Operations Research Program Area Report

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Introduction

Mathematics departments that offer courses or concentrations in Operations Research (OR) often find that these courses attract different types of students to the department. OR, which applies analytical methods (e.g., probabilistic queueing models, discrete optimization) to ~~help make~~ inform better decision-making, appeals to students who want to influence decision-making in business or policy. Solving concrete problems (e.g., designing better transportation networks, optimally sequencing emergency room patients) naturally engages students' curiosity and creativity, and develops skills in communication and critical thinking. Of course OR also develops standard mathematical skills. In linear programming, for example, linking the geometry of a feasible region to a linear systems of constraint equations requires connecting geometric and algebraic viewpoints.

OR courses frequently offered to undergraduates include linear programming, discrete optimization and networks, Markov chains and queueing theory, simulation, and overview courses like math modeling. Prerequisites for most operations research courses include probability and statistics and linear algebra. Departments should also seek to design courses that fit their particular audiences. Examples include the University of Nebraska's Advanced Farm Management and Linear Programming course and the U.S. Naval Academy's Analysis of Naval Tactics course, which applies decision analysis and game theory to naval warfare.

Modeling should take center stage in undergraduate OR courses

What makes OR fundamentally different from most areas of mathematics is *modeling*. While any OR model or algorithm has a rich mathematical theory at its core, OR itself is the study of real-world *operations*: systems, processes, decisions that can be made more effective and efficient through careful analysis. It is usually this grounding in real-world modeling that captures student interest. Thus an introductory operations research course that does not emphasize modeling over theory ill serves its students and loses a critical opportunity to demonstrate the power and beauty of mathematical thinking to solve problems that affect us daily.

The process of OR modeling has several parts: (Hillier and Lieberman)

1. defining the problem;
2. formulating the model;
3. gathering data;
4. solving the model;
5. assessing and refining the model;
6. implementing results.

The process typically requires multiple iterations: too confirm that the proposed modeling approach meets stakeholders' objectives; to collect, distill and interpret messy data; and to assess the sensitivity of the model output to assumptions about model input. Learning this process in a hands-on manner develops students' critical thinking, problem-solving and communication skills, and engages their creativity and curiosity.

Strategies. Putting modeling at center stage sometimes requires an instructor to leave the comfort zone of the traditional theorem-and-proof-driven mathematics course. Following are several recommended strategies for including modeling in an introductory OR course.

1. Word problems that teach students to distill a real-world scenario into mathematical language should be prominent in classroom activities and homework assignments.

2. Case studies and projects should be assigned to help students learn the process of formulating solving a real-world decision problems and communicating results to decision-makers.

3. Peer-reviewed articles in publications such as *OR/MS Today* and *Interfaces* should be assigned to demonstrate to students the wide applicability of OR and the role theory plays in making complex problems tractable.

A wide range of real-world scenarios can be modeled in an operations research course. These include public transportation network design, queueing systems, facility location, inventory management, airline scheduling, asset management, and even radiation therapy. After only a brief introduction to the field, students often quickly identify problems of personal relevance that can suggest class projects: dormitory room assignments, campus bookstore inventory management, course scheduling, etc.

Theory should be used as a tool for more effective application of OR techniques

Undergraduate OR courses should acquaint students with the essential reciprocity among theory, modeling, and computation in this field. In a traditional mathematics course, a primary objective could be to learn a mathematical theory and develop skills of mathematical proof. In an OR course, theory should serve applications. We offer some examples.

1. Duality theory plays an important role in sensitivity analysis of solutions to linear programming problems. Sensitivity analysis permits an operations researcher to determine and communicate the range of applicability of recommended solutions.

2. The shape of the feasible region and objective function (e.g., convex versus non-convex) in a mathematical program determines whether a local optimum is guaranteed to be a global optimum, and informs the sorts of algorithms that can be used.

3. The unimodularity property of the constraint matrix of network flow problems guarantees that network problems with integer data will have integer solutions to their linear programming relaxations. Integer programs that can be modeled in this way are therefore as easy to solve as linear programs.

4. In stochastic processes, the eigenvalues of the transition probability matrix of an ergodic Markov chain guarantee steady-state behavior that permits the analysis of queues.

It is less important that students prove many theorems rigorously than that they understand the statements and implications of a few fundamental theorems, and how they relate to the solution of real problems. Follow-on courses or graduate work can then fill in the details of the rich mathematical theory underlying OR methods.

Students need transferable familiarity with computational tools

Computation is indispensable to the practice of OR; enormous datasets and models containing thousands of variables and constraints are routine. A typical workflow requires using one type of software (say, R) to clean and summarize data, another (say, AMPL) to generate an optimization model from a set-based description, another (say, CPLEX) to solve the model, and perhaps yet another to display a solution graphically. All software has a short shelf-life, so graduates must be prepared to learn and integrate new tools as needed.

Following are some recommendations concerning computation.

1. Instructors should teach, and students should use, appropriate specialized software for each project or course: statistical or database software for data-intensive tasks, set-based modeling software for expressing optimization models, particular solvers for different families of constraint and objective functions.

2. Regarding Microsoft Excel: Students can build rich mathematical models, including simulations, decision trees, discrete dynamical systems, and linear and non-linear optimization models, using Excel and add-ins. Although it has drawbacks as a teaching tool, Excel has low barriers to entry for students in a first OR course, and is more readily available than specialized operations research software.

3. Computational tools can help students understanding theory, as when we use a MATLAB script or calculator program to do single pivots of the simplex algorithm. Reliable pivots are conducive to quick demonstrations of cycling in the simplex algorithm, or reviewing the connection between basic solutions and the geometry of the feasible region.

4. Students should learn the limits of computational tools and understand that software is not a black box that gives incontrovertible answers.

5. **Students pursuing a concentration or major in operations research should learn general programming skills (loops, conditional execution) using a widely-used language like C++, Python, Java, or MATLAB.** Writing and debugging software forces students to "debug" their thinking about algorithms and optimality conditions. Students who can program are prepared to learn about computational complexity, which sometimes derails large models. Nevertheless, lack of programming skills should not rule out students who want to take only one or a few OR courses.

Curricula in OR should build a strong foundation in probability and statistics

Uncertainty permeates real-world decision-making, and large data sets are becoming ubiquitous. Without a strong foundation in probability and statistics, a student continuing in either graduate work or industry in OR will be at a severe disadvantage. Basic familiarity with probability models and stochastic processes permits students to study queuing theory, inventory management and other stochastic operations research topics. Statistics, with a focus on large-scale data analysis or process improvement, prepares students for opportunities in business analytics. Prior to beginning a graduate program in operations research, a student should have taken at least one course that goes beyond the material of a traditional introduction to probability and statistics.

Students should have the opportunity to solve problems for real decision-makers, companies, or organizations

Model formulation exercises from textbooks are useful to build proficiency with modeling and to increase recognition of common problem structures. But there is no substitute for solving a real problem in the messy real world. Students learn quickly that an unreasonable amount of simplification might be needed for a problem to fit neatly in the form of problems seen in class. They will experience first-hand the computational limitations that arise when solving problems with many constraints and variables and will be forced to think creatively to develop heuristics to solve them.

Real-world problem-solving opportunities can arise in several ways:

1. As a course project, students might identify problems of personal relevance to them or that arise in student life where an operations research approach could be useful. Examples include dorm room assignment, course scheduling, and dining hall inventory management.

2. As a senior capstone (e.g., a thesis or team project) experience, an organization can propose a problem needing an OR perspective; student(s) communicate regularly with liaison from the organization to ensure that the project is achieving the desired objectives.

3. As a summer research or independent study project, a student might partner with an organization or administrative office on campus to solve an important problem.

Real-world problem-solving experiences like these challenge students mathematically and encourage them to think analytically about decision problems around them. Seeing the power of mathematics in solving important problems while they are still in college can inspire students to continue learning both mathematics and OR.

Students should practice communication throughout the modeling process, from model specification to presenting solutions in the form of recommendations to decision-makers.
Because the results of an OR study are rarely used by mathematicians but instead by decision-makers and stakeholders, effective communication to a non-technical audience is a critical skill for an operations researcher.

OR courses offer many opportunities to encourage communications skill development:

1. In homework assignments, students should be expected to explain in words the variables, constraints, objectives, and underlying assumptions of their models. They should also be expected to consider, describe, and choose among alternative formulations of a problem.

2. Students should learn to communicate the solution to an optimization or modeling problem in the language of the original problem. The decision-maker should be left with a clear understanding of the implementable decision, the existence of multiple optima, and the effect that minor modifications to problem inputs can have on the recommended solution.

3. In projects, students should be encouraged to select problems having an identifiable stakeholder (e.g., the campus registrar for a course scheduling project, or the manager of a local store for an inventory management project) with whom they can communicate throughout the modeling process.

4. Written reports should have clear grading rubrics that reward effective writing for a non-technical audience.

Conclusion

We encourage math departments to offer OR courses that help students make meaning of the mathematics they learn. OR is fundamentally about modeling: translating and adapting real organizational problems (e.g., designing train schedules or radiation therapy treatment plans) into suitable mathematical models that can guide decision-making. Both theory—the body of beautiful mathematics that elucidates solution structures—and computation are indispensable in using those models to make recommendations to decision-makers. OR students should be encouraged to take creative approaches and to articulate assumptions when tackling under-specified problems, to learn computational tools, to know which theoretical results apply in particular problem settings, and to communicate about mathematics throughout the modeling process.