? Why are we measuring things in radians in the first place? In this session, we will discuss a number of hands-on activities that address these sorts of questions. All we'll need is some graph paper, a measuring tape, and a ten-inch embroidery hoop.

**Oai Ha** Utah State University (oai.ha@aggiemail.usu.edu)

#### The Use of 3D Multi-Sectional, Interlocking Geometric Models and Magnetic Nets as Teaching Aids for Spacial Ability Training and Middle School Geometry Education

Middle school students constantly struggle with 3D geometry that requires them to visualize a three-dimensional (3D) object from a twodimensional (2D) picture or to identify the shapes of 2D cross sections of 3D objects with a plane. These visualization abilities are sub-factors of spatial ability that enables people to generate, recall, and manipulate mental representations of actual and imagined shapes, objects, and structures. There is now considerable evidence that spatial abilities contribute to performance in science, technology, engineering, and mathematics (STEM) fields. Studies investigating the relationship between spatial ability training and STEM achievements found that low spatial performers are better trained with concrete objects from which they can manipulate and receive spatial information via their haptic and visual communication channels. Manipulating geometric solids, learning how to develop a net from a geometric solid object or constructing a geometric solid from a net can help students explore the properties of various geometric solids and develop spatial ability.

This project develops: 1) Tangible geographic models (TGMs) and their magnetic nets for net development skills training; 2) TGMs that are precut into pieces at different locations and directions for cross section identification skill training. All pieces of precut solids are joined together by unique magnetic systems embedded beneath the cutting surfaces. TGMs can be regular 3D geometry solids, platonic solids, or engineering graphics models. The models developed in this project will be used in an experiment in the next step to evaluate how they affect students' spatial abilities and geometry learning outcomes.

Tanaka Noriko (tanaka-nagoya@y5.dion.ne.jp)

#### What is the NORISHIRO? - Plane Development of a Polyhedron with the Tabs

I will introduce the job of my students and me. It is the problem about NORISHIRO. When we make a strong polyhedron from a sheet of paper, we need to cut out a piece of paper, put some glue on the tabs for sticking and stick the tabs on the other sides of paper. The tabs are NORISHIRO in Japanese. We considered whether it is possible to make a tab on every other edge of the piece of paper.

#### Charlotte Ann Knotts-Zides Wofford College (knottszidesca@wofford.edu)

#### Visualizing Multivariate Functions in a Desktop-sized 3-D Coordinate System

Students in multivariable calculus often have trouble visualizing the three-dimensional surfaces or regions formed by multivariate equalities, inequalities or functions. An early assignment in my class is to create a three-dimensional coordinate system using two manila folders, one placed flat on the desk and the other perpendicular to the desk and open at a 90-degree angle. Students use a ruler (conveniently reproduced along the side of the assignment paper) to label the three axes and identify the three coordinate planes. The assignment consists of locating points on the coordinate planes, finding projections for points not on the planes, and visualizing the surfaces or regions formed by various equalities and inequalities. Colored dot stickers are used to represent points and talk about projections of points onto coordinate planes while colored sheets of paper can be bent to create surfaces in three-dimensional space. Students are encouraged to create sketches of the surfaces and regions and are welcome to take photos with an appropriate mobile device. The goal of this early hands-on exposure to a three-dimensional coordinate system is to develop the students' intuition for visualizing functions; students are encouraged to use this hands-on model while doing homework and we return frequently to it in class whenever a student asks a question about visualizing in 3-D. Samples of the activity will be shared during the talk along with copies of the assignment itself. The assignment can also be downloaded from webs.wofford.edu/knottszidesca.

Nicole FiderUniversity of California - Irvine (nfider@uci.edu)Casey KelleherUniversity of California - Irvine (clkelleh@uci.edu)Alessandra PantanoUniversity of California - Irvine (apantano@uci.edu)Ryan SullivantUniversity of California - Irvine (rpsulliv@uci.edu)

#### Hands-on Exploration of Topological Invariants

In this talk we describe a novel hands-on workshop where students of all ages can learn topology through guided exploration.

The workshop begins with a brief lecture, where students are introduced to the notion of topologically equivalent spaces. Students are then asked to determine if objects are topologically equivalent (e.g., a coffee cup and a donut), and are invited to mold the objects in clay and simulate continuous deformations. After this guided activity, without the help of any manipulative, they compare other objects in terms of topology.

This preliminary exploration is followed by an investigation of topological invariants. Various properties of topological spaces are presented, and students again use clay to investigate which properties are preserved under continuous deformations. With washable markers, they draw triangulations of the surfaces and rediscover the relation between the Euler characteristic and the genus. They also explore how the Euler characteristic is independent of how "fine" the triangulation is. With cotton string, they draw circles around different surfaces and figure out which circles can be contracted to a single point by pulling the string.

This proposed active learning workshop is extremely versatile. It is suitable for an introductory lesson on topology, but can be expanded to include an investigation of the relation between Euler Characteristic and Fundamental Group for higher-level students. Properly adapted, it can be the focus of an outreach event for K–12 students.

This talk will describe the workshop activities in detail and the experience from the first implementation at UC Irvine.

#### **David Ely** The Ohio State University (ely65@osu.edu)

**Jeanette Palmiter** Portland State University (palmiter@pdx.edu)

#### **Problem Solving through Computer Simulations**

A television program shows players competing in a Texas Hold'em Poker Tournament where the players' hands are revealed along with percentages showing the probability of each hand winning. The on-screen percentages change as the game evolves. How are these percentages determined? It clearly is not through theoretical probability computations, but, instead, through Monte Carlo or simulation techniques.

How does one simulate these probabilities? Manipulatives are useful, but limiting. The fancy and expensive software packages are inflexible. Instead, consider as one's preferred tool, the building blocks of technology itself, computer programming.

Although most mathematics majors are required to take a computer science class, seldom do they apply their experience to mathematical problem solving. The fault lies in the way computer programming is introduced. A new "as-needed" pedagogy will be illustrated showing how to explore advanced mathematical problem solving and computer programming together in the classroom. Only when one encounters situations that call for more commands or syntax is it introduced.

In this presentation, classical probability problems will be explored by soliciting audience input to develop the needed simulation and then we will build together the ensuing computer program, thereby modeling a typical class group session. Student response data will be shared along with a discussion as to the feasibility of this approach to mathematical problem solving.

#### **Joshua Lioi** University of Arizona (jlioi@math.arizona.edu)

#### Modeling Biology in the Classroom: Birds, Bacteria, and Disease

Many students coming out of high school are not aware of the interdisciplinary possibilities of mathematics. Biocomplexity is one such possibility that lies at the intersection between mathematics, biology, computer science, and physics. I have worked with several high school teachers, and together we have developed a number of activities that explore biocomplexity. Using NetLogo, we developed a number of fun and interactive applets that allow students to study biological phenomena such as the flocking of birds, the swarming of bacteria, and the spread of disease. These can be used to provide excellent hands on examples for applications of calculus, differential equations, or linear algebra. Going more in depth with these topics, students could become mathematical biologists by adding to the mathematical model. Also, since NetLogo is free and fairly easy to learn, these students could add their new ideas to the model programs. For example, a student could incorporate predators to see how the flocking birds would react and explore the best strategies the flock would have to take to avoid those threats. Students who have interests in other sciences get very excited to see that math has this sort of power.

#### Steve B. Zides Wofford College (zidessb@wofford.edu)

#### "Field" & Stream: Experiencing a Vector Field

Students often have great difficulty bridging the gap between single and multivariable calculus. When taking vector analysis this conceptual divide expands into a full-blown crevasse, especially when the notion of a three-dimensional vector field is introduced.

In this talk, I will describe a successful off campus activity designed to make the vector field idea more concrete and experiential. Armed with velocitimeters and student designed directional indicators, students were taken to the Lawson Fork Creek and asked to wade out into the water. Small peer groups collected velocity data in various regions of the creek and submitted that data to a class database. Upon return to campus, each student was asked to make a formal report which included their directional indicator design, a complete velocity vector field of the creek and a personal reflection of the experience. Once all the details of this embodied activity are described, the talk will then shift to the strengths and weaknesses of the experience. Although many of the students point to this event as one of the highlights of the class, problems with incomplete and imprecise data reduce the quality of the result. Also, integration of the experience into the course material needs to be more overt and consistent.

#### Luke Wolcott Lawrence University (luke.wolcott@gmail.com)

#### Report on the Bodies of Data Workshops

In July 2014, I'm running a *Bodies of Data* workshop with Portland-area dancers, clowns, and theater performers. The workshop introduces the subject of topological data analysis — a fascinating and accessible approach to understanding the "shape" of data, and its persistent qualitative structure. The point of the workshop is to use TDA — the mathematical ideas and metaphors, and their vernacular counterparts — as a jumping-off point for movement activities and exploratory performance. My talk will report on this workshop. Furthermore, during MAA MathFest I will be running the workshop again, recruiting mathematicians to also learn TDA and collaborate with the performers.

## Mathematics in Honors Programs

#### Thursday, August 7, 1:00–3:55 PM, Plaza Level, Broadway I & II

#### Organizer: Jacci White Saint Leo University

Honors Colleges and Programs look for unique opportunities to reach out to bright and capable students who may not be mathematics majors. This session will focus on courses, strategies, or activities, that have been used for non major mathematics classes designed for honors students. Speakers should provide evidence of the success of and/or challenges involved with the courses they have taught.

#### Paul E. Seeburger Monroe Community College (pseeburger@monroecc.edu)

#### Applying Calculus Techniques to Analyze the Motion of Single and Double Ferris Wheels

One of the goals of an honors mathematics course should be to give students an opportunity to use the tools they learn in the course to analyze something interesting in real-life. This was the goal of a project I created for my Honors Calculus 1 course in the fall 2013 semester. In addition to applying calculus to real-life, I also wanted to help students use a number of different calculus concepts in the process. This project requires parametric equations, derivatives applied to motion, average rates of change, optimization, and related rates. First students were asked to use parametric functions to describe the motion of a rider on an ordinary single Ferris wheel. Then they found the rider's horizontal and vertical velocities as functions of time, and used optimization and related rates to analyze various aspects of the rider's motion. A component of the project also required students to use Excel to approximate the horizontal and vertical instantaneous velocities and the instantaneous speed of the rider over half-second time intervals using average rates of change. Once this portion of the project was completed, graded, and returned to the students, they were given the second half of the project which required them to similarly analyze the much more interesting motion of a rider on a double Ferris wheel. I will share my experience coaching my students through this project and show a computer simulation I created to help my students better visualize the motion of the double Ferris wheel.

#### **Brian Camp** Saint Leo University (brian.camp@saintleo.edu)

#### Creating a Freshman Honors Mathematics Course (for Non-Majors)

Many honors students may be interested in Mathematics but choose other majors so they may never have the opportunity to explore mathematics at the college level. A freshman mathematics course for non-majors allows them to explore some topics that are not traditionally taught at the freshman level. Potentially this may pique the curiosity of some students and encourage them to want to explore more advanced areas within the discipline. In this talk, some experiences will be shared about how and why such a class has been created, some hurdles in creating it as well as some discussion of the topics that might be included in such a class.

#### David Clark Randolph-Macon College (davidclark@rmc.edu)

#### Dimension and Direction: A Journey Through Mathematical Space

Some of the most beautiful ideas in mathematics reside in the realm of geometric topology, which is rarely taught even to undergraduate math majors. Yet the objects and techniques of this field can be made accessible to a motivated English major, if pitched properly. This honors course developed the notions of "dimension" and "direction" to explore shapes of possible universes (i.e., manifolds), knots, and fractals. There was a heavy emphasis on the manipulation of physical models, creativity, and open-ended inquiry; we investigated topics from a variety of perspectives, including literature, psychology, and physics.

#### Dan C. Kemp South Dakota State University (dan.kemp@sdstate.edu)

#### Honors Calculus at South Dakota State University

The standard calculus sequence is taught as an Honors course at SDSU. Qualified students from all majors are invited. The majority of current students are engineering majors. Distinguishing features include limited size (25), outside readings on mathematics topics, and Projects that involve students in significant mathematics that is not typically found in elementary calculus. Readings for Calculus I & II are typically from a History of Mathematics book while Calculus III readings center around the Philosophy of Mathematics. Readings are assessed by Reflection Papers that students write throughout the semester. Projects include work from Euler: Find four distinct positive integers such that the sum of any two is a perfect square and Wallis' infinite product for  $\pi$ . Students work on Projects outside of class and in assigned groups. Reports from students via midterm and end of course surveys indicate that they like both additions to the course, but the readings are the most popular.

#### **Jacqueline Jensen-Vallin** Lamar University (jajensen681@gmail.com)

#### **Honors Elementary Statistics**

In Spring 2013, I was invited by the head of the Honors Program at Slippery Rock University to teach an Honors version of Elementary Statistics. The population was students in the Honors program, who were generally not mathematics majors, and a select group of mathematics majors. I designed an inquiry-based learning course, which gave the students problems to solve and asked them what tools they would need to answer those questions. The course concluded with a group project–each group was to design a question, find data online which would help answer their question, analyze the data, and then compare their conclusions to the published conclusions online. We will discuss strengths and drawbacks of this format, including student comments and feedback.

## Philip B. YasskinTexas A&M University (yasskin@math.tamu.edu)Douglas B. MeadeUniversity of South Carolina (meade@math.sc.edu)

#### Maple in Honors Calculus

For over 15 years, Dr. Yasskin has been teaching honors sections of Engineering Calculus at Texas A&M University (although there are some math majors) in which students have learned the Maple Computer Algebra System. It is used to solve problems and complete projects which are more extensive than the usual exercises.

Since 2002, they have been writing Maplets which are Maple based applets which perform some computation. The ones our students write are designed to teach other students how to do calculus computations. Many have been subsequently enhanced and included in the Maplets for Calculus collection written by the Yasskin and Meade. To write such Maplets, students need to learn the basic programming constructs of "if" statements, "do" loops and procedures. (This is the first programming for some students, especially math majors.) Writing Maplets that teach is more intensive, than just solving the problems, because the students need to be able to generate and solve the problems with general variables and functions which will later be randomly selected.

This semester, some students have done a project (written by Dr. Meade for students at the University of South Carolina) in which they use Maple to design a goblet satisfying certain restrictions on volume, thickness and center of mass. Then we used a new feature of Maple 18 to export the file for 3D printing.

This talk will present some of the Maplets written by honors students (and discuss how they are written) and show some of the goblet projects.

#### Mark Bollman Albion College (mbollman@albion.edu)

#### **Searching For Great Issues In Mathematics**

The honors curriculum at Albion College includes four Great Issues seminars: in science, social science, humanities, and fine arts. Most students graduating with all-College honors complete one seminar from each division, as well as an independent honors thesis. The seminars are required to be discussion-oriented, interdisciplinary, writing-intensive, and exam-free, but the content of a Great Issues seminar is left to the individual instructor. As Albion has no mathematics requirement either for admission or graduation, it has been challenging to find a place for meaningful mathematics within the current seminar structure. This talk will describe two ongoing efforts to introduce mathematical topics into Honors seminars: a Great Issues in Science course, "8 Big Ideas That Shaped Science", that studies non-Euclidean geometry and a Great Issues in Humanities course, "Perspectives On Gambling", that includes a significant mathematics component.

Sarah L. Mabrouk Framingham State University (smabrouk@framingham.edu)

#### Why Statistics??? An Opportunity for Exploration and Reflection

At Framingham State University, most honors courses are specially designed 100- and 200-level course sections satisfying general education requirements. Each semester, interested faculty members submit Honors course proposals describing how the proposed Honors version of a course differs from the standard version; the Honors Council reviews the submissions and approves worthy proposals. Since 2009, I have included faculty interviews among the assignments for my Honors sections of MATH 117 Introduction to Statistics. The faculty interviews, a component of my Why Statistics?? Assignment, allow students to explore the application of statistics in various disciplines and to consider how they may use statistics in their course of study as well as in future careers. In this presentation, I will discuss the Why Statistics??? Assignment, the faculty interviews, and online discussions regarding the usefulness of statistics lead by students in the Honors section and involving students in the standard section. In addition, I will discuss how the Why Statistics??? Assignment enriches the end-of-semester Honors reflection paper.

#### Lisa Marano West Chester University (Imarano@wcupa.edu)

#### "To Be Honorable is to Serve" How to Align with this Motto in a General Education Honors Mathematics Course

"To Be Honorable Is to Serve" is the motto of West Chester University's Honors College. This succinctly summarizes the aim of the program which is to prepare students to be a force of positive change through leadership, scholarship, service and teamwork. How does one infuse these ideals into a general education mathematics course for those who are not already required to take mathematics by their academic major? The answer: Social Justice and Service Learning. There are many challenges that can affect the extent to which one can incorporate these ideas. For example, is there flexibility to alter existing courses? Are there hurdles in place on campus which hinder creating new courses? Once the course is in place, how does one deal with student resistance? Is it risky to get involved when untenured? In this talk, I will discuss my experiences with teaching Social Justice and Service Learning to the Honors College population and how I dealt with these same challenges. Anecdotal evidence of their effectiveness will be provided.

## Flipping Pedagogy in College Mathematics Courses

#### Session 1: Thursday, August 7, 1:00-5:55 PM, Plaza Level, Broadway III & IV

Organizers: Jean McGivney-Burelle University of Hartford Larissa Schroeder University of Hartford John Williams University of Hartford Fei Xue University of Hartford Mako Haruta University of Hartford Ben Pollina University of Hartford

While the expression "flipping a course" is relatively new, this pedagogical strategy has been around for a number of years. Some tenets that underlie this type of pedagogy are that: (1) out-of-class time should be highly structured to best prepare students for in-class activities; (2) it is useful to evaluate students' pre-class preparation and for instructors to have access to this information; (3) class time is better spent having students engage in cooperative problem solving and discussions rather than listening and taking notes; and, (4) students benefit from more frequent structured practice and feedback in the classroom from a knowledgeable teacher. In this session participants will present and discuss examples of flipped mathematics courses and share the benefits and challenges of this type of pedagogy. Descriptions of unique models of flipped classes are welcome as are results of research on flipping pedagogy.

## **Perry Y.C. Lee** Kutztown University of Pennsylvania (plee@kutztown.edu)

Padraig McLoughlin Kutztown University of Pennsylvania (mcloughl@kutztown.edu)

#### Flipped/Inquiry-Based Learning Approach in a 'Large' College Algebra Classroom: An Interim Report

Inquiry-Based Learning (IBL) in a mathematics classroom seems to be very effective for engaging students in the understanding of course material. That is, students interact with peers and the instructor by asking questions and conjecturing (by doing mathematics) in the classroom. Recently, the 'flipped' or inverted approach to teaching courses has received considerable attention, and in these flipped courses, students come to class prepared before the class meeting. The common denominator in the flipped and IBL methodologies seems to be an accent on active student involvement in the classroom.

One author implemented a method which uses both the flipped and the IBL methods (or the F/IBL method) into his 'large' College Algebra classroom for managing and engaging students both inside and outside the classroom setting. Such was done in the Fall 2013 semester and the Spring 2014 semester.

During this past 2014 Spring semester, student-learned outcomes were assessed to determine the effectiveness of this F/IBL approach in his 'large' College Algebra classroom compared to two other 'large' College Algebra classrooms that were both taught using the traditional lecture-style methods.

This talk will address the differences in the F/IBL approach used in the Fall 2013 semester versus the Spring 2014 semester; will also present preliminary data of student learned outcomes from the F/IBL and two other College Algebra large classes during the 2014 Spring semester; and how the next stage of this research shall be designed.

Alison Reddy University of Illinois (aahlgren@illinois.edu)

## Flipping College Algebra: A Blended Approach

At the University of Illinois all students scoring below 50 on the placement exam are placed into College Algebra. Thus students enter the course with very diverse mathematical needs and background knowledge. The challenge was to maximize student outcomes with minimal resources within the context of a single course. College Algebra was redesigned to a blended course: once a week large lecture, heavy use of ALEKS for learning and practice outside of class, and Piazza for communication. As ALEKS adapts to each individual student's learning and needs some students may spend 120 hours in the system over the course of the semester, while others may spend 30. Piazza has been added for communication and class discussion. There are 200 students in lecture and they often feel anonymous and removed from the instructor. Piazza has provided a place for whole class discussions and for students to communicate directly with each other. The redesign has been extremely successful in meeting the individual needs of all the students and we have seen improved success rates and student satisfaction. Collected data will be shared.

## Jerry Overmyer MAST Institute (jerry.overmyer@unco.edu)

#### Flipping College Algebra: What Affects Student Achievement?

This quantitative research compares five sections of college algebra using the flipped classroom methods and six sections using the traditional lecture/homework structure and its effect on student achievement as measured through common assessments. In the traditional sections, students spent class time receiving lecture and reviewing homework and exams. Outside class time was spent on traditional homework. In the flipped sections, students viewed short video lectures and submitted basic homework solutions online outside of class time. Students then completed their homework assignments in class with the instructor. Some flipped sections instructors also used collaborative group work, inquiry-based learning and active whole-class discussions. All sections took common assessments for their final exam and completed a pre/post algebra readiness exam.

The exam data from the sections were analyzed and compared using regression and ANOVA methods with instructional method, gender and ACT mathematics scores as independent variables. Final exam scores and pre/post algebra readiness exam scores were the dependent variables. The findings of this research show that there was not a statistically significant difference in the scores of students in the two groups, however students in the flipped sections did score better than student in the traditional sections. Instructors of flipped sections who had previous classroom experience with inquiry-based and cooperative learning methods had sections with statistically significant higher common final exam scores. The results are followed by implications for teaching and recommendations for practice and further research.

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## Procedural and Conceptual Thinking in a Flipped College Algebra Classroom

One of the more recent advents of the use of technology in traditional learning settings is the "flipped classroom", in which course content delivery is time-shifted through the use of online video or lecture and then non-lecture based activities related to the videos are done in the classroom. Having taught using a flipped classroom in a previous semester, my goal when teaching College Algebra in a flipped setting in Spring 2014 was to improve in which the specific content was being delivered to students in my classroom. In the past, I found that procedural video lectures followed by in-class activities centered on procedures was effective, if anything, because students were spending more time with course material. However, my goal was to place emphasis on conceptual thinking, especially within the context of mathematical modeling.

I redesigned my College Algebra flipped classroom to have mostly procedural ideas in online video lectures and used class time for more conceptual ideas and modeling problems. Together with another colleague, we flipped three out of four sections, one of which I did not flip and taught in a traditional format. We collected artifacts from our classes such as quizzes, exams, group labs, and attitudinal surveys. All of the collected student data, together with teaching reflections, are being analyzed using mixed methods to explore the details of student learning through this new pedagogy. In this presentation, I will present our results and implications for future teaching and research.

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Daniel Pinzon	Georgia Gwinnett College (dpinzon@ggc.edu)
Matt Stackpole	Georgia Gwinnett College (mstackpo@ggc.edu)

#### Re "modeling" College Algebra: A Flipped, Inquiry-Based Approach

We will discuss a flipped, inquiry-based approach to college algebra at Georgia Gwinnett College. This approach was used in 9 sections of College Algebra in the Fall 2013 semester taught by the presenters. Students work in small, structured groups on guided inquiry activities after watching 15-20 minutes of videos before class. We discuss a portion of an in-class activity and a writing project used in the course. The results after one semester are that the students in this model did just as well, even slightly better, than students in the traditional lecture sections on common final exam questions.

Heidi Staebler-WisemanMontana State University (h.staeblerwiseman@montana.edu)Jocelyn ShortMontana State University (short@math.montana.edu)Kelsey KochMontana State University (kelsey.koch@msu.montana.edu)

#### TEAL (Technology Enhanced Active Learning) College Algebra at Montana State University

The following aspects of TEAL College Algebra at Montana State University will be described: course outline, online-video mini-lectures, structured student lecture notes, in-class group worksheets, online homework and grading structure. Data will be provided that compares student success in TEAL sections versus traditional lecture sections over three semesters. Lastly, the co-presenters will discuss the "benefits and challenges" of the TEAL format from their various perspectives; e.g. as a graduate student, as an experienced faculty member, and as a developer of course materials.

**Rikki Wagstrom** Metropolitan State University (rikki.wagstrom@metrostate.edu)

## Integrating Sustainability into Algebra Courses: A Flipped Classroom Model

Metropolitan State University offers a course entitled Mathematics of Sustainability. This course is designed as a pre-requisite for College Algebra that also fulfills the university's general education requirement, People and the Environment. Since first offering the course in 2008, a primary challenge in teaching the course is finding sufficient time in class to both cover the required mathematics curriculum and also the sustainability-related math curriculum, both of which were developed specifically for the course. To address this issue, a flipped classroom pedagogy was introduced in Fall 2013. In this presentation, I will discuss how this pedagogy was implemented in the Mathematics of Sustainability course, the benefits we've observed from implementing a flipped classroom approach, and the challenges of adopting such a pedagogy for use in a non-standard course.

Karen O'HaraHigh Point University (kohara@highpoint.edu)Adam Graham-SquireHigh Point University (agrahams@highpoint.edu)Laurie ZackHigh Point University (lzack@highpoint.edu)Jenny FuselierHigh Point University (jfuselie@highpoint.edu)Ron LambHigh Point University (rlamb@highpoint.edu)

## **Flipping Freshman Mathematics**

The flipped classroom has been studied on a small scale for a variety of upper-level mathematics courses. We were interested in a largerscale study of flipped methodology on lower-level math courses. Our study compared a flipped class with a standard lecture class in four introductory courses: Finite Mathematics, Precalculus, Business Calculus, and Calculus 1. Each professor recorded videos of their lectures using screencast software. The flipped sections watched the videos outside of class and then used time in class to actively work problems, whereas the traditional sections had lectures in class and did all homework outside of class. No statistical difference was found in the test scores of the students, though there was a fair amount of qualitative data to indicate potential problems with implementing flipped pedagogy. In particular, we found that a number of students had a negative opinion of the flipped model, and that attitudes toward math in general tended to decline, comparatively, for students in the flipped class.

Larissa Bucchi Schroeder University of Hartford (schroeder@hartford.edu) Jean Marie McGivney-Burelle University of Hartford (burelle@hartford.edu) Fei Xue University of Hartford (xue@hartford.edu)

## How does flipping affect students' perceptions about learning Calculus?

In flipped Calculus I courses at the University of Hartford, students spend the majority of the class period working in small groups or engaged in whole group discussions. Preliminary data suggests that students who spend time doing and discussing mathematics develop different

perceptions about learning mathematics than those enrolled in lecture-based classes. In this presentation, we will discuss the results from a recent qualitative study of students' attitudes towards mathematics after taking a discussion-oriented flipped Calculus I course.

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## Flip the Calculus Classroom: What Works, for Whom and in What Context?

Taking advantage of web technology, the "flipped classroom" model is becoming increasingly popular. Students watch recorded video lectures before coming to class. In class they review and assimilate materials through peer instruction and just-in-time teaching. Since fall 2012 two mathematics instructors at Simon Fraser University, Burnaby, B.C., have been using this model in their first year calculus courses. A team that includes both instructors, an educational developer, a graduate student in a mathematics education program, and a statistician has been established to collect and analyze various data sets with the goal to understand the perceived impact of this pedagogical model on student learning and instructor's teaching. Data were collected from student questionnaires, student interviews, instructor interviews, classroom observations and the educational developer's personal reflections. In this session, we will present some of our results from instructor's point of view. We will discuss the lessons learned in terms of instruction preparation, student preparation, and classroom implementation.

Lori Beth Ziegelmeier Macalester College (lziegel1@macalester.edu) Chad Topaz Macalester College (ctopaz@macalester.edu)

## A Study of Flipping vs Not Flipping in Applied Calculus

Educators realize that the time spent in the classroom is a limited commodity. Thus, uncovering the most effective use of class time is of utmost importance. The relatively new trend of flipping the classroom — or moving lecture outside of the classroom and more problem solving, discussion, and student-driven activities into the classroom — has recently gained traction as a way to center the learning on students. In an effort to better understand the efficacy of this approach, a controlled study at the small liberal arts college Macalester College was implemented. Two sections of the entry level Applied Calculus course — requiring no prerequisites and focusing on modeling and computation from both a single and multivariable perspective — were compared. One professor taught both sections, with one section following a more traditional lecture approach and the other following a flipped approach. Data related to student performance, as well as perceptions of the approach and attitude toward mathematics in general were collected and analyzed. This talk will provide an overview of the structure of the two sections, set-up of the study, and analysis of the data.

## Jean Marie McGivney-Burelle University of Hartford (burelle@hartford.edu) Larissa Bucchi Schroeder University of Hartford (schroeder@hartford.edu)

## Challenges and pitfalls of assessing the effectiveness of flipped mathematics courses

As flipping pedagogy becomes more widespread, it is important to evaluate the impact of this approach on student engagement, learning, and retention in mathematics courses. A significant challenge in studying the efficacy of any instructional method is controlling for a range of variables that influence student learning. In this presentation, we will discuss the pitfalls we have encountered while assessing the success of our flipped Calculus I courses, as well as some of the research tools that have provided us with useful data about student perceptions and performance.

## Gulden KarakokUniversity of Northern Colorado (gulden.karakok@unco.edu)Emilie NaccaratoUniversity of Northern Colorado (emilie.naccarato@unco.edu)

## Meta-analysis of Flipped "Pedagogy" in Undergraduate Mathematics Courses

The flipped "pedagogy" or "classroom" model is being used more frequently in many undergraduate courses. In this model, content delivery in the form of lecture is time-shifted through the use of online video, and then in-class, face-to-face time is centered around non-lecture based activities. As with any new teaching model, some immediate questions are whether the use of such an online learning system is helpful in improving student learning and, considering that students' learning is at stake, whether the flipped classroom model is comparable to traditional classroom instruction. Depending on instructor goals, any comparison or assessment may be greatly influenced by the implementation. An understanding of the flipped classroom encompassing all possibilities is necessary to gain more information about student learning. While research exists that describes various implementations of the flipped classroom, investigating the instructional goals when creating online videos and in class activities, as well as various implementations of these (videos and in-class activities) together with student outcomes, requires in-depth exploration. For these reasons, we conducted a qualitative research study to describe and characterize the implementations and expectations of a flipped classroom from the perspective of post-secondary math instructors who have experience with a flipped classroom. We believe that before reporting on any student learning outcomes, such a meta-analysis of various instructional implementations (which includes curriculum development and student assessment) is required. In this presentation we will report on the themes we observed within various implementations of this model.

## Mindy Capaldi Valparaiso University (mindy.capaldi@valpo.edu)

## Flipping Calculus II: Did it improve this infamous course?

Calculus II material, as many students will tell you, is not easy. What helps, in this course as in any math class, is practice. By flipping calculus II, students can get extra "lecture" material, in-class practice, and out-of-class practice. The course was also made more conceptual, in an inquiry-based learning style. This talk will describe the nuts and bolts of a flipped calculus II class, including what a typical day entailed, example problems, and how the pre-class material was presented. I will also discuss its success based on student grades and surveys.

Jim RolfYale University (jimrolf@yahoo.com)Yu-Wen HsuYale University (yu-wen.hsu@yale.edu)Susie KimportYale University (susie.kimport@yale.edu)Jennifer FrederickYale University (jennifer.frederick@yale.edu)

### Flipping the Integral Calculus Classroom with Multiple Instructors

We report on results from an experiment to assess the use of an Inform/Confirm/Extend (ICE) framework to flip several sections of an Integral Calculus course with multiple instructors. In particular we share student performance and attitude data from the use of pre-class videos to provide students basic Information, online prep problems to Confirm student understanding, and in-class activities to Extend student understanding. We also provide data from related to our attempts to transfer this approach to multiple instructors teaching multiple sections using the same ICE framework in the same course.

## Session 2: Friday, August 8, 8:30–11:45AM, Ballroom Level, Parlor AB

Mary D. Shepherd Northwest Missouri State University (msheprd@nwmissouri.edu)

## **Reading Guides in a Flipped Classroom**

Unlike a "traditional" flipped classroom where students watch a video lecture by the teacher before coming to class, I give my students reading guides that describe how one might read a textbook for understanding. For more than 10 years, I have been teaching, researching, and encouraging students to read their textbook before coming to class, but it is only this past year that I made the commitment to 'flip' entirely with daily reading guides in Precalculus and Calculus 2 to help students read their textbook outside of (and before) class. I will discuss the development of reading guides; the types of problems the students work on before class, during class and after class; the feedback I am able to give students; and some of the reactions of the students to this non-lecture type of class.

Christine Ann Shannon Centre College (christine.shannon@centre.edu)

## A Measured Approach to Flipping the Analysis Classroom

While the "flipped classroom" can conjure images of making videos of the proof of the Mean Value Theorem for students to watch before class, there is a more measured approach to getting students prepared for active involvement in the development and proof of the theorems of real analysis. As attention spans have shortened, it has become more challenging to keep the class engaged in a course where so much of the work revolves around proofs. By expecting students to read the material before class and providing them with some sign posts to guide that reading, class can be centered on worksheets which direct small groups of students through a proof by outlining steps and asking pertinent questions along the way. This combination can encourage critical investigation of the meaning of a formal definition or the need for a particular hypothesis by looking at examples and counterexamples. These insights can fuel the students' more active participation in the proof writing process in class.

## Reza O. AbbasianTexas Lutheran University (Rabbasian@tlu.edu)John T. SiebenTexas Lutheran University (JSieben@tlu.edu)

### A Day in the Life of an Inverted Classroom

In this presentation we will discuss lessons learned in designing and conducting an inverted classroom. We have practical advice concerning the production of passive learning materials (videos), and the active learning assignments (in-class work). In the first segment of our presentation we will summarize various approaches to creating short videos, avoiding potential pitfalls and means of encouraging students to watch the videos. In the second part we will talk about the most important part of flipping the classroom, namely the group and individual projects assigned during the class time. We will discuss the reference material, types and frequency of assignments and assessment of individual and group work. Our presentation is intended for mathematics and/or statistics educators with interest in using instructional videos, online resources and flipped classroom approach to teaching. Our talk will be appropriate for the new comer to the flipped classroom and we welcome discussion with the more seasoned practitioner.

## Emily Cilli-Turner Salve Regina University (emily.cilliturner@salve.edu)

## Flipping the Classroom in Introductory Statistics

Although there is research studying the effects of flipping the classroom in several educational contexts, thus far there is little research focused on using this teaching technique in the introductory statistics classroom. This talk will describe a project to study the effectiveness of using a flipped classroom teaching intervention in an introductory statistics course. Data was collected educational outcomes, such as course grades

and performance on final exam questions, as well as measures of students' attitudes toward the flipped classroom methodology. Performance data was be compared against a control group of students in the same course in the prior semester where the flipped classroom was not used. Results on student performance on questions about important topics in introductory statistics as well as the impact of student attitudes toward the flipped classroom on performance will be shared

## **Jessica Knoch** Lane Community College (knochj@lanecc.edu)

## Introductory Statistics in a Flipped Format for Community College Students

This spring, I taught an introductory (200-level) statistics course using a flipped instructional format. The students read the textbook or watched instructional videos before coming to class, using prep sheets to assist with their learning and for accountability. Then in the class-room, I used a collection of structured group and individual activities that were designed to make use of the knowledge they were supposed to have gained. Using this format, what I had previously changed from a traditional lecture course to a combination of lecture and activities, now became a series of activities with targeted instruction given as needed before and after group or individual work. Where I used to assign several homework problems for each concept, in this class the homework became a mix of reading and learning, completing the prep sheets, and only one or two "focus" problems per chapter.

In my presentation, I will outline the materials I created to use in this flipped format, the instruction methods in class and outside, and the potential time outlay for anyone considering creating a flipped classroom. In addition, I'll share what worked well and what didn't, what I would do differently, what students thought of the flipped format, and how outcomes compared to other similar classes.

## **Eric Eager** University of Wisconsin - La Crosse (eeager@uwlax.edu)

#### Math Bio or BioMath? Flipping a Mathematical Biology Course

Mathematics has become an indispensable set of tools for modern biology and chemistry. Biological applications are creating problems needing the development of new mathematics, and mathematics is providing the biological community with previously unattainable solutions and perspectives. Because of this, courses in Mathematical Biology have become commonplace in university curricula around the country. However, not all Mathematical Biology audiences are created equal. We currently have one Mathematical Biology course at UW – La Crosse. This course is a requirement for BioChem majors. Because of this I decided, in the spring of 2013, to implement a modified flipped classroom based entirely on case studies, with the mathematics introduced outside of class via video lectures. In this talk I provide the layout for the course, some example case studies and video lectures, as well as the results of a CLASSE survey describing students' reactions to the course.

#### **Rebecca Diischer** South Dakota State University (rebecca.diischer@sdstate.edu)

#### An Activity-Based Approach to Flipping Quantitative Literacy

A year ago, South Dakota State University's online Quantitative Literacy course transitioned to an on-campus course. A flipped instruction approach was utilized with great success. Prior to each class period, students completed a graded learning guide that led them through lecture videos, their e-textbook, and "You Try It" problems. Class time was spent primarily on working in teams designated by major to complete activities featuring hands-on, real-world problems. Students utilized a wide range of resources including their smart phones, social media, and government websites. This talk will discuss the development, implementation, and success of an activity-based flipped instruction approach to Quantitative Literacy.

### Benjamin V.C. Collins University of Wisconsin - Platteville (collinbe@uwplatt.edu) James A. Swenson University of Wisconsin - Platteville (swensonj@uwplatt.edu)

#### Flipping the Discrete Math Classroom

We describe our experience with flipping the classroom — making lectures available as online videos, so that students can engage actively with the course content during the regularly scheduled classroom hours.

At UW-Platteville, the Discrete Mathematics course marks a critical transition for sophomore-level mathematics majors. Every semester, an unacceptably high proportion of students are unsuccessful in Discrete Math, even though these are students who have already shown the ability to succeed in mathematics at a college level. Since our Discrete Math course is specifically intended to prepare students for our proof-centered courses at the junior and senior level, we hoped that improving Discrete Math might lead to better outcomes for many of our majors.

After four semesters, we regard the project as a success, and intend to continue with the flipped paradigm. Unfortunately, but predictably, the new course structure did not lead to complete success for all students. Still, we have enjoyed the new opportunity to interact with the students, and for the students to interact with each other, in a more active way. We are encouraged by the students' feedback on the flipped classroom, which was generally positive, and we believe that students are coming out of our new Discrete Math course better prepared to succeed in their future work.

#### **Fei Xue** University of Hartford (xue@hartford.edu)

Larissa Bucchi Schroeder University of Hartford (schroeder@hartford.edu)

## Technology Tips for Creating Videos in a Flipped Mathematics Course

Most models of flipping pedagogy include the expectation that students watch short videos of course content before coming to class. While there are freely available mathematics videos on websites (e.g., YouTube), some faculty are interested in creating more personalized videos

that reflect their unique style and approach to the content. However, the creation of such videos can be challenging. In this session we will discuss the various hardware and software (e.g., Camtasia, Maple, GeoGebra, iPad apps) we used in creating videos for our project entitled Flipping Calculus. We will also discuss TrACE, a video server used to house videos and facilitate faculty-student interaction. Sample videos will be shown.

Alex Capaldi Valparaiso University (alex.capaldi@valpo.edu)

#### Selling the Concept - a Primer on Salesmanship of the Flipped Classroom Model

So you tried a flipped classroom, but ended up with poor teaching evaluations. Students wondered why they had to "teach themselves" and "knew" they learned more from traditional lectures. Now you're too scared to try again. Don't be! Students are often afraid to go outside their comfort zone. This presentation will give a number of tactics that can be employed to help persuade students that, while the flipped classroom may be uncomfortable at first, you're on their side!

## **Active Learning in Mathematics**

## Session 1: Friday, August 8, 8:30–11:45 AM, Ballroom Level, Galleria II

Organizers: David Taylor Roanoke College

Robert Allen University of Wisconsin, La Crosse

**Lorena Bociu** North Carolina State University

Active learning is the process where students engage in activities such as reading, writing, or problem solving that encourage analysis, synthesis, and evaluation of class content. It has been well-known that active learning strategies increase student learning and have long-lasting effects on student success (Braxton, et al, 2008). For this session, we invite instructors of mathematics to discuss ways to promote this hands-on learning in the classroom. In particular, techniques that involve short reading, writing, or problem-solving prompts and exercises that are designed to reinforce classroom material are encouraged. Both examples of individual student active learning strategies and successful uses of group- related strategies (such as "think, pair, share" ideas) are welcome. The session is designed for instructors to share their experiences and provide useful tips and tricks on implementing these strategies and overcoming obstacles to active learning in general. Examples and ideas can come from any type of course, from undergraduate non-major service courses and early-major mathematics courses to late-major and even graduate-level classes. Speakers are encouraged to include assessment data on the effectiveness of their active learning strategies or empirical feedback from students and/or faculty about their strategies. Talks that focus on embodied activities that connect cognition with physical action in the classroom should submit talk proposals to the Embodied Activities in the Teaching and Learning of Mathematics session.

Krista FoltzOregon State University (foltzkr@onid.oregonstate.edu)Mary BeisiegelOregon State University (mary.beisiegel@oregonstate.edu)Scott L. PetersonOregon State University (speter@science.oregonstate.edu)

#### Active Learning in Redesigned College Algebra: Lessons Learned from Implementation

At Oregon State University, the College Algebra course was redesigned to make at least 50% of class time consist of active learning. Specifically, during two out of four class days each week students are organized into small groups where they complete exploratory activities involving new material while instructors facilitate student engagement and interactions to promote deeper learning of the material. In addition to this restructuring of the course, another goal was to understand how students are experiencing the redesigned nature of the course. Bi-weekly interviews were conducted over the term with students who were enrolled in the redesigned college algebra course to uncover what aspects of active learning are useful and challenging to students. In this session, we will share the following: (1) the rationale and process for redesigning the course based on active-learning principles, (2) the exercises used to get students actively engaged in mathematical thinking, and (3) our findings from understanding the students' experiences in the redesigned course, which include what students found most useful (such as interacting with peers during group activities and deepening their understanding by teaching others); as well as what students found most challenging (for example, the transition from a procedural to a conceptual approach to learning mathematics). Finally, we will provide tips for implementation which include: introducing active learning on the first day of class, offering opportunities for students to familiarize themselves with material prior to activity days, and providing students with a wrap-up of the "big ideas" from the exploratory activities.

## Bernadette Mullins Birmingham-Southern College (bmullins@bsc.edu)

## Active Learning for Pre-service and In-service Teachers

We describe hands-on learning activities used in a Numerical Reasoning course for undergraduate elementary education majors and a summer professional development workshop for in-service teachers. Sample tasks involve both individual and group problem solving. We also describe data from a rubric-scored, performance-based pre- and post-assessment.

## Suzanne Ingrid Doree Augsburg College (doree@augsburg.edu)

## **Turning Homework Problems into Inquiry Based Classroom Activities**

How do we actively engage students in thinking about mathematics? Active learning pedagogies can be quite successful, but the mathematics of the activity matters too. Many of us are convinced that activities which inspire (and require) student inquiry produce high levels of student engagement and, consequently, student learning. Luckily anyone can learn how to turn homework problems into inquiry based classroom activities. And, it is easier than you might think. In this talk I'll share a couple of examples of activities from Precalculus and Discrete Mathematics classes. Moreover, I'll describe the process I use to create the activities and some secrets to successful implementation.

#### Peter Banwarth Oregon State University (banwartp@math.oregonstate.edu)

#### Active Learning in Mathematics: The Math Telephone Game

In a recent redesign of its College Algebra course, instructors and researchers at Oregon State University developed a number of pedagogical materials intended to provide opportunities for active learning in the classroom. In this paper I will discuss research that I have conducted regarding the Math Telephone game, an activity in which students work together to translate information about different functions across multiple representations.

Research has shown that developing the ability to use multiple representations is critical for students to increase their conceptual understanding of mathematical ideas (Goldin, 1998). Experts in a variety of scientific fields naturally use multiple representations in their work (Arcavi, 2003). Furthermore, active and collaborative learning has been shown to increase student success in mathematics terms of both demonstrating greater conceptual understanding and improving grade outcomes (Triesman, 1992).

In this study, a focus group of six students worked on the Math Telephone game in a series of five video-recorded interviews. The group setting and engaging activity engendered a great deal of discussion of ideas and peer feedback. By the second interview, students displayed a natural propensity to graph the functions and use multiple representations. However, when presented with an application incorporating the ideas of the game, the students did not gravitate toward a graphical approach, instead relying on a guess-and-check method. In this paper I will describe and interpret the students' interactions and explain how I think how the Math Telephone activity has impacted these students' learning of key mathematical concepts.

## Mary D. Shepherd Northwest Missouri State University (msheprd@nwmissouri.edu)

## Active Algebra

Abstract Algebra (or Modern Algebra) has typically been a "Definition Theorem Proof" type lecture class. Desiring a much more active classroom, a year ago, I experimented with a Moore Method Algebra class that focused on Groups. But, after what appeared to me to be a dismal failure (on my part) to have the students respond and enjoy the class, I knew something different was needed. During the spring 2014 semester, I decided on a different approach, starting with rings, specifically the integers, and engaging the students in what I called "playing" with the rings, we explored many different rings. I had a much more active and engaged class. I will give some sample tasks to show how this "play" can lead students to the proofs of some theorems/properties. If available, I will include some student comments from this past semester along with comparisons of grades and progress made.

### **Emma Norbrothen** Plymouth State University (emn1010@plymouth.edu)

#### Making Abstract Algebra Less Abstract

"Abstract Algebra is abstract" is one of the most accurate statements I have ever received in a student evaluation. In an effort to make abstract algebra more concrete for undergraduates, I created a curriculum that engages students in several active learning techniques. Whenever we started a new topic, we had a "Discovery Day" in which groups of students discovered the new material for themselves. Before ending our discussions about a topic, we had a "Workshop Day" in which students would teach, critique, and help each other through the homework problems. This presentation will include examples of discovery activities, reflections on how to run workshops, and the schedule of the class. Additionally, at the end of the semester, students were asked to evaluate and reflect on these active learning techniques, and their evaluations and suggestions will be included in the presentation.

#### Alessandra Pantano University of California, Irvine (apantano@uci.edu)

#### Strategies to Progressively Increase Students' Intellectual Engagement in the Learning of Abstract Algebra

In this talk, I will describe a number of instructional strategies designed to maximize the students' intellectual engagement in a one-year-long abstract algebra sequence, starting with Group Theory in the Fall, and moving to Ring Theory in the Winter and Galois Theory in the Spring. In the Fall quarter, students participate in group study sessions where they work on thought-provoking assignments (e.g., "prove or disprove" problems); in the Winter quarter, they are exposed to interactive lectures to "reverse engineer" proofs and discover the reasons behind the definition of key concepts like maximal or principal ideals; finally, in the Spring quarter, they are offered a full IBL course. Naturally, the level of involvement in higher order thinking increases as students proceed through the sequence. The goal is to balance the increase in complexity of the class material with the increase in the mathematical maturity of the students and their augmented comfort with active learning techniques. By the end of the academic year, students are required to fill out incomplete pre-lecture notes outlining the skeleton of

the proofs, present problems and proofs to the class, reflect upon their learning by drawing concept maps, and write-up a concise description of the highlights of the course for a future abstract algebra student.

## **Donna Flint** South Dakota State University (donna.flint@sdstate.edu)

## **Actively Learning Real Analysis**

South Dakota State University has recently developed an "active learning classroom" modeled after similar classrooms around the country. The Undergraduate Real Analysis Class was taught in this classroom in Spring 2013 and Spring 2014. For a professor who always had the inclination to include student participation and discussion, the new classroom offered all kinds of new opportunities. The result was a class where students unabashedly say "Real Analysis is my favorite class!" This presentation will discuss class activities and pedagogy designed to engage students in a traditional classroom before the active learning classroom was available, how the active learning classroom enabled expansion of these activities, and how teaching in the active learning classroom empowered the professor to engage students with active learning strategies even when not using the specially designed room.

## **Ben Galluzzo** Shippensburg University (bjgalluzzo@ship.edu)

## Pull Out Your Phone: A Quick Search for Relevant Statistics

The use of real world data is commonplace in the undergraduate non-major statistic course. However, static presentation often makes this same "real" data appear to be less authentic in the classroom. In this talk we will discuss how a one-day online search activity, and subsequent short searches, can motivate student engagement in statistics throughout the semester.

## Paul E. Seeburger Monroe Community College (pseeburger@monroecc.edu)

## **Exploring Velocity and Acceleration Vectors Visually**

In multivariable calculus, we ask students to calculate vector-valued functions for velocity and acceleration, given a position function. Students often find it easy to visualize the velocity vector being tangent to the space curve describing the motion, but they rarely have a clear picture of the acceleration vector and its relationship to the motion and to the corresponding velocity. Using a freely available online multivariable calculus applet called CalcPlot3D, students can complete a guided exploration of velocity and acceleration. As part of this guided activity, students complete a pre-test, answer exploration questions, and then complete a post-test. The pre- and post-tests measure what improvement occurs in their conceptual understanding of velocity and acceleration by completing the visual exploration. After students have completed this activity, there is often a lively class discussion about the interaction between the acceleration and velocity vectors they observed in the dynamic examples from the exploration. In addition to demonstrating this online exploration, analysis of the pre- and post-test results and student comments on their own learning will be shared. CalcPlot3D is part of an NSF-funded grant project called Dynamic Visualization Tools for Multivariable Calculus (DUE- CCLI #0736968). See http://web.monroecc.edu/calcNSF/.

## Session 2: Friday, August 8, 1:00–4:55 PM, Ballroom Level, Galleria II

Jerry Dwyer<br/>Levi JohnsonTexas Tech University (jerry.dwyer@ttu.edu)<br/>Texas Tech University (levyjohnson@gmail.com)Brock WilliamsTexas Tech University (gwilliams@math.ttu.edu)

### **Surviving Active Learning in Mathematics**

Active learning opportunities often look very different in mathematics from efforts in areas such as lab sciences or pre-professional programs. For example, undergraduate students often lack the necessary course experience to tackle true research problems, and service learning opportunities can often be difficult to integrate into mathematics coursework. This session will focus on relaying some of the creative strategies ranging from zombie super computers to cardboard canoes for engaging students and surviving the administration of these programs.

Matt Boelkins Grand Valley State Unniversity (matt.boelkins@gmail.com)

## Activities for Calculus

Calculus offers ample opportunities to engage students in active learning rather than passive listening. In this talk, I will share some examples from a free collection of over 100 activities that I've developed for use in calculus that are designed to actively engage students in the development of deep personal understanding. Along the way, I will discuss class structure and style to facilitate regular use of in-class activities, students' reactions to the activities and this approach, and direction to additional supporting resources.

Elizabeth Thoren University of California - Santa Barbara (ethoren@math.ucsb.edu)

## Student Conjecturing in Linear Algebra

Students struggle to make the transition from lower division to upper division mathematics courses — where the emphasis shift to abstraction, generalization, precision and proof can be overwhelming. To help prepare students in my lower division inquiry-based linear algebra course

for this jump, I removed all theorems from the course notes and had my students make their own conjectures. Not only did they learned a great deal about mathematical precision and proof in order to support and refine their conjectures, they also began to understand how the process of mathematical inquiry works. The talk will outline the essential elements of this conjecturing activity, as well as some of the surprising results.

## William Abrams Longwood University (abramswp@longwood.edu)

## **Discovering Concepts in Calculus II**

I will share a series of group labs I developed to help students discover both techniques and concepts in Calculus II. Some of these labs were based on examples that I had traditionally used as part of lecture. All of these labs were influenced by the POGIL Calculus training I took recently, although these are not official POGIL labs. I will also talk about difficulties involving student motivation that resulted from open ended and challenging questions. In particular, I had to change the way that I interacted with the groups during lab and I had to change the way I traditionally graded labs. A simple statistical comparison of this class with the last Calculus II class I taught does not show a significant improvement in grade, but I will argue that these labs benefit student learning anyway.

## Randall E. Cone Virginia Military Institute (conere@vmi.edu)

Angie Hodge University of Nebraska - Omaha (amhodge@unomaha.edu)

## Opening Up the Space: Creating Collaborative Learning Environments Outside of the Classroom

As educators, some of our most important interactions with students happen in environments outside of the classroom setting. Such environments include: math tutoring labs, break rooms, workshops, study spaces, review sessions, and recitations. When incorporating Active Learning (AL) and Inquiry-Based Learning (IBL) methods into these settings, we must be particularly sensitive in understanding human interactions with each other and with their environment. In this session, we describe the development and creation of two such learning environments at our home institutions: one being a comfortable and informal study space for students, and the other a formal mathematics education and tutoring center.

## Lew Ludwig Denison University (ludwigl@denison.edu)

## Test Tuesday

In the past, we found that students struggled with the first test. Students would often underestimate their own knowledge and/or the difficulty of the material. To prepare students for tests, we developed a strategy which is referred to as "Test Tuesday;" an active learning technique that engages the students in the material while providing formative feedback in real time. In addition, as a think, pair, share technique was used, students were better able to gauge their understanding of the material compared with the rest of the class.

## **Michael Nathanson** Saint Mary's College of California (man6@stmarys-ca.edu)

## Mathematics without the Math: Using Group Worksheets to Circumvent Math Anxiety

Like many instructors, I want my students to see mathematics as a language which can be used creatively and collaboratively to communicate ideas and construct solutions to problems. I exhort them to "put their whole self in" and tap into their curiosity, common sense and social skills. Unfortunately, this approach is often in direct conflict with deeply ingrained habits of mind which students have developed in earlier courses. To circumvent these learned habits, I often use group worksheets to preview material before we discuss it formally. These worksheets are specifically designed to avoid setting off students' Pavlovian math responses. The questions are written in plain language, and students are not expected to use algebra or complex formulas to solve them. Different questions are designed to develop mathematical communication skills (oral and written); willingness to experiment with ideas; tenacity in addressing difficult problems; and an ability to string several simple steps together. In the process, they develop their own conceptual vocabulary around the new material without awakening the Ghost of Math Class Past.

I will describe my methods and goals and discuss the inherent challenges.

Brandy S. WiegersNational Association of Math Circles, Central Washington University (brandy@msri.org)Addie EvansSan Francisco State University (adde@sfsu.edu)Servando PinedaSan Francisco State University (spineda@mail.sfsu.edu)Matthew KimSan Francisco State University (mcadier@mail.sfsu.edu)

## **Pre-Calculus Lab Book**

In Biology and Physics Lab classes students are asked to record their observations and questions while interacting with engaging approaches to learning their course materials. The PreCalculus Lab-book brings this approach to supplemental math instruction, moving beyond home-work worksheets and creating a written record by students of hands-on-activities and observations of mathematical patterns. The activities included graphing height vs volume of water in bottles, solving linear problem mysteries, spaghetti trig graphs, and Desman on Desmos, and many mores. These "lab" activities highlighted crucial mathematical connections between PreCalculus and Calculus. They also provided opportunities for instructors to nurture students' mathematical and academic skills. Mathematical skills included focus on graphing, using notation, and understanding the difference between equality and equivalence. Student study skills included creating achievable academic goals,

test taking skills, and skills for communicating with peers and professors. Presentation will include assessment of success for the approach and plans for future development.

## Edmund A. Lamagna University of Rhode Island (eal@cs.uri.edu)

## Algorithmic Thinking Unplugged with Puzzles and Games

Puzzles and games are the basis of a liberal arts math course designed to develop mathematical and computational problem solving skills. Desired outcomes include: 1) motivating and creating excitement for math and computer science, 2) working in small groups on fun, interesting problems, 3) instilling the necessary confidence and persistence to solve complex, difficult problems, 4) encouraging "out of the box" thinking and applying alternative problem solving strategies, and 5) developing an ability to communicate mathematical ideas both orally and in writing.

With the proliferation of computational devices, algorithmic thinking is an important problem solving skill for today's students, particularly those in STEM disciplines. While there is a tendency to think of computation in terms of languages and programming, algorithms and algorithmic puzzles long predate the appearance of computers. Puzzles and games provide an "unplugged" programming-free way for students to learn about algorithms while improving their mathematical problem solving skills.

The presentation focuses on engaging puzzles and games promoting algorithmic thinking that the presenter has successfully used with students in an activity-based freshman seminar. Examples include sequential movement puzzles, Collatz sequences, sorting weights and spotting counterfeit coins with a balance scale, puzzles that demonstrate the use of recursion and mutual recursion, and board games on graphs. Developing summations and solving them through "proofs without words" are used to illustrate the analysis of algorithms.

## Tim Gegg-Harrison Winona State University (tgeggharrison@winona.edu)

Nicole Anderson Winona State University (nanderson@winona.edu)

## Using Games to Engage Students in Discrete Mathematics

Discrete mathematics is essential to computer science (CS) students. Unfortunately, in prior years our CS students were not performing well in their discrete mathematics class and were not retaining the material. In order to help our students see the relevance of discrete mathematics, we created an algorithms-based discrete mathematics course. We attempted to get the students more actively engaged by turning the homework assignments into social events where we encouraged students to work through the homework assignments in groups. Rather than having them turn in their solutions to these problems, they were given a 10-15 minute (individual) quiz at the beginning of the each class period that consists of a subset of the homework problems. We saw a drastic improvement in their performance and their retention of the material. However, some of our students were still viewing the course as something they had to "get over with" in order to get to the interesting concepts of computer science.

In this talk, we will present our most recent attempt to provide an active learning environment for our discrete mathematics students. We are creating mobile game apps to help introduce each of the major topics. We will talk about one example of this where we have created a game that is based on some variants of Nim that we use to motivate mathematical induction. Students play the game with their classmates using mobile devices, think about strategies for winning, and ultimately identify a strategy that they prove is optimal using mathematical induction.

## Silvia Saccon The University of Texas - Dallas (silvia.saccon@utdallas.edu)

## Learning Math by Doing Math: Problem-Solving Workshops in Calculus

In an effort to promote students' active engagement in their learning process, I started to run my calculus classes as problem-solving workshops. Students experience an immersion in problem-solving activities by working in teams at the board on problems designed to build and stretch their conceptual understanding of calculus. In this learner-centered environment, my role is to facilitate and guide students' interaction with concepts and with each other. In this talk, I will provide an overview of the course structure and associated active learning strategies, discuss benefits and challenges of this approach, and include an informal review of assessment data and feedback from students.

## Steven Klee Seattle University (klees@seattleu.edu)

## Active Exploration of Graphs and Graph Theory

This talk is motivated by the desire to empower students to read, parse, and comprehend the statements and proofs of complicated mathematical theorems and algorithms. We will describe techniques that have been used to actively engage students in an introductory graph theory class through group work, in-class activities, and self-reflection with the goal of instilling them with practices that will benefit them in future math classes. We will conclude with an exploration of how these techniques can be applied to courses other than graph theory.

## **Project-Based Curriculum**

## Session 1: Friday, August 8, 8:30–11:25 AM, Plaza Level, Broadway III & IV

# Organizers: Emek KoseSt. Mary's College of MarylandCasey DouglasSt. Mary's College of MarylandAngela GallegosLoyola Marymount University

One of the goals of mathematics teaching is enabling the learner to apply their mathematical knowledge to other disciplines and to real-world problems. One method to achieve this goal is project-based learning, which involves students attempting to solve interdisciplinary problems arising outside of the traditional classroom. The problems may arise from general social concerns or from within business, non-profit, or government organizations. Project-based learning can encourage inquiry, problem solving, collaboration, reasoning, and communication skills. We invite papers that address how project-based learning is facilitated at any level and the content of such projects. Evidence should be included as to the effectiveness of such projects and/or the system by which students engage in such projects.

## Daniel Showalter Ohio University (showaltd@ohio.edu)

#### Possibilities and Challenges for Place-Based Mathematics Education

Place-based mathematics education (PBME) links classroom mathematics with the mathematics of the local community, land, and culture. Not only can PBME make learning mathematics more relevant and coherent for students, but it can also address authentic needs in the community. In this session, results will be presented from two recent studies on PBME.

In the first study, seven sites were chosen from around the U.S. as being exemplars of PBME. Sites included a lutherie (instrumentmaking) class in Kentucky, an aquaculture program in Alabama, a forestry-based Algebra course in Vermont, and an entire PBME-centric school on an island in Maine. Members of our research team then spent one week at each site interviewing students, teachers, parents, and administrators.

In the second study, I interviewed 15 graduates of a NSF-funded doctoral program with a heavy emphasis on PBME. The graduates shared their attempts to implement PBME theory in the high schools and universities where they were teaching.

Themes elicited from the interview transcripts of both studies will be presented and interpreted in terms of what they mean for the future of PBME.

## Victor lan Piercey Ferris State University (piercev1@ferris.edu)

#### A Project-Based General Education Math Course

A terminal, general education math course for a highly math anxious and skeptical population can be a real challenge. In this talk I will discuss a project-based version of this course that included service-learning. In addition to describing the projects, I will address the selected outcomes, assessment, and techniques for student engagement and buy-in. Finally, I will share data regarding the impact on math anxiety and student beliefs about math.

**Bruce Piper** Rensselaer Polytechnic Institute (piperb@rpi.edu) **Kristin Bennett** Rensselaer Polytechnic Institute (bennek@rpi.edu)

### High Dimensional Data Analysis Projects in a Freshman Mathematics Class

The "Data Analytics Throughout Undergraduate Mathematics" (DATUM) project at Rensselaer Polytechnic Institute is an integrated education/research program aimed at training students to become versatile in the philosophy and tools of data analysis and data modeling. The anchor of the educational program is a new Freshman level, project centered course about analyzing high-dimensional data that arises in realworld applications. Sample projects include using Fisher Linear Discriminant Analysis in classifying bio-degradable chemicals and Principal Component Analysis in face recognition. Students are formed into teams of "Data Analytic Consulting Companies" and real data is used for the projects. The social and business significance of the applications provide context and motivation for the students and many of them do work above and beyond the project descriptions. The only prerequisite for the course is Calculus, and the class is designed to develop little more than the bare minimum of the mathematics needed for the projects. The projects help equip the students with an exploratory mindset to engage in the mathematics they will study in subsequent courses. The design and structure of the class and the projects will be discussed. Results on the effectiveness of the class as measured by pre and post survey assessments as well as the success of students in subsequent summer research will be presented.

David Jay Graser Yavapai College (David.Graser@yc.edu)

#### How to Sustain Projects in College Algebra and Finite Mathematics

Using projects in lower level classes is more than assigning a problem and collecting the solutions from students several weeks later. For a project to succeed, it needs to be carefully constructed from realistic applications and monitored to ensure that students stay on task. In

#### Project-Based Curriculum

addition, the student's understanding of the project needs to be assessed in a manner that is meaningful for the student and useful to the instructor. In this session, I'll introduce two series of projects that I use in College Algebra and Finite Mathematics. In the series of projects for College Algebra, students model college costs to determine the savings they might accrue from attending two-year college to get a fouryear degree. The series of projects for Finite Mathematics examines how a water provider might blend water from different sources to reduce the contaminants to an acceptable level. I'll discuss how these project were constructed as well as logistical concerns that might arise in using the projects. Scaffolding activities are key to monitoring each student's progress so I will show several examples of these activities. I'll also present several research posters that students have created to document their solutions to each project.

## Dianna SpenceUniversity of North Georgia (djspence@ung.edu)Brad BaileyUniversity of North Georgia (brad.bailey@ung.edu)

### Researching the Effectiveness of Project-Based Learning in Elementary Statistics

We describe the findings to date of a long-term investigation into the effectiveness of student-directed projects in elementary non-calculus statistics courses. The researchers developed curriculum materials and instructional methods to foster authentic discovery learning through projects, with a focus on multiple disciplines; these disciplines include the social sciences, health sciences, business, government, criminal justice, and education. Using these materials, instructors have participated nationwide in a quasi-experimental pilot study to compare student outcomes in project-based classes to those in "traditional" classes. We articulate distinguishing features of the projects and describe how their use modifies the instructional approach to the course; share the curriculum materials developed to facilitate these projects; give examples of student work; outline the mixed methods research design used to measure the impact of this approach; and summarize results of analyses to date based on quantitative and qualitative data collected from the pilot instructors.

## **G. Daniel Callon** Franklin College (dcallon@franklincollege.edu)

## **Community-Based Projects Using Real-World Data**

Almost all nonprofit organizations have lots of unanalyzed data or a need for specific data but don't have the expertise to address the situation and can't easily afford to hire someone to do so. As a capstone experience for our applied mathematics and quantitative analysis majors, our students work in teams to complete semester-long statistical consulting projects for local nonprofit agencies. The course is cross-listed between economics, psychology, sociology, and mathematics, and starts with preparatory instruction in the consulting process, group dynamics and leadership, and project management conducted by a panel of expert advisors who also volunteer to consult with the teams on an as-needed basis throughout the semester. The course has been offered for twelve years and has been cited by a number of alumni as the most powerful experience of their undergraduate education. It has also been cited by our department and by Franklin College as a distinctive and vital aspect of our curricular offerings directly aligned with our college's mission.

We will discuss how the course is structured, supervised, and assessed, along with examples of current and previous projects. We will also present evidence of its impact on the individual nonprofit organizations and on the community as a whole.

## Judith E. Canner California State University, Monterey Bay (jcanner@csumb.edu)

## Understanding Mathematics for Good: Undergraduates, Ethical Consulting, and Service Learning

Data increasingly drives policy, business and politics. Many mathematics and statistics students enter the workforce with significant knowledge, but have little experience applying that knowledge to solve real-world problems. In addition, mathematical educators often neglect the teaching of ethics in the application of statistical and mathematical methods. I will present my efforts to design an upper-division, project-based, service learning course for mathematical consultants at California State University, Monterey Bay (5500 students). I will discuss the structure of the course, assigned readings and discussion topics. I will present student responses to the classroom discussion and the relationship between classroom discussions and student projects. I will outline the professional and mathematical preparation necessary in the course curriculum to ensure the success of student projects. In addition, I will present examples of student projects for community agencies and address both student and agency responses to the service learning projects, the successes of the course, and areas in need of improvement. Finally, I will discuss ethical and professional dilemmas and considerations the students faced in their projects as they experienced their first foray into professional consulting. I hope to provide a foundation on which other programs can build a project-based consulting course for undergraduate students so that we sufficiently prepare our students to enter the workforce after graduation.

## Ksenija Simic-Muller Pacific Lutheran University (simicmka@plu.edu)

### Mathematizing Social Justice: Bringing University Events into the Mathematics Classroom

While social justice topics have become more prominent in university and college curricula in the recent years, mathematics classes still often remain detached from these topics, perpetuating the idea of mathematics as neutral and culture-free. This session will present a project assigned annually in mathematics content courses for preservice teachers that attempts to bridge the gap between mathematics and social justice. Students enrolled in the courses attend the Tunnel of Oppression, an annual interactive event that explores different forms of injustice through different scenes (e.g. rape in the military, child labor in Asia, and racial profiling). Following the event they write reflections about the presence of mathematics in the scenes; conduct additional research and write mathematical reports about scenes of their choosing; and give recommendations to the event organizers for strengthening their arguments through the use of mathematics. Over the last three years, the

assignment has become more carefully structured and better aligned to the event and its organizers. As a consequence, students have shown greater ownership of their work and have expressed greater enthusiasm about the assignment.

We will look at the assignment guidelines; examples of student work; and samples of student project evaluations, and discuss ways in which community or university events can be connected to the mathematics curriculum in meaningful ways. While this project is assigned in courses for preservice K-8 teachers, it can easily be modified to fit any lower-division mathematics class.

Mariah BirgenWartburg College (mariah.birgen@wartburg.edu)Brian J. BirgenWartburg College (brian.birgen@wartburg.edu)

## Modeling Calculus: A Project-Based, First Term Calculus Class

To solve the numerous problems with a traditional Calculus sequence, Wartburg College has re-structured the curriculum to start with Modeling Calculus. In this course, students learn to develop, and numerically solve, differential equations models of interdisciplinary problems. The course is designed to introduce students to the skills necessary to tackle increasingly more challenging questions as the course moves through the study of various standard models and asks the students to improve the model in order to make it more realistic. The final activity asks students to find their own journal article in a different discipline and re-create the author's analysis and, if possible, extend the results of the model.

This approach introduces all students to new material immediately, whether or not they have taken Calculus in High School. The use of published problems for projects puts to rest the ever-present question, "where will I ever use this?" Students gain essential programming skills in a non-threatening environment and their confidence in mathematical logic is strengthened without relying on their skills in Algebra. They work on a challenging project for an extended period of time and develop the ability to write a scientific journal article and present their research to their peers. Successful students are then tracked to either a fast-paced techniques of Calculus course, Statistics, or directly to Linear Algebra depending on what they need for their current major.

## Session 2: Friday, August 8, 1:00–3:55 PM, Plaza Level, Broadway III & IV

**Nora Strasser** Friends University (strasser@friends.edu)

#### Annexation Question Leads to Applied Project

During the Fall semester of 2012, a group of students were enrolled in a special projects course designed to specifically research a problem presented to them by the Zoning and Planning Commission of Garden Plain, Kansas. The students were asked to research and present their findings on the question: "What are the total marginal and fixed benefits/costs for annexation of adjacent property into the city?" The students met with the teacher once a week and the teacher acted as a facilitator. However, the students took the lead and worked collaboratively. The students did all of the research and created a written report. The students also created a formal presentation that they presented to the Zoning and Planning Commission during one of the Commission's regular meetings. The project will be described and student evaluation data will be presented to indicate the success of the project.

## Andres Abelardo Padilla-Oviedo Faculty (apadillao@utpa.edu)

#### Challenge-Based Instruction: Analysis of Bullet Proof Vest

Part of inquiry-based learning, Challenge-Based Instruction helps develop students' adaptive expertise by allowing them to discover and apply concepts by introducing a challenge and involving them in the steps of a Legacy Cycle. Two groups of students in a statistics and chemistry course were provided with an interdisciplinary challenge to analyze a bullet proof vest. The challenge was designed to increase students' interaction and engagement by providing a real world application that required chemistry and statistics. Groups were formed across the two courses and students shared their knowledge in fibers and conducted t-tests to analyze the fibers to create a bullet proof vest. In this talk, the challenge, curriculum, instructor collaboration, and group interaction will be discussed more in detail.

Jean Marie Linhart Texas A&M University, Central Washington University (jmlinhart@math.tamu.edu)

#### Building a Successful Project-based Mathematical Modeling Course

A mathematical modeling course provides an ideal setting for an interdisciplinary project based curriculum. The projects we undertake arise from mathematical biology, investing, and weather prediction. Students are also encouraged to come up with their own final project. Giving students choices and options in how they complete projects is one of the key components that encourages buy-in and personal investment in their outcome. Careful management of group work and participation is another. Writing and presenting about projects is a third. Successful outcomes have ranged from publications in undergraduate journals, winning writing and video competition entries, and award winning MathFest presentations.

## **Corban Harwood** George Fox University (rharwood@georgefox.edu)

## Encouraging Deeper Understanding Through Mathematical Modeling-Focused Projects

Over the past two years, I have incorporated project-learning activities in Differential Equations, Linear Algebra, Numerical Methods, and Liberal Arts Math classes. I have encouraged modeling-focused projects by having students research their own data, compare problem formulations and question their results at every step. I will describe some of the individual and group projects my students have done to connect with other disciplines like business, political science and engineering and share how I set up my courses to support them in connecting to real data and problems.

## **Michael Dorff** Brigham Young University (mdorff@math.byu.edu)

## PIC Math: Preparing Students for Careers in Business, Industry, and Government

PIC Math is a new program to prepare students in the mathematical sciences to succeed in careers in business, industry, and government (BIG). Funded by a \$2 million NSF grant, this program (a) helps students be aware of their choices for non-academic careers and opportunities for internships, (b) helps faculty be more fully aware of non-academic career options for their students, make connections with people working for local BIG organizations, and develop internship opportunities for their students, (c) offers students the opportunity to have a research experience related to real-world problems from BIG during a spring semester course, and (d) provide training to students and faculty in how to successfully work on problems from BIG and develop the needed communications skills. To accomplish these objectives, we are developing a set of educational and informative videos, conducting summer training workshops for faculty, and preparing materials for a semester-long course in which students learn skills and work on research problems from BIG.

## Shawn Chiappetta University of Sioux Falls (shawn.chiappetta@usiouxfalls.edu)

## Embedding Undergraduate Research in a Senior Capstone Course

In 2003, the University of Sioux Falls mathematics program started a senior capstone course with dual purpose. One purpose was to capture assessment information from the mathematics and computer science students regarding their undergraduate experience. The other significant component of the capstone course is to have majors complete an undergraduate research project and presentation. This talk will highlight objectives USF's faculty have for the research project and some changes instituted strengthening the research experience for students. Some changes include making the course a two-semester sequence and, recently, requiring presenting their research at a conference.

## **Sukanya Basu** Wentworth Institute of Technology (sukanyabasu@yahoo.com)

## Implementing Project-Based Learning in the Differential Equations Curriculum

In Fall 2013, I taught a course on Differential Equations to a group of 25 engineering students at Wentworth Institute of Technology in Boston, MA. This course was taught in the spirit of Wentworth's hands-on project-based learning principle. As part of this teaching principle, I designed a set of six final group projects involving differential equations for my students along the following lines. I divided the class into six groups of four students each and assigned to each group one of the following real-world problems as mini-research topics:

- 1. Effect of Forcing and Resonance in the 1940 Tacoma Narrows Bridge Collapse
- 2. Multiple Compartmental Analysis: The 1989 Exxon Valdez Oil Spill
- 3. Effect of Forcing and Resonance in the 2002 Millennium Bridge Oscillations
- 4. Active Shock Absorbers: the Motion Master Ride Management System
- 5. Dynamics of a System Consisting of Two Magnets and a Spring
- 6. Multiple Compartmental Analysis: Cleaning up the Great Lakes

My students' job was to research each topic thoroughly by reading actual news clips dating back to the event in question, read research articles on efforts by research mathematicians to explain the event, set up a differential equations model based on the knowledge they gathered, solve it on their own using techniques learned in class and in some cases beyond, by filling in details in research papers with my help if necessary, and then give a well-coordinated group presentation in front of the entire class. In addition, they were expected to answer audience questions on their feet in an intelligent manner for full credit.

I was very impressed to find that my undergraduate students greatly exceeded my expectations. In this talk, I would like to share with my audience their presentations along with tips on how I supervised these projects including what went well and what I would like to change in future based on my experiences and my students' feedback. Hopefully this talk will serve as a guide for faculty members at other institutions wishing to implement project-based learning in their own Differential Equations curricula.

## Agendia Timothy Atabong Madonna University Nigeria (agendia@yahoo.com)

### Undergraduate Curriculum on the Relationship Between Mathematics and Computer Science with other Disciplines

The demand for Mathematical/Computer knowledgeable skill workers across different disciplines in the world today in on the rise. There is no non vacuous mathematical as well as computational job in the universe today. The two disciplines are in a growing involvement in

all recent developments in Science, Technology, Arts and Humanities. Ironically, in Nigeria, the enrollment in undergraduate mathematics is a last resort. In Africa, 90% of secondary school graduates (The parents will influence their decision in most cases) will like to study medicine and surgery not because of their passion, but because employment is guarantee. This generally leads to un qualified medical practitioners. Diversifying Mathematicians and Computer scientists into relevant disciplines will be a semantic advancement in the fight against unemployment especially in the under developed countries. We introduce in this paper a curriculum for a full semester undergraduate course for Mathematics and Computer science majors and minors, the relationship between Mathematics and Computer science with other disciplines. A course of this nature will improve on the enrollment of students in fields of computer science and mathematics who will specialized in any discipline of their choice after graduation. The contents of the course and the strategic plan to start the course in the 2014/2015 academic year is presented in the paper.

## **Emma Smith Zbarsky** Wentworth Institute of Technology (smithzbarskye@wit.edu)

## Using Matlab to Present Multidimensional Information

This project was done in a multivariable calculus class, but it only focused on understanding, presenting, and interpreting multidimensional information. Following a week-long introduction to two- and three-dimensional plotting in Matlab, the students were asked to find a multidimensional dataset with at least 100 datapoints. They had to represent this data graphically in a clear and comprehensible fashion and discuss their findings in a short paper. I will present information on the introduction I gave as well as an overview of the students' results in a variety of fields including sports, business, climate and geography.

## **Open and Accessible Problems in Real or Complex Analysis**

## Friday, August 8, 1:00–2:55 PM, Plaza Level, Broadway I & II

## Organizers: Lynette Boos Providence College

Su-Jeong Kang Providence College

Undergraduate research is more popular than ever, and there is a high demand for open and accessible problems for students to tackle. Analysis is an area particularly suited for this research because it builds off of the foundational material that students learn in calculus. In addition, analysis is rich with problems that are easily stated, but more difficult to solve, and often lead to further questions for investigation. We invite presentations about open problems in real or complex analysis suitable for undergraduate research or joint faculty and undergraduate research. Presentations concerning results about these types of problems, preferably with open questions remaining, are also welcome.

Stephan Ramon Garcia Pomona College (stephan.garcia@pomona.edu)

## **Quotient Sets**

Given a subset  $A \subseteq \mathbb{N}$ , what can be said of the set of quotients obtained from A? For instance, is the set of quotients of prime numbers dense in the positive reals? We discuss a host of interesting and surprising results about quotient sets. Much of this work is suitable for student involvement. Partially supported by NSF Grants DMS-1001614 and DMS-1265973.

Robert W. Vallin Lamar University (rvallin425@gmail.com)

## The Sum of Golden Ana Sets

This idea starts with the "verbal" sequence *a*, *ana*, *anaanana*, ..., where to go from step N to step N + 1 the *a*'s are replaced with *ana* and the *n*'s with *an*. This can be translated into a Cantor-like set, G, in the unit interval called the Golden Ana Set. In this talk we present the origins of the Ana Set and the Golden Ana Set and discuss questions about adding copies of these to get an interval à la the well-known  $C_{1/3} + C_{1/3} = [0, 2]$ . The problem is ideal for undergraduates with an interest in real analysis or fractals.

## Jeffrey Clark Elon University (clarkj@elon.edu)

## A Topology of Subdivision for the Real Numbers

The set of real numbers can be recursively split into disjoint subsets that can serve as a base for a non-standard topology on the set. A sequence of division points is created recursively by mapping the real numbers injectively into the set of all finite and infinite sequences of +'s and -'s; the numbers mapping to the finite sequences serve to divide the real numbers into nested intervals that form a base for the topology. This talk will explore elementary descriptions of this topology and open questions related to its properties.

Andrzej Piotrowski University of Alaska Southeast (apiotrowski@uas.alaska.edu)

## Linear Operators, Zeros of Polynomials, and Orthogonal Polynomials

This talk will give an introduction to the theory of diagonalizable complex zero decreasing operators (CZDOs), survey several interesting and accessible open problems regarding CZDOs, and highlight selected new results from an MAA sponsored National Research Experience

for Undergraduates Program at the University of Alaska Southeast. The topics will be comprehensible to anyone with a solid foundation in differential calculus, yet deep enough to be of interest to specialists in the field.

## Michael BrilleslyperU.S. Air Force Academy (mike.brilleslyper@usafa.edu)Beth SchaubroeckU.S. Air Force Academy (beth.schaubroeck@usafa.edu)

## Locating the Roots of a Family of Polynomials: Three Open Questions

We explore and visualize the roots of  $p(z) = z^n + z^k - 1$ , where  $n \in \mathbb{N}$  and  $1 \le k \le n - 1$ . In a recent paper, we determined when p(z) has roots on the unit circle. This investigation led to numerous questions concerning how many roots occur inside or outside the unit circle as a function of n and k. In this talk, we conjecture explicit formulas for these values; use technology to investigate limits of our formulas as  $n \to \infty$ ; and discuss the behavior observed relative to the  $n^{th}$  roots of unity when animating the roots of p(z) for fixed n, as k varies. Proofs of our conjectures and investigations related to this problem are suitable for undergraduate research projects.

## Donald Leigh Hitzl Lockheed Palo Alto Research Lab, Retired (domarltd@comcast.net)

Frank Zele Lockheed Martin Advanced Technology Center, Retired (fzele@juno.com)

## The Two Body Problem Elevated to the Complex Domain

In this paper, elliptical orbits of the Two Body Problem in 2 dimensions are elevated to the Complex Domain of 4 dimensions. The two dependent variables x(t) and y(t) are elevated to  $x(t, \tau) + ip(t, \tau)$  and  $y(t, \tau) + iq(t, \tau)$  where all variables x, y, p, q, t and  $\tau$  are real. The time variables t and  $\tau$  are coupled through a single parameter  $\theta$  so  $\tau = \tan(\theta)t$  and  $\theta$  is to be a small angle (2 to 10 degrees say). Here the absolutely simplest case of the two body problem is developed, which we call the RESTRICTED TWO BODY PROBLEM. We start, at t = 0 at perigee, with both the Eccentric Anomaly E and True Anomaly f both = 0. Following earlier work given in References 6 to 11, it is found that p and q are automatically second order "perturbation sensitivities" of x and y respectively. The second order Perturbation Derivatives are shown to give the time variable natural frequency  $\omega_n(t, \tau)$  and the time variable damping  $\zeta_D(t, \tau)$  for the x oscillation. For the y oscillation, which has no "symmetry breaking", these two quantities are constant at 1 and 0 respectively. Elevating any problem such as this, which features only trigonometric functions, to the Complex Domain naturally brings in hyperbolic terms for the complex time  $\tau$ . Thus the exponential divergence of neighboring trajectories is quantified exactly and completely.

## **Curriculum Development to Support First Year Mathematics Students**

## Session 1: Friday, August 8, 1:00–4:55 PM, Ballroom Level, Parlor AB

Organizers: Donna Flint South Dakota State University Rebecca Diischer South Dakota State University Charles Bingen University of Wisconsin, Eau Claire

A common focus of university administration is student retention and graduation. First year mathematics courses, both general education and major specific, have comparatively high drop/fail/withdraw rates. This means that they are often scrutinized in regard to their effect on retention and graduation rates. In this session, we would like to hear what you have been doing to respond to this scrutiny. We hope to focus on departmental-wide efforts, rather than specific classroom approaches. Presentations could include complete course redesign, co-requisite support courses, restructure of curriculum, departmental efforts to standardize, etc. Note that we would like to hear about successful, in process, and unsuccessful initiatives. Presentations that include a description of the initiative along with data supporting the success or failure of these initiatives are especially encouraged.

#### Mary Ann Barbato Fitchburg State University (msaadi@fitchburgstate.edu)

#### Developmental Mathematics Redesign at Fitchburg State University

For the past two years Fitchburg State University has been making significant changes to their developmental mathematics program in an effort to improve student retention and success in mathematics courses. We are at the end of year two of a three year grant under the Massachusetts Department of Higher Education Vision Project which enabled us to pilot a self-paced modular based instruction with in class tutors, in our developmental mathematics courses. This presentation will address our experiences with this pilot, data collected thus far, and future plans. It will also address a new policy we have developed to get students into their gateway mathematics course early in their academic careers.

Lanee Young Fort Hays State University (lyoung@fhsu.edu)

Jeff Sadler Fort Hays State University (jsadler@fhsu.edu)

## Comparing Student Attitudes and Successes in College Algebra using Emporium, Problem Solving, and Traditional Methods

Many students take College Algebra because it is required but they never see the application of the content to their lives or potential careers. Using three different methods of instruction, the researchers compared student attitudes toward mathematics at the beginning and end of three

semesters. Student success was also measured in the three classes to determine any relationships between method of instructions, student attitude, and student success in this general education course.

Charles Bingen University of Wisconsin - Eau Claire (bingencw@uwec.edu)

## Taking Over an Existing Developmental Math Program: What Works and Determining What to Improve

This talk will address the experiences of taking over an existing developmental math program. The discussion will include a description of the developmental courses, instructional methods used in the courses, success rates, and future changes in the courses.

## Craig Miller University of New Haven (cmiller@newhaven.edu)

## The Math Zone: An Open Emporium-Style Model Attempting the Fast Track

The first proposal for an Emporium-style program at the University of New Haven occurred between the Fall 2012 and Spring 2013 semesters. With a new Department Chair who had worked in a former institution with a similar, successful program, we charged forth. By the end of the Spring 2013 semester, course outlines and syllabi had been produced and course technology adapted. In May, the ground was broken on a new facility while summer sessions were used to develop course content and procedures. By the end of August a full 60 and then some computer terminal lab space had been built all for a new Emporium-style program. Up next was to open the doors for over 700 students into our program. What came afterwards during the first year of the program was unique.

## Senan Hayes Western Connecticut State University (hayess@wcsu.edu)

### The Startup of a Math Emporium—Trials and Tribulations

The Emporium Model for remediation has proven successful at the postsecondary level. The model increases student success while at the same time reducing institutional cost (NCAT). The model uses adaptive learning technology, along with videos, animations, and practice exercises, to help students master required material. This model requires the student to take responsibility for their own learning and progress through the use of a custom workbook, multiple practice tests, and formal chapter assessments. Meanwhile, the teacher provides individualized and small group instruction. The traditional "chalk and talk" approach of lecturing, where the student is a receptacle of information, changes to one where the student is constantly learning.

This presentation will focus on how the Emporium Model was developed at WCSU from conception to implementation. Results of the pilot and the first year successes and failures will be discussed. The process of how improvements were made via email, formal and informal meetings, and student surveys will be highlighted. Ongoing course modifications and the future outlook of the Emporium will conclude the talk.

## **Donna Flint** South Dakota State University (donna.flint@sdstate.edu)

#### Restructuring of the Remedial Program at South Dakota State University (SDSU)

After a long trend of declining performance in remedial and first year students at SDSU, the time came in 2010 to reconsider the standard method of bringing underprepared students up to the level of expectation for a college level credit bearing course. Based on research provided by the National Center for Academic Transformation, the Complete College America Consortium, and our own faculty, SDSU developed a placement policy and course structure which enables students to complete credit bearing courses more quickly and more successfully. After 4 years of development, the structure is in place and has had one year of complete implementation. This talk will report on the placement policy and the overall structure of the remedial program at South Dakota State University which includes intense remedial courses for the least prepared, a co-requisite structure for students who are better prepared, but are still at risk of being unsuccessful, and an alternative course for students who do not need the traditional College Algebra course.

## **Carri Hales** South Dakota State University (carri.hales@sdstate.edu)

#### Improving Remedial Success Using an Enhanced Mastery-Based Format

The South Dakota State University mathematics department switched to a mastery-based, emporium style format in an effort to bring underprepared students up to the level of expectation for a college level credit bearing course. While analyzing the success rate of students in the remedial course, we identified that students with a lower placement score continually struggled in remediation. To provide more structure and support for these students, an enhanced mastery-based course was developed. This talk will discuss the features of the enhanced course which include activities, group work, learning guides, lectures, and structured work days and will also report on the success of the course.

#### **Rebecca Diischer** South Dakota State University (rebecca.diischer@sdstate.edu)

### A Co-Requisite Model for College Algebra

According to Complete College America, only 35.1% of remedial students at 4-year institutions graduate within 6 years. In order to help better prepared remedial students begin immediately in a credit bearing math course, a co-requisite remedial College Algebra Lab was created to

accompany the College Algebra course. The lab utilizes "just-in-time" remediation to strengthen pre-algebra skills as they are needed in the College Algebra course. This talk will discuss the development, implementation, and success of a College Algebra co-requisite model.

Cheryl Jarrell McAllisterSoutheast Missouri State University (cjmcallister@semo.edu)Daniel DalySoutheast Missouri State University (ddaly@semo.edu)Tamela RandolphSoutheast Missouri State University (trandolph@semo.edu)

## **Rethinking First Year Mathematics to Improve Student Retention**

The Department of Mathematics at Southeast Missouri State University, in response to a charge from administration, is preparing to launch a new First Year Mathematics program for fall 2014. The former 2(or 3)-course developmental mathematics program has been dropped and redesigned, new placement guidelines have been adopted and nearly all of the first year mathematics course have undergone extensive redevelopment. Co-requisite labs supporting the general education mathematics courses will be required for underprepared students. The session will focus on what we are trying to do, how we organized the work load to rethink the curriculum of all of the courses, and the unknowns that can't be answered until we have taught the new curriculum at least one year.

Mary BeisiegelOregon State University (mary.beisiegel@oregonstate.edu)Krista FoltzOregon State University (foltzkr@onid.oregonstate.edu)Scott L. PetersonOregon State University (speter@science.oregonstate.edu)

## It's not just about the Content: Holistic Change in a First-Year Mathematics Course

At Oregon State University, College Algebra has often been singled out as the least passed and least favored course on campus, with low success and high attrition rates as common problems. To remedy these issues, we developed a three-pronged approach to revitalizing the course across the department, which included redesigning the: (1) curriculum materials, (2) students' learning experiences, and (3) teaching of the course. The goals of the redesign were to: increase student engagement in mathematical practices, present a conceptual approach to teaching and learning mathematics, increase success rates in the course, and promote interest in STEM fields. To meet these goals, changes were first made to curriculum materials and assessment of learning that would promote and evaluate student engagement in mathematical practices and reasoning from both conceptual and applied perspectives. Second, the structure of the classroom was completely redesigned by implementing hands-on learning, rather than lecturing during class time, and by reducing class sizes in order to increase student contact time with instructors and teaching assistants. Third, professional development modules were created and offered to instructors and graduate teaching assistants on a weekly basis to support their transitions to the redesigned course, as well as to offer an opportunity to talk about course content, pedagogical strategies, and student engagement. In this session, we will present changes that have been successful and unsuccessful, the data showing some improved student outcomes, student and instructors' responses to the redesign, and our vision for the future.

**Camille A McKayle** University of the Virgin Islands (camille.mckayle@uvi.edu) **Robert Stolz** University of the Virgin Islands (rstolz@uvi.edu)

## Peer Led Team Learning in Foundation Mathematics for College Students: A University Approach

Peer Led Team Learning (PLTL) was implemented in zero credit pre-algebra courses at the University of the Virgin Islands-an open enrollment, Historically Black University. Initially implemented as a pilot, the approach was scaled up and implemented for every section of developmental math courses during academic year 2013-2014. During fall 2013, UVI had the highest pass rates for developmental mathematics courses in 6 years. During the pilot, the average perception of confidence to solve mathematics problems was higher for students who participated in PLTL sessions than students who did not. We will present how the University was able to administer PLTL in all courses, and present the preliminary results. Though this is a preliminary study, it is expected that increased success in these courses will lead to increased retention and persistence. We will explore ways in which to track the progress of students longitudinally.

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Douglas B. MeadeUniversity of South Carolina (meade@math.sc.edu)Philip B. YasskinTexas A&M University (yasskin@math.tamu.edu)
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#### Improving Student Success in Calculus at the University of South Carolina

Improvements in the success rate in the calculus course for STEM majors at USC have been made in several different areas: placement, prerequisites, projects, and practice. The foundation for all of these changes can be traced to the presenters' collaboration on what is now known as the Maplets for Calculus (M4C) project. Each "maplet" provides a convenient environment for practicing a specific topic in calculus (or precalculus). Next came the restructuring of the weekly meetings that replaced one of the two weekly recitation sessions with a computer lab in which students learned to use Maple to reinforce calculus concepts, and to complete more in-depth projects. The real improvements were not realized, however, until the Mathematics Placement Test was updated and all course prerequisites were actively enforced. The results have been truly exceptional: a 33% increase in success (ABC) rate in Calculus I without reducing the number of success students.

## Session 2: Saturday, August 9, 8:30–11:45 AM, Ballroom Level, Parlor AB

**G. Daniel Callon** Franklin College (dcallon@franklincollege.edu)

#### A Multi-tiered Support System

At Franklin College we have implemented a variety of innovative curricular structures at all levels of our freshman course offerings in keeping with our departmental focus on active learning.

For students in majors requiring Calculus and Calculus II, including mathematics majors and minors, we teach a one-hour activity course called Introduction to the Mathematical Sciences which supplements first-semester calculus. We will describe the unique features of this course, which is designed to give students a taste of what pursuing a mathematics-related profession entails, and which has been described as the math equivalent of co-ed volleyball.

Students required to take calculus, but needing additional preparation, enroll in a course titled Functions and Models. The course itself emphasizes families of functions, their properties, and their uses in modeling and applications. An additional day is added to the weekly course schedule for each student to integrate an individualized self-paced algebra review, with no more than 13 students at each session. Students are required to attend the review only until a sequence of mastery exams is successfully completed.

For students whose mathematical preparation is marginal, we offer a summer bridge program focused on financial mathematics, with those students moving into a section of our liberal arts quantitative reasoning course taught by one of the bridge program professors. Similar students unable to participate in the bridge program are enrolled in a weekly one-hour supplemental lab which reinforces the more challenging aspects of the quantitative reasoning course.

Data from all these initiatives will be provided.

Mazen Shahin I	Delaware State University (mshahin@desu.edu)
Andrew Lloyd	Delaware State University (alloyd@desu.edu)
Tomasz Smolinsk	i Delaware State University (tsmolinski@desu.edu)
Melissa Harringto	<b>n</b> Delaware State University (mharrington@desu.edu)

## An Effective Approach to Increase Mathematics Readiness of Freshmen STEM Students

At Delaware State University, about 89% of incoming freshmen take a commercial mathematics placement test and are placed into a developmental mathematics course, Introduction to Algebra. In the summer of 2012, an NSF-funded project at DSU, the Science and Mathematics Initiative for Learning Enrichment (SMILE), offered Introduction to Algebra as a free online course for incoming freshmen who were SMILE participants. To insure the integrity of the online course, students were required to take an on-campus departmental common final exam. Nearly 81% of these students passed the course and progressed to College Algebra in the Fall of their freshman year.

The success of this pilot project encouraged us to expand the free online offerings to two mathematics courses in summer 2013, Introduction to Algebra, and College Algebra. In this year the mathematics placement of SMILE participants was determined by a combination of students' high school transcripts and their placement exam scores. Again, in each course, around 80% of these students passed. Taking their introductory mathematics course on-line during the prefreshman summer allowed the SMILE students to start their freshman year one to two levels ahead in mathematics compared to their placement. In the summer of 2014, these two online courses will be offered again to about 125 incoming STEM students.

We will discuss this model including the recruitment of the participants, the development of the online mathematics courses, and program assessment and outcomes. We will also discuss the possibilities of expanding this model to all University incoming freshmen.

## Frederick Butler York College of Pennsylvania (fbutler@ycp.edu)

## **Creating a Mathematics First Year Seminar Course**

This talk details the author's progress so far in developing a mathematics first year seminar course as part of a college wide general education revision. The course is being designed both for declared math majors and interested undeclared students, to try to give students a picture of what advanced mathematics and mathematical research is like early on in their academic careers. A preliminary topic list and syllabus outline will be shared, including how the course will meet the required general education outcomes of critical thinking, communication, interdisciplinary problem solving, and professional standards. The author welcomes suggested additions to the topic list as well as ways to improve the overall course proposal.

Emma Smith ZbarskyWentworth Institute of Technology (smithzbarskye@wit.edu)Amanda HattawayWentworth Institute of Technology (hattawaya@wit.edu)Ophir FeldmanWentworth Institute of Technology (feldmano@wit.edu)

#### **Designing a Mathematical Support Structure for Entering Students**

As a department, we have been specifically working on developing additional support for first year students for three years. Nearly all of our students take one of two mathematics courses in their first term: about 460 students in calculus 1 and 430 students in college math 1. In

addition to the standard office hours and tutoring center help, we have hired undergraduate mentors to help out both in classes and running recitation sections in addition to lecture. This past year we also offered an optional one credit class with additional peer tutors and a professor to discuss applications of the material in each class. We also began offering a summer bridge program to transfer students and traditional incoming first years who were identified as at-risk. Finally, for the last three years we have run several sections of a second chance "Reboot" hybrid calculus class that lets students get back on their feet while starting over at the midterm and finishes with a final exam just before the start of spring term classes. I will discuss our successes and failures, as well as presenting the changes we plan to make to our program this coming year.

#### Michael Allen Lundin Central Washington University (lundin@cwu.edu)

#### Embedded Tutoring in First Year College Mathematics Classes

We describe building a tutoring project in which prospective secondary mathematics teachers in their first seminar course learn how to tutor students in a variety high-volume, freshman mathematics courses. Assessment data indicate benefits for tutors, tutees, and survey course faculty, as we learned the hard way how to improve the program. The evolution of the embedded tutoring model led to an economical, versatile, and viable learning community, an example of how changing the curriculum of one course out of necessity can benefit many in the mathematics community.

## Jennifer Hegeman Missouri Western State University (hegeman@missouriwestern.edu)

## Requiring Instructor-Generated Learning Activities in Online College Algebra Can Reduce Failure and Withdrawal Rates

When a university's state appropriations are determined by its students' retention and graduation rates, faculty teaching freshman-level mathematics are the first to be visited by administrators seeking lower failure and withdrawal rates. These same university administrators have been known to ask faculty to develop more online courses. Can these administrative requests be addressed simultaneously in the area of general studies mathematics? Online general studies mathematics courses are becoming more widespread, in part due to the development of text-based, interactive multimedia tools that not only provide students with a complete set of online course materials but also provide faculty with an online course management system. While the text-based, interactive multimedia tools have much to offer both students and mathematics educators, simply providing students with these tools is not enough to ensure their success. During the past year, the online College Algebra courses offered by the presenter's university were reorganized to include structured learning activities that students were required to complete prior to engaging with the text-based multimedia tools. Preliminary results suggest that the implementation of instructor-generated learning activities, in conjunction with text-based materials, enhances student learning in the online environment, and consequently, reduces failure and withdrawal rates. Data collected also suggests that making these instructor-generated learning activities available in face-to-face offerings of College Algebra can further reduce failure and withdrawal rates.

## Sharon Vestal South Dakota State University (sharon.vestal@sdstate.edu)

## How a Co-Requisite Calculus I Lab Can Improve Student Success in Calculus I

How a Co-Requisite Calculus I Lab Can Improve Student Success in Calculus I Students taking first semester calculus don't seem to have trouble with the calculus concepts – they typically struggle in the course because of a weakness in their algebra and trigonometry skills. We will describe how we were able to create a co-requisite Calculus I lab focused on the specific algebra and trigonometry skills needed for first semester calculus. We will include the structure of the lab, some problems from the lab manual, changes we have made, and data on how we have improved student success in first semester calculus. We will also discuss future ideas for the course and the course materials.

Malgorzata DubielSimon Fraser University (dubiel@math.sfu.ca)Justin GraySimon Fraser University (jgray@math.sfu.ca)Natalia KouzniakSimon Fraser University (kouzniak@sfu.ca)Cameron MorlandSimon Fraser University (cmorland@sfu.ca)Jamie MulhollandSimon Fraser University (j\_mulholland@sfu.ca)

## Remedial Efforts in Calculus Classes at Simon Fraser University: Results and Challenges

In the past decade the gap between knowledge of high school mathematics retained by the students entering SFU and the foundations that are required to ensure a successful start in calculus classes at the university, has widened. Currently, there are four calculus I courses offered in the Department of Mathematics at SFU. In the second class of the semester all students write multiple-choice in-class Diagnostic Test which is worth 5% of the final course mark. Topics include major concepts studied in grades 11 and 12. They are posted on the department web site and the students receive a welcome message from the Coordinator with all pertaining information and links to the topics and Practice Test prior to the semester. Based on the statistical data collected over the years, it became clear that the outcome of the Diagnostic Test serves well to predict grades in the first midterm and, most importantly, the final course grades. Students who have achieved 60% and above on the Test, are assigned those 5% towards the final mark; students who scored lower are assigned zero mark and they are required to participate in additional classes and write on-line quizzes, to recover the mark. We will present data about performance in remedial classes and discuss our efforts to aid students. Also, we are actively involved in the dialogue with high school teachers to reflect recent changes in BC high school mathematics curriculum both, in the Diagnostic Test and in the topics covered in remedial classes.

## Bonnie Gold Monmouth University (bgold@monmouth.edu)

## Concepts, not Calculations: Helping First Year Mathematics Students Learn What Mathematics Is

At Monmouth University, we went from 55 mathematics majors spread through the four years to over 125. Part of this was due to awareness on the part of incoming students of the need for mathematics K-12 teachers, but we believe part was due to two curricular changes we made. Both had the effect of letting students know that mathematics is about concepts, and that calculations, while an important part of mathematics, are not all there is. One was introducing a transitions course, Introduction to Mathematical Reasoning, in the first year rather than later on. The second was moving to a calculus reform curriculum, which enabled students whose symbolic calculation skills are weak to nonetheless succeed in first-year calculus. We learned over time that we needed to ensure that students do develop those computational skills at some point in the major, and introduced gateway exams for that purpose. I will discuss these changes and their effect.

Alison Reddy University of Illinois (aahlgren@illinois.edu)

Marc Harper (marcharper@gmail.com)

## Precalculus Redesign: The Influence of a Placement Program and the Power of a Name

The University of Illinois redesigned its Precalculus course using information from its placement program and the name was changed to "Preparation for Calculus" to accurately represent the purpose of the course. Entry and exit student data will be shown.

The data shows the mathematical strengths and weakness of the students entering and exiting the Precalculus course. The entry data was used to tweak curricular emphasis in the course. For example students enter the course with a strong foundation in polynomial functions and a weak foundation in exponential and logarithmic functions. The exit data shows improved DFW rates and is compared to student data of those who enter Calculus I directly their first year to show internal prerequisite preparation.

We are able to show that the redesigned course is preparing students to compete with those who enter Calculus I directly. The name was changed to address campus wide misconceptions of the course; many students thought that Precalculus was a review of all mathematics prior to calculus and not a course within itself and thus it was viewed as a review and terminal course.

## Session 3: Saturday, August 9, 1:00–2:35 PM, Ballroom Level, Parlor AB

**Robin A. Cruz** The College of Idaho (rcruz@collegeofidaho.edu)

**Dave Rosoff** The College of Idaho (drosoff@collegeofidaho.edu)

**Nicole Seaders** Willamette University (nwebb@willamette.edu)

### A Collaborative Transition to Applied Calculus with Modeling

Over the last two years, faculty at Willamette University and The College of Idaho made broad changes in calculus instruction at their institutions following a model developed at Macalester College. Here we describe the ideas that led to the changes, the process of transition, and discuss differences in student outcomes between traditional Calculus I and the Calculus with Modeling courses.

## **Darci L. Kracht** Kent State University (darci@math.kent.edu)

## Experiments with Large-Lecture/Lab Hybrid Models for Business Calculus

Kent State University offers a brief applied calculus course primarily for business majors. We face competing pressures to save money, increase the number of undergraduates taught by tenured faculty, and increase student success rates. To address the first two, class sizes were doubled. We want to find a class structure that will address the third concern and mitigate the effects of the larger class size. It must work for all faculty, from graduate students, to non-tenure track faculty experienced in teaching this course, to tenured faculty who have never taught it. We have experimented with four approaches, including large lectures and three different lecture/lab hybrids. I will report on the degree of success of these attempts along with lessons learned.

## Paul N. Runnion Missouri University of Science and Technology (prunnion@mst.edu)

## Interactivity and Intervention: An Overview of Calculus Redesign at Missouri S&T

In August 2013, the Department of Mathematics and Statistics at Missouri University of Science and Technology was asked to redesign our large enrollment three-semester calculus sequence. Two core components of our redesign are enhanced interactive labs and a new intensive intervention program for underprepared students. This talk will give an overview of these core components and the current status of the four-year redesign initiative, which is being funded by the campus and the University of Missouri System.

## Nathan P. Clements University of Wyoming (clementsuwyo@gmail.com)

## ALEKS in Calculus I at the University of Wyoming

ALEKS is seen as an effective placement and algebra learning mechanism by some. At the University of Wyoming, it was implemented in all Calculus I sections in an attempt to help our students review algebra skills necessary for calculus. In this talk, I will discuss how it was implemented, what changes we saw in student performance, and what we foresee for how it will inform our calculus program in the future.

## Allison Henrich Seattle University (henricha@seattleu.edu)

J. McLean Sloughter Seattle University (sloughtj@seattleu.edu)

## Improving Student Success in Calculus

Over the last few years at Seattle University, we have implemented a multi-faceted strategy for improving student success in calculus. A successful program that was pioneered at CU Boulder inspired our model. We now offer oral reviews for exams, course-specific study groups led by undergraduate Learning Assistants, and a two-quarter Calculus with Algebra sequence for STEM majors who are underprepared to enter Calculus I when they begin college. Statistics show that these interventions have had a positive impact on student success in Calculus. In this talk, I will describe our interventions in greater detail and share our successes and challenges.

## More Favorite Geometry Proofs

## Saturday, August 9, 1:00-4:15 PM, Ballroom Level, Galleria I

## Organizer: Sarah Mabrouk Framingham State University

This session invites presenters to share their favorite undergraduate geometry proofs. These proofs should be suitable for Euclidean and non-Euclidean geometry courses as well as for courses frequently referred to as "modern" or "higher" geometry but not those related to differential geometry or (low-level) graduate courses. Proofs must be for theorems other than the Pythagorean Theorem and should be different from those presented during the MAA MathFest 2013 paper session (see http://www.framingham.edu/~smabrouk/Maa/mathfest2013/ for more information). Presenters must do the full proof, discuss how the proof fits into the course, provide information regarding prerequisite topics for the proof, and discuss associated areas with which students have difficulty and how such concerns are addressed so that students understand the proof. Presenters are invited to discuss how they have modified the proof over time as well as to share historical information for "classic" proofs and explorations/demonstrations that they use to help students understand the associated theorem. Abstracts should include the theorem to be proved/discussed as well as brief background information.

## Deirdre Longacher Smeltzer Eastern Mennonite University (deirdre.smeltzer@emu.edu)

## A Proof of Ptolemy's Theorem via Inversions

Ptolemy's theorem, attributed to second century Greek mathematician Claudius Ptolemaeus, gives necessary and sufficient conditions for one to be able to inscribe a given quadrilateral in a circle. A standard proof involves using inscribed angles and similar triangles. A more elegant and modern proof utilizes an inversion in the plane and resulting properties to establish a generalization of the theorem.

## Dan C. Kemp South Dakota State University (dan.kemp@sdstate.edu)

## Archimedes' Twin Circles in an Arbelos

Proposition 5 from Archimedes' Book of Lemmas was popularized in 1954 by Leon Bankoff as a surprise in an arbelos. An arbelos consists of three mutually tangent semicircles with diameters on a common line and lying on the same side of that line. Archimedes asserts that in an arbelos, the two circles that are tangent to two of the semicircles and the common tangent of the smaller semicircles are congruent.

Archimedes' synthetic proof, suitable for presentation in a geometry course, will be given. Archimedes' proof is historically interesting because it contains the first known reference ('...by the properties of triangles...') of the altitudes of a triangle being concurrent. Also a modern proof using analytic geometry will be presented. If time permits, further discussion of the twin circles of Archimedes will be given.

## **Alvin Swimmer** Arizona State University (aswimmer@asu.edu)

#### Euler's Famous Line: Gateway to The Harmonic 2:1 Centroid Concurrency

It was Archimedes of Syracuse (287–212 B.C.) that first proved that the three medians of every triangle are concurrent at the centroid, G, which divides each median in the ratio 2:1. Two millenia later, in 1763, Leonhard Euler, the most prolific mathematician of all time, discovered the line determined by the orthocenter, O, and the circumcenter C, of any (non-equilateral) triangle also contains the centroid G which divides the interval [O, C] in the ratio 2:1.

In more recent times, Tom Apostol and Mamikon Mnatsakanian, discovered in 2004, that the incenter B and the 1-dimensional center of mass D determine a line which contains the centroid and G divides the interval [B, D] in the same 2:1 ratio.

In 2006, it became clear to me that, the 5 lines and intervals mentioned above, associated with each (non-equilateral) triangle are part of an infinite family of lines all concurrent at G, Each of these lines is determined by 2 points which determine an interval divided by G, in the 2:1 ratio. I call this family The Harmonic 2:1 Centroid Currency.

## **David Marshall** Monmouth University (dmarshal@monmouth.edu)

## **Reflections in Geometry**

Our junior level geometry course provides a study of Euclidean and non-Euclidean geometries, but leaves much of the specific content up to the instructor, and the variation can be large. My course emphasizes transformations, starting with the classification of isometries of the

Euclidean plane. The Three-Reflections Theorem plays a central role in this classification and helps with several other pedagogical and content goals. It provides students with an opportunity to (1) seek generalizations later in the course, (2) experiment with a "proof by generic example", (3) make good practical use of available technology, and (4) do a little group theory. The theorem (and related results) also serves as an entry way into more general discussions of classification theorems, Klein's Erlanger Program, and the role of group theory in geometry.

## Thomas Q. Sibley St. John's University (tsibley@csbsju.edu)

## **Reflections on Reflections**

The Common Core State Standards emphasize geometric transformations, highlighting the Three Mirror Theorem: Every Euclidean plane isometry is the composition of at most three mirror reflections. The proofs for this theorem and the theorems leading to it also generalize beautifully to hyperbolic and spherical geometries. Even more, they generalize to higher dimensions for all three of these geometries with only minor modifications. Future secondary teachers, indeed all mathematics majors, can gain important insight by considering these theorems from this unifying perspective.

## Justin Allen Brown Olivet Nazarene University (jbrown7@olivet.edu)

## The Shortest Path Between Two Points and a Line

For two points on the same side of a line L, what is the shortest path beginning at one point, intersecting L, and ending at the second point? The answer, sometimes called Heron's Theorem, is to find the point of intersection of L and the line connecting the first point to the reflection of the second point across L.

In a geometry classroom, asking the question in the open-ended form above often leads to interesting ideas from students. Many of their ideas do not yield the shortest path in general, but result in an interesting discussion. And once students realize that their initial idea is incorrect, they are invested in finding or at least seeing the correct proof. We will discuss some of these incorrect ideas, as well as the proof of the theorem, which uses similar triangles.

## **Genghmun Eng** (genghmun.eng@aero.org)

#### The Perfect Heptagon from the Square Hyperbola

It is well known that the perfect 7-sided heptagon cannot be constructed with a compass and a straight edge alone, but it can be done with an angle trisector. However, virtually all perfect heptagon constructions include a "magic" step, which presents some angle  $3\theta$ , followed immediately by presenting the angle  $\theta$ , then continuing on with the construction. It is also well known that angle trisection cannot be done with a compass and a straight edge alone, because it requires solving an irreducible cubic, but that it can be done with the help of a noncircular conic section. The first conic section we usually learn about is the inverse relationship Y = C/X, which makes it a nice item to use for implementing a trisection construction within a heptagon construction. Starting with an unmarked X - Y coordinate system and a single square hyperbola **H**, we show that adding 4 circles and 10 lines gives us the  $(2\pi/7)$  angle for the perfect heptagon. The proof of why it works, and the motivation for how a square hyperbola naturally arises, is a nice example of combining both constructive geometry and elementary trigonometry.

## Ruth I. Berger Luther College (bergerr@luther.edu)

#### The many Shapes of Hyperbolas in Taxicab Geometry

Taxicab geometry is a good topic for open-ended explorations in non-Euclidean geometry. My undergraduate students are always surprised at the many different looking hyperbolas they discover. A simple geometric argument can be given to classify the different hyperbola shapes, as well as the other conic sections. The shape depends on the slope of the line connecting the foci. The underlying reason for this is that in taxicab geometry circles have sides of slope 1 and -1.

## Jeff Johannes SUNY Geneseo (johannes@member.ams.org)

#### Geometry Knows Topology: The Gauß-Bonnet Theorem

Last year I spoke in this session about angle sum of spherical triangles. And I think now "I cannot have two favourite geometry theorems." On the other hand, what if we think of geometry in a different context? What if we work with surfaces of non-constant curvature for a change? Now curvature is a variable quantity. We cannot speak any longer of the simple situation of the sphere, but instead we talk about a more sophisticated view of changing geometry. In this context, we cannot ask what the curvature of the whole surface is, but we can consider the integral of the curvature over the entire surface. From this perspective we can zoom out from those triangles with their angle sums and see global topological properties of our surface. In this talk we will move from angle sum through a sequence of generalizations and new ideas to find the famous and powerful Gauß-Bonnet Theorem.

## **Philip Todd** Saltire Software (philt@saltire.com)

### Finding the Fermat Point by Physics and by Transformation

My new favorite way of finding the point with minimum sum of distances to the three vertices of a triangle involves a mechanical thought computer described in Mark Levi's "The Mathematical Mechanic". An interactive model of this mechanical system can motivate the conjecture that for some triangles the solution lies at the point which subtends equal angles to the sides. It can also motivate accurate conjectures on conditions for this solution to apply. An alternative exploration tool for students with no physics background will also be presented. Again, this will allow for both the solution and the conditions under which the solution applies to be induced. When minimizing the length of a path, I like proofs which apply transforms to create a path equal to the one to be minimized, but where the minimal path is clearly a straight line. We can use reflections in this way to solve Fagnano's problem of finding the inscribed triangle with minimal. Can we apply transforms to reduce the Fermat Toricelli Point problem to finding the shortest distance between two points? Our conjecture suggests that 120 degree rotations may be the transforms of choice. Application of these rotations yields a visual proof both of the result for triangles which have no vertices over 120 degrees, and of its breakdown when there are angles greater than 120 degrees.

## **General Contributed Paper Sessions**

Organizers: Lynette Boos Providence College Susan Callahan Cottey College

## **Interdisciplinary Topics in Mathematics**

## Thursday, August 7, 8:30–10:25 AM, Parlor C

Paul R. McCreary The Evergreen State College - Tacoma (pmccrear@gmail.com)

## \*Other\* Liberal Arts Disciplines Taught Together with Mathematics

Taking to heart the view that mathematics is the study of patterns, there is no limit to what other disciplines can be linked and studied together with mathematics. For the past 18 months we have been investigating how mathematics and media literacy can be integrated in other disciplines in the liberal arts curriculum. Each academic quarter one course in a six-course mathematics sequence for math teachers was co-taught with one other course, ranging from film arts to music to environmental biology to literature. In addition, a group of students participated in a project to identify "acts of math" carried out by students and faculty in "non-math" courses. We report on the results and insights uncovered in the process.

## Michael Gagliardo California Lutheran University (mgagliar@callutheran.edu)

## **Flipping Linear Algebra**

I will discuss the motivation and format of my flipped linear algebra course. The primary goals of flipping the classroom were to continuously engage the students with the material and allow students of varying mathematical backgrounds to contribute to the course. This was accomplished through pre-work, homework presentations and a wiki glossary.

## **Prof. Peter Olszewski** Penn State Erie, The Behrend College (pto2@psu.edu) **Jessica Resig** Penn State Erie, The Behrend College (jjr27@psu.edu)

## A Flipped Math for Nurses

In the fall 2013 semester, the mathematics department at Penn State Erie, The Behrend College, designed and taught a College Algebra I class with specific focus for nursing students. This course was taught in a hybrid format over a seven-week period for students who are enrolled in the University's Bachelor of Science accelerated nursing program. By utilizing applications specific to the nursing profession, the course provided each student with a deeper understanding of algebra as it related to their field of study. In short, it provided a way for students to learn the foundational principles of algebra within the context of the healthcare field using applications such as growth rates, the elimination of cancer from the bloodstream, and life expectancy. Due to the demands of a condensed schedule, content was presented using a series of instructor-created video lectures in addition to face-to-face sessions. The incorporation of online course content also allowed for an increased focus on problem solving during class sessions. Various technologies were used including ANGEL, Drupal, Doceri for the iPad, and My Math Lab. In this talk, we will describe how the course was taught, the applications presented, the various technologies used, feedback from the students, lessons learned, goals for improvements, and the future outlook of offerings of the class.

## Daniel R. Huber University of Tampa (dhuber@ut.edu)

Leslie Braziel Jones University of Tampa (lbjones@ut.edu)

## Bones, Muscles and Math: Biology and Geometry Working Together

We describe an integrated high school geometry lesson which uses a mathematical model based on data collected by students to estimate the forces generated by their triceps muscles via the elbow joint. This lesson is part of a greater body of work in which the development of biological context for mathematical instruction and mathematical modeling are emphasized. We also report on the reception that these ideas have received over a three-year period of teacher-training workshops for secondary educators.

## Amy Ackerberg-Hastings University of Maryland University College (aackerbe@verizon.net)

## Harold and the NMAH Object Groups: Young Children's Responses to Crockett Johnson's Mathematical Paintings

Crockett Johnson (1906-1975), best known for the children's book Harold and the Purple Crayon, painted over 100 mathematical diagrams and figures in the last decade of his life. Eighty of those works are held by the Smithsonian's National Museum of American History and digitized at http://americanhistory.si.edu/collections/object-groups/mathematical-paintings-of-crockett-johnson. This talk reports on my experience of discussing the concepts in three of these paintings with the pre-K, kindergarten, and 1st grade classes at the Mustard Seed School in Hoboken, NJ. I suggest some lessons learned about sharing the history of mathematics and mathematical art in early childhood and elementary education contexts.

## **B. Lynn Bodner** Monmouth University (bodner@monmouth.edu)

## Frieze Patterns of the Mamluks

A distinguishing feature of Islamic art is the covering of planar surfaces with elaborate geometric patterns, both one-dimensional frieze and two-dimensional 'wallpaper' patterns. There is a profusion of original and restored examples of frieze patterns found on the walls and ceilings of monuments in Egypt and Syria dating from the Mamluk Sultanate (1250 to 1517), considered the most important Muslim empire in the latter Middle Ages. Since certain pattern types are preferred and intuitively recognized as being "right" by various cultures, it is of interest to classify Mamluk patterns based on the symmetries they possess. From mathematical group theory, we know that there are only seven different possible symmetry groups for one-dimensional patterns, each of which is a collection of the four rigid motions (or distancepreserving transformations, called isometries) of the plane that leave a frieze pattern invariant. This paper will illustrate, discuss, analyze and classify a few of the many beautiful examples of one-dimensional frieze patterns found today on extant Mamluk monuments of Egypt and Syria.

## Magdalena Luca MCPHS University (magdalena.luca@mcphs.edu)

## **Mini-Flipping Biostatistics**

Biostatistics is the second upper level course in the statistics sequence required for Public Health students at our university. The emphasis of the course is on advanced scientific reasoning: reading, writing, interpreting and validating statistical analyses found in journal articles. Teaching this course is challenging because students have either forgotten most of the topics they have learned in the first course, or, even worse, they have serious misconceptions about statistics. To improve and accelerate students' basic statistical knowledge, I have successfully "mini-flipped" the course. In this presentation, I will discuss an effective model of flipping the biostatistics course, and explain the word "mini". Furthermore, I will describe what topics and assignments were offered out-of-class in order to allow students to engage in higher-level discussions during class time. Class time activities demonstrated that mini-flipping the biostatistics course led to a dynamic, more active learning environment in which students thrived and learned a lot.

Julian Michael Buck Francis Marion University (julianmbuck@gmail.com)

## **Designing a Successful Capstone Course**

Many undergraduate mathematics programs culminate in a "Capstone" type class for their graduating senior students. I will discuss the challenges, difficulties, and successes that I experienced in my first time teaching such a course, and suggest possible improvements for future years.

## **Probability or Statistics**

## Thursday, August 7, 8:30–10:10 AM, Galleria III

John Charles Wierman Johns Hopkins University (wierman@jhu.edu)

## A Disproof of Tsallis' Conjecture for the Exact Percolation Threshold of the Kagome Lattice.

In 1982, Tsallis derived a formula which proposed an exact value of 0.522372... for the bond percolation threshold of the kagome lattice. We use the substitution method, which is based on stochastic ordering, to compare the probability distribution of connections in homogeneous bond percolation on the kagome lattice to those on an exactly-solved inhomogeneous bond percolation model on the martini lattice. The lower bound obtained is  $p_c$  (kagome) > 0.522394, which shows that the value conjectured by Tsallis is too small to be correct. The method also provides an improved upper bound of  $p_c < 0.740880$  for the (3, 12<sup>2</sup>) lattice bond percolation model.

## Michael Z. Spivey University of Puget Sound (mspivey@pugetsound.edu)

## **Probabilistic Proofs of Some Binomial Coefficient Identities**

A combinatorial proof of an identity is frequently a beautiful thing. It involves interpreting the identity as counting the number of ways an event can occur but using two different approaches. However, identities that do not involve integers do not naturally lend themselves to combinatorial proofs. Some of these identities can instead be interpreted probabilistically - as expressing, in two different ways, the probability that a particular event occurs. In this talk we use inclusion-exclusion and the drawing-balls-from-jars paradigm to prove some common binomial coefficient identities that involve fractions.

David DiMarcoNeumann University (dimarcod@neumann.edu)Ryan SavitzNeumann University (savitzr@neumann.edu)Fred SavitzNeumann University (fsavitz@neumann.edu)

### The M-tile Deviation, A New Class of Measures of Dispersion: Assessing Learner Achievement

In an era where assessment guides many of the policies and practices implemented in higher education, colleges and universities are compelled to accumulate information containing large sets of data whose analysis necessitates applications of measures of central tendency. To address the demand for accurate and practical interpretations of these data, we introduce a new class of measures of dispersion, called the m-tile

deviation (MD). The quartile deviation is a special case of the MD and the mean deviation using the median equals a special case of the MD. The breakdown points of the MD are derived and compared to the breakdown points of the standard deviation and the median absolute deviation. In addition, the property of the stability of a measure is also used to compare the measures under discussion. The ease of computing measures will also be discussed. M-tile deviations are flexible, as their breakdown points and stability are functions of m. Hence a user may choose the value of m that best suits their application.

## Azar Khosravani Columbia College Chiacago (akhosravani@colum.edu)

**Constantin Rasinariu** Columbia College Chicago (crasinariu@colum.edu)

## An Investigation of Benford's Law Characterizations

One intriguing quality of many empirical data sets is that their first digit distribution is not uniform but rather logarithmic. Thus, the probability of occurrence of d as a first digit is given by  $P(d) = \log(1 + d^{-1})$ . Benford random variables are known to have this property. We will explore the basic properties that characterize Benford distributions such as scale-, base-, and sum-invariance as well as the uniform distribution mod one. Using Mathematica, these properties are demonstrated through examples of Benford random variables and sequences.

## Stephen A Sedory Texas A&M University - Kingsville (kfsas00@tamuk.edu)

Michael Lee Johnson Texas A&M University - Kingsville (mike21john@yahoo.com) Sarjinder Singh Texas A&M University - Kingsville (sarjinder.singh@tamuk.edu)

## Efficient Use of the Negative Hypergeometric Distribution in Randomized Response Sampling

Many questions, whose answers are of interest to sociologists and government policy makers, are sensitive, and people being surveyed are often hesitant to respond, or are reluctant to respond truthfully. Randomized response techniques are methods to estimate, through face to face surveys, such things as the proportions of persons in a population who are members of a (sensitive) subgroup, while protecting the individual's privacy. In this talk we indicate how these methods work and look at a newly proposed estimator, based on the negative hypergeometric distribution, that turns out to be more efficient than several standard estimators.

## Tania M. Lopez Hipple MUSE School (tml90028@aol.com)

## Data Literacy Skills—A Pathway to Bringing Quantitative Reasoning and Real-World Topics Into the Classroom

Data Literacy skills such as analyzing, visualizing, and interpreting data have become gateway skills to full participation in the workforce and civic engagement in the 21st century. In this session, participants will learn about ways to drive engagement and inquiry into their classrooms by bringing in real data around a variety of curriculum topics.

Participants will get a chance to explore activities, projects, and education technology tools that exemplify this approach and how it can help them align their instruction to the Common Core Standards. Participants will also get an opportunity to create their own activities and investigations that they can implement with their students.

## **James Helmreich** Marist College (james.helmreich@marist.edu)

K. Peter Krog Marist College (peter.krog@marist.edu)

## Project-Based Approach to Understanding Quantile Regression

Quantiles are a generalization of the notions of quartiles, deciles, and percentiles that can be used to describe non-central positions in a distribution. Quantiles and quantile regression are not generally covered in undergraduate calculus-based statistics courses, but they are useful tools for analyzing distributions when non-central positions are of primary importance. For example, social-science researchers investigating such topics as income inequality, educational inequality, or health outcomes inequality might find the center of the relevant distribution to be of less interest than the tails of that distribution.

We present a project-based approach to understanding quantiles and quantile regression for students interested in calculus-based statistics. The student first develops familiar concepts such as the sample mean, sample median, and ordinary least squares (OLS) regression as solutions to minimization problems, and then uses similar techniques to develop minimization approaches to quantiles, least absolute deviation (LAD) regression, and quantile regression. Project resources use real data to illustrate the usefulness of LAD and quantile regression in situations that do not lend themselves well to analysis using conditional-mean models, and interactive Maple and R worksheets allow the student to explore both the statistical models and the underlying mathematics.

## **Assorted Teaching Topics**

## Thursday, August 7, 8:30–10:25 AM, Parlor A & B

## Jerry Dwyer Texas Tech University (jerry.dwyer@ttu.edu)

## Mathematics Teaching Transformed — Lessons to be Learned

Early career teaching is often dominated by traditional lecture style delivery and heavy emphasis on end of course exams. For one instructor, mid career transformation resulted in the adoption of a wide variety of different pedagogical approaches and means of assessment. This talk

explores the nature of learning and assessment in mathematics at both secondary school and college level and provides strategies for others who strive for continued growth in their teaching.

## David Jay Graser Yavapai College (David.Graser@yc.edu)

## Passive and Active Activities in the Flipped Classroom

Learning mathematics in a college setting is a series of passive and active experiences that take place inside and out of the classroom. Depending on the faculty member, what takes place in the classroom may be more passive (such as lecture) or more active (such as solving problems collaboratively). In my version of a flipped classroom, passive activities take place outside of the classroom. This leaves more time during class for activities that are designed to result in successful learning. In this session, I will describe how I designed these activities and their basis in brain science.

## Mahmud Akelbek Weber State University (makelbek@weber.edu)

## Flipping Precalculus-Incorporating Online and In-Class Activities

Precalculus at Weber State University is a five credit course. Students in this class range from with little mathematical background to highly prepared and motivated. With this in mind, we designed and delivered a beginning to flip precalculus class. In this talk, I present online and in class activities that we incorporated in this beginning to flip precalculus class.

Victoria Kofman Quality Engineering Education, Inc. (drKofman@comcast.net)

## **Using Analogous Problems**

The students learn to use models that can help getting to the core of the original problems and solve them. The method can be applied for solving a variety of the problems including the following:

Find the last digit in 13 to the 2014-th power.

Find the sum of 50 consecutive numbers starting from -25.

Find the product of 50 consecutive numbers starting from -25.

Find the sum of 200 consecutive numbers starting from 200.

The approach allows the students who are not on good terms with Math to understand complex Math ideas and enjoy a Eureka moment.

## Matthew Lewis Utah State University (matthew.lewis@aggiemail.usu.edu)

## Zombie Models: A Sexy Approach to Improving Mathematics One Brain at a Time

Aristotle once said "For the things we have to learn before we can do them, we learn by doing them." As mathematics educators we understand that many aspects of our subject can be taught while others can only be learned. Similar to learning how to swim or play a musical instrument, students of mathematics benefit from using their theoretical knowledge in "real-world" circumstances that interest them. However, in most undergraduate mathematics courses students are subjected to contrived examples moonlighting as authentic applications. Students who participate in engaging, lively mathematics are certainly more likely to enroll in future mathematics courses. This study develops an example of an effective Project-Based Learning experience based around the Humans vs. Zombies moderated tag game played on the USU campus. Results indicate that participating students achieved course objectives at higher learning levels and increased in mathematical appreciation.

## Tanaka Noriko (tanaka-nagoya@y5.dion.ne.jp)

## Making Problem

I will introduce "Making Problem". This is an active learning strategies in Mathematics. Each group of 4 or 5 students makes the mathematics problem and solves it under a one-hour discussion, and presents it in the classroom. They teach and learn from each other. This learning-style was done in my geometry classroom in Japan. Using this style, most students understand mathematics more deeply.

## Melissa A. Desjarlais Valparaiso University (melissa.desjarlais@valpo.edu)

## **Reconceptualizing Mathematics for Elementary Teachers**

The purpose of my Scholarship of Teaching and Learning project is to develop classroom interventions and assessment methods to improve student dispositions and minimize student misconceptions related to mathematics among prospective elementary school teachers. Elementary education majors often experience math anxiety or lack of mathematical confidence, and they may have misconceptions about the mathematics they should be prepared to teach. These can affect their understanding of and attitudes towards mathematics (which will in turn influence their students' abilities and attitudes towards mathematics). In this talk I will briefly describe some evidence-based practices for changing student misconceptions and a few common mathematical misconceptions among elementary education students. Then I will describe what I have developed to address these misconceptions: the interventions and assessments, a rubric to assess changes, and an initial concept map to identify how the level of understanding progresses through developmental stages.

## Bryan Nankervis Texas State University (bn10@txstate.edu)

## How Admissions Cutoff Scores Favor Affluent Students and Act as a Barrier for Many Minority Students

SAT scores directly correlate with family income, while Hispanics and Blacks traditionally score significantly lower than Whites and Asian Americans. As a result, when institutions utilize a cutoff score on the SAT as part of the admissions process it serves as a benefit to higher-

income students and an obstacle to students of color. This study utilizes data from the College Board to demonstrate by example how a cutoff score at a prestigious public institution discriminates against college-bound Black, Hispanic, and low-income students.

## **Modeling or Applications**

## Thursday, August 7, 8:30–10:25 AM, Broadway III & IV

 David Metzler
 Albuquerque Academy (metzler@aa.edu)

 Edward Earl
 Independent Researcher (esquared@earthlink.net)

## Measuring Mountain Impressiveness with New Topographic Functionals

Mountainous terrain is more impressive than low, rolling hills or a flat plain. Much of that impressiveness is personal and subjective, but there are commonalities as well. In particular, large local relief and steep drops correlate strongly with most people's idea of visual impressiveness. We introduce three functionals that evaluate terrain based on local relief and steepness, which have good formal properties, and which match well with visual impressiveness. We will present numerous examples from the world's mountain terrain.

Nathanial Burch Gonzaga University (burchn@gonzaga.edu)

## The Second-Order Lanczos Derivative and a Nonlocal Flux

The one-dimensional classical diffusion equation, its simplest form, postulates that the rate of change in time of a field u(x, t) is proportional to the second spatial derivative of the field. There are many cases, however, in which the classical diffusion equation is inadequate, e.g., the diffusion process is anomalous or the field does not admit two spatial derivatives. Alternate models of diffusion must then be considered. The second-order Lanczos derivative of a function f at a point x,  $f''_L(x)$ , can be viewed as a weighted average of classical second-order derivatives and, thus, gives rise to a viable such model. A closer investigation of  $f''_L(x)$  reveals a method for establishing a generalized, nonlocal, flux and an ensuing nonlocal model for diffusion. We explore this model via its connections to stochastic processes, relationship to classical diffusion, and potential applications, e.g., to movement ecology.

## Brian Winkel US Military Academy, Emeritus (brianwinkel@hvc.rr.com)

#### SIMIODE – Systemic Initiative for Modeling Investigations and Opportunities with Differential Equations

We present details of a learning community for teachers and learners of differential equations, SIMIODE – Systemic Initiative for Modeling Investigations and Opportunities with Differential Equations, in which collaboration across and between campuses is afforded, peer reviewed and colleague produced teaching materials are at the heart of the course literature, and modeling scenarios and technology drive and facilitate teaching and learning. SIMIODE uses the NSF sponsored HUBZero platform on the Internet to provide a rich environment for teaching and learning. See www.simiode.org.

## George Moss Union University (gmoss@uu.edu)

#### The Collapse of the Tacoma Narrows Bridge

Almost 75 years ago, the Tacoma Narrows Bridge collapsed during a time of moderately high winds. Video of the bridge vibrating, twisting, and falling have captivated students for decades. Several attempts have been made by engineers, physicists, and mathematicians to model the behavior of the bridge leading up to its collapse. We will examine several of these attempts to see what questions have been answered and what questions remain.

Yan Hao Hobart and William Smith Colleges (hao@hws.edu)

## How Popular do You Want to Be?—A Mathematical Model of College Friendships

Many people want to be popular. Still, statistics show that U.S. adults have only four close friends on average. The dynamics of real life friendship networks are of interest to sociologists for decades, however, very few mathematical models have been set up to explain statistical data. In this project, we propose a mathematical model that use quality time shared between friends as the major measure to evaluate college friendship wellness.

We assume the obligation of helping each other is reciprocal between each pair of friends, ie. students request company from their friends when needed (upset, stressed, etc) and the requests will always be fulfilled as long as the friend has enough time. Also, we assume fulfilled requests enhance friendships and unfulfilled requests harm friendships. Using Watts-Strogazt random networks – where the nodes represent students and the edges represent friendships between them – we quantitatively studied the dynamics of college friendship networks. Our model showed that friendships can effectively help college students de-stress. Further, we found that the lengths of college friendships are closely related to how popular each individual is. We also studied whether keeping many friends is beneficial.

## Lynn Foshee Reed Einstein Educator Fellowship (lynn.foshee.reed@gmail.com)

## Weddell Seal Morphometrics: An Example of Mathematical Modeling to Solve a Polar Science Problem

Weddell seals are the world's southernmost mammals and have been intensively studied in the Ross Sea area of Antarctica for over 40 years. One problem that researchers encounter is how to measure the amount of subcutaneous fat of a seal from simple, non-invasive measurements

#### Assorted Mathematics Research Topics

taken in the field. Morphometrics is the term used to describe such a quantitative analysis of size and shape. The problem is a good one to assign to students learning to create mathematical models, because it does not require a high level of mathematics yet it is quite challenging to design a model that provides an answer with reasonable accuracy. Once the students have built a mathematical model, they can test it by using real data collected by scientists studying Weddell seals. This is one of many interesting examples from polar science that the presenter has gathered while an Einstein Fellow in NSF's Division of Polar Programs.

### Alexander Atwood SUNY Suffolk County Community College (atwooda@sunysuffolk.edu) Andrea Blum SUNY Suffolk County Community College (bluma@sunysuffolk.edu)

#### Deducing the Age of an Ancient Natural Nuclear Reactor in a Pre-Calculus Class

A natural nuclear fission reactor was operational approximately 2 Billion years ago in Africa. At that time, the Uranium in this reactor was naturally enriched in Uranium-235 at a level where the reactor could be moderated by ground water flowing through the soil containing the Uranium. By looking at the radioactive decay of Uranium-235 (Half-Life of 0.7 billion years) and of Uranium-238 (Half-Life of 4.5 billion years) and of other radioactive elements created in the reactor, students in a pre-calculus class can calculate the age and operational characteristics of this ancient nuclear reactor.

## Sol Garfunkel COMAP (sol@comap.com)

#### The Mathematical Contest in Modeling

This talk will give a brief history of the now 30 year old MCM competition, its origins and its growth. In particular we will discuss the increase in chinese participation and the implications for undergraduate curriculum and instruction in China and in the U.S.

## Assorted Mathematics Research Topics

## Thursday, August 7, 1:00-4:25 PM, Parlor C

**Michael Nathanson** Saint Mary's College of California (man6@stmarys-ca.edu)

#### Interesting Matrix Problems from Quantum Information Theory: Locally Distinguishing Quantum States

Suppose we wish to use a quantum system to encode some information. The set of codewords corresponds to a set of complex unit vectors, and the information may be perfectly recovered if and only if the corresponding vectors are mutually orthogonal. The situation becomes much more complicated if we assume that our states are shared by two physically separated parties, represented by a tensor vector space. This set-up generates lots of interesting matrix-theoretic questions; I will sketch some basic results and open problems.

## Christopher R. LeeUniversity of Portland (leec@up.edu)Valerie J. PetersonUniversity of Portland (petersov@up.edu)

#### The Rank of a Recurrence Matrix

A recurrence matrix is defined as a matrix whose entries (read left-to-right, row-by-row) are sequential elements generated by a linear recurrence relation. We show that the maximal rank of such a matrix is determined by the order of the corresponding recurrence. In the case of order-two recurrences, the associated matrix fails to have maximal rank whenever the ratio of the two initial values of the sequence is an eigenvalue of the relation. Methods require only introductory linear algebra and differential equations.

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James LongLehigh University (jtl209@lehigh.edu)Lee StanleyLehigh University (ljs4@lehigh.edu)
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#### A Busy Beaver Problem for Infinite-Time Turing Machines

In 1962, T. Radó introduced the so-called busy beaver function, which came to occupy an important place in classical (finite-time) computability. Among other things, Radó showed that it asymptotically dominates every classical total computable function from  $\mathbb{N}$  to  $\mathbb{N}$ .

We introduce a generalization to the infinite-time Turing machine (ITTM) setting and show that it asymptotically dominates all ITTM total computable functions from  $\mathbb{N}$  to  $\mathbb{N}$ . We argue that this analogue of one of Radó's main results in the classical setting is strong evidence for the naturality of the generalization. We also take the opportunity to illustrate the generalization's astonishing growth rate by comparing it with some the fastest-growing functions which are currently known.

## Jeff Johannes SUNY Geneseo (johannes@member.ams.org)

#### On the Other Side of the World, or Right Next Door?

There are some simple and some sophisticated theorems about what happens on the other side of the world. In this talk we will consider some topological reinterpretations of these results by asking simple questions like "what do we mean by the opposite side?" and "what do we mean

## Errata

We are very sorry that the following abstract was inadvertently left out of the abstract booklet.

## Akhilan Boopathy Lakeside School (s\_boopathy@hotmail.com)

## A Simplified Approach to the Cubic Formula

A unique derivation of a method to solve a polynomial of degree three with real coefficients is presented. Depth of extrapolative analysis is not emphasized. Rather brevity and elegance as emphasized. Vieta's Substitution is avoided, and substitutions in general are minimal. This derivation is viable for all variances of cubic equation.

by a circle?" By adjusting our expectations, we can have some curious effects of moving our opposite side to things more like opposite time or changing our circles into other closed curves. No prerequisites for exploring simple changes to important ideas.

**Jeremiah Bartz** Francis Marion University (jbartz@fmarion.edu)

## **Exact Values of Trigonmentric Functions**

The exact values of trigonometric functions are well-known for  $\pi/n$  where n = 1, 2, 3, 4, and 6 along with their integer multiples. What about  $\pi/5$ ? In this talk, we discuss two ways to determine the exact value of trigonometric functions for  $\pi/5$ . One method uses techniques from college algebra and trigonometry. The other utilizes Euler's Formula from complex analysis.

Adrian Gentle University of Southern Indiana (apgentle@usi.edu)

## Perplexing Factorizations: An Undergraduate Research Project

We report on an undergraduate research project that explores the number theory of perplex (or hyperbolic) integers, a generalization of the integers of the form a + bh, where a and b are integers and the hallucinatory number h satisfies  $h^2 = 1$ , but h is not real (and therefore  $h \neq \pm 1$ ). The natural properties of the integers are so familiar and deeply ingrained that it can be difficult for students to imagine that alternatives are possible. We demonstrate that the perplex integers provide an excellent setting for students to explore several unfamiliar ideas, including zero divisors, non-prime irreducibles, and the failure of unique factorization.

Charlie Smith Park University (charlie.smith@park.edu)

## **Beautiful Integer Patterns**

This talk will feature the amazing beauty which lies within the structure of the integers. Starting with simple concepts such as adding, squaring and cubing, stunning designs can be constructed. Several examples will be shown, each followed by a rigorous mathematical justification. This talk will be accessible to undergraduate students.

 Lauren Cassell
 Ohio Northern University (l-cassell@onu.edu)

William Roger Fuller Ohio Northern University (w-fuller@onu.edu)

## **Guarding a Koch Fractal Art Gallery**

This paper presents a generalization of the standard art gallery problem to the case where the sides of the gallery are continuous curves which are limits of polygonal arcs. The allowable limiting processes for such generalized art galleries are defined. We construct an art gallery in which one side is the Koch fractal and the other sides are three sides of a rectangle. The appropriate measure of coverage by guards is not the total number of guards but, rather, the guards-to-side ratio. We compute this ratio for the cases of shallow and deep versions of the Koch fractal art gallery.

## Lori McCune Missouri Western State University (Imccune@missouriwestern.edu) David McCune William Jewell College (mccuned@william.jewell.edu)

## Active Context Free Games with Finite Target Language

An active context-free game is a two player game of perfect information on strings over a finite alphabet. In each move, player 1 selects a position of the current word and player 2 rewrites the corresponding letter according to the rule of a context-free grammar. Player 1 wins if a string of the regular target language is reached. In this talk, we will examine such games with finite target language and give an algorithm to decide which player has a winning strategy in this setting.

## Robert Russell Molina Alma College (molina@alma.edu)

### Expressing Recursively Defined Polynomial Sequences in Terms of Extended Fibonacci Polynomials

Fibonacci Polynomials are well studied, have many interesting properties and diverse applications. In this talk we define Extended Fibonacci Polynomials and use them to solve a certain type of recurrence relation. More specifically, let  $\alpha$ ,  $\beta$ , f, g and h be polynomials and define the polynomial sequence  $P_n$  by  $P_0 = \beta$ ,  $P_1 = \alpha$ , and  $P_n = fP_{n-1} + gP_{n-2} + h$  for  $n \ge 2$ . Our main result shows that  $P_n$  can be expressed non recursively in terms of Extended Fibonacci Polynomials.

## Kendra KillpatrickPepperdine University (kendra.killpatrick@pepperdine.edu)Naiomi CameronLewis and Clark College (ncameron@lclark.edu)

### Inverson Polynomials for Permutations Avoiding Consecutive Patterns

In this talk, we use the language of Fibonacci tableaux to give the inversion generating functions for permutations that avoid  $\Pi$  where  $\Pi$  is a set of three or more consecutive permutation patterns. In addition, we introduce the more general notion of strip tableaux which are a useful combinatorial object for studying consecutive pattern avoidance. We give the inversion generating functions for  $\Pi$  a subset of 4 or 5 consecutive permutation patterns and for all but one of the cases where  $\Pi$  is a subset of three consecutive permutation patterns.

## Lee N. Collins County College of Morris (ncollins@ccm.edu)

## Extending the Tables of Wallis: Conjectures on Catalan Numbers and the Gamma Function

English mathematician John Wallis published his prized work, *Arithmetica Infinitorum*, in 1656. It was in this publication that he studied tables of values of the reciprocal integral  $1/\int_0^1 (1-x^{1/Q})^P dx$ . By method of interpolation, Wallis eventually achieved his most famous result, the infinite product for  $\pi$  (the Wallis Product). After a careful study of these historic tables, it is obvious they contain a wealth of underlying information. We use these tables to conjecture a well known formula for  $\Gamma(2z)$ , commonly referred to as *Legendre's Duplication Formula*, and derive a reciprocal integral representation of Catalan numbers. By exploring the connections within these derivations, we develop a closed form relationship expressing the *n*th Catalan number,  $C_n$ , in terms of the gamma function. The derivations of these results are non-trivial and provide an insightful look at some interrelations in mathematics.

## Nick Huo Han Huang (nhyellowribbon@gmail.com)

### **Repeating Fractions and Primes**

In base-10 number system, the repeating fractions possess quite lot properties then could be thought, including natures of primes.

## **Teaching or Learning Introductory Mathematics**

## Thursday, August 7, 1:00–5:40 PM, Galleria III

## Kayla Bradley Dwelle Ouachita Baptist University (dwellek@obu.edu)

## **Evangelizing for Mathematics**

Ever heard the expression "preaching to the choir"? Well, when it comes to teaching mathematics to liberal arts majors, I am definitely not preaching to the choir. These students come to me loaded with mathematical baggage. They are the "mathematically agnostic" and I have a choice: I can try to drag them, kicking and screaming, into the world of mathematics or I can bring the world of mathematics into theirs. This session is about how I have learned to accept and embrace these students for who they are and seek ways to make mathematics attractive to them. I accomplish this by allowing them to touch, see, and feel mathematics through the use of active learning, projects, and social media. Employing this approach transforms their understanding of and deepens their appreciation for mathematics, bringing about many successful "conversions".

## Crystal Lorch Ball State University (jlorch@bsu.edu)

John Lorch Ball State University (clorch@bsu.edu)

## Rethinking Ball State University's Liberal Arts Math Course

We discuss the renovation of Ball State University's main core curriculum math course. Flipping the classroom is a prominent feature of this redesigned course. We survey the redesign process, how the redesign was implemented, and what it has accomplished, with an emphasis placed on flipping pedagogy. We also give an overview of the resources we have amassed for this project, including the developmental course website.

Jason Molitierno Sacred Heart University (molitiernoj@sacredheart.edu)

## Writing Projects in a First Year Seminar Class in Mathematics

The First Year Seminar Class in Mathematics, titled "Mathematics without Calculations—It's a Beautiful Thing!" is a class that is meant for non-math majors. The goal of this class is to teach students college level writing and speaking. In this class, I assign daily writing assignments where students must read about a mathematical topic from the textbook (*Heart of Mathematics*, Burger & Starbird) and write a two-page reflection. Topics that have assigned are the infinitude of prime numbers, comparing different levels of infinity, one-to-one correspondences, platonic solids, four dimensionality, and fractional dimensions. In this talk, I will discuss the specifics of these writing assignments and how they improved student writing. I will also discuss lengthier research papers and presentations assigned.

## Michael Weimerskirch University of Minnesota (weim0024@umn.edu)

#### **PreCalculus Flipped Classroom and Active Learning**

The University of Minnesota has undertaken a project to redesign the PreCalculus course with three goals is mind: 1) Better prepare students for Calculus 2) Provide resource materials for incoming students wishing to test into Calculus 3) Provide review materials for current Calculus students. Results of the first year of the project will be presented.

## **Kay Geving** Belmont University (kay.geving@belmont.edu)

## College Algeba: Improving Student Success Using a Hybrid Approach

In the spring of 2013, I conducted both hybrid and traditional College Algebra courses using the same textbook, exams, and grading criteria. Comparison of test scores, final exam scores, evaluation comments, and other observations indicate the hybrid format greatly improves performance and comprehension of otherwise troublesome concepts. Discussion will include my initial reluctance and how I was able to

transform my approach to teaching. Students actively participate, work numerous problems, and learn from their own mistakes during and after class.

## **Gus Greivel** Colorado School of Mines (ggreivel@mines.edu)

## Active Learning Strategies to Improve Student Attitudes and Outcomes: The Studio Statistics Model at CSM

The department of Applied Mathematics and Statistics at the Colorado School of Mines (CSM) has initiated significant changes to the delivery of our introductory course in statistics to accomplish several goals, including (i) more efficient delivery staffing and delivery, (ii) expanded graduate student support, and (iii) an improved student experience in the course. While these goals may appear to be in opposition with one another, we have exploited the potential for student-centered active learning experiences in Statistics to develop a hybrid lecture/studio delivery of this course that aims to accomplish all three. This model has been used to great success in the Physics department at CSM. This talk considers the motivation to change our instructional model, outlines a new model for statistics instruction and provides an analysis of the successful initial pilot of this model.

## Amy Wheeler Hondros College (amywheeler@fuse.net)

## **Project Based Activities in Online Statistics**

Student acquisition of conceptual understanding of statistics can be challenging in any introductory course. The presentation will describe the process of transitioning from a standard online undergraduate statistics course to one which engages students in active learning projects. Rational for the course change as well as activities will be presented. These activities can also be presented in a traditional classroom.

## David Robert Gurney Southeastern Louisiana University (dgurney@selu.edu)

## Bringing Variety to Elementary Statistics Problems Based on Real Data

At first glance, elementary statistics problems based on real data seem almost immutable, but there are some steps one can take to alter these problems while keeping a firm foundation in real data. This paper will provide a few examples of how to do this.

## Victor lan Piercey Ferris State University (piercev1@ferris.edu)

## Quantitative Reasoning for Business: An Inquiry-Based Approach

Quantitative Reasoning for Business is a new course that brings together inquiry-based learning, quantitative reasoning, and writing across the curriculum. The course is targeted to the "intermediate algebra" level. In this talk I will discuss the approach to inquiry-based learning in the classroom, approaches to promote student buy-in, the problem-based nature of the course materials, the role of reading and writing in the course, and data on the impact on math anxiety and student beliefs about math.

## **Gregory V. Bard** The University of Wisconsin - Stout (bardg@uwstout.edu)

### Macroeconomics in Finite Math: Rediscovering and Recreating Leontief Analysis

Developed by Wassily Wassilyovich Leontief (1906–1999), the technique of "Input-Output Analysis" is often an example of applying linear algebra to macroeconomics, and appears in many Finite Mathematics and Linear Algebra textbooks. The idea is to divide the economy of a province/region into sectors each producing one commodity. Equations are written where  $T_{ij}$  dollars of commodity *i* are required to make 1 dollar of commodity *j*, with an external demand of  $d_j$ . Find a production schedule  $x_j$  so that both external demand and internal consumption are exactly satisfied.

The problems in classic textbooks are often oversimplified and this results in an unexciting model. Presumably this was done to ensure that the underlying matrix was  $3 \times 4$ , to enable the RREF to be computed by hand. However, with technology, this restriction is no longer necessary. Using Sage, the author revitalized this topic. First, an interactive webpage was created to explore the assumptions and macroeconomics of the problem, without requiring knowledge of the mathematics. Second, Sage can do the RREF for students, allowing problems of 5–7 variables. Third, the author discovered some simple follow-on analysis steps in economics textbooks that students can perform, which shed further light on the model and its mathematical properties.

During this talk, the author will display Stages 1 and 3. As a result of this work, matrices in general (as well as Leontief Analysis in particular) went from being a disliked topic to a favorite topic, as recorded by an anonymous survey of 60 Finite Mathematics students.

## **Rebecca Metcalf** Bridgewater State University (rmetcalf@bridgew.edu)

### Reaching More – A hybrid Mathematics course for Early Childhood, Elementary, and Special Education Majors

Online and hybrid courses have recently become a way to reach more students, but is it a good fit for all subjects? This session will focus on considerations and decisions made in the development of a hybrid course for Early Childhood, Elementary, and Special Education majors by a novice at online curriculum redesign. Topics will include review of course outcomes, determining which content might best be delivered in an face-to-face setting and that which can be transitioned to online delivery, development of discussion questions and rubrics, identifying ways to support students in need of assistance with mathematics literacy, and ways to assess the effectiveness of the delivery method.

## Daniel J. Heath Pacific Lutheran University (heathdj@plu.edu)

## Euclidean & non-Euclidean Origametry

Most students in our College Geometry course are pre-service high school math teachers. Given the Common Core's focus on transformations, our students need to understand this more than in past years, so we develop geometry axiomatically from the viewpoint of transformations.

Writing the axioms in terms of origami (as opposed to straightedge and compass) allows us to begin most classroom conversations with hands-on activities involving only a piece of paper. Here we present a few of our activities, and offer our course notes and software to other geometry teachers at all levels.

Rachel Frankel University of Cincinnati, Blue Ash College (frankere@ucmail.uc.edu)

## After the Test, What Now?

This talk will address the issue of post-assessment analysis and action. Often a class does poorly or below instructor expectations on an exam. The questions of "why?" and "what can be done to make sure students have the knowledge necessary to continue and succeed in the course?" arise and need to be addressed. To answer the first question, the metacognition technique of prompted reflection will be presented. Prompted reflection has been shown to help students reassess their study habits, learn new study and organizational skills, and take responsibility for their learning. Various examples of reflection worksheets will be shown as well as an analysis of their efficacy based on student feedback. The next step is to ensure students relearn the material that was being assessed. Since topics in mathematics build one on top of the other this is crucial for student success. Several strategies will be described and the effectiveness of each technique will be shown based on student results.

Xuan Hien NguyenIowa State University (xhnguyen@iastate.edu)Ian MouzonIowa State University (imouzon@iastate.edu)Alicia CarriquiryIowa State University (alicia@iastate.edu)Ulrike GenschelIowa State University (ulrike@iastate.edu)Elgin JohnstonIowa State University (ehjohnst@iastate.edu)Andee KaplanIowa State University (ajkaplan@iastate.edu)Wolfgang KliemannIowa State University (kliemann@iastate.edu)Kenneth KoehlerIowa State University (kkoehler@iastate.edu)

## Characteristic of Students During and After Introductory College Level Mathematics and Statistics Training

All participants in the ongoing STEM education discussion agree that, in addressing national priorities, a key concern is the critical transition of students from high school (or community college) to a four-year college program in the mathematical sciences in particular. Failure in college-level mathematics and statistics courses may discourage students from pursuing STEM majors or perhaps lead to complete college drop-out. We present the results of a statistical investigation that explores the characteristics of students who stay in STEM versus the ones who leave STEM or drop out of college. Attention is paid particularly to gender and high school preparation. Preliminary results indicate that incoming female students come less prepared mathematically. However, the same proportion of male students and female students stay in STEM in their first academic year.

## **Thomas Fryer** Asian University for Women (tom17fryer@gmail.com)

## Developing an Introductory Mathematics Course in a South Asian Context

The Asian University for Women is an institution that aims to educate the next generation of female leaders in Asia and beyond. The Introductory Mathematics course is a central feature of the university's foundational year that aims to help students adjust and thrive within an academic American liberal arts environment. This talk shall discuss the challenges of developing an Introductory Mathematics course in this diverse South Asian context. The context itself is firstly explored through case studies of mathematics teaching in Bangladesh, and student interviews from several other countries. It will then be discussed how the learning outcomes, content and teaching pedagogy of the Introductory Mathematics course has changed to respond to the challenges posed by such a context.

Ashley Johnson University of North Alabama (ajohnson18@una.edu)

## Supplemental Instruction at the University of North Alabama

Supplemental Instruction is offered at a variety of schools and comes in many different formats. The University of North Alabama piloted an SI program beginning last fall in a handful of courses, many with traditionally low success rates. In this talk, I will discuss the format of the supplemental instruction program at UNA, and some results and observations from this first year.

Todd Cadwallader OlskerCalifornia State University - Fullerton (tcadwall@fullerton.edu)Martin BonsangueCalifornia State University - Fullerton (mbonsangue@fullerton.edu)Kathy LewisCalifornia State University - Fullerton (kathylewis@fullerton.edu)Ashley Thune-AguayoCalifornia State University - Fullerton (athune@fullerton.edu)Jolene FlemingCalifornia State University - Fullerton (jofleming@fullerton.edu)

## Supplemental Instruction: Closing the Achievement Gap for Underrepresented Minorities

California State University, Fullerton is a Hispanic-serving institution with a diverse population that includes many first-generation students. Like many other institutions, our first-year math classes have had higher than average D/F/W rates. In response, the math department, along

with other departments in natural sciences, has implemented Supplemental Instruction (SI) workshops for these courses, including college algebra, precalculus, and the mainline calculus sequence. These workshops are optional, one-unit credit/no-credit courses that are linked to specific sections. Each workshop is led by an upper-division mathematics major, who facilitates the workshop by creating activities for students to work on in small groups.

Since beginning our SI workshops, we have seen significantly increased pass rates for SI participants, especially for underrepresented minorities (URM). In Fall 2010, we saw pass rates in first-semester calculus of 62% for non-URM who did not participate in SI, but a pass rate of 80% for those who did. Among URM students, the pass rates went from 35% for non-participants to 73% for participants. While students self-select into SI, there are no differences in academic predictors (e.g., high school GPA, entrance exam scores) between SI and non-SI participants, suggesting a "value added" effect to the program.

Other benefits to these students include a decreased need to repeat classes and a decreased time to graduation (saving time and money). The institution has benefited by retaining and graduating increased numbers of mathematics majors.

Our presentation will discuss how we implemented SI in our department, the challenges we faced, and our ultimate successes.

Melanie Butler Mount St. Mary's University (mbutler@msmary.edu)

### **Using Reading Guides in Mathematics Courses**

This talk will describe the use of Reading Guides in mathematics courses, including Math for Elementary Education and Abstract Algebra. In these courses, Reading Guides have been used to help students read and understand the textbook and to help students assess their own understanding. In addition, Reading Guides have been used to cut down on lecture and allow more time for in class activities.

## **Teaching or Learning Calculus**

## Friday, August 8, 8:30–11:55 AM, Parlor C

John Thoo Yuba College (jthoo@yccd.edu)

## Making Waves in Vector Calculus

In (vector) calculus textbooks, a common exercise is to verify that u(x,t) = f(x-ct) satisfies the wave equation. The mathematical theory of waves in fact is a rich source of PDEs that are accessible to vector calculus students. We present suitable examples — that include traveling waves, solitons, shocks, and tsunamis — that can enrich a vector calculus class or be used in an entire adjunct weekly workshop for students. Students would see how what they are learning is applied, and how some of the techniques they have learnt in previous courses (such as partial fraction decomposition) are used to solve important problems in applied mathematics. Students at this level generally do not see such examples even though they are accessible with help from the instructor.

Nathan P. Clements University of Wyoming (clementsuwyo@gmail.com)

#### Impact of a Large Lecture Model in Multivariable Calculus

Due to recent budget cuts at the University of Wyoming, classroom sizes has crept up from 30 to 45–50 in the Calculus classes. In an effort to provide more sections for Calculus I, at a smaller size, we changed Multivariable Calculus (Calculus III) into large lectures of 100 with recitations of 30–35. This will be a report on impacts of the change. This report will include data on change in student performance as well as their perception of usefulness of the change.

Michael AxtellUniversity of St. Thomas (axte2004@stthomas.edu)Dwyer DaveUniversity of Evansville (dd4@evansville.edu)Mark GruenwaldUniversity of Evansville (mg3@evansville.edu)Ken LutherValparaiso University (ken.luther@valpo.edu)

## **Resequencing Calculus: An Early Multivariable Approach**

The presenter will describe the development and implementation of a reformulated three-semester calculus sequence created at the University of Evansville and being piloted at various other institutions. The sequence is constructed so that the first two semesters constitute an appropriate two-course sequence for students in the life sciences and certain other STEM disciplines. To accomplish this, certain topics are introduced in the second semester, notably some multivariable topics, while others are delayed until the 3rd semester. This early treatment of multivariate calculus enables students who complete Calculus 2 to enter directly into courses in differential equations, calculus-based probability, or linear algebra. The presenters will discuss their approaches for dealing with multiple challenges, including those posed by course transfers and AP credit. This project was supported by NSF DUE CCLI grant #0836676 and is currently supported by NSF TUES grant DUE-1225566. Details can be found at http://www.resequencingcalculus.com.

John Frohliger St. Norbert College (john.frohliger@snc.edu)

## **Flipping My Calculus Classes**

In the Fall of 2013, I offered my first-semester calculus class as a "flipped" course. Lectures were given as narrated PowerPoint presentations, available in PowerPoint and HTML5 formats. The bulk of in-class time was spent on small teams of students working on problems. This

approach succeeded well enough that I flipped my second-semester calculus class in the spring. I will share the mechanics of what was done, with my impressions of what worked and what didn't, as well as feedback from the students.

## Bob Sachs George Mason University (rsachs@gmu.edu)

#### A Successful Week 1 in a Flipped Calculus 1 Course

The first week of any course establishes a tone and expectations for the rest of the term. I will report on a carefully designed set of IBL activities that worked well for me this past fall. My context is Calculus I at a research university in an active learning classroom (72 students, 2 learning assistants). The first week established that students would write on the whiteboard walls and share responsibility for their learning. It also got the attention of the large fraction of the class that was calculus-aware. Some data on this experiment will also be presented.

## **Derek Thompson** Trine University (theycallmedt@gmail.com)

#### Writing About Continuity

Recently, I have begun the discussion of limits in Calculus I by asking students what they knew about continuity. From there, they learn the definition of continuity and the three ways to be discontinuous: a hole, an asymptote, and a jump. I asked students to write a 500-word paper on the topic. They were to discuss three graphs of their own creation, which had to have physical meaning. Each graph was to have one of these types of discontinuities, and they had to explain the reason for the discontinuity. In this talk, I will discuss the results of the assignment—both good and bad.

Ken Collins Charlotte Latin School (kcollins@charlottelatin.org)

#### **Classroom Investigations to Prepare Students for Calculus**

This session will share several investigations we have written and used to help develop students' understanding of limits and instantaneous rate of change. We will discuss some ideas for developing these investigations and offer reproducible copies to participants to use in their classes.

#### **Cynthia Northrup** University of California - Irvine (cynthianorthrup@gmail.com)

#### Math Circle Problems in the Calculus Classroom

Math Circles are problem-solving workshops in which students use any combination of methods from their own mathematical experience. In the traditional calculus classroom, students are taught methods and formulas and ask to solve problems until the methods are mastered. By asking students to think about more complex situations in calculus, they must combine the information they have in new ways and think about problems they have not seen before. The hope is that students will come away with a better understanding and appreciation of calculus.

## Stacy Marie Musgrave Arizona State University (stacy.musgrave@asu.edu)

#### Introductory Calculus with Meaning and without Limits

Mathematics is about ideas and concepts, not just formulas and algorithms. At least, that is what I tried to convey to my introductory calculus class in Spring 2014. In my third experience with teaching calculus at the undergraduate level, I welcomed my students by discussing, not limits, but rather the four arithmetic operations. Many students were horrified, some bemused, and others high-fived thinking they found an easy A.

The purpose of beginning an introductory calculus course with a discussion of arithmetic—particularly the meaning of quotient—is to convince students that mathematics is about ideas, and not just formulas and algorithms. In order to support this goal, I further introduced a graphing program, aptly called Graphing Calculator. The program supports the development of meaning for mathematical notation (e.g., function, summation, and integral notation) and concepts (e.g., average rate of change, generating accumulation from rate of change). Creating mental images of the mathematical ideas being discussed often proves challenging for students; Graphing Calculator supports this process for students.

In my presentation, I will share one of the activities for making meaning for quotient to demonstrate what I mean by "calculus with meaning". I will also share some animations from Graphing Calculator that will illustrate how this program supports student development of mathematical ideas, and how it permitted me to teach introductory calculus without limits. Audience feedback is welcome, particularly with suggestions on how to measure success with the class in fall semester.

#### Brandy Benedict Merrimack College (benedictb@merrimack.edu)

#### Effectiveness of Teaching From a Bound Set of Lecture Notes in a Calculus Course

This paper describes an experience teaching a first-semester calculus course using a printed set of instructor lecture notes and exercises, bound together with blank pages for student notes and work. Analysis of end-of-semester survey results on student perceptions and opinions showed

that students overwhelmingly enjoyed working in class from a manual containing the lecture notes. For the instructor, using the lecture notes in this manner facilitated the preparation and delivery of course content in later semesters, allowing for more discussion and in-class problem solving opportunities. Although mean final exam scores from past semesters showed no significant difference in grades between classes who used the lecture notes and those who did not, both the instructor and the majority of students felt that the use of the notes during lecture facilitated student engagement and encouraged their continued use in future semesters.

# Corinne Casolara Montana State University - Bozeman (corinne.casolara@montana.edu)

Veronica Baker Montana State University - Bozeman (baker@math.montana.edu)

# Strategies to Promote Student Success in Calculus

Montana State University is a large, public, research-intensive institution, and like many large, public universities, has been searching for the best approach to support student success in the calculus sequence. In Fall 2013, the Department hired Student Success Coordinators (SSCs) for Calculus I and II, two large, multi-section courses serving approximately 12% of all students on campus each year. Responsibilities of the SSCs include course coordination, instructor and GTA mentoring, curriculum development, and supplementary instruction for identified "at-risk" students, with an overall goal to improve the DFW rate for those courses. Historically at MSU, Calculus I and Calculus II have had average DFW rates of 47% and 42% respectively, which has had a direct impact on four- and six-year graduation rates among STEM majors. Preliminary findings have shown an improvement in overall student success since the hiring of the SSCs. Additionally, at-risk students who took advantage of supplementary instruction also showed improvement in their performance. This report will discuss the roles of the SSCs as well as address strategies such as using online homework, dealing with prerequisite deficiencies, and utilizing real-world applications during class time. Preliminary performance data indicate these teaching initiatives are achieving the goal of improving overall student performance.

Heather BollesIowa State University (hbolles@iastate.edu)Adrian JenkinsIowa State University (ajenkins@iastate.edu)Elgin JohnstonIowa State University (ehjohnst@iastate.edu)Xuan Hien NguyenIowa State University (xhnguyen@iastate.edu)

#### The Effectiveness of Clickers in a Large-Enrollment Calculus Classroom

We present the results of a study about clickers in a large-enrollment engineering Calculus I course. Three instructors each taught a largeenrollment class of Calculus I in Fall 2012 and again in Fall 2013, one semester with clickers and the other semester without them. We tested for the existence of an effect of clickers on student learning by comparing student performance on the same three quizzes in both semesters. The results are so far mixed, without pronounced improvement for all three instructors. We will discuss the analysis controlling for high school preparation and standardized test scores.

# Paul Sisson LSU Shreveport (paul.sisson@lsus.edu)

 Tibor Szarvas
 LSU Shreveport (tibor.szarvas@lsus.edu)

#### History and Exploration in the Teaching of Calculus

Calculus is still too often presented as a collection of tools and theorems devoid of human connections and relationships to other topics. This tendency is understandable, given the sheer amount of material many departments try to cram into their calculus sequence, but learning usually suffers as a result. Drs. Sisson and Szarvas, both of whom have many years of experience as professors of mathematics and as university administrators, describe their approach to teaching calculus, both in the classroom and in their textbook, with a special focus on using historical examples and anecdotes as non-mathematical "hooks" and inspiration for exploration. This approach allows students to learn calculus by making connections with what they already know and what they can discover through the use of technology and each other.

## Edgar Fuller West Virginia University (ef@math.wvu.edu)

Jessica Deshler West Virginia University (deshler@math.wvu.edu)

#### The Impact of Placement and Curriculum Reform on Success in First-Year Courses

Each year approximately 5000 students enter West Virginia University as first-year students. In this talk we will give an overview of this student intake process and the first-year course structures in mathematics designed to offer options to as wide a variety of students as possible while fostering student success in these first-year and subsequent courses. We will discuss some of the difficulties we have encountered and present an analysis of some trends within the data for these courses over the last five years including results from placement testing given to determine algebra and calculus readiness at student orientation during the summer prior to their arrival on campus. We will also give an analysis of student performance as they subsequently enter developmental mathematics, college algebra, precalculus and calculus in a number of course structures.

# **Mathematics and Technology**

# Friday, August 8, 8:30–11:55 AM, Galleria III

# Andrea Blum SUNY Suffolk County Community College (bluma@sunysuffolk.edu)

Alexander Atwood SUNY Suffolk County Community College (atwooda@sunysuffolk.edu)

# Inspiring Critical Thinking Through Programming Projects in a Precalculus Class

A common complaint from students is, "I do all my online homework and understand everything, but I can't do well on tests." One reason for this frustration is that students neglect to organize their thoughts about the process they need to follow in order to solve a problem. They are doing homework, but they are not really studying. Anyone who has ever written a computer program knows that they must consider the initial information they have, where they want to end up and what pathways they can follow to get there. Group projects that require students to write simple programs for the TI-84 calculator, can help them focus on the steps they need to take a problem to completion. Projects must be designed carefully to ensure every student has a role to play and detailed rubrics must be created for fair grading. The biggest problem in implementing these projects is in trying to instill confidence in students so they can master this level of critical thinking and are able to learn new technology. In this presentation I will share some projects that my precalculus students have produced.

# Matthew Leingang New York University (leingang@nyu.edu)

# Using an Online Homework System for Written Homework

Online homework systems provide good practice for repetitive, computation problems. Can they also be used for deeper problems emphasizing writing? Systems are adding features allowing essay questions to be added to homework assignments, and for those essays to be graded by human graders. We will discuss lessons learned from administering written homework over the WebAssign system.

## John C. Miller The City College of New York, Emeritus (jcmill@mindspring.com)

#### Flipped Classrooms Require—and Should Inspire—Better Software

Software bears a heavy burden when used to present new mathematical topics to students outside the classroom. After introducing a topic and showing at least one sample problem, programs should minimally provide feedback emulating a competent instructor in response to each student's attempts to solve similar problems.

Most math faculty routinely say "show your work" or "show all your steps" when accepting solutions for feedback. Flipped classroom software should do the same. However, most current commercial programs allow a student to enter only a completely simplified final answer. If that answer is incorrect, just one correct solution method is then typically presented, not influenced by the student's particular incorrect response.

This presentation will include a brief history of math practice software, via examples, from early multiple-choice solution formats to today's short-answer materials. It will conclude (1) that it is our profession's responsibility to insist on more responsive computer-based materials that better emulate best faculty practices and (2) that we have largely failed our students in this regard. Relying on computers to introduce topics using the flipped classroom model makes the issue even more important, and should inspire faculty to campaign more energetically for better materials.

# Steven Klassen Missouri Western State University (klassen@missouriwestern.edu)

## **Evolution of a Statistics Classroom**

With evolving technologies, the expectation for a student to learn introductory concepts outside the classroom is now more realistic and the experience of a flipped classroom is more effective. Complementing an open source Statistics textbook with Panopto videos, Doceri demonstrations, and WeBWorK assignments, our elementary statistics course is in the process of moving to a flipped classroom design. Previously, this course has been a traditional face-to-face lecture course paired with a publisher's online homework system. With the possibility of the course becoming part of the general studies curriculum in the future, these changes may also help to handle increased enrollment and enable a transition to hybrid and/or online sections of the course.

#### Mili Shah mishah@loyola.edu (mishah@loyola.edu)

#### Using Online Technologies to Create Journal Articles in Numerical Analysis

This talk will focus on the online technologies used in Numerical Analysis to formulate journals. Specifically, students used the website http://writelatex.org to create and LaTeX journal articles. In addition, students were able to save and share versions of their articles for peer review amongst their classmates in the course. Reviews were submitted online using GoogleDocs and reviewer response emails were automatically generated using FormEmailer. Instructions documenting each of these processes will be presented along with case examples.

### **Eugene Boman** Penn State, Harrisburg Campus (ecb5@psu.edu)

#### Euclid 21: Euclid's Elements for the 21st Century

In 2012 I began a project with several undergraduate students to map out the explicit dependencies between the Propositions, Postulates, Definitions and Common Notions in Euclid's "The Elements." Our goal was to render those connections visually and interactively via a

computer code. Our project is now nearly completed. I will describe how the project was organized, what we did, what we learned, and what is left undone at this point. I will also demonstrate the capabilities of the software and describe possible future projects along the same lines.

# Jeff Randell Knisley East Tennessee State University (knisleyj@etsu.edu)

## The Sophisticated Pencil: Computation as Transformation of the Traditional Mathematics Curriculum

Technology has transformed instruction, allowing for instance the use of lecture videos and interactive content, which in turn allows us to "flip" the classroom. Moreover, computational approaches not only in mathematics but throughout the sciences will only increase in importance and are already dominating the prevailing approaches in some fields. Indeed, the computer is in many ways the mathematician's new pencil, but it is a very sophisticated pencil. Though powerful, teaching students in a course to use computational tools often conflicts with the conceptual goals of the course, primarily because computation versus mathematics requires instruction in two disparate contexts. How successful we are in "flipping" a course that has been enriched by computational approaches is often highly dependent on how well the software used ameliorates the conflict between different contexts.

For example, the IPython notebook allows both rich, web-oriented interactive content and powerful mathematical computational tools, including the ability of the two contexts to interact with each other. Similarly, the Sagemath cloud combines computation with presentation, as do tools such as Rstudio, Geogebra, and similar. Moreover, the learning curve for using these tools can be quite low for someone familiar with a web browser, which means that a single assignment can proceed from introductory lecture to tutorial to exercises to coding activities, and so on – even ultimately to assessment. In this presentation, we focus on the IPython notebook as an implementation of an assignment that allows an instructor to flip their in-class efforts from a focus on introduction of new material to a focus on interaction with students during the middle and end of an assignment. We also discuss limitations to this approach and how future versions of the software tools mentioned could address some of these limitations.

#### **Brian Katz** Augustana College (briankatz@augustana.edu)

#### An Active Introduction to Sage

This year, my department started to transition from Mathematica to Sage, and I was challenged to help the students learn this new tool in an active, inquiry-based approach. Personally, I learned to use these kinds of tools (a) by modifying existing code (b) to accomplish a new goal, and I tried to offer experiences with these properties to students. In this talk, I will share the tasks I developed with the gracious support of many other instructors, focusing on the relationship between the tasks and the active, inquiry-based learning opportunity they create.

# Nathan C Carter Bentley University (ncarter@bentley.edu)

Kenneth G Monks University of Scranton (monks@scranton.edu)

#### Advances in Lurch, a Word Processor that can Check Students' Proofs

The *Lurch* project has been able to check the mathematical reasoning in students' proofs for several years. This talk will review those capabilities briefly, and then show the latest features, including high-quality typesetting of mathematical expressions courtesy of *MathJax*. See http://lurchmath.org for more information.

#### Brian Douglas Sharp Indiana University of Pennsylvania (bds@iup.edu)

#### Multimedia Mathematics: Motivating Students With Digital Photographs and Videos

For many students their interest in mathematics increases when they can see connections between the mathematics being taught in the classroom and the world in which they live. Often students' exposure to these connections is limited to 'real-world applications' printed in a textbook or presented on a PowerPoint slide. While these problems may attempt to bridge the gap between the conceptual and the applied, many students need more than a written word problem to peak their curiosity. This is where digital photographs and videos come into play. By having students discuss and analyze carefully selected digital media, teachers can help students understand that mathematics is a powerful tool they can use to understand the world around them. In this session participants will learn three various ways to incorporate digital media into mathematics classrooms. Digital media can be used to illustrate problem conditions (When will the world's population reach maximum capacity?). Digital media can be used for data collection (Based on your timing data, determine if a car is speeding). Digital media can also be used to add humorous or interesting contexts to problems (Gabriel's Horn can be filled with paint but not painted.) Participants will also learn how to set up a YouTube channel to manage video clips and how to manipulate photographs in Geogebra.

James Anthony Mendoza Epperson The University of Texas - Arlington (epperson@uta.edu) Andrew Paul Byrns Dallas Independent School District (andrew.byrns@mavs.uta.edu)

# Dynamic Visualization's Effect on Mathematics Graduate Student and Inservice Teachers' Views of Transformations of Functions

This study uses technology-generated (e.g., Geometer's Sketchpad, GeoGebra) dynamic visual representations of transformations of functions in order to determine how dynamic visualizations influence mathematics instructors' content knowledge and pedagogical content knowledge on the topic of transformations of functions. We used smart pen technology to individually interview and record the work of three secondary school mathematics teachers from a large urban school district in the southwestern United Stated and three mathematics graduate students from a mid-sized (33,000 students) state university in the southwestern United States. We presented participants with elementary functions

#### Mentoring

encountered in high school Algebra I and Algebra II along with student questions that arise when viewing dynamic visualizations of simple parameter changes. After viewing animations, participants were invited to revise, if necessary, previous resolutions to the questions posed. Findings suggest that the dynamic visual images introduced misconceptions regarding some transformations and the graduate students tended to acquire the misconceptions with higher frequency than the secondary school teachers.

#### **Rebekah Gilbert** University of Illinois - Urbana-Champaign (rgilber1@illinois.edu)

#### **Experience-Driven Evolution of Technology-Based Courses**

For the past twenty-five years, the University of Illinois at Urbana-Champaign has offered technology-based math courses that use Mathematica. This talk will highlight the recent improvements implemented to this program, specifically in Calculus I, Calculus II, and Differential Equations. First, these changes include new homework assignments in Mathematica that teach students how to use Mathematica to solve standard problems in a step-by-step format. Second, I have created modular worksheets that accompany interactive demonstrations in Mathematica to emphasize the visual aspect of the concepts presented in the course. These new curriculum components provide examples of how to effectively utilize technology to improve student learning. Future plans to incorporate Mathematica demonstrations in large-lecture discussion sections will also be described.

## Ginger Harper Kaplan University (harper891@yahoo.com)

#### Staying In Touch with Students with Technology

The on ground classroom is limited to scheduled interactions. Office hours are limited and often times not convenient, and the ability to request support in a relatively immediate manner is usually not an option. Online mathematics students face unique environments and challenges as well. Some online environments can leave students feeling "out of touch" or make them feel loosely connected. However, there are proactive approaches that motivate and encourage student participation with a focus on strategies for increased student success through learner and instructor interactions and effective discourse in online distance learning environments.

The online mathematics classroom has a dynamic much different than the traditional classroom. Through creative interactions, the online environments can offer innovative approaches that are successful, creative and effective. Technology has allowed us to break down the barriers preventing effective communications and contact. Utilizing various modes of technology and interactive communication, we can create a learning environment that promotes learning in multiple learning styles so that all learners feel that they can participate, contribute, and be successful.

Jana TalleyJackson State University (jana.r.talley@jsums.edu)Lecretia BuckleyJackson State University (lecretia.a.buckley@jsums.edu)Jessica Buck MurphyJackson State University (jessica.l.buck@jsums.edu)Shontrice GarrettJackson State University (shontricegarrett@att.net)

#### Exploring the Use of Mobile Devices as Student Response Systems in Undergraduate Mathematics Courses

The accessibility of mobile devices to students and instructors at the post-secondary level creates a multitude of avenues for which classroom instruction can be enhanced. As components of a student response system, these devices may be used to engage students in meaningful classroom discussions through participation in poll questioning activities. This study is designed to investigate how mobile devices can be used in undergraduate mathematics courses to engage students in mathematical dialogue, which has been shown to facilitate conceptual understanding. The investigation will be conducted at Jackson State University, an urban minority-serving university, and the participants will be enrolled in a freshman level mathematics course. The following research questions will be addressed:

- 1. What are benefits of utilizing mobile devices as student response systems to (a) facilitate mathematical discourse, (b) encourage student participation in class activities, and (c) manage class time?
- 2. In what ways do students believe the use of mobile devices as student response systems enhances their learning experience(s)?
- 3. What instructional practices facilitate successful implementation of mobile devices as student response systems?

# Mentoring

# Friday, August 8, 1:00–2:25 РМ, Galleria III

Jenna Carpenter Louisiana Tech University (jenna@latech.edu)

#### **Creating Sustainable Programs to Support Women Faculty in Mathematics**

For the last five years, Louisiana Tech University has had an NSF ADVANCE Project designed to create a culture of success for women faculty in engineering and science. Many of the eight programs were designed from the outset with an eye toward sustainability for the long-term. As the project moves to post-grant status, we will report on the first year of this transition, with recommendations on building sustainable programs to support the success of women in mathematics.

**Megan Wawro** Virginia Tech (mwawro@vt.edu)

Jessica Ellis San Diego State University (jess.ellis84@gmail.com) Hortensia Soto-Johnson University of Northern Colorado (hortensia.soto@unco.edu)

#### MPWR: Mentoring and Partnerships for Women in RUME

Mentoring and Partnerships for Women in RUME (MPWR) was an NSF sponsored seminar designed to (a) provide an environment for women in the undergraduate mathematics education research community to discuss issues specific to them; and (b) informally provide or expose women to mentorship and partnership opportunities among graduate students, junior and senior faculty within that research community. Seminar participants had the opportunity to engage with prominent women who shared their experiences with: Mentoring + Partnerships, Roles + Identities, Overcoming Obstacles + Meeting Challenges, and Personal + Professional. As part of the presentation we will share the structure of the seminar and the participants' reactions to the seminar. Participants' responses to a follow up survey show that participants walked away (a) feeling as though they were part of a community, (b) recognizing that women have similar experiences in balancing roles and identities in academia, (c) acknowledging that there are numerous paths to tenure and promotion, and (d) feeling empowered.

# Gary MacGillivray University of Victoria (gmacgill@math.uvic.ca)

#### **Mentoring Undergraduates**

Mentoring students is something that touches every aspect of what we do as instructors. The intent to have a discussion about mentoring students in general, and then shift the focus to working with undergraduate students on research projects not related to any course. Topics include:

- (i) What exactly is 'mentoring', and how does it compare to 'coaching'?
- (ii) What is appropriate for undergraduate research?
- (iii) What is the desired outcome and what is the assurance the project is at the right level?
- (iv) What constitute good practices for working with the student(s) through the life cycle of the project?
- (v) What determines whether there has been success?

# Noureen Khan University of Northern Texas - Dallas (noureen.khan@unt.edu)

#### The Intentional Mentoring

As a mentor, faculty member has the opportunity to have a life-long impact on her students. However, providing quality mentorship to undergraduate research students has recently emerged as an important strategy for successfully recruiting and retaining students in the mathematics programs. We explore the value of mentoring, the developmental profile of young mathematicians, and the traits of a good mentor.

#### Lina Sanchez leal Rutgers University (linas@rci.rutgers.edu)

Gabriela Garcia Cliffside High School, NJ (gabriela\_garcia1@aol.com)

#### The Power of a Good Mentor: Lessons Learned from a Four Year AfterSchool Mathematics Program

During the past four years, I worked with a group of students in an Afterschool mathematics enrichment program. The initial goal of the program was to introduce calculus concepts via simulation software to middle school students-high school students. As time progressed, a personal relationship was built with the students in order to help engage them in the topics. Once the personal bonds were made, students began to require more advisement about classes and life. As effective professors, we are proponents and advocates of strong mentoring and advising of students at all levels. The program goal shifted to include a mentoring component to support and guide the development of the students in order for he or she to gain the knowledge, values, and skills integral to forging a strong sense of personal identity. Salient values, mathematical calculus concepts and practices were learned through collaboration within the program. In this session, the mentor will discuss the conceptual and methodological understandings gleaned, students' progress and obstacles and future activities. Then the mentee will discuss her progress, successes and obstacles as well as why she wants to become a mathematician.

# Daniel Patrick WisniewskiDeSales University (daniel.wisniewski@desales.edu)John T. GareyDeSales University (jg7500@desales.edu)

#### Measuring Educator Effectiveness & Pre-Service Teacher Supervision

The Pennsylvania Department of Education (PDE) has begun implementing a process for measuring educator effectiveness based on student achievement of content standards; in particular, PDE has created the Student Learning Objective (SLO) Process as part of a multiple-measure, comprehensive system of Educator Effectiveness (authorized by Act 82) throughout the state of Pennsylvania. In anticipation of this new procedure on the horizon, pre-service teachers (undergraduate and graduate) at DeSales University (DSU) were required to conduct an SLO Project which follows the template of this PDE assessment process.

The focus of this presentation will be a brief outline of this newly created form of measurement, but more importantly, concentrated attention will be given to the personal experience of the supervision process of a student-teacher in the mathematics secondary classroom. As a first-time mathematics supervisor collaborating with DSU's education faculty, the speaker will share his perspective on the procedure, highlighting the mutual learning of effective pedagogy and the challenges of proper assessment of remedial, at-risk high school students, with an eye on measuring the effectiveness of the educator.

Through the lens of a new, objective participant in the preparation process of future secondary mathematics teachers, reflections will be shared about specific challenges of the SLO process and the adaptation of instructional planning based upon data-driven assessment, as well as how this fits into the wider view of the certification process: from preparation for standardized certification examinations to implementing effective differentiated instruction to diverse learners at the high school level.

# Outreach

# Friday, August 8, 1:00–2:40 РМ, Parlor C

Philip B. Yassk	in Texas A&M University (yasskin@math.tamu.edu)	
Alex Sprintson Texas A&M University (spalex.tamu@gmail.com)		
Kaitlyn Phillipson Texas A&M University (kaitlyn@math.tamu.edu)		
Trevor Olsen	Texas A&M University (trevorolsen3@gmail.com)	
Frank Sottile	Texas A&M University (sottile@math.tamu.edu)	

#### **Texas A&M Math Circle: Structure and Activities**

Math Circles bring mathematicians into direct contact with pre-college students, fostering a passion for mathematics. The TAMU Math Circle began in 2011 as an after-school club at a local 5–6th grade school and moved to the A&M campus in 2012. It meets 3–5 PM most Saturdays of the school year with about 40 5–8th graders attending. The Circle starts with a 30-minute "game" activity, then splits into two groups, based on algebra experience, for a 90-minute structured activity.

We are fortunate to have graduate and undergraduate students helping with the Circle. They help with set-up and clean-up, manage supplies, assist the speakers and work with the students. Olsen organizes the "games" and Phillipson has led several structured activities. It's great when young mathematicians get involved in the culture of outreach.

Our Circle has an interesting twist in that many of the parents of our students are TAMU faculty in other STEM fields. In fact, our circle grew out of the TAMU Summer Educational Enrichment in Math when one of the parents, Sprintson, in Electrical and Computer Engineering, offered to help organize a circle. So, in addition to presentations by mathematicians, we have had several presentations by STEM faculty on how mathematics is used in their disciplines.

The circle received an MSRI Grant for the 2011–12 year and a Dolciani Grant for the 2014–15 year, enabling us to pay travel costs for prestigious speakers. In this talk, we will discuss the organization, activities, funding and sustainable of our Circle.

Leesa Anzaldo University of California - Irivine (lanzaldo@uci.edu) Timmy Ma University of California - Irivine (timmym@math.uci.edu) Cynthia Northrup University of California - Ivine (cynthianorthrup@gmail.com) Alessandra Pantano University of California - Irvine (apantano@uci.edu)

#### The UCI Math Circle: Afternoons of mathematical investigations for middle and high school students

There are so many interesting topics in mathematics, which are accessible to students of all ages, yet are not part of the normal K–12 school curriculum. Filling this gap and initiating K–12 students to abstract mathematics is often challenging, as the emphasis on computations and problem solving in the school curriculum often makes students feel intimidated by advanced theoretical concepts. The UCI Math Circle—a wonderful outreach program sponsored by the Mathematics Department at UCI with the support of MAA, MSRI and NSA—has overcome this obstacle by grouping students in a 'workshop-style' environment where they can learn a variety of topics (from fractals to isometries of periodic patterns) through guided exploration. The UCI Math Circle for talented high schoolers focuses on promoting higher-level research-type skills, much like an inquiry based college-level abstract mathematics course. Emphasis is placed on improving proof writing skills, rather than speed and performance in math contests. The UCI Math Circle for middle school students has a similar exploratory flavor (discovery without lecturing), in a friendly, dynamic and non-competitive environment. Students join their efforts in solving problems without rushing for an answer, and provide logical arguments for their answers. Although the mathematics discussed is often difficult, students feel it is "safe" to make mistakes; they are proud and excited to present their solutions on the board. This talk will present a selection of the most successful activities, discuss important logistical aspects of the Circle and share a range of enthusiastic comments from dozen of students.

Brandy S. Wiegers National Association of Math Circles, Central Washington University (brandy@msri.org)

#### National Association of Math Circles, First National Survey Results

The National Association of Math Circles has grown over the last ten years from 12 known Math Circles in 2003 to more than 170 today. We are currently conducting the first study of the impact of these programs and the initial survey results will be first presented at the 2014 MathFest session. Earlier surveys of a small set of Circles have demonstrated that Math Circles impact the mathematical attitudes of participating students. The presentation will discuss the creation of the survey, the current national study results, and plans for future studies.

Additional information about Math Circles is available at mathcircles.org and Math Circle representatives will be available at the NAMC booth for the conference to provide additional details to follow up the presentation.

#### Klay Kruczek Southern Connecticut State University (kruczekk3@southernct.edu)

# Southern Connecticut State University's GEAR UP Summer Mathematics Program

We will discuss the mathematical portion of the five-week summer program for New Haven (CT) youth, run by Southern Connecticut State University, and funded by a U.S. Department of Education GEAR UP (Gaining Early Awareness and Readiness for Undergraduate Programs) grant. This grant program is designed to increase the number of low-income students who are prepared to enter and succeed in post-secondary education. The theme for the 2013 summer camp (for rising 8th grade students) was "Bridges to the Future," and the theme for the 2014 summer camp (for rising 9th graders) will be "Soaring Higher." We will talk about the structure, mathematical content, and successes of each year's program, as well as what we have learned about the students and how to more effectively run this summer program.

# Daniel Teague NC School of Science and Mathematics (teague@ncssm.edu)

# Supporting Mathematics Research Projects for Advanced High School Students

The PREP workshops from the summer of 2012 and 2013 on supporting mathematics research for high school and two-year college students has resulting in a network of teachers sharing research problems and student work. Examples of the research problems and student work will be described as well as information on the website and course organizations being used. The group is only two years old, but there have already been some significant successes in student competitions and increased interest in mathematics.

# Stephanie Anne Salomone University of Portland (salomone@up.edu)

# Encouraging STEM Majors to Consider a Career in Teaching Through Nonprofit Partnership

In this presentation we will describe and discuss the effectiveness of a Noyce-sponsored summer internship program to encourage undergraduate STEM majors to consider teaching as a profession.

Paired with master teachers and mentors, Noyce Interns work for eight weeks in the summer to gain practical experience working with grade 2-12 students to improve interns' understanding of STEM instruction both through a traditional tutoring program and through Saturday Academy STEM classes and camps. The tutoring and SA classes provide programming for underserved students as about 40% of the students participating in these are low income, under represented minorities, or First Generation College bound.

Through reflections, group meetings, and formal evaluations, we will determine whether and how these informal teaching experiences are influencing these STEM majors to consider teaching as a profession. We will have one years' worth of internship data, and will be completing year two of the program. We will present what we feel have been successful components of the internship program and what improvements we are continuing to make.

# Gulden KarakokUniversity of Northern Colorado (gulden.karakok@unco.edu)Brian ChristopherUniversity of Northern Colorado (brian.christopher@unco.edu)

# Mathematics Summer Research Program for Undergraduates: The National Research Experience for Undergraduate Programs

The National Research Experience for Undergraduate Programs (NREUP) is a rigorous summer mathematics research program for underrepresented undergraduate minorities supported by the Mathematical Association of America (MAA) and its Strengthening Unrepresented Minority Mathematics Achievement (SUMMA) program with funding being provided by the NSF and the NSA. The 6-8 week long session allows undergraduates a research experience in mathematics that includes the sharing of their research through conferences and/or a journal. The NREUP has the goal of increasing interest in obtaining advanced degrees and careers for underrepresented minorities in STEM fields. During its life span of 10 summers (2003-20013), the program reached 465 people (218 female, 247 male). 59% were African American, 23% were Hispanic Americans and 10% were Latino Americans with the remaining 8% consisted of Asian Americans, Pacific Islanders, American Indians and Middle Easterners. Each participant was requested to participate in evaluating their NREUP program at its end and at one-, two-, and four-year intervals. We will share the results of the NREUP evaluation and our future plans for exploring other aspects of the NREUP.

# **Research in Applied Mathematics**

# Friday, August 8, 3:00-5:10 РМ, Parlor C

Steve Szymanowski Northeastern Illinois University (steveszy@gmail.com)

# The Study of Complex Dynamics of Methamphetamine Use and Markets in California

Methamphetamine use has grown into a serious problem in several areas of the USA. Methamphetamine marketers exhibit unique dynamics which influence the persistence of a methamphetamine market. We have developed two mathematical frameworks that capture (1) the dynamics of methamphetamine use and control and (2) the complexities inherent to local methamphetamine markets in the State of California. Our goal is to identify and understand the underlying mechanisms that drive the dynamics of methamphetamine use in various local communities.

The model analysis involves the system dynamics and uncertainty analysis through treatment and incarceration data. The first model focuses on the influence of social, demographical, and intervention factors that may control the patterns of methamphetamine use in California. The model stratifies the population into two sub-populations: at-risk and not-at-risk populations. The source of the at-risk population is the not-at-risk population. The at-risk population is further divided into susceptible-to-initiate methamphetamine use, current meth users, users under treatment, individuals incarcerated for possession, and temporally recovered individuals. The goal of this model is better understanding of social, demographical, and intervention factors that may impact the trends of methamphetamine use in a local community.

The second model focuses on local and nonlocal methamphetamine marketers (i.e. gangs, international cartels). Mechanisms in the model highlight the main differences between gangs and outside cartels. The organization and structure of the marketers is an important characteristic which influences their behavior. An initiation function is utilized to show that initial methamphetamine use is dependent on the gang population. Social interaction and influence is a critical component to methamphetamine marketing and is also built in to the model. The goal of this model is to study the cooperation and competition dynamics between two distinct types of marketers, and identify factors with the potential to destabilize coexistence—leading to extinction of one or both.

The preliminary results show that improving socio-economic conditions such as poverty and education can help lower the vulnerability and eradicate the at-risk population as expected. We found that reducing treatment time and efficacy of treatment programs (in comparison to improving law enforcement programs) will drastically reduce the size of the methamphetamine user population.

#### **Javier Garza** Tarleton State University (garza@tarleton.edu)

#### **Optimal Control of the Spread of Cholera**

Effective treatment and prevention strategies as they relate to the spread of cholera following an outbreak are of significant interest to the global health community. We present a model that describes the environmental and human-to-human transmission dynamics of the disease within the human population, as well as the growth of the bacterial concentrations, and perform a sensitivity analysis. Treatment and prevention controls are analyzed to suggest optimal control protocols.

#### Elizabeth Anna Evans Rose-Hulman (evansea@rose-hulman.edu)

#### Getting Fed Faster with a Six-Sigma, Production System Analysis

Optimal processes are an ideal state for any company. Typically, companies are equipped to handle a normal capacity, but issues arise when the demand spikes above the normal capacity levels. Keeping customers happy, the inefficient processes are usually hidden or not on display when dealing with the customers. This is not the case with our college's on-campus Subway. This Subway, part of a nationally-known chain sandwich shop, undergoes a daily, cyclical demand pattern. Right at the "Meal-Exchange" time, the line for a sandwich goes from the normal queue length of less than four people to over forty. The process of making the sandwiches does not change from the normal situation to the spiked situation which leads customers to wait long periods of time and grow unhappy while the business loses out the opportunity to profit from the customers who decide to get their meals from elsewhere. This production system analysis is about determining the ways for Subway to become better equipped to handle the spike in demand while beginning to optimize the processes presented to their customers. Following the six-sigma's DMAIC process, Minitab and Promodel were used in order to analyze the current situation and make possible improvements to correctly address the system bottleneck.

# Jesus Pascal The American University of Afghanistan (pascal\_jesus@yahoo.com)

#### Computing the Value Function for a Singular Optimal Control Problem

This works refers to the construction using the dynamic programming approach of the candidate value function for a one-dimensional singular optimal control problem with controls taking values in a closed cone in R. Then a verification theorem allows us to prove that this candidate turns out to be the value function for the considered control problem.

# Robert Thompson Macalester College (robcth@gmail.com)

#### Assembling Broken Surfaces Using Differential Invariant Signatures

We present a strategy for reassembling a broken surface based on boundary curve matching. The matching process relies on the invariant signature, a curve parameterized by curvature and torsion. These ideas are applied to 3d scans of a broken ostrich egg. This project is joint work with students at Macalester College and University of St. Thomas in St. Paul, MN.

#### Louis Essien Effiong (louisn66@yahoo.com)

#### Effect of Thermal Diffusion And Chemical Reaction on Heat And Mass Transfer in Micropolar Fluid

Unsteady heat and mass transfer by free convective MHD micropolar fluid flow bounded by a semi- infinite porous plate in a rotating frame under the action of transverse magnetic field with chemical reaction in the presence of heat generation is considered. The plate is assumed to oscillate in time with constant frequency so that the solutions of the boundary layer are the same oscillatory type. The governing system of partial differential equations are transformed to dimensionless equations using dimensionless variables. The dimensionless equations are then solved analytically using perturbation technique. With the help of graphs, the effects of the various important parameters entering into the problem on the velocity, microrotation, temperature and concentration fields within the boundary layer are discussed. Also the effects of the pertinent parameters on the local skin friction coefficient and rates of heat and mass transfer in terms of the local Nusselt and Sherwood numbers are presented numerically in tabular form. The results show that the observed parameters have significance influence on the flow, heat and mass transfer.

#### **Oahimire Imumolen Jonathan** Fluid dynamics (imumolen@yahoo.co.uk)

#### Heat and Mass Transfer in a Micropolar Fluid With Thermal Radiation Over a vertical Plate

Heat and mass transfer effects on unsteady flow of a micropolar fluid over an infinite moving permeable plate in a saturated porous medium in the presence of a transverse magnetic field are studied. The governing system of partial differential equations is transformed to dimensionless equations using dimensionless variables. The dimensionless equations are then solved analytically using perturbation technique to obtain the expression for velocity, microrotation, temperature and concentration. With the help of graphs, the effects of the various important parameters entering into the problem on the velocity, microrotation, temperature and concentration fields within the boundary layer are discussed. Also the effects of the pertinent parameters on the skin friction coefficient and rates of heat and mass transfer in terms of the Nusselt number and Sherwood numbers are presented numerically in tabular form. The results shows that the observed parameters have significance influence on the flow, heat and mass transfer.

### **Stanley Ryan Huddy** State University of New York - New Paltz (huddys@newpaltz.edu) **Joseph Skufca** Clarkson University (jskufca@clarkson.edu)

#### Complete Synchronization on Networks of Identical Oscillators with Diffusive Delay-Coupling

Synchronization behaviors on networks of coupled oscillators are of great interest due to their many applications in physics, biology, sociology, etc. These behaviors include, but are not limited to, complete synchronization, phase synchronization, lag synchronization, and generalized synchronization. Over the past few decades, an extensive amount of work was accomplished to analyze these types of synchronization under the framework of instantaneous coupling. More recently, coupling delay has been added to the communication between nodes to account for finite propagation times of the coupling signals in real world settings. In this work, we present necessary conditions for complete synchronization on networks of identical oscillators with diffusive delay-coupling and a single constant delay. It is found that complete synchronization is possible if at least one of the following conditions is met: (1) the network is regular, (2) the system solution is tau-periodic, or (3) the synchronized solution is a fixed point. Numerical simulations of five-node networks with chaotic node dynamics are presented as examples of synchronization on such networks.

#### **David Kenric Hammond** Oregon Institute of Technology - Wilsonville (david.hammond@oit.edu)

Yaniv Gur SCI Institute, University of Utah (yanivg@sci.utah.edu)

#### Measuring Distances Between Weighted Graphs by Graph Diffusion.

Many interesting scientific problems involve collection of data that can be described as connectivity networks, which can be represented as weighted graphs. An important tool for analyzing such data is a notion of the distance, or dissimilarity, between weighted graphs. In this work we develop a novel difference metric between weighted graphs, called the graph diffusion distance (GDD), that is appropriate for the case when the vertices of the weighted graphs are in correspondance. Out approach is based on the insight that two graphs should be similar if they enable similar patterns of communication between vertices. We model communication generically by heat diffusion across the graph. Our metric is based on the graph Laplacian exponential kernel matrices  $e^{-tL}$ , where L is the graph Laplacian. This kernel encapsulates the patterns of heat diffusion formed by initial conditions located to single vertices. The GDD is then calculated by taking the Frobenius norm of the difference of the exponential kernels from each graph, which corresponds to averaging the sum-of-squares differences of the patterns of heat diffusion. This yields a family of distance metrics corresponding to different diffusion times, a unique metric is obtained by finding the diffusion time giving the maximum difference. Examples of using the GDD to compare synthetic graphs, and on real-data graphs representing human anatomical brain connectivity will be presented.

# **Research in Graph Theory or Combinatorics**

# Friday, August 8, 3:00–5:25 РМ, Galleria III

#### Karin Saoub Roanoke College (saoub@roanoke.edu)

#### **Dynamic Storage Allocation using Tolerance Graphs**

Dynamic Storage Allocation is a problem concerned with storing items, such as processes in a computer's memory, that each have weight and time restrictions. Approximate algorithms have been constructed through online coloring of interval graphs. We present a generalization that uses online coloring of tolerance graphs and utilizes online algorithms in which the corresponding tolerance representation of a vertex is also presented. We apply the results on the online-with-representation chromatic number of various classes of tolerance graphs to a generalization of Dynamic Storage Allocation, giving us a polynomial time approximation algorithm with linear performance ratio.

Mark AndersonRollins College (manderson@rollins.edu)Shiying GuRollins College (sgu@rollins.edu)Charles Evans HedgesRollins College (chedges@rollins.edu)Felipe QuirogaRollins College (fquiroga@rollins.edu)

## Eternal Colorings and k-eternal Graphs

Given a proper coloring of the vertices of a graph, a local recoloring at a vertex v is a new proper coloring for which v is assigned a new color and vertices which are not local to v are not assigned new colors. This has applications in event scheduling, where the time at which some event is scheduled must change without causing a major disruption to the whole schedule. Eternal colorings are those proper colorings that repeatedly allow for local recolorings and k-eternal graphs are those graphs for which every proper k-coloring is eternal. We survey what is known about such colorings and graphs and provide some open questions.

John Engbers Marquette University (john.engbers@marquette.edu)

#### Extremal *H*-Colorings of Trees and Forests

Given a finite graph H, an H-coloring of a finite, simple graph G (or graph homomorphism) is a map from the vertices of G to the vertices of H that preserves edge adjacency. H-colorings generalize many important graph theoretic notions, such as proper q-colorings (via  $H = K_q$ ) and independent sets (via H as an edge with a loop on one endvertex).

A natural extremal question is the following: given a family of graphs G and a graph H, which graph(s) in G have the largest number of H-colorings? We consider this question for the family of n-vertex forests and trees. Numerous open questions remain.

Cayla McBee Providence College (cmcbee@providence.edu)

#### Prime Labelings of Graphs

A graph G on v vertices has a prime labeling if and only if there exists a labeling of the vertices with the numbers 1 through v such that any two adjacent vertices have labels which are relatively prime. It is an interesting problem to determine which graphs have prime labelings. I will discuss the prime labeling problem and examine under what conditions the ladder graph,  $P_n \times P_2$ , has a prime labeling.

Richard Low San Jose State University (richard.low@sjsu.edu)

Dan Roberts Illinois Wesleyan University (drobert1@iwu.edu)

#### **Group-antimagic Labelings of Graphs**

A labeling of a graph is an assignment of values to its vertices, edges, or both. Let A be a non-trivial abelian group and let G be a simple connected graph. We say that G is A-antimagic if there exists a labeling, f, of the edges of G with non-zero elements of A such that the induced vertex labeling,  $f^+$ , given by  $f^+(v) = \sum_{uv \in E(G)} f(uv)$  is injective.

In this talk we will examine some new results on the group-antimagic property for various classes of graphs.

**Christina Graves** The University of Texas - Tyler (cgraves@uttyler.edu) **David Milan** The University of Texas - Tyler (dmilan@uttyler.edu)

#### Inflection Points of Reliability Polynomials

We show that for every n, there exists a multigraph whose reliability polynomial has at least n inflection points.

Timothy B. FlowersIndiana University of Pennsylvania (flowers@iup.edu)Shannon R. LockardBridgewater State University (Shannon.Lockard@bridgew.edu)

#### Insights into *m*-ary Partitions from an *m*-ary Tree

In 1999, Neil Calkin and Herb Wilf "recounted" the rationals on a binary tree. They also connected this tree to the hyperbinary integer partition function. Several authors have explored connections between this tree and other interesting binary trees as well as introduced generalizations of the tree. There is also a natural extension of hyperbinary partitions to hyper m-ary partitions. In this talk we introduce an m-ary tree that is a generalization of Calkin and Wilf's tree. We will discuss characteristics of this tree and show how the tree reveals properties of the hyper m-ary partition function.

Michael Tiemeyer Armstrong Atlantic State University (michael.tiemeyer@armstrong.edu)

#### On C<sub>z</sub>-Factorizations with Two Associate Classes

Let  $K = K(a, p; \lambda_1, \lambda_2)$  be the multigraph with: the number of vertices in each part equal to *a*; the number of parts equal to *p*; the number of edges joining any two vertices of the same part equal to  $\lambda_1$ ; and the number of edges joining any two vertices of different parts equal to  $\lambda_2$ . Necessary and sufficient conditions for the existence of *z*-cycle decompositions of this graph have been found when  $z \in \{3, 4\}$ . The existence of *C*<sub>4</sub>-factorizations of *K* has been settled when *a* is even; when  $a \equiv 1 \pmod{4}$  with one exception; and for very few examples when  $a \equiv 3 \pmod{4}$ . In this paper, we give a general construction for  $C_z$ -factorizations of *K* when  $a \equiv 1 \pmod{2}$  and  $\lambda_1$  is even, and some specific results for z = 6.

**Cathy Kriloff** Idaho State University (krilcath@isu.edu)

**Terry Lay** Idaho State University, Retired (layterr@gmail.com)

#### Hamiltonian Cycles in Cayley Graphs of Complex Reflection Groups

A Cayley graph of a group G has vertices given by the elements of the group and edges joining g to sg and g to  $s^{-1}g$  for each s in a specified set S that generates G. An intriguing conjecture states that every such Cayley graph of a finite group with respect to any generating set has a Hamiltonian (spanning) cycle. We show the conjecture holds true for Cayley graphs of imprimitive complex reflection groups with respect to standard sets of generating reflections. This is joint work with Terry Lay and generalizes a similar result for real reflection groups due to Conway, Sloane, and Wilks in 1989.

# **Research in Analysis**

# Saturday, August 9, 8:30–9:25 AM, Parlor C

Kevin Rion Bridgewater State University (krion@bridgew.edu)

#### Hypercyclicity and the Range of an Operator

A bounded linear operator T on a topological vector space X is said to be hypercyclic if there is a vector x in X for which the orbit  $\{x, Tx, T^2x, T^nx, \ldots\}$  is dense in X. In this talk, we address where "most" hypercyclic vectors for an operator T are located relative to the range of T. If x is a hypercyclic vector, then so is  $T^n x$  for every n. Moreover, the range of T is dense in X, so one might expect that in some sense, most hypercyclic vectors for an operator T are in the range of T. Consistent with this expectation, we show the set  $\bigcap_{n=1}^{\infty} Range(T^n)$  contains a dense subset that consists of vectors that are hypercyclic for T; and then contrary to this expectation, we show for every non-surjective hypercyclic operator T on a Banach space X, the set of hypercyclic vectors for T that are not in the range of T is large in that it is a set of category II. This was joint work with Juan Bes.

Hamid Semiyari James Madison University (msah.sem@gmail.com)

#### A Solution to Boundary Calue Problems and Volterra Integral Equations with Parker and Sochacki Method

G. Edgar Parker and James Sochacki, of James Madison University (JMU) developed a method based on Picard's iteration method to solve ordinary differential equations (ODEs). The method is an algorithm that generates Maclaurin series solutions to Initial Value Problems (IVPs). The method converts an IVP into a system of first order ODEs, where the right hand side is a polynomial. The advantage of this method is that the method requires only addition and multiplication which makes it a good choice for computation.

We demonstrate this method for two point boundary value problems (ODE) and Volterra integral equations.

#### Xiao-Xiong Gan Morgan State University, Baltimore (xiao-xiong.gan@morgan.edu)

#### Behavior of Boundary Convergency for Power Series

The behavior of power series on boundary of convergence domain has been an interesting topic since power series were introduced. For example,

$$f(x) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n} x^n$$

converges on (-1, 1] but diverges at x = -1. It is more complicated on  $\mathbb{C}$ .

The composition of formal power series has been an important part of the formal power series theory. We introduce new results about the boundary convergence of a power series by applying certain results of formal power series.

#### **Donald Leigh Hitzl** Lockheed Palo Alto Research Lab, Retired

#### Analytical and Numerical Investigations of the Riemann Hypothesis

The Zeta Function, in the real domain, is remarkably simple.  $\zeta(x)$  is just the sum  $\sum_{n=-\infty}^{\infty} n^{-x}$ . This function is defined for x > 1 and has a pole at x = 1. However, the search is for zeroes of  $\zeta$  for they correspond to prime numbers. Elevating x to  $s = \sigma + it$  leads to  $\zeta(s) = \zeta_1(\sigma, t) + i\zeta_2(\sigma, t)$  in the Complex Domain. All the known zeroes have been found at  $\sigma = 1/2$ . The Riemann Hypothesis is that no sets of  $\zeta$  zeroes occur off the critical line  $\sigma = 1/2$ .

For all complex analytic functions, the Cauchy Riemann equations are fundamental first-order coherences. Our new discovery is that there are second-order coherences for all complex analytic functions. These are named the Perturbation Derivatives. Furthermore, if the functions involved are infinitely differentiable, like the Riemann Zeta Function, the two Perturbation Derivatives can be cascaded yielding a fourth order pde representing a Damped Quartic Oscillator. This is the general solution behavior for all infinitely differentiable (i.e., not power series) complex analytic functions.

The Riemann Zeta function is particularly simple in that every single term for n > 1 represents an Undamped Quartic Oscillator, merrily bouncing around for all time t without converging to any steady condition. Understanding the meaning of the Perturbation Derivatives enables a very easy proof of the famous Riemann Hypothesis. In particular, our new Discovery opens the door for all sorts of new results if one takes a problem in the real domain of N dimensions and elevates the problem to the complex domain of 2N dimensions.

# **Teaching or Learning Developmental Mathematics**

# Saturday, August 9, 8:30–9:25 AM, Galleria III

**Ben Ntatin** Austin Peay State University (ntatinb@apsu.edu)

#### Using Pictures to Study Students' Mathematical Beliefs and Attitudes.

This study describes student's beliefs and attitudes towards mathematics learning. The study uses pictures about mathematics drawn by the students themselves. An analysis of these pictures for encoded mathematical beliefs showed in general, that we could divide the belief structure and attitudes of students into categories that were consistent with established categories. However, another category emerged that showed a belief system inconsistent with known categories.

Ann Hanson Columbia College Chicago (ahanson@colum.edu)

#### Activities to Reinforce Fraction Concepts in the Developmental Math Classroom

In this presentation, the speaker will demonstrate several activities that are designed to help students understand fraction concepts. One of the activities is to cut a sheet of paper in half while focusing on what it means to divide something in half. After the students have done the activity, the students are to prove why this method works. This activity can be extended to other geometric shapes. Another activity has students using fraction tiles to add and subtract fractions. The relationship between fractions, decimals and percents will also be demonstrated with another activity. The speaker has used these activities successfully for many years.

Mary B. Walkins The Community College of Baltimore County (leolawalkins@yahoo.com)

#### Using Critical Thinking Skills in Developmental Mathematics

As a new full-time faculty member at the Community College of Baltimore County, I participated in the New Faculty Learning Community (NFLC) 2013–2014, an induction program required by the college. In the spring 2014, we had to fulfill the requirements of a mini-project designed to increase student engagement and learning, so I decided to use the results of the project which were blended into this paper. To increase understanding and retention in my students, I used critical thinking skills in two developmental mathematics courses which focused on introductory algebra. Thus, to facilitate the process, I enhanced the courses using pre- and post-quizzes in one course or quizzes in another course. I will present the assessment strategy/activity, the results of the comparative data, and potential modifications for future implementation.

**Christina Tran** California State University - Fullerton, Fullerton Mathematical Circle (cnt\_tran@csu.fullerton.edu) **Benjamin David Blazak** California State University Fullerton (benblazak@gmail.com)

# "Is it time to go home yet?": Student Engagement in Extracurricular Mathematics, Grades 2-4

In our Fullerton Math Circle, we have the privilege of helping to mathematically shape the minds of some of the brightest students in Orange County—some of them even brighter than our professors (though, lucky for us, not yet quite as experienced). Grades 2–4 pose a unique challenge in that, while unusually bright, the students are also normal kids: if you don't quickly make them part of a fun conversation, scribbling on the whiteboard will become infinitely more interesting than the beautiful concepts you sought to unfold.

In our talk we will focus on some of the strategies we use to promote and encourage engagement, while (with an appropriate mixture of sneakiness and honesty) boldly leading them into the joys we know from learning.

# **Research in Number Theory**

# Saturday, August 9, 8:30–11:10 AM, Galleria I

**Bud Brown** Virginia Tech (ezbrown@math.vt.edu)

#### Difference Sets, Singer Designs, and Singer Difference Sets

A  $(v, k, \lambda)$  difference set is a set of k elements in a group G of order v such that the set of nonzero differences  $d_i - d_j$  contain  $\lambda$  copies of the set of nonidentity elements of G. For example,  $\{1, 3, 4, 5, 9\}$  is an (11, 5, 2) difference set for the group of integers mod 11. This talk is

about an interesting connection between certain difference sets (called Singer difference sets) and certain block designs that appear in finite projective geometries (called Singer designs) – and about James Singer, for whom these combinatorial objects are named.

**Renilson Adriano Silva** Federal Institute of Education, Science and Technology of Sao Paulo and University Center Modulo (ras@usp.br)

#### Odd Numbers, their Relation to Primitive Pythagorean Suits and Traingulares Numbers—Theorem Ren

The odd numbers are the main set of integers. The behavior of these numbers has been of great interest to mathematicians in recent years, particularly with respect to odd primes. These numbers have an interesting behavior with respect to primitive Pythagorean suits, this relationship has been studied by some mathematicians, such as Sierpinski. This paper presents this relationship from the perspective of those odd and their positions in one dimension, the position of your square and its representation in dimension 2, can be demonstrated geometrically.

Demonstrating this relationship with the equation  $m^2 + I_0^2 = (I_0 + 1)^2$ . Where  $I_0$  is the position of the square of any odd m.

## David Penniston University of Wisconsin Oshkosh (pennistd@uwosh.edu)

#### Arithmetic of k-Regular Partition Functions

Perhaps the most famous results in the theory of partitions are the congruences proved by Ramanujan. In recent years there has been a great deal of research devoted to the behavior of k-regular partitions, which are partitions none of whose parts are divisible by k. In this talk we discuss new results on the arithmetic of k-regular partition functions, including families of Ramanujan-type congruences.

Jiayuan Wang George Washington University (j453w588@gwmail.gwu.edu) Max Alekseyev George Washington University (maxal@gwu.edu)

#### A Computational Method for Solving Exponential-Polynomial Diophantine Equations

Combining number theory with computer programming, we developed a novel computational method for solving Diophantine equations of the form  $f(n) = b^m$  with respect to integers m and n, where b > 0 is a fixed integer and f(x) is a second-degree polynomial. Our method involves solving generalized Pellian equations and computing the solution periods modulo prime powers and employs computer algebra system PARI/GP. We use our method for systematic study of such equations and present many numerical results. As an example, we prove that the only solutions to the equation  $2n^2 + 1 = 3^m$  are  $(m, n) = (0, 0), (1, \pm 1), (2, \pm 2), \text{ and } (5, \pm 11).$ 

**Don Vestal** South Dakota State University (donald.vestal@sdstate.edu)

# A Set of Two-color Off-diagonal Rado Numbers for $x_1 + x_2 + \cdots + x_m = ax_0$

We establish some values for the off-diagonal 2-color Rado numbers for a system of two equations. For positive integers a, b, m, and n, let S = S(m, a; n, b) denote the smallest positive integer S, provided it exists, such that for any 2-coloring of the integers in the set  $\{1, 2, ..., S\}$  using red and blue, there exists a solution to the equation  $x_1 + x_2 + \cdots + x_m = ax_0$  using only red integers or a solution to the equation  $x_1 + x_2 + \cdots + x_m = ax_0$  using only red integers or a solution to the equation  $x_1 + x_2 + \cdots + x_m = ax_0$  using only red integers or a solution to the equation  $x_1 + x_2 + \cdots + x_m = ax_0$  using only red integers or a solution to the equation  $x_1 + x_2 + \cdots + x_m = ax_0$  using only red integers. Then we have S(1, 1; 1, 2) = 2 and  $S(m, 1; m, 2) = m \lceil \frac{m}{2} \rceil + \lceil \frac{m}{2} \rceil - 1$  for  $m \ge 2$ .

David Krumm Claremont McKenna College (david.krumm@gmail.com)

#### **Squarefree Parts of Polynomial Values**

Let C be a hyperelliptic curve defined over the rational numbers, and consider the set S of all squarefree integers d such that the quadratic twist of C by d has a rational point. In this talk we will discuss the question of whether, given a prime number p, the set S contains representatives from all congruence classes modulo p. When C has genus 0 this question can be answered using elementary number theory, but for higher genera it seems to require the use of big conjectures in diophantine and arithmetic geometry.

## Jason Preszler University of Puget Sound (jpreszler@pugetsound.edu)

#### **Emergent Reducibility in Polynomial Dynamics**

Given an irreducible polynomial f(x), when can iterates (i.e., repeated composition of f with itself) of f(x) become reducible? This talk will focus on recent progress with cubic and quartic polynomials and computational methods to search for examples.

#### Nathan Carlson California Lutheran University (ncarlson@callutheran.edu)

#### Connections Between Furstenberg's and Euclid's Proofs of the Infinitude of Primes

In 1955 Furstenberg gave a surprising topological proof of the infinitude of primes using arithmetic progressions. At first glance this proof seems unusual and unlike other proofs of this famous result. Cass and Wildenberg (2003) and Mercer (2009) gave non-topological versions of Furstenberg's proof that uncovered the essential number theory used in that proof. Yet on the surface neither version seems to bear much resemblance to Euclid's original proof. In this talk we give a modification of the Furstenberg/Mercer proof that in fact looks much like that classical proof. This demonstrates that while Furstenberg's proof seems unusual, at its core it is in fact quite similar to the first and most well-known.

# Anthony Shaheen California State University - Los Angeles (ashahee@calstatela.edu)

#### The Gaussian Moat Problem

If we start near the origin of the complex plane and take steps of bounded length along the Gaussian primes can we make it out to infinity? No one knows the answer to this question. Some recent papers give partial results. In this talk I will first give a description of the Gaussian primes and then will discuss some of the known results. This talk is for a general audience. No knowledge of Gaussian primes is assumed.

#### Jay Lawrence Schiffman Rowan University (schiffman@rowan.edu)

#### Some Palatable Morsels, Integer Sequences and Number Theory Trivia

The year 2014 celebrates the one hundredth anniversary of the birth of the legendary mathematician Martin Gardner. In this paper, I present a number of fun and hopefully fascinating morsels that serve as mathematical food for thought in the sense of bordering on the unusual as well as injecting humor. For example, we furnish several magic squares including one consisting of nine consecutive primes, the initial pair of primes at a four digit distance from one another, Albert Wilansky's brother-in-law's telephone number and its unusual property, the curious and interesting role played by the prime 7919 and the composite integer 2584 as well as an integer sequence to paraphrase Martin Gardner as a pattern that is curiously fascinating but leads nowhere. Please join us as we embark on a brief journey of number theoretic heaven.

#### Edwin O'Shea James Madison University (osheaem@jmu.edu)

#### Divisibility Tests Unified: Stacking the Trimmings for Sums

Divisibility Tests are algorithms that can quickly decide if one integer is divisible by another and can be roughly broken into two types, "trimming" and "summing." We will derive a trimming test for every integer and will seek to unify the trimming and summing families of tests by showing that the best summing tests can be derived directly from the trimming tests via a "stacking" procedure. "Stacking" is nothing more than the claim that a six year old (but not an adult) would always choose 10 pennies over a dime.

In the spirit of this session, we use only the most basic of divisibility properties to develop these tests. In particular, we do not use the binomial theorem or modular arithmetic. The well known sum and alternating sum of digits tests for 9 and 11 follow as corollaries. This work was motivated by discussing the well known tests with an "Introduction to Proofs" class and wondering if there was a more coherent narrative to presenting these algorithms.

# **Research in Algebra**

#### Saturday, August 9, 8:30–11:10 AM, Broadway I & II

Papiya Bhattacharjee Penn State Behrend (pxb39@psu.edu)

#### Various Extensions of Commutative Rings

If *R* and *S* are two commutative rings with identity, and  $f: R \to S$  is an injective ring homomorphism, then we can consider *R* as a subring (unital) of *S*, and we say that  $R \hookrightarrow S$  is a ring extension. An extension of rings  $R \hookrightarrow S$  is a *p*-extension if given any  $s \in S$ , there exists an  $r \in R$  such that sS = rS; that is, the principal ideals in *S* are generated by elements in *R*. Again, an extension of rings  $R \hookrightarrow S$  is a *pg*-extension if given any nonzero  $s \in S$ , there exists an  $r \in R$  such that  $sS \cap R = rR$ . In general, neither one of the two extensions implies the other. In this talk the speaker will discuss these various extensions of commutative rings, describe under what conditions one extension implies the other, and how these *p*- and *pg*-extensions are related to contractions and extensions of ideals. Furthermore, if time permits, the speaker will mention other types of ring extensions, such as, essential extension, rigid extension, and regular localization, which are related to *p*-extension and *pg*-extension.

# Hannah Robbins Roanoke College (robbinshr@gmail.com)

#### **Calm Ring Extensions and Associated Primes**

Let *R* and *S* be commutative Noetherian rings and *M* a finitely generated *R*-module. If  $R \rightarrow S$  is flat, then  $S \otimes M$  has only finitely many associated primes whenever  $Ass_R M$  is finite. In this talk we generalize flat extensions to a new type of ring extension, called calm, which also preserves finiteness of the associated primes when we move from a module to its extension. We will discuss properties of calm extensions and give some examples of rings which have only calm extensions.

Irawati Irawati Institut Teknologi Bandung (irawati@math.itb.ac.id)

#### The Generalization of HNP Ring, 2 Bezout Ring and P-Bezout Ring

We generalize HNP ring to M-HNP module, 2 Bezout ring to 2 Bezout module and P-Bezout ring to P-Bezout module. We also see the properties

## **Reyes Matiel Ortiz-Albino** University of Puerto Rico - Mayaguez (reyes.ortiz@upr.edu) **Cesar Serna-Rapello** University of Puerto Rico - Mayaguez (cesar.serna@upr.edu)

#### $\tau$ -Factorizations, when $\tau$ is an Equivalence Relation

We discuss a framework for a general theory of factorization in integral domains called  $\tau$ -factorization, where  $\tau$  denotes an equivalence relation on an integral domain. We define properties of relations that were characterized by Anderson and Frazier in 2006, such as: divisive, associated-preserving and multiplicative relation. We show the existence type of relation with structural properties very similar as the usual product. Also, we show a type of an equivalence relation  $\tau$  and its associated-preserving extension  $\tau'$ , where an element x is  $\tau$ -irreducible element if and only if is a  $\tau'$ -irreducible element and some other facts. Finally, we show how this help to understand the theory of  $\equiv_n$ -factorizations.

# Suwanda Hennedige Yasanthi Kottegoda Southern Illinois University Carbondale (yasanthi\_k@yahoo.com)

#### The Number of Zeros of Linear Recurring Sequences over Finite Fields

In this talk, I discuss the possible number of zeros of a homogeneous linear recurring sequence over a finite field  $\mathbb{F}_q$ , based on an irreducible minimal polynomials of degree d and order m as the characteristic polynomial. I prove upper and lower bounds on the cardinality of the set of number of zeros. The set is determined when  $t = (q^d - 1)/m$  has the form  $q^a + 1$  or  $q^{2a} - q^a + 1$  where  $a \in \mathbb{N}$ . The connection with coding theory is a key ingredient.

Jeffrey Stuart	Pacific Lutheran University (jeffrey.stuart@plu.edu)
Minerva Catral	Xavier University (catralm@xavier.edu)
Leila Lebtahi	Universitat Politecnica de Valencia (leilebep@mat.upv.es)
Nestor Thome	Universitat Politecnica de Valencia (njthome@mat.upv.es)
<b>James Weaver</b>	University of West Florida (jweaver@uwf.edu)

#### Two Groups Associated with an $\{R, s + 1, k\}$ -Potent Matrix

We present selected results from a pair of recent papers by M. Catral, L. Lebtahi, J. Stuart, N. Thome and J. Weaver on  $\{\mathbf{R}, s + 1, k\}$ -potent matrices. Such matrices **A** satisfy  $\mathbf{R}^{k-1}\mathbf{A}^{s+1}\mathbf{R} = \mathbf{A}$  for some integer  $s \ge 0$ , some integer  $k \ge 2$ , and some matrix **R** such that  $\mathbf{R}^k = \mathbf{I}$ , the identity matrix. These matrices generalize several broadly studied classes of matrices. Specifically, when s = 0 and  $\mathbf{R}$  is the standard, irreducible,  $n \times n$  circulant permutation matrix (so k = n), the class of  $n \times n$  circulant matrices is obtained. When s = 0 and **R** is the "cross diagonal identity" matrix (so k = 2), the class or centrosymmetric matrices is obtained. When s = 1 and  $\mathbf{R} = \mathbf{I}_n$ , the class of idempotent  $n \times n$  matrices is obtained. When s > 1 and  $\mathbf{R} = \mathbf{I}_n$ , the class of s-potent matrices is obtained. When s > 1 and  $\mathbf{R} = \mathbf{I}_n$ , the class of s-potent matrices is obtained. We explore the spectral properties of  $\{\mathbf{R}, s, k\}$ -potent matrices, the relationships among the spectral projectors of such a matrix, and the Jordan form of such a matrix. Additionally, we discuss  $\mathbf{A}^{\#}$  when **A** is not invertible, and the nature of the classical matrix group associated with **A** and  $\mathbf{A}^{\#}$ . Finally, we explore a noncommutative subgroup of the classical matrix group with order  $k((s + 1)^k - 1)$  that consists of all products of the form  $\mathbf{A}^j \mathbf{R}^p$  where j and p are positive integers and  $j \le (s + 1)^k - 1$ .

Lydia Kennedy Virginia Wesleyan College (lkennedy@vwc.edu)

#### An Algebra with Characteristic Dependent Associativity

Let K be a field. We will construct an algebra V over K with the property that the associativity of multiplication is dependent on the characteristic of K. We explore other properties of the algebra including the annihilator ideals, an associated Lie algebra and the ideal lattice.

#### Jeremy Thompson USAFA (jeremy.thompson@usafa.edu)

# The Frobenius Number of Balanced Numerical Semigroups

We investigate the Frobenius number of numerical semigroups of the form  $S = \langle pD, pD + n, qD - n, qD \rangle$  where p, q, D, and n are integers such that p < q, gcd(p,q) = 1,  $D \ge 1$ ,  $1 \le n < \frac{q-p}{2}D$ , and gcd(D,n) = 1 (with some additional conditions on D to be explained as we go).

Robert Wayne BenimPacific University (rbenim@gmail.com)Loek HelminckNorth Carolina State University (loek@ncsu.edu)Farrah JacksonElizabeth City State University (fjchandler@mail.ecsu.edu)

#### Isomorphy Classes of Involutions of Sp(2n, k)

A first characterization of the isomorphism classes of k-involutions for any reductive algebraic groups defined over a perfect field was given in a paper by Helminck in 2000 using 3 invariants. In this talk, we build on these results to develop a detailed characterization of the involutions of Sp(2n, k). We use these results to classify the isomorphy classes of involutions of Sp(2n, k) where k is any field not of characteristic 2.

# Christopher Kennedy Christopher Newport University (christopher.kennedy@cnu.edu)

# Directly Finite Modules of sl<sub>2</sub>d

The Lie algebra  $sl_2d$  is an ( $\mathbb{N}$ , max)-graded infinite dimensional analogue of  $sl_2$  that arises naturally in deep matrix Lie algebras and their equivalent formulations ( $C^*$ -algebras, Leavitt algebras). We use direct limits of modules to define directly finite modules, determine necessary and sufficient conditions for such a module to be cyclic and construct irreducible highest weight modules of  $sl_2d$ .

#### Margaret T. Kinzel Boise State University (mkinzel@boisestate.edu)

#### Characteristics of Algebraic Symbol Sense

Fluency with algebraic symbols significantly influences students' ability to reason abstractly about quantitative relationships. The development of symbol sense is an ongoing research focus (Arcavi, 2005) as well as an instructional focus as indicated within the Standards for Mathematical Practice of the Common Core State Standards (CCSSO, 2010). In particular, the second mathematical practice refers to students' ability to both decontextualize—work with abstract symbols while allowing the referents to shift to the background—and contextualize—to reconnect with those referents as needed in order to appropriately interpret the relationships within the situation. The notation can thus be the focus of one's attention, or the means through which one's attention on quantitative relationships is mediated (Kinzel, 2000).

Clinical task-based interviews were conducted and the data analyzed to establish evidence-based constructs in the development of algebraic symbol sense. Interview participants included both "experts" (practicing mathematicians) and "beyond novice students" (students enrolled in an introductory foundations of mathematics course). Comparisons across the levels of participants shed light on characterizations of symbol sense and on the potential for supporting its development.

Arcavi, A. (2005). Developing and using symbol sense in mathematics. For the Learning of Mathematics, 25(2), pp. 42-48.

Kinzel, M. T. (2000). Characterizing ways of thinking that underlie college students' interpretation and use of algebraic notation. Unpublished dissertation, The Pennsylvania State University.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). Common Core State Standards-Mathematics. Washington DC.

# **Research in Geometry**

# Saturday, August 9, 10:00–11:10 AM, Parlor C

#### **Steven Edwards** Southern Polytechnic State University (sedwards@spsu.edu)

#### **Fibonacci and Logarithmic Spirals**

The logarithmic spiral was first described by Descartes. Fibonacci spirals are more recent creations. The Fibonacci spiral has usually been studied via a construction with squares whose sides are the Fibonacci numbers. We consider the spiral that arises from a construction with nested golden triangles. We compare the geometry of these and related spirals.

#### **Frank Morgan** Williams College (frank.morgan@williams.edu)

#### The Convex Body Isoperimetric Conjecture

The conjecture says that the least perimeter to enclose given volume inside an open ball in  $\mathbb{R}^n$  is greater than inside any other convex body of the same volume. We'll discuss progress, including some work by undergraduates.

#### Corey Shanbrom California State University - Sacramento (corey.shanbrom@csus.edu)

#### Periodic Orbits in the Heisenberg-Kepler Problem

The Kepler problem is among the oldest and most fundamental problems in mechanics. It has been studied in curved spaces, such as the sphere and hyperbolic plane. Here, we formulate the problem on the Heisenberg group, the simplest sub-Riemannian manifold. We take the sub-Riemannian Hamiltonian as our kinetic energy, and our potential is the fundamental solution to the Heisenberg sub-Laplacian. The resulting dynamical system is known to contain a fundamental integrable subsystem. We will discuss the use of variational methods in proving the existence of periodic orbits with k-fold rotational symmetry for any odd integer  $k \ge 3$ , and show approximations for k = 3.

# Alvin Swimmer Arizona State University (aswimmer@asu.edu)

#### An Intrinsic Relationship Beween Finite Projective Planes and Finite (Galois) Fields

There is a method, due to Wesson, for beginning the construction of a finite projective plane of order *n*. This method allows one to list 2n + 1 of the lines completely and two of the points on each of the remaining  $(n^2 + n + 1) - (2n + 1) = (n(n - 1))$  of the remaining lines.

The remaining points must be chosen in some way to satisfy the projective plane axioms.

I will provide a (non-coordinate) method for completing this process using the additive and multiplicative group tables of the Galois Field of order *n*.

**Jack Mealy** Austin College (jmealy@austincollege.edu) **Samantha Le** Austin College (sle10@austincollege.edu)

#### New Directions in Staircase Metric Geometry

Further results in Staircase Metric Geometry (formerly, Snell Geometry). After a short overview of this category, its natural associated methodology, and a brief mention of a couple of earlier results, in this presentation we focus on a new direction for the work. Specifically, we extend the scheme to include situations wherein the parameter space consists, rather naturally, of unions of non-convex subsets of Euclidean planes, each with a different 'index of refraction'. The appropriate angle change law (analogous to Snell's Law of optics) associated to a given connection scheme is discussed. Then, after extending the construction to countably infinite unions of these parameter space subsets, we investigate various complete geodesics that result, and consider related asymptotic objects.

# Assessment

#### Saturday, August 9, 10:15–11:25 AM, Galleria III

#### William Branson St. Cloud State University (wbbranson@stcloudstate.edu)

#### Local Information and Assessment in Business Math at St. Cloud State University

Teaching involves taking pedagogy and tailoring it to your own circumstances. Knowing who your students are and how they behave is essential. St. Cloud State University is a regional comprehensive university, and Math 196 at SCSU is required for business majors, but no other students. In Spring 2014, I initiated what will be a long-term study of these students, aimed at improving student outcomes by understanding both the backgrounds of these students, and how they behave while taking the course. Via surveys, I investigated their backgrounds in mathematics, their expectations of the course, and how they used the pedagogical tools (text, on-line homework system, etc.), and then tied these results to outcomes on tests and quizzes. The method of assessment I am following has more to do with anthropology and ethnography, than with statistical methods. The data I gather is more qualitative than quantitative, based on self-reporting by students and my own observation in the classroom. The nature of the course and the university restricts the variation in the student population, allowing for a long-term study of this particular course. Such a study can allow me to identify persistent groups of students who share particular characteristics (re-entry students, say) and then tailor pedagogy to address their weaknesses and reinforce their strengths.

#### **Solomon Abogunde lyekekpolor** Federal University, Wukari-Nigeria (iyekesol2010@yahoo.com)

# University Students' Perception on Poor Achievement of Undergraduates in Introductory Mathematics-Related Courses in Taraba State - Nigeria

This study sought to investigate students' perception on undergraduate students' poor achievement in introductory Mathematics related courses in universities in Taraba State of Nigeria. A descriptive survey research design which used both quantitative and qualitative aspects of research was used in the study. Specifically, Mathematics Achievement Perception Structured Questionnaire (MAPSQ) was used to collect data as an instrument in the study. The reliability coefficient of 0.81 of the instrument obtained after computation using test-retest method, indicate its stability. Undergraduate students (N=220) made up of 110 males and 110 females drawn from the three universities in Taraba State form the sample in the study. While frequency table was used to present quantitative data that answered the research questions, the research hypotheses were tested using Chi- square statistic at 0.05 level of significance. Some external factors like poor foundation in elementary mathematics, inadequate human and material resources, bad teaching, poor teaching methods and negative teacher attitudes were implicated as being responsible for students low mathematics achievement. Laziness, phobia, lack of interest and learners absenteeism were some of the internal forces spotted by the students as being responsible for their low achievement status. Recommendations to improve learners' achievement in mathematics were consequently made.

Gail TangUniversity of La Verne (gtang@laverne.edu)Milos SavicUniversity of Oklahoma (milossavic@outlook.com)Gulden KarakokUniversity of Northern Colorado (gulden.karakok@unco.edu)Houssein El TurkeyUniversity of Oklahoma (houssein@ou.edu)Molly StubblefieldUniversity of Oklahoma (mstubblefield@math.ou.edu)

# A Research-Based Rubric To Assess Students' Creativity in Proof and Proving

Creativity is important to the growth of students' learning of mathematics. When teaching, avoiding the acknowledgment of creativity could "drive the creatively talented underground or, worse yet, cause them to give up the study of mathematics altogether" (Mann, 2005, p. 239). Furthermore, not exposing students to creative proofs or solutions to problems could lead them to believe that the study of mathematics is about procedures or recollections of a correct proof technique. Despite the importance of thinking creatively in mathematics, there currently is no assessment tool that measures students' mathematical creativity or their growth in thinking creatively with regards to proof.

Our research group was intrigued by a creative thinking rubric created by the American Association of Colleges and Universities (Rhodes, 2010). Two researchers on our team (Drs. Karakok & Savic) interviewed mathematicians about creativity in proof, and in particular, allowed them to examine the aforementioned rubric and use it to value creativity in three sample proofs. We utilized these experts' ideas, along with

#### History or Philosophy of Mathematics

another creativity rubric by Leikin (2009), to eliminate some categories and revise others to make it suitable for mathematical proofs and proving. Our session will report on the rubric, highlighting certain aspects of it that will be beneficial to professors and instructors teaching upper-level proof-based courses. We believe that such a rubric, if used in the classroom, may aid students in thinking meta-cognitively about proof in general, which then can help students improve their proving process.

Sijie Liu University of Alabama - Tuscaloosa (sijieliu@outlook.com)

#### An Improved Mixed Clustering Approach for Teaching Evaluation

We studied that clustering is one of the primary techniques in the field of data mining with an unsupervised model of pattern recognition. Clustering analysis is a division of data into similarity groups according to the given rules. C-means algorithm is a classical clustering algorithm based on the clustering objective function. PSO-clustering-algorithm which imitates the behavior of particle swarm has already been applied in the data mining. In this paper, a mixed clustering algorithm was proposed which combined fuzzy c-means clustering method with adaptive PSO algorithm that can be used to evaluate the teaching quality automatically. As we known that the quality of class teaching is a very important to the education system nowadays. Making a correct and comprehensive evaluation to teachers will stimulate their enthusiasm for teaching. In this paper, we applied this improved method for teaching quality evaluation and the result shows this new mixed method therefore gave us a more objective and accurate explanation for the teaching performance.

#### Brian Christopher University of Northern Colorado (brian.christopher@unco.edu)

#### The Relationship between Calibration, Anxiety and Achievement in Preservice Elementary Teachers Mathematics

This study aims to determine the connection between calibration, math anxiety and achievement. Calibration is defined to be the measure of a person's perceived performance on a task compared to his or her actual performance on that task. The literature on calibration has quantitatively linked calibration's effect on student achievement, but has yet to examine this relationship for preservice elementary teachers. Furthermore, there needs to be more research on the relationship between calibration and math anxiety. The participants for the study were preservice elementary teachers enrolled in the first course in a three-course math sequence at a Ph.D. granting institution. With 89 eligible subjects that agreed to participate, the cooperation rate was 77%. Math anxiety and calibration surveys, and a graded assessment were collected during the spring semester of 2014. Linear regression and independent t-tests were used in data analysis. Calibration accuracy was found to be a statistically significant predictor of math anxiety, while both of these variables were found to significant predictors of achievement. Also, calibration accuracy was significantly different between students repeating the course and the first-time students, while calibration postdiction was significantly different between those who were allowed to have homework out on for the assessment and those who were not.

# **History or Philosophy of Mathematics**

# Saturday, August 9, 1:00–2:55 PM, Galleria III

Meighan Irene Dillon Southern Polytechnic State University (mdillon@spsu.edu)

#### What is Algebra and Where did it Come From?

The word "algebra" is Arabic in origin but in the west, activities we would agree to be algebraic in nature pre-date the Arabic period, at least to the time of Diophantus. On the other hand, the symbol slinging that we tend to think of as the essence of classical algebra was only hinted at until Viète's work appeared during the sixteenth century. In this talk, we touch on the debunked notion that Euclid was doing algebra in Book II of the Elements, consider what Diophantus was doing with equations, look at some of al-Kwarizmi's work and move on to Cardano, Viète, Descartes, and other moderns in an attempt to understand where classical algebra came from and how it arrived at so prominent a place in modern mathematics.

# **Deborah Bennett** New Jersey City University (dbennett@njcu.edu)

#### Venn-Euler-Leibniz Diagrams

Having remained unpublished for over two hundred years, the 1686 manuscripts of the universal genius Gottfried Wilhelm Leibniz illustrated the four types of Aristotelian propositions and all of Aristotle's valid syllogisms through the use of drawings of groups of circles. In 1761, the much-admired master mathematician Leonhard Euler used the same diagrams for the same purpose without reference to Leibniz. One hundred and twenty years later, John Venn ingeniously altered what he called "Euler circles" to become the diagrams that we now attach to Venn's name. This talk will explore the history of the Venn diagram, created by Leibniz.

#### Keith M Dreiling Fort Hays State University (kdreilin@fhsu.edu)

# Mechanical Solutions to the Three Construction Problems from Antiquity

Even though it is impossible trisect an angle, square a circle, and double the volume of a cube by construction methods, it is possible to solve these problems when mechanical means are used. Geometer's Sketchpad will be used to demonstrate solutions to these problems.

# Shigeru Masuda RIMS, Kyoto University (hj9s-msd@asahi-net.or.jp)

# The Toil and Moil in Proving the Describability of the Trigonometric Series

After Lagrange expressed the theory of propagation of sound (1759-61) by the trigonometric series, Fourier (1822) proposed the analytical theory of heat, including trigonometric series without proving their convergence. Since then many mathematicians, Poisson (1823), Cauchy (1823), et al., tried to prove the describability of trigonometric series until success by Carlson (1966) in  $L^2$  and by Hunt (1968) in  $L^p$ . We focus on the earlier attempts, such as those of Lagrange, Fourier, Poisson, Cauchy, Dirichlet, and Liouville, of proving the describability of trigonometric series as three steps of mathematical history in the following categories of interest:

- A : periodicity in algebraic equations for the string/cord,
- B : applicability of trigonometric series in the heat analysis,
- C : demonstration of describability or convergence on the heat problems.

We can see the point of contrarieties and discussions are sifted from A into B and C, as follows:

- on A between Euler and d'Alembert,
- on B and C between Fourier and Poisson,
- on C after Cauchy, according to Dirichlet, or after Poisson, according to Liouville. We document on these topics.

# Weiping Li Walsh University (wli@walsh.edu)

#### Ming Antu's Influence on Chinese Mathematics in Qing Dynasty

Ming Antu was an astronomer, mathematician, and topographic scientist of Mongolian nationality in Qing dynasty. He worked in the Imperial Astronomical Bureau all his adult life. His mathematical work was mainly on infinite series and was a combination of Western mathematics and traditional Chinese mathematics. He actually was the first Chinese mathematician who studied systematically the theory of infinite series. We will show how his work and in particular his book "Ge Yuan Mi Lu Jie Fa" (The Quick Method for Obtaining the Precise Ratio of Division of a Circle) influenced several generations of Chinese mathematicians in 18th and 19th centuries.

# Eileen Donoghue City University of New York, CSI (efdonoghue@gmail.com)

#### Truman H. Safford: A Nineteenth Century Astronomer's Views on School Mathematics

Truman H. Safford (1836-1901) made his mathematical reputation early as "the Vermont boy-calculator," performing impressive feats of computation. After study at Harvard, he obtained an appointment to its College Observatory. In 1865, Safford became director of the Dearborn Observatory in Chicago and, in 1876, Field Memorial Professor of Astronomy at Williams College. Subsequently, Safford was invited to join Simon Newcomb and other mathematicians and educators as a member of the Mathematics Conference of the Committee of Ten, whose report was completed in 1893. This paper will explore Safford's views on teaching mathematics, concentrating on his observations about and prescriptions for the schools of his era. Sources examined will include his monograph, Mathematical Teaching and Its Modern Methods (1887), the New York Mathematical Society Bulletin article, "Instruction in Mathematics in the United States" (1893), and two unpublished essays, Notes on Gymnasium Studies (Germany) and The Probable Future of Our School System. Connections will be drawn to current-day discussions of these issues within the mathematics community.

#### Walter Meyer Adelphi University (meyer1@adelphi.edu)

#### Was There Curricular Modernism Also?

In Plato's Ghost, Jeremy Gray sets out the case that there was, in the late 19th and early 20th centuries a worldwide tendency for mathematical research to become inward-looking, and in particular to give little attention to applied mathematics. Gray calls this tendency modernism. Did such a thing also occur in the teaching of undergraduate mathematics? This talk will study the case of Stanford's short-lived Applied Mathematics Department to see whether curricular modernism is an appropriate way to think about events at Stanford in the first 3 decades of the 20th century.

# Jeremy Sylvestre University of Alberta, Augustana Campus (jeremy.sylvestre@ualberta.ca)

#### Can a Mathematician Write a Proof So Complex Even He Cannot Believe It?

Azzouni (2004) has put forward the argument that a proof of a mathematical statement typical of modern practice indicates the existence of a formal logical derivation of the statement that can be mechanically verified, and that confidence in the correctness of the proof is partially derived from the supposed existence of this derivation. Pelc (2009) counters that the potential for the length of such a derivation to reach a size making mechanical verification a physical impossibility implies that no contribution to confidence in correctness can be ascribed to its potential existence. I argue that the contribution to confidence lies in the conceivable achievability of the mechanical verification of a derivation rather than the practical achievability, and I use mathematical induction as an example of this distinction. Furthermore, the basis of Pelc's claim lies in the physical limitations of the known universe. But I hold that a mathematical statement, being of the universe, cannot be more information-dense than the universe itself, and so a corresponding derivation for the statement of minimal length cannot exceed these physical limitations.

# **Teaching or Learning Advanced Mathematics**

# Saturday, August 9, 1:00–3:55 PM, Parlor C

#### Alan Alewine McKendree University (jaalewine@mckendree.edu)

#### The Best Tasting Basis Ever!

The idea of a vector space basis in linear algebra is difficult for many undergraduates to master. We will present a pedagogical tool to aid students in their understanding of a basis by using a gastronomical "example."

#### Timothy Edward Goldberg Lenoir-Rhyne University (timothy.goldberg@gmail.com)

#### More Geometry with SET

The card game SET can be viewed as a model of a finite affine geometry, and both its structure and its educational usefulness as a concrete example have been well-studied. The most common way to define this geometric structure is by assigning coordinates to the cards according to their characteristics (shape, color, shading, and number). In this presentation, we will describe the geometric structure of SET without coordinates, using the definition of an affine space as a set of points on which a vector space acts simply transitively. This approach clarifies the different roles played by points and vectors in affine geometry in general, which is largely obscured when using coordinates. This presentation will be accessible to undergraduates.

# Bonnie Amende<br/>Carol OverdeepSaint Martin's University (bamende@stmartin.edu)Saint Martin's University (coverdeep@stmartin.edu)

#### Chemistry, Legos, and Proofs

At Saint Martin's University, we introduce our students to proofs in Discrete Mathematics. We use many concepts from number theory which seem natural and familiar to the students. The difficulty this creates is that these concepts seem "obvious" to the students who then struggle to learn how to organize and formulate their proofs. To aid in their understanding, we adapted a creative group activity involving Legos from an article in an American Chemical Society publication. The purpose of this talk is to describe the activity, how we have used it, and its effectiveness.

### David McCune William Jewell College (mccuned@william.jewell.edu)

#### Introducing Mathematical Induction Using Combinatorial Games

Mathematical induction can be a difficult concept for students to learn, and students often have trouble using the technique in contexts other than combinatorial formulas and recursion. Furthermore, while many students can write correct induction proofs, the same students may have a poor grasp of why induction is a valid proof technique. In order to address these issues, we argue for an introduction to induction that incorporates combinatorial game theory. Using these games allows students to see the power of induction outside of its normal introductory context of counting formulas. In addition, students who master induction proofs using game theory seem to have a better grasp of why induction is a valid technique. In this talk, we give examples of games used in the classroom, we discuss the students' reactions to the games, and we explain why the games enhanced the students' understanding of induction.

# **David Nacin** William Paterson University (nacind@wpunj.edu)

#### Liedoku for Abstract Algebra

We discuss the use of puzzles and games in abstract algebra. In particular, we present an exercise over the Lie algebra sl(2) intended for a standard undergraduate course. Students are not expected to have seen any form of Lie theory. The exercise is designed to expose them in a fun way to a new type of algebraic structure, and test their combinatorial reasoning.

Scott R. Kaschner The University of Arizona (skaschner@math.arizona.edu)

#### More Bang From Your Book: A Simple Strategy to Promote Active Reading

One cannot overstate the difficulty of getting students to read mathematics text books in a productive way. This preliminary report will describe a simple intervention to instruct and motivate students in the effective use of their textbook as a learning tool. While the intervention could be used in any course, the data presented will be from its implementation in an Introduction to Linear Algebra course. The successes and future improvements for the intervention will also be presented.

Michelle GhristU.S. Air Force Academy (michelle.ghrist@usafa.edu)Dale PetersonU.S. Air Force Academy (dale.peterson@usafa.edu)Ralph BoedigheimerU.S. Air Force Academy (ralph.boedigheimer@usafa.edu)Benjamin KallemynAir Force Institute of Technology (benjamin.kallemyn@afit.edu)

#### Using Individual Oral Exams in Mathematics Courses

Over the last ten years, we and other faculty members at the U.S. Air Force Academy have incorporated individual oral exams into various mathematics courses. We have experimented with various approaches, shared results and ideas with other department members, and refined

our techniques. We have found that this alternative assessment gives the instructor considerable insight into students' understanding and that most students value the opportunity to demonstrate their abilities via this kind of assessment. In this paper, we provide suggestions on successfully administering oral exams and offer evidence supporting their use in mathematics courses.

# Lisa Oberbroeckling Loyola University Maryland (loberbroeckling@loyola.edu)

#### Programming Mathematics as an Advanced Math Course

In this course, mathematical topics such as numerical integration and Taylor polynomials are covered as a bridge for students between calculus and upper level courses. Other topics, such as complex numbers and fractals, expose students to topics they may not see unless they take a specific upper level course. Assignments that have both theoretical questions and programming are created to aid their understanding of the mathematics in addition to giving them basic programming skills in MATLAB. The course design, example assignments, and final projects will be discussed.

# Ximena CatepillanMillersville University of Pennsylvania (ximena.catepillan@millersville.edu)Cynthia TaylorMillersville University of Pennsylvania (cynthia.taylor@millersville.edu)

#### An Ethnomathematics Graduate Course

In 2013 we developed the graduate course Ethnomathematics, Math 605, as part of the Master of Mathematics Education courses at Millersville University of Pennsylvania. Math 605, a 3-credit course, introduces Ethnomathematics as a field by examining mathematics across and within cultures. In addition, the course is designed to strengthen and expand students' understanding of mathematical topics (e.g., number systems, geometry, combinatorics, group theory) through study of the mathematics of world cultures. Students will discuss ways in which mathematical concepts covered in the course may be used to refresh or augment grade 7-12 mathematical concepts and develop classroom materials in Ethnomathematics for use in their classroom. In this talk, a complete description of the course, and specific examples in which mathematical ideas and culture interplay, will be presented.

#### Jan Cameron Vassar College (jacameron@vassar.edu)

#### A Capstone Seminar on the Mathematics of Voting

In Spring 2014, I taught a seminar on voting theory for our twenty four Senior mathematics majors. The seminar involved weekly meetings of about two hours, consisting of student presentations, mini-lectures, and various group activities. In the second half of the semester, students completed independent capstone projects, in which they read current research papers in voting theory and (in some cases) did a little bit of original research. In this talk, I'll discuss the format of the course, some of the specific activities and discussions that took place, and share some ideas for potential projects.

#### **Lew Ludwig** Denison University (ludwigl@denison.edu)

#### The Over-Easy Classroom

A modification of the flipped-classroom technique is presented which is referred to as the "over-easy" method. Using this method, the students engage with the material before coming to class. A detailed description of the technique including several examples from different types of classes are provided, as well as several references from cognitive psychology supporting its advantages. The presentation concludes with the benefits and challenges of the over-easy method.

#### Lenny Ornas McNeese State University (gornas@mcneese.edu)

#### **Flipping Differential Equations**

We decided to try "Flipped Classrooms". We chose a class to start with, DiffEq. We discuss why we wanted to flip classes (not getting the results we wanted, shorter semester with longer classes, classes getting larger,...), why DiffEq, what we did (videos, using software ubiquitously, group work,...), how it all worked out, and how we will change for next time.

# **Graduate Student Session**

# Great Talks for a General Audience: Coached Presentations by Graduate Students

# Saturday, August 9, 1:00–5:30 PM, Plaza Level, Broadway I & II

Sharif Ibrahim Washington State University (mathfest@sharifibrahim.com)

# Marijuana and the Mathematics of Randomness

How do you distribute licenses to sell marijuana? Washington state recently legalized the use and sale of recreational marijuana. There were more prospective retailers than available licenses so the state needed to run a lottery to see who would be eligible. The Social & Economic Sciences Research Center at Washington State University was chosen to run the lottery and came to the math department to figure out how it all should work. How do we generate a random ordering of applicants in a double-blind fashion and ensure the process is fair, robust, and auditable? How do we deal with the inevitable complications of the real world? What does random even mean? Find out about true randomness, pseudorandom numbers, and how to shuffle a deck of cards with a pair of dice.

**Amy Streifel** Washington State University (amystreifel@gmail.com)

# **Skew-Adjacency Matrices of Graphs**

In traditional adjacency matrices of graphs, if there is an edge between the *i* th and *j* th vertices of the graph, then the matrix has 1s in the (i, j) and (j, i) positions. In my research as a grad student I switch things up by asking what happens if you make one of those entries a -1 instead. These are called skew-adjacency matrices. With  $2^m$  possible skew-adjacency matrices for a graph with *m* edges, does this lead to an equal explosion on the number of skew characteristic polynomials? When does a graph have only one skew characteristic polynomial? Can we make graphs to get any number of possible skew characteristic polynomials we want? And how exactly does one calculate a characteristic polynomial without looking at a matrix at all?

# Roberto C. Soto The University of Iowa (roberto-soto@uiowa.edu)

## Working in Groups

Consider the remainders when dividing all of the integers by a fixed integer n. Now think about a regular polygon with n sides and the "symmetries" of this polygon. Finally, consider all of the permutations of n distinct objects. How are the remainders, the "symmetries" of a regular polygon, and the permutations of n objects related? What are the similarities between these situations? How can we use linear algebra to study these different scenarios and why would we desire to do so? Please join us in this dynamic session as we explore this group of situations.

# **Poster Sessions**

# **Poster Session on IBL Best Practices**

# Thursday, August 7, 3:30–5:00 PM, Plaza Level, Plaza Foyer

# Sarah Wright Adelphi University (swright@adelphi.edu)

# A Flipped IBL Linear Algebra Class

We discuss the ins and outs and ups and downs of a linear algebra class I taught in the spring. The class made use of a variety of IBL techniques and followed a version of the flipped classroom model. I will break down the structure of the course, give stories of success and areas of improvement, and provide samples of materials.

Victor lan Piercey Ferris State University (piercev1@ferris.edu) Alyssa Finch Ferris State University

#### Affective Implications of Inquiry-Based and Project-Based Learning in General Education

You are a social work major but you have to take a math class. You are a management major that has to succeed in accounting and finance courses. These courses require more math than your management courses. You are math anxious. You believe math is useless and painful. If you don't solve a math problem immediately and quickly, you are lost. Might an inquiry-based learning or a project-based learning course be right for you? In this poster we will see what data suggests based on two semesters of an inquiry-based course for business students and project-based course for general education.

# Violeta Vasilevska Utah Valley University (violetta.vasilevska@uvu.edu)

# Comparing Effectiveness of IBL Strategies in Different Classroom Settings

The poster will outline what worked/did not work in implementing IBL in two upper level math classes: Abstract Algebra and Topology. Moreover, it will be illustrated how the different class structures and settings affected the implementation of this method. In addition, various active learning strategies (modifications from IBL) used frequently in the classes I teach will be presented. Student responses will be provided that demonstrate the effectiveness of these implementations.

**Dora Matache** University of Nebraska - Omaha (dmatache@unomaha.edu) **Angie Hodge** University of Nebraska - Omaha

#### IBL Approach in an Upper Division/Graduate Class on Boolean Networks

We report on a recent experience with an existing special topics course that has been revised under the inquiry-based learning (IBL) paradigm in Spring 2014. This course focuses on an introduction to dynamics of Boolean networks in which nodes can take on two states, on/off, and whose states over time change based on logical relationships with other nodes. This course is targeted to advanced undergraduate students as well as graduate students. It prepares them for further research in mathematics by requiring them to work on authentic research projects in the area of Boolean networks and their dynamics. The course has been restructured so that the students start working independently or in teams from the first day of class, so they become more oriented towards learning on their own and reading the course material by themselves. This is absolutely essential for any type of mathematical research, which starts with reading and understanding various publications. We report on the structure of the course, the activities and tasks performed by students, the assessment of their work, and the student feedback on a tailored questionnaire.

# Susan Crook Loras College (susan.crook@loras.edu)

#### My Best Advice for Teaching a First IBL Course

When first planning to teach a course in an IBL fashion, it can be daunting to cull through all the advice given. I have distilled what I believe to be the most crucial advice to a beginning IBL user. Some is advice I was given; some are things I have learned from teaching my own IBL courses. All of the advice is supported by student feedback from my Introduction to Proofs and Real Analysis courses and my personal observations of other Introduction to Proofs courses. Advice will address course planning, material development, and course management among other areas of issue in a college classroom.

#### Charles Funkhouser California State University - Fullerton (cfunkhouser@fullerton.edu) Miles R. Pfahl Turtle Mountain Community College

#### Native American-based IBL Mathematics Materials for Undergraduate Courses

This project develops and researches undergraduate mathematics IBL materials based in the culture and mathematics of Native American Peoples for integration into undergraduate courses. Mathematics topics include probability, number theory, transformational geometry, and

pre-service elementary and secondary education-related content. These materials-both paper and electronic-are classroom ready, and have been developed and piloted in consultation with Tribes in the Rocky Mountains, the Plains, the Pacific Northwest, and the Southwest. This work is an NSF DUE TUES Type 2 funded project

#### Robert Klein Ohio University (kleinr@ohio.edu)

#### **Pushing Barbie Farther**

The Barbie Bungee activity is fairly well known and involves data collection and linear regression. For a secondary math methods course, this activity was enhanced to include far more concepts, more open-ended and open-middled student pathways, and a critical reflection component. The poster will describe the activity and highlight different student approaches.

#### **Javier Garza** Tarleton State University (garza@tarleton.edu)

#### Teaching with IBL: Successful Strategies That Can Yield Desired Outcomes

A model will be presented for the implementation of inquiry based learning in the form of a modified Moore method approach. The balance of emphasis on critical reading as well as proof writing/development, and approaches to effective assessment and student engagement will be discussed. Quantitative and qualitative assessment data will be shared to demonstrate the effectiveness of the approach.

# Randall E. Cone Virginia Military Institute (conere@vmi.edu)

#### The 2014 Summer Mathematics Institute on IBL: Grades 3–12

Now in its second year, the 2014 Summer Mathematics Institute (SMI) on IBL, for K–12 public school teachers in Virginia, saw structural and pedagogical changes over the previous year's 2013 SMI. Six professors and two graduate students from around the United States came together to develop and facilitate a week-long institute on the inclusion of IBL into the K–12 mathematics curriculum. In advance of the 2014 SMI, the administration for these public schools estimated that 50% of their grades 3–5 teachers were math-phobic. In this poster, we discuss the successes of the 2014 SMI, which include: the effective strategic choices made during the institute's development, the techniques used to address "math phobia", and the philosophical reasons behind including material from George Polya's IBL-oriented book "How to Solve It".

# Suzanne Ingrid Doree Augsburg College (doree@augsburg.edu)

#### Using IBL to Teach Conjecturing: Examples from Discrete Mathematics

Why teach conjecturing? Conjecturing is at the core of what mathematicians do — it is our research experiment, our way of thinking through an abstract question, and prelude to developing a theory. Teaching conjecturing helps students grow their inner mathematicians, preparing them for advanced courses and undergraduate research. But the benefits extend beyond. Conjecturing motivates students to write proof; improves students' reasoning and writing skills; and builds student confidence. Plus, it's fun and leaves students wanting more. The MAA CUPM 2004 Curriculum guide agrees: students should "approach problem solving with a willingness to try multiple approaches, persist in the face of difficulties, assess the correctness of solutions, explore examples, pose questions, and devise and test conjectures." How teach conjecturing? Discrete mathematics and inquiry-based learning, of course! To me this is the ideal content-pedagogy pair for teaching conjecturing. For the past fifteen years I have been teaching sophomore level discrete mathematics using inquiry-based methods — a little more each time I teach the course. This poster will briefly describe the practical elements of the IBL course structur — schedule of topics, types of activities, and assessment of student learning, and will highlight a few mathematics activities/assignments that have been particularly effective in teaching conjecturing. Our results couldn't be better — students love the course, they spend a lot of time working on classwork in and out of class, they are prepared for advanced proofs courses, and we have increased the number of students going on in mathematics.

# PosterFest 2014: A Poster Session of Scholarship by Early Career Mathematicians and Graduate Students

Friday, August 8, 3:30-5:00 PM, Plaza Level, Plaza Foyer

Yanping MaLoyola Marymount University (yma@lmu.edu)Lisette dePillisHarvey Mudd CollegeErica GrahamNorth Carolina State UniversityAmi RadunskayaPomona CollegeJulie SimonsTulane University

#### A Model-Based Comparison of Clinical and Biological Determinants for Anticoagulation Therapy

We combine temporal models of coagulation and fibrinolytic pathways with warfarin pharmacokinetics and pharmacodynamics to study anticoagulation therapy. We compare two modeling frameworks to determine treatment efficacy: a traditional clinical assessment based on a simulated prothrombin time test and a biological assessment influenced by platelet activation and simplified chemical/cellular transport dynamics. We then discuss the roles of these assessment methods in the development and success of individualized treatment strategies.

Briana Forster-Greenwood Idaho State University (fostbria@isu.edu)

#### **Distance Spectra of Cayley Graphs of Complex Reflection Groups**

We consider Cayley graphs of complex reflection groups with vertices labeled by group elements and edges corresponding to multiplication by any reflection in the group. Extending the work of P. Renteln for real reflection groups to all finite complex reflection groups, we show the distance matrix (recording lengths of shortest paths between any two group elements) has all integer eigenvalues. The key Fourier transform perspective from the real case persists, but in the general complex case, a discrepancy between geometry (codimension of fixed point spaces) and group structure (reflection length) prompts an analysis of reflection-preserving group automorphisms, and, in a finite number of cases, computer calculations with the software GAP. This is joint work with Cathy Kriloff.

Sharif Ibrahim Washington State University (mathfest@sharifibrahim.com)

#### Distance Between Surfaces Via the Flat Norm: Discretization, Computation, and Integral Decomposition

The flat norm is a natural way to measure currents (generalized surfaces with orientations and multiplicities) by decomposing a *d*-dimensional current optimally into pieces that are (a) boundaries of (d + 1)-dimensional currents and (b) any *d*-dimensional pieces that are left. Here, the optimal decomposition has the minimum mass sum of the two parts where part (a) uses (d + 1)-dimensional mass. In general, direct computation of the flat norm is difficult. However, it can be computed for boundaries of top-dimensional currents via the TV- $L^1$  functional. In this case, if the input current has only integer multiplicities, then the optimal decomposition can be taken to be integral as well. We extend feasible computation (of a suitable discretization) and integrality results to a more general setting. We can approximate currents with chains on a simplicial complex by means of our simplicial deformation theorem. In this discretized setting, the problem of computing the flat norm and finding the corresponding decomposition can be solved efficiently with linear programming. When certain conditions hold (most notably, when the complex has the same dimension as the ambient space and the chains have codimension 1), the LP has an integral optimal solution. Thus integral simplicial chain inputs have an integral simplicial flat norm decomposition. By working with both the simplicial and continuous flat norms and suitable approximation tools, we extend the discrete result to the continuous case in some useful settings and provide a framework for future extension.

# Servando Pineda-Carranza San Francisco State University (spineda6239@gmail.com)

#### Hopf Random Walks on the Faces of Permutohedra

Recently, Diaconis, Pang, and Ram defined a random walk on the elements of two types of combinatorial Hopf algebras. Aguiar and Ardila defined the Hopf monoid of certain polytopes called generalized permutohedra, which gave rise to the Hopf algebra of generalized permutohedra. We use both algebraic structures to define two random walks on the faces of the standard permutohedra; one being the rock-breaking random walk described by Diaconis, Pang, and Ram. We give an exact formula for the expected time of absorption, and find the probability of absorption. In addition, we find that by applying our random walk to the faces of *n*-dimensional cubes we get similar results.

#### Lucas Waddell Clemson University (lwaddel@clemson.edu)

#### Identifying Solvable Instances of the Quadratic Assignment Problem

The quadratic assignment problem is an NP-hard discrete optimization program that has been extensively studied for over 50 years. It has a variety of applications in many fields, but has proven itself extremely challenging to solve. As a result, an area of research has been to identify special cases which admit efficient solution strategies. This work examines four such cases, and shows how each can be explained in terms of the dual region to the continuous relaxation of a classical linear reformulation of the problem known as the level-1 RLT representation. These explanations allow for simplifications and/or generalizations of the conditions defining the special cases.

#### Cynthia Taylor Millersville University of Pennsylvania (cynthia.taylor@millersville.edu)

#### Mathematics Teacher Educators' Practices in K-8 Content Courses

In the U.S., there is a national debate about how to best prepare future teachers (Boyd et al., 2008; Levine, 2006). This debate has been fueled by claims that teacher education programs do not sufficiently prepare teachers for the classroom (Grossman, 2008). Furthermore, we know little about the practices of university faculty who prepare mathematics teachers, as these practices are not widely studied, documented or disseminated (e.g., Bergsten & Grevholm, 2008). The goal of this study is to describe how K-8 mathematics teacher educators provide the opportunity for prospective teachers to develop pedagogical content knowledge in their mathematics content courses for future teachers. The research question underlying this study is: What instructional practices do mathematics teacher educators use to help improve prospective teachers' Pedagogical Content Knowledge in an elementary mathematics content course? Findings from this research will contribute to the call for the need to research mathematics teacher educator practices, develop a shared professional curriculum, and build a usable knowledge base for mathematics teacher educators (Floden & Philipp, 2003). Finally, this study will join others (e.g., Van Zoest, Moore, & Stockero, 2006) in providing new insights regarding content and images of practices to enrich the preparation of mathematics teacher educators – practices they could attend to in order to enrich the learning experience of prospective teachers.

#### **Courtney Gibbons** Hamilton College (crgibbon@hamilton.edu)

#### Modules, Hilbert Series, and... Continued Fractions?

This poster will describe the surprising way in which the study of continued fractions is useful in describing which Hilbert series can be realized for modules over a very special ring.

**Emily Herzig** University of Texas - Dallas (elh042000@utdallas.edu) Viswanath Ramakrishna University of Texas - Dallas Mieczyslaw Dabkowski University of Texas - Dallas

#### On Reversion, Rotation, and Exponentiation in Dimensions Five and Six

The explicit matrix realizations of the reversion anti-automorphism and the spin group depend on the set of matrices chosen to represent a basis of 1-vectors for a given Clifford algebra. There are iterative procedures to obtain bases for higher dimensional Clifford algebras from lower dimensional ones. We use such a procedure to find a basis of 1-vectors for Cl(0,5) from a basis of 1-vectors of Cl(0,3) consisting of

the Pauli matrices, and find that the explicit form of reversion involves neither  $J_4$ , where  $J_{2n} = \begin{bmatrix} 0_n & I_n \\ -I_n & 0_n \end{bmatrix}$ , nor  $\tilde{J}_4 = J_2 \oplus J_2$ . However, we use the relation between  $4 \times 4$  real matrices and the quaternion tensor product  $\mathbf{H} \otimes \mathbf{H}$  to obtain the matrix form of reversion for this basis. Next we find the matrix form of reversion for a basis of 1-vectors for Cl(0, 6) obtained from Cl(0, 0). We then apply this to compute the exponentials of  $5 \times 5$  and  $6 \times 6$  real skew-symmetric matrices in closed form by reducing the problem to the computation of exponentials of certain  $4 \times 4$  matrices. For this we obtain closed form expressions for the minimal polynomials of these  $4 \times 4$  matrices without computing their eigenstructure. Finally, we provide a novel representation of Sp(4). Consequences include natural interpretations for some members of an orthogonal basis for  $M(4, \mathbb{R})$  provided by the isomorphism  $\mathbf{H} \otimes \mathbf{H}$ , and a first principles approach to the study of the spin groups in dimensions five and six.

#### Sarah Anderson Clemson University (sarah5@g.clemson.edu)

#### **QR Code and Error Correction**

In today's society of text messages, emails, and video chats, people are more connected than ever. As a consequence, emergent technology is constantly changing the landscape of the modern classroom, where students are exposed to new technology and communication methods at an early age. For example, elementary students are using QR codes to access homework and answer keys. But how much of the mathematics behind these communication methods do students understand? We will give examples of how to implement engaging coding theory activities to explain QR (quick response) codes as well as how to introduce related technology. Students will explore the structure of QR codes (finder and timer patterns, format information, etc.) as well as different error correction levels of QR codes and how Reed-Solomon codes are used with QR codes to correct errors that may arise from smudges and tears. They also allow QR codes to be customized and for companies to insert their logos. Reed-Solomon codes are algebraic geometry codes, a large family of error correcting codes used in a variety of applications and a topic of current research.

Matt Stackpole Georgia Gwinntt College (mstackpo@ggc.edu) Daniel Pinzon Georgia Gwinntt College Kathy Pinzon Georgia Gwinntt College

## Re"modeling" College Algebra: A Flipped, Inquiry-Based Approach

This poster describes a flipped, inquiry-based approach to college algebra at Georgia Gwinnett College. This approach was used in 9 sections of College Algebra in the Fall 2013 semester taught by the authors. Students work in small, structured groups on guided inquiry activities using iPads after watching 15-20 minutes of videos before class. We discuss a portion of an in-class activity and a writing project used in the course. The results after one semester are that the students in this model did marginally better than students in the traditional lecture sections on common final exam questions.

#### Sandi Lavito California State University - Stanislaus (slavito@csustan.edu)

#### Spring Math Camp: A Hands-On Approach to Mathematics and Critical Thinking for Young Students

The Spring Math Camp is an annual program dedicated to raising interest in mathematics and science among 2nd–6th grade students. In 2014, it completed its third year. This program was developed as a method of giving young students a hands-on approach to understanding fundamental mathematical properties such as fractions, decimals, place values, and geometry. In doing so, the program was able to address key components of the Common Core Standards while increasing over all enthusiasm and confidence in mathematics and science subject areas.

**Elizabeth Lane-Harvard** University of Central Oklahoma (emlane@jacks.sdstate.edu)

Tim Penttila Colorado State University

Stanley Payne University of Colorado - Denver

### **Strongly Regular Graphs from Arcs**

In 1968, Tits constructed generalized quadrangles from ovals in Desarguesian projective planes. In 1969 and 1971, Ahrens-Szekeres and Hall constructed generalized quadrangles from hyperovals in Desarguesian projective planes. Similarly, Payne, in 1985, constructed generalized quadrangles from q-arcs in Desarguesian projective planes. The concurrency graph of a generalized quadrangle is a strongly regular graph. Thus, each of the above generalized quadrangles is associated with a strongly regular graph. Removing the hypothesis that the plane is Desarguesian, we construct strongly regular graphs with the same parameters as the concurrency graphs of the generalized quadrangles arising from ovals, hyperovals, and q-arcs.

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# Washington, D.C.

