Carl B. Allendoerfer Awards

Steven J. Brams and Peter S. Landweber

"Three Persons, Two Cuts: A New Cake-Cutting Algorithm," *Mathematics Magazine,* 95:2, 110–122. doi.org/10.1080/0025570X.2022.2023300

How do you divide limited resources among multiple parties in an equitable, envy-free manner? This is the question of fair division, and it is as relevant today as it has been for centuries. Fortunately for us, many brilliant minds are on the case and have explored the concepts of fair division in various settings. In this delightful article "Three Persons, Two Cuts: A New Cake-Cutting Algorithm," Brams and Landweber add their improved analytic twist to a known cake-cutting algorithm.

The authors' exposition is very accessible. They first invite the reader through an intriguing and thorough recounting of the more recent developments in various cake-cutting algorithms. They carefully describe the nuts and bolts of these algorithms, together with their constraints and their pros and cons. From the start, the reader has a strong sense of how the authors' contribution improves upon the existing tools in fair division.

The authors describe their algorithm in five steps. They model the cake using the interval [0, 1] and the algorithm begins by having the three players submit their value functions (a probability density function) to a referee. Using integration, the referee uses these functions to determine two cutpoints that yield connected allocations to the players that are maximin envy-free or maximally equitable. The authors end their main contribution by illustrating their algorithm over two examples that are both clear and enlightening.

This well-organized and engaging article exemplifies the invitational tone one can convey through a generous discussion of the rich and complex mathematical history of cake-cutting algorithms. It serves as a valuable resource for those interested in pursuing further study and research in the fascinating world of fair division.

Responses

Steven J. Brams: It is a privilege and honor to receive this award, especially because I am a political scientist who enjoys working with mathematicians. Since beginning research on fair division with Alan D. Taylor at Union College in the early 1990s and writing two books with Alan on different aspects of this topic in the late 1990s, I have worked with mathematicians, statisticians, computer scientists, economists, and lawyers on both theoretical models and real-life applications, from divorce settlements to international dispute resolution. My article with Peter Landweber in *Mathematics Magazine* shows how, in dividing divisible goods like cake or land—or indivisible goods like the marital property in a divorce—one can solve equations to determine where to make cuts that give three players portions that are envy-free (each player thinks he or she got the most valuable piece) and equitable (all players think they received the same percentage of the total). Previously, the only solution to this problem was to use simultaneously moving knives and have the players say when they create ties for the most valuable pieces. (This approach was not considered quite kosher, because it required the players to choose among infinitely many cuts.) Unfortunately, our solution does not extend to a greater number of players than three. So, it remains an open question to generalize cake-cutting algorithms like ours. But just as important as the mathematical theory underlying the fair division of divisible as well as indivisible goods is how it might be applied to practical problems of fair division, which will increasingly require the collaboration of mathematicians and researchers in other disciplines, which is a challenge I commend to mathematicians interested in using their skills to make the world a fairer place.

Peter S. Landweber: I am delighted to share the Allendoerfer Award with Steven Brams for providing assistance with the mathematics in the study of a very natural cake-cutting problem for 3 players. I have followed Steve's work on game theory and fair division for many years, in numerous books and papers, and so am especially delighted to join him as a co-author. Some time ago I taught a couple of undergraduate courses on game theory, which were good fun. For the first of these I took up Steve's offer to give a talk to my class, which gave us a valuable perspective on the subject. Noting that Professor Allendoerfer was a topologist, I hope it is not amiss to mention that topology has rebounded nicely from the period two decades ago when the Poincaré conjecture was proved, leading some researchers (and alert students) to feel that topology was no longer an active field. Indeed, topology is now playing a prominent role in topological data analysis, a growing field with impressive applications, as well as in other fields such as topological robotics in which I have written several joint papers.

Biographical Sketches

Steven J. Brams is professor of politics at NYU. He is author, co-author, or co-editor of 19 books and more than 300 articles. He has applied game theory and social-choice theory to voting and elections, bargaining and fairness, international relations, and the Bible, theology, and literature. He is a former president of the Peace Science Society (1990–91) and the Public Choice Society (2004–2006). He is a fellow of the American Association for the Advancement of Science (1986), a Guggenheim Fellow (1986–87), and was a visiting scholar at the Russell Sage Foundation (1998–99).

Peter S. Landweber is currently emeritus professor of mathematics at Rutgers University. His principal field of study is algebraic topology, especially bordism and cobordism theory of manifolds. He discovered the Exact Functor Theorem, sometimes known as the LEFT.